EXPERIMENT 5 DESIGNING OF MILK COLLECTION ROUTE

structure

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5.1 INTRODUCTION

Milk being a perishable commodity must reach at milk plant without any delay once it is produced at village level. The resources with the milk plant are limited. The optimization tools are used to design the routes for milk in a minimum time and cost.

5.2 OBJECTIVE

- To apply one of optimization tools for designing milk collection route.

5.3 EXEPRIMENT

i. Principle

Milk plant has to collect milk from different co-operative societies located at different places. Most of these societies may not have a truck or tanker load of milk. Therefore, a single truck or tanker has to collect milk from several societies on route. These routes must be decided in such a way that the minimum resources are spent for the collection of milk.

ii. Requirements

- Paper
- Geometrical box
- Computer or Calculator
- Necessary writing material

iii. Procedure

If it is desired to find route shortest in distance (or time or cost) from one origin to another destination, among the alternative paths which are available to a milk tanker for collecting (or distributing) the milk; the problem can be handled with the help of assignment problem.

The problem is called routing problem which is a special case of Assignment Problem and are solved by using Hungarian Algorithm.

If the Milk Tanker has to visit n cities, and the distance from ith city to jth city be C_{ij} such that $C_{ij} = C_{ji}$ for all i and j = 1 to n, then it can be represented as in Cost effectiveness table (Table 1)

Table 5.1 : Cost Effectiveness Table of Routing Problem

		To Cities						
		1	2	3	…	J	…	n
From Cities	1	∞	C_{12}	C_{13}	…	C_{lj}	…	C_{1n}
	2	C_{21}	∞	C_{23}	…	…	…	C_{2n}
	3	C_{31}	C_{32}	8	…	…	…	C_{3n}
	…	…	…	…	…	…	…	…
	i	C_{i1}	C_{i2}	C_{i3}	…	C_{ij}	…	C_{in}
	…	…	…	…	…	…	…	…
	n	C_{n1}	C_{n2}	C_{n3}	…	C_{nj}	…	∞

Hungarian Method of Assignment Problem can be summarized as follows:

Step I: In the given matrix, subtract the smallest element from each row from every element of that row.

Step II: In the reduced matrix obtained from Step I, subtract the smallest element in each column from every element of that column.

Step III: Draw the least possible number of horizontal and vertical lines to cover all the zeros of the starting table. Let the number of lines be N. Now, if N = n, the order of the cost matrix, than the optimum assignment has been attained. Moreover, if N < n, then go to the next step.

Step IV: Determine the smallest cost in the starting table not covered by the N lines. Subtract this cost from all the uncovered elements of the starting table and add the same to all those elements of the starting table which are lying at the intersection of horizontal and vertical lines, thus obtaining the second modified cost table.

Step V: Repeat steps I, II, III, IV until we get N = n.

Step VI: Examine the rows successively until a row with exactly one unmarked zero is found. Enclose this zero inside a circle and an assignment will be made here. Mark a cross (x) in the cells of all other zeros lying in the column of the encircled zero. Continue in this manner until all the rows have been taken care off.

Step VII: Repeat step VI for column similarly.

Step VIII: Repeat step VI and VII successively until either (i) No unmarked zero is left or (ii) there lie more than one unmarked zeros in one column or row. In case (ii) encircle one of the unmarked zeroes arbitrarily and mark a cross in the cells of remaining zeroes in its row and column. Repeat the process until no unmarked zero is left in the cost table.

Example

A milk tanker has to lift milk from five milk supply centers A, B, C, D and E. The distances between the five centers are given in the table (Table 2). If the tanker starts from the centre A and has to return centre A which route the driver should select so that total distance traveled is minimized.

Table 5. 2 : Routing Problem Data

To

From		A	B	C	D	E
	A	-	50	60	20	30
	B	50	-	40	50	30
	C	60	40	-	70	40
	D	20	50	70	-	80
	E	30	30	40	80	-

Step I: The given problem is solved as assignment problem. Assignment cannot be made along the diagonal cells. It means milk tanker is already standing at any supply centre, say B. its further movement to the same centre B cannot be made and therefore for this obvious reason infinite value is given to the diagonal cells (Table 3).

Table 5.3 : Assignment Algorithm

To

From		A	B	C	D	E
	A	∞	50	60	20	30
	B	50	∞	40	50	30
	C	60	40	∞	70	40
	D	20	50	70	∞	80
	E	30	30	40	80	∞

Step II: Subtract minimum element of each row from all the elements of the respective row (Table 4).

Table 5.4 : Assignment Algorithm

To

From

A	B	C	D	E	
A	∞	30	40	0	10
B	20	∞	10	20	0
C	20	0	∞	30	0
D	0	30	50	∞	60
E	0	0	10	50	∞

Step III: As the column C does not have a zero value subtract the minimum element of that column form every other its element and it will give a reduced matrix as in Table 5. Draw the least possible number of horizontal and vertical lines to cover all the zeroes of the starting table. Let the number of these lines be N. Now N ≠ n 'n' is the order of the matrix (i.e. number of the cities). Here the number of lines drawn are (L_1, L_2, L_3, L_4) N = 4, where the number of cities (or the order of the matrix) is 5.

Table 5. 5 : Assignment Algorithm

To

From

	A	B	C	D	E	
A	∞	30	30	0	10	
B	-20 -	- ∞ -	- 0 -	- 20 -	- 0 -	L_1
C	20 -	- 0 -	- ∞ -	- 30 -	- 0-	L_2
D	0	30	40	∞	60	
E	0 –	- 0 -	- 0 -	- 50 -	- ∞	L_3
	L_4					

Step IV: For obtaining a complete set, determine the smallest element (cost/distance) in the starting table, not covered by the N lines. It is observed that, this value is 10 amongst of values of cost elements in unmarked lines. Subtract this cost, 10 from all the surviving (uncovered) elements of the starting matrix (i.e. Table 5) and add the same to all those elements of the starting matrix which are lying at the intersection of horizontal and vertical lines, thus obtaining the second modified cost matrix (i.e. Table 6).

Table 5.6 : Assignment Algorithm

To

From

	A	B	C	D	E	
A	∞ -	- 20 -	- 20 -	- 0-	- 0 -	L_1
B	-20 -	- ∞ -	- 0 -	- 20 -	- 0 -	L_2
C	20 -	- 0 -	- ∞ -	- 30 -	- 0-	L_3
D	0	20	30	∞	50	
E	0 –	- 0 -	- 0 -	- 50 -	- ∞	L_4
	L_5					

Step V: Until N = n has not been obtained repeat this procedure of adding further zeroes. For obtaining the optimum assignment, first of all only consider the zero elements of the matrix as shown in Table 7.

Table 5.7 : Assignment Algorithm

To

	A	B	C	D	E
A				0	0
B			0		0
C		0			0
D	0				
E	0	0	0		

Step VI: Examine the rows successfully until a row with exactly one zero is found. Enclose this zero inside a square (e.g., [0]) and an assignment has been made. Then mark a cross (x) in the cells of all other zeros lying in the column of marked [0] zero to show that crossed zero cells cannot be considered for future assignment. Use this procedure for each column as well. By examining Table 7. It is observed that row D and column D both have single zeros and hence put cross in the respective column and row as shown in Table 8.

Table 5.8 : Assignment Algorithm

To

	A	B	C	D	E
A				0	0
B			0		0
C		0			0
D	0				
E	0	0	0		

Step VII: If we now examine the rows and columns of table 8 carefully we find pairs of zeros and not one single zero. Now there is a tie for the selection of the cell for assignment. Under such a situation arbitrarily select EB cell and mark square around it. Cross off (x) other corresponding zeros. It will give us cost matrix in Table 9.

Step IX: In Table 9 there is single zero in row C. Put a square around it and cross off the zero in the cell DA. Then in the D row, there is left a single zero in the cell DB.

Table 5.9 : Assignment Algorithm

To

	A	B	C	D	E
A				0	0
B					
C					
D	0				
E	0				

Step X: Taking information from Table 6 fill up rest of the entries in Table 9. We will have Table 10.

Table 5.10 : Assignment Algorithm

To

	A	B	C	D	E
A	∞	20	20	0	0
B	20	∞		20	
C	20		∞	30	
D	0	20	30	∞	50
E	0			50	∞

Step XI: The zeros in Table 10 give solution to the assignment problem, but it is not a solution to the routing problem why? The tanker has to start from A centre.

In the A row marked 0 is under D column. Thus tanker will move from centre A to D supply centre. Again from D (in D row) its next move is to A centre (as there marked 0 in DA cell). Now it is not a solution to the routing problem because it tells us to go from A to D, D to back to A. Thus the route obtained is: A → D → A but the other supply centre has been ignored.

Step XII: Next to find best solution to the problem try to find the smallest non-zero element i.e. 20 (Table 11). The cells of smallest non-zero element are tried in Table 11.

Table 5.11 : Assignment Algorithm

	A	B	C	D	E
A	∞	20	20	0	0
B	20	∞		20	0
C	20	0	∞	30	
D	0		30	∞	50
E	0	0	0	50	∞

Step XIII: Now two sets of assignment with feasible solution to the routing problem are obtained.

(i) From		To	(ii) From		To
A	→	D	A	→	D
D	→	B	D	→	B
B	→	C	B	→	E
C	→	E	E	→	C
E	→	A	C	→	A
Total Distance		**180 km**			**200 km**

5.4 RESULT/ SOLUTION

Thus the best route for the salesman is (i) Option : A → D → B → C → E → A and the total distance (as a proxy of cost) traveled would be 20+50+40+40+30=180 km.