UNIT 12 AUTOCORRELATION *

Structure

12.0 OBJECTIVES

After going through this unit, you should be able to:

- outline the concept of autocorrelation in a regression model;
- describe the consequences of presence of autocorrelation in the regression model;
- explain the methods of detection of autocorrelation;
- discuss the procedure of carrying out the Durbin-Watson test for detection of autocorrelation;
- elucidate the remedial measures for resolving autocorrelation; and
- outline the procedure of dealing with situations where autocorrelation exists in models with a lagged dependent variable.

12.1 INTRODUCTION

In the previous unit, you studied about heteroscedasticity. You saw that heteroscedasticity is a violation of one of the assumptions of the Classical Linear Regression Model (CLRM), viz., homoscedasticity. If the variance of the error term is not constant across all observations, then we have the problem of heteroscedasticity. In this unit, we discuss about the violation of another assumption of the CLRM. Recall that one of the assumptions about the error

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terms is that the error term of one observation is not correlated with the error term of another observation. If they are correlated, then the situation is said to be one of autocorrelation. This is also called as the problem of serial correlation. This can be present in both cross-section as well as time series data. Let us discuss the concept of autocorrelation in a little more detail.

12.2 CONCEPT OF AUTOCORRELATION

The classical linear regression model (CLRM) assumes that the correlation among various error terms is zero. We know that heteroscedasticity is associated more with cross sectional data. Autocorrelation is usually more associated with time series data. Of course, autocorrelation can be present even in cross-section data. Some authors use the term autocorrelation only for time-series data. They use the term 'serial correlation' for describing autocorrelation in cross-section data. Many authors use the terms autocorrelation and serial correlation as synonyms. They use the term across both cross-section as well as time-series data.

Autocorrelation occurring in cross-sectional data is also sometimes called spatial correlation (correlation in space rather than in time). In CLRM we assume that there is no autocorrelation. This implies:

$$E(u_i, u_j) = 0 \qquad i \neq j \qquad \ldots(12.1)$$

Equation (12.1) means that the stochastic error term associated with one observation is not related to or influenced by the disturbance term associated with any other observation. For instance, the labour strike in one quarter affecting output may not affect the output in the next quarter. This implies there is no autocorrelation in the time series. Similarly, in a cross-section data of family consumption expenditure, the increase in one family's income on consumption expenditure in not expected to affect the consumption expenditure of another family. In the example of output affected due to labour strike above, if $E(u_i, u_j) \neq 0,\ i \neq j$, this implies a situation of autocorrelation. This means the disruption caused by the strike in one quarter is affecting the output in the next quarter. Similarly, increase in consumption expenditure of one family may influence the consumption expenditure of other families in the neighbourhood due to the 'demonstration effect' (cross-sectional data). It is thus more a case of spatial correlation. It is therefore important to analyse the data carefully to bring out what exactly is causing the correlation among the disturbance terms. Let us see more carefully the different situations or cases of autocorrelation as depicted in Fig.12.1. In panels (a) to (d) of Fig. 12.1 we find distinct pattern among u_t . In panel (e) of Fig. 12.1 we do not see any such pattern. Note that since autocorrelation is seen mostly in time series data, we use the subscript 't' in place of 'i' to indicate individual observations. Let us now study the reasons of autocorrelation with some specific examples from economics.

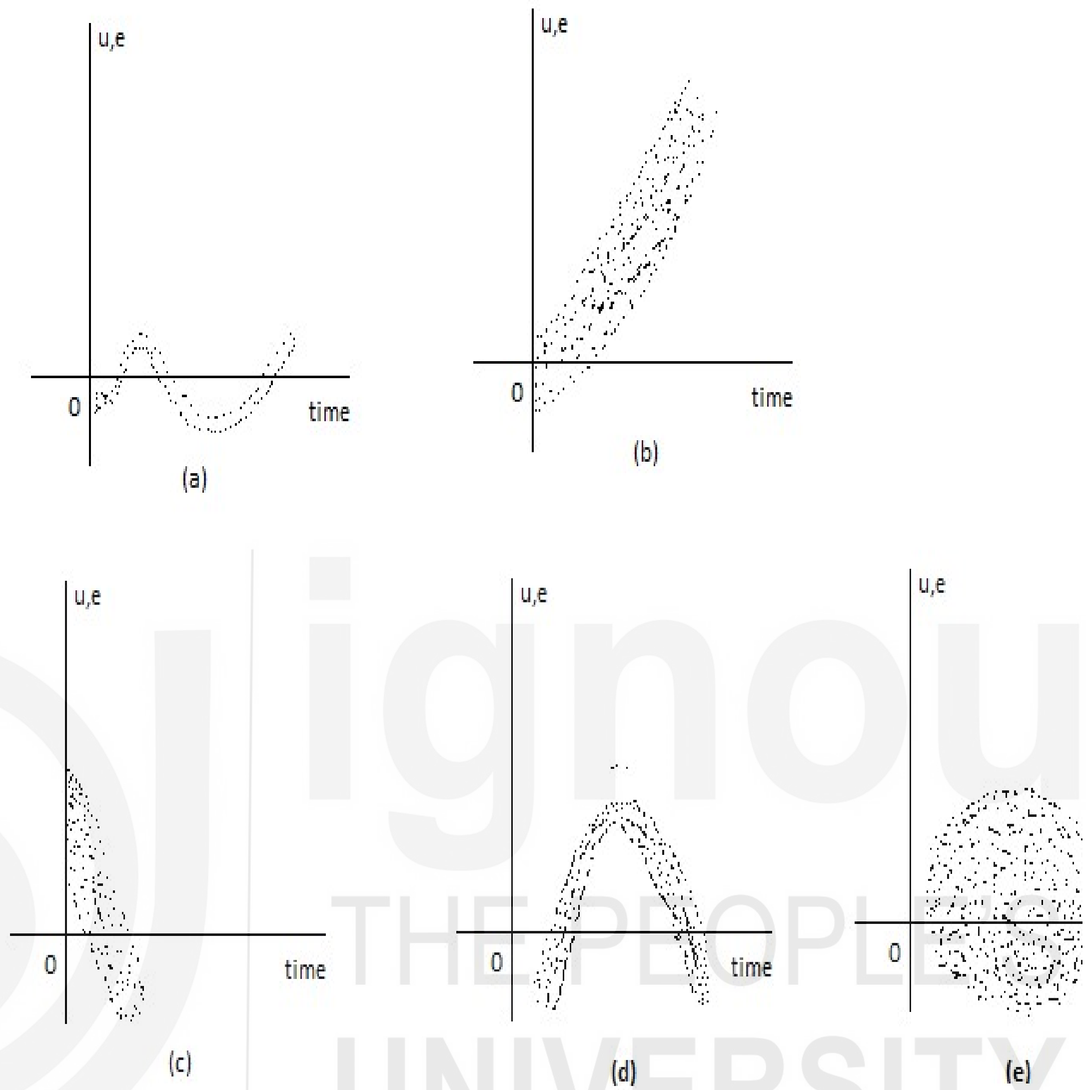

Fig. 12.1: Cases of Autocorrelation

12.3 REASONS FOR AUTOCORRELATION

The various reasons for the presence of autocorrelation can be discussed under the following broad heads.

(a) Inertia or Sluggishness

Most of the economic time series data displays inertia or sluggishness. For instance, gross domestic product (GDP), production, employment, money supply, etc. reflect recurring and self-sustaining fluctuations in economic activity. When an economy is recovering from recession, most of the time series will be moving upwards. This means any subsequent value of a series at one point of time is always greater than its previous time value.

Such a momentum continuous till it slows down due to, say, a factor like increase in taxes or interest or both. Hence, in regressions involving time series data, successive observations would generally be inter-dependent or correlated. Such an uptick effect is termed as 'inertia' which literally means a situation that continues to hold in a similar manner for many successive time periods. We see its opposite effect in periods of recession when most of the economic activity will be suffering, i.e., will be sluggish.

(b) Specification Error in the Model

By an incorrect specification of model, certain important variables that should be included in the model may not be included (i.e. a case of under-specification). If such model-misspecification occurs, the residuals from such an incorrect model will exhibit systematic pattern. If the residuals show a distinct pattern, it gives rise to serial correlation.

(c) The Cobweb Phenomenon

Many agricultural commodities reflect what is called as a 'cobweb phenomenon'. In this, supply reacts to price with a lag of time. This is mainly because supply decisions take time to implement. In other words, there is a gestation period involved. For instance, farmers' decision to plant crop might depend on the prices prevailing in the previous year's supply position or function. This can be written as:

$$S_t = \beta_1 + \beta_2 P_{t-1} + u_t \qquad \dots (12.2)$$

In (12.2), the error term u_t may not be purely random. This is because, if the farmers over-produce in year *t*, they are likely to under-produce in year (t + 1) since they want to clear away the unsold stock. This usually leads to a cobweb pattern.

(d) Data Smoothing

Sometimes we need to average the data presented. Considering averages implies 'data smoothing'(see Unit 5 of BECC 109 for an example). We may prefer to convert monthly data into quarterly data by averaging the data over every three months. However, this smoothness, desired in many contexts, may itself lead to a systematic pattern in disturbances, resulting in autocorrelation.

Autocorrelation may be positive or negative depending on the data. Generally, economic data exhibits positive autocorrelation. This is because most of them either move upwards or downwards over time. Such a trend continues at least for some time i.e. some months, or quarters. This means, they are not generally expected to exhibit a sudden upward or downward movement unless there is a reason or a shock.

12.4 CONSEQUENCES OF AUTOCORRELATION

When the assumption of no-autocorrelation is violated, the estimators of the regression model based on sample data suffers from certain consequences. More specifically, the OLS estimators will suffer from the following consequences.

a) The least squares estimators are still linear and unbiased. In other words, the estimated values of parameters continue to be unbiased. However, they are not efficient because they do not have minimum variance. Therefore, the usual OLS estimators are not BLUE (best linear unbiased estimators).

b) The estimated variances of OLS estimators (b_1 and b_2) are biased. Hence, the usual formula used to estimate the variances, and their standard errors underestimate the true variances and standard errors. Consequently, the decision of rejecting a parameter on the basis of *t*-values, concluding that a particular coefficient is statistically different from zero, would be an incorrect conclusion. In other words, the usual *t* and *F* tests become unreliable.

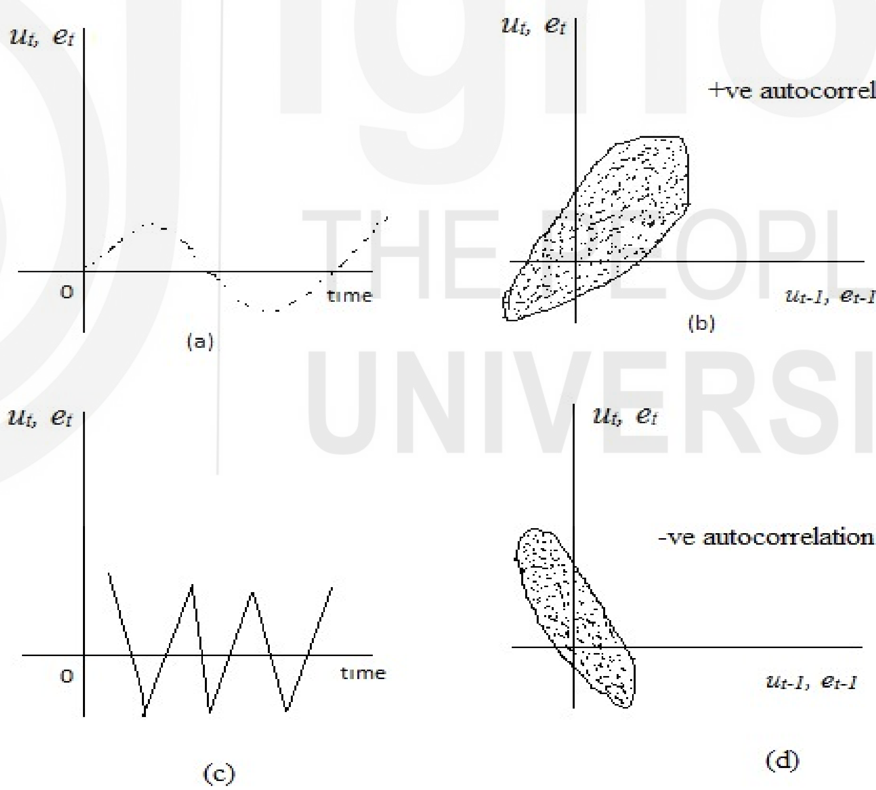

Fig. 12.2: Patterns of the Error Term in Autocorrelation

a) As a direct consequence of the above, the usual formula for estimating the population error variance, viz., $\hat{\sigma}^2 = (\text{RSS}/\text{df})$ yields a biased estimator of true σ^2. In particular, it underestimates the true σ^2. As a consequence, the computed R^2 becomes an unreliable measure of true R^2.

Fig. 12.2 shows the pattern of error terms under different situations of autocorrelation. Note that since the population error terms (u_t) are not known, we are plotting the sample residuals (e_t).

Check Your Progress 1 [Answer the questions in 50-100 words within the space given]

1) What is meant by autocorrelation in a regression model?

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2) In which type of data the problem of autocorrelation is more common? Why?

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3) State the broad reasons for autocorrelation.

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4) Enumerate the consequences of autocorrelation.

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12.5 DETECTION OF AUTOCORRELATION

There are many methods of detecting the presence of autocorrelation. Let us discuss them now.

12.5.1 Graphical Method

A visual examination of OLS residuals e_t quite often conveys the presence of autocorrelation among the error terms u_t. Such a graphical presentation (Fig. 12.3) is known as the 'time sequence plot'. The first part of this figure does not show any clear pattern in the movement of the error terms. This means there is an absence of autocorrelation. In the lower part of Fig. 12.3, you will notice that the correlation between the two residual terms is first negative and then becomes positive. Therefore, plotting the sample residuals gives us the first indication on the presence or absence of autocorrelation.

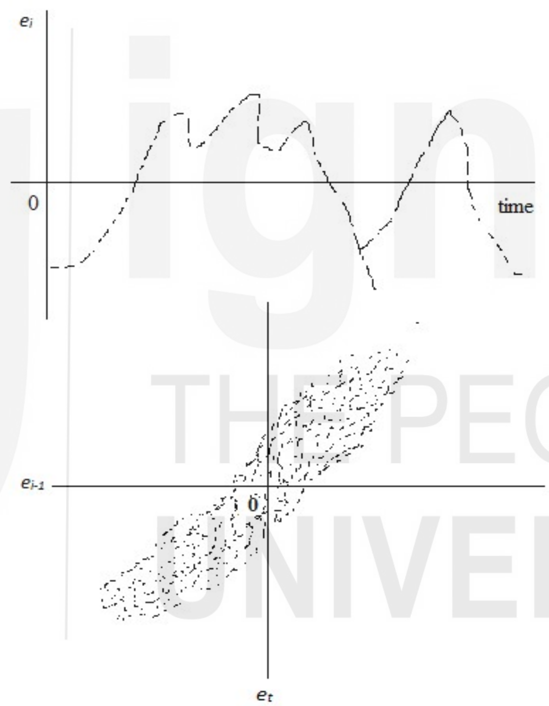

Fig. 12.3: Graphical Method for Detection of Autocorrelation

12.5.2 Durbin-Watson Test

The Durbin-Watson test, or the DW test as it is popularly called, is an analytical method of detecting the presence of autocorrelation. Its statistic is given by:

$$d = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} \quad \ldots (12.3)$$

Equation (12.3) defines the *d*-statistic suggested by Durbin-Watson as the ratio of the sum of squared differences in the successive residuals to the residual sum of squares. For computing the *d*-statistic, we take the sample size to be (*n*–1) since one observation is lost in taking the successive differences. There are certain assumptions underlying the *d*-statistic. These are:

(a) The regression model includes an intercept term. Therefore, this method cannot be used to determine autocorrelation in regression models without the intercept term (i.e. regression equation which passes through the origin).

(b) The *X* variables are non-stochastic, i.e., their values are fixed in repeated samples.

(c) The error term evolves as follows :

$$u_t = \rho u_{t-1} + v_t, \qquad -1 \leq \rho \leq 1 \qquad \ldots (12.4)$$

Equation (12.4) states that the value of error term at time period *t* is dependent on the value of the error term in time-period (*t*–1) and a purely random term v_t. The extent of dependence on past value is measured by ρ which lies between –1 and 1.

The regression model given in equation (12.4) is referred to as the first-order auto-regression scheme. It is denoted by *AR(1)*. The usage of the term 'autoregressive' implies that the error term u_t is regressed on its own lagged value of one period, i.e., u_{t-1}. It is therefore called the first-order autoregressive scheme. If we include 2 lagged values ($\text{i.e.}, u_{t-1}$ and u_{t-2}) then we have the *AR(2)* scheme. Likewise, when we extend the number of lagged values to '*p*', we have the *AR(p)* scheme.

(d) The regression model does not contain any lagged value of the dependent variable as one of the explanatory variables. In other words, the test is not applicable to models like:

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + u_t \qquad \ldots (12.5)$$

where Y_{t-1} is the one-period lagged-value of the dependent variable *Y*. Models of the above type are known as auto-regressive (AR) models. For such cases, the *d*-statistic cannot be used.

We can estimate ρ from equation (12.4) as follows:

$$\hat{\rho} = \frac{\sum_{t=2}^{n} e_t e_{t-1}}{\sum_{t=1}^{n} e_t^2}$$

[Recall that the estimator of b_2 in the two variable regression model is $b_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$. We apply the same logic to derive $\hat{\rho}$ above]

We can expand equation (12.3) to obtain

$$d = \frac{\sum e_t^2 + \sum e_{t-1}^2 - 2\sum e_t e_{t-1}}{\sum e_t^2}$$

The above can be approximated to

$$d \approx 2\left(1 - \frac{\sum_{t=2}^{n} e_t e_{t-1}}{\sum_{t=1}^{n} e_t^2}\right)$$

We can take an approximate value of d as:

$$d \approx 2(1-\hat{\rho}) \quad \ldots (12.6)$$

where the symbol $\approx$ denotes 'approximately'. In equation (12.6), $\hat{\rho}$ is an estimator of the first order autocorrelation scheme. Table 12.1 presents the value of the d-statistic for different values of $\hat{\rho}$.

From Table 12.1 we find that $0 \le d \le 4$. The Durbin-Watson statistic thus provides a lower limit d_L and an upper limit d_U. The computed value of d is therefore a value between 0 and 4. From such a value, we can infer on the nature of autocorrelation as follows:

a) If d is closer to 0, there is evidence of positive autocorrelation.

b) If d is closer to 2, there is evidence of no autocorrelation.

c) If d is closer to 4, there is evidence of negative autocorrelation.

Table 12.1: Value of *d*-Statistic according to $\hat{\rho}$

Value of $\hat{\rho}$	Implication	Value of d-statistic
$\hat{\rho} = -1$	Perfect negative autocorrelation	4
$\hat{\rho} = 0$	No autocorrelation	2
$\hat{\rho} = 1$	Perfect positive autocorrelation	0

The steps in applying the DW test are therefore the following:

1. Run the OLS regression and obtain the residuals e_t.
2. Compute d as:

$$d = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

3. Find out the critical Table values d_L and d_U for given sample size and given number of explanatory variables.

Follow the decision rule, as depicted in Fig. 12.4.

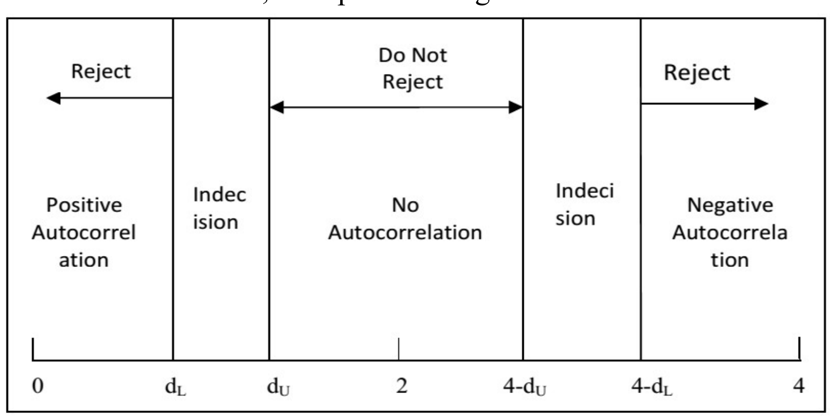

Fig. 12.4: Range of Values of Durbin-Watson Statistic

One drawback of the d-test is that it has two zones of indecision viz. $d_L < d < d_U$ and $(4 - d_U < d < 4 - d_L)$.

12.5.3 The Breusch-Godfrey (BG) Test

To avoid the pitfalls of the Durbin Watson d-test, Breusch and Godfrey have proposed a test criterion for autocorrelation that is general in nature. This is in the sense that:

(a) It can handle non-stochastic regressors as well as the lagged values of Y_t ;

(b) It can deal with higher-order autoregressive schemes such as AR(2), AR(3) … etc.

(c) It can also handle simple or higher order moving averages.

The BG-Test is also referred to as the LM (Lagrange Multiplier) Test (see Unit 8). Let us now consider a two-variable regression model to see how the BG test works.

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad \text{… (12.7)}$$

where u_t follows a P^{th} order auto regressive scheme AR(P) like:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \ldots + \rho_\rho u_{t-p} + v_t \qquad \text{… (12.8)}$$

where v_t is the white noise or the stochastic error term. We wish to test:

$$H_0: \rho_1 = \rho_2 = \ldots \rho_p = 0 \qquad \text{… (12.9)}$$

The null hypothesis says that there is no autocorrelation of any order. Now, the BG test involves the following steps:

(a) Estimate the model $Y_t = \beta_1 + \beta_2 X_t + u_t$ by OLS method and obtain the residuals e_t.

(b) Regress the residuals e_t on the p-lagged values of estimated residuals obtained in step (a) above, i.e., $e_{(t-1)}$, $e_{(t-2)}$, ……., $e_{(t-p)}$ [as in equation (12.8)]. Here we take the residual e_t which are estimate of the error u_t, as the error term is not known.

(c) Obtain R^2 from the auxiliary regression (12.8) in the step (b) above.

(d) Now, for large samples, the Breusch and Godfrey test statistic is computed as:

$$(n-p)R^2 \sim \chi^2_p \qquad \text{… (12.10)}$$

It is called LM test, as it has a similar form to the LM test described in Unit 8. The BG test statistic follows chi-squares distribution with p degrees of freedom where p is the number of regressors in the auxiliary regression (equation (12.8)).

We draw inferences from the BG test as follows:

(i) If $(n-p)R^2 > \chi^2_{critical}$, we reject H_0 and conclude that at least one ρ is statistically different from zero, i.e., there exists autocorrelation.

(ii) If $(n-p)R^2 < \chi^2_{critical}$, we do not reject H_0 and conclude that there exists no autocorrelation.

Check Your Progress 2 [Answer the questions in 50-100 words within the space given]

1) State the methods of detecting autocorrelation.

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2) Specify the test statistic applied in the DW test.

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3) State the assumptions under which the DW test is valid.

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4) Point out the limitations of the DW test.

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5) In what ways the BG test for autocorrelation is an improvement over the DW test?

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12.6 REMEDIAL MEASURES FOR AUTOCORRELATION

To suggest remedial measures for autocorrelation, we assume the nature of inter-dependence in the error term u_t in a regression model like:

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad \dots (12.11)$$

and that the error term is following an AR (1) scheme like:

$$u_t = \rho u_{t-1} + v_t \qquad -1 \leq \rho \leq 1 \qquad \dots (12.12)$$

where v_t is assumed to follow the OLS assumptions. We first consider the case where ρ is known. Here, transforming the model in a certain manner (called as the Cochrane Orcutt procedure) will reduce the equation to an OLS compatible model. When ρ is not known, we need some simple approaches which help us in overcoming the situation of autocorrelation. Let us study these approaches now.

12.6.1 Autoregressive Scheme is Known: Cochrane-Orcutt Transformation

Suppose we know the value of ρ. This helps us to transform the regression model given at (12.11) in a manner that the error term becomes free from autocorrelation. Subsequently, we apply the OLS method to the transformed model. For this, we consider a one-period lag in (12.11) as:

$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1} \qquad \dots (12.13)$$

Let us multiply equation (12.13) on both the sides by ρ. We obtain:

$$\rho Y_{t-1} = \rho\beta_1 + \rho\beta_2 X_{t-1} + \rho u_{t-1} \qquad \dots (12.14)$$

Let us now subtract equation (12.14) from equation (12.11) to obtain:

$$(Y_t - \rho Y_{t-1}) = \beta_1(1-\rho) + \beta_2(X_t - \rho X_{t-1}) + v_t \qquad \dots (12.15)$$

Note that we have used v_t for the new disturbance term above. Let us now denote:

$$Y_t^* = (Y_t - \rho Y_{t-1})$$

$$X_t^* = (X_t - \rho X_{t-1})$$

$$\beta_1^* = \beta_1(1-\rho)$$

The transformed model will be

$$Y_t^* = \beta_1^* + \beta_2 X_t^* + v_t \quad \dots (12.16)$$

Now, the transformed variables $Yt^*$and Xt^*will have the desirable BLUE property. The estimators obtained by applying the OLS method to (12.16) are called the Generalized Least Squares (GLS) estimators. The transformation as suggested above is known as the Cochrane-Orcutt transformation procedure.

12.6.2 Autoregressive Scheme is not Known

Suppose we do not know ρ. Thus, we need methods for estimating ρ. We first consider the case where ρ = 1. This amounts to assuming that the error terms are perfectly positively autocorrected. This case is called as the First Difference Method. If this assumption holds, a generalized difference equation can be considered by taking the difference between (12.11) and its first order autoregressive schemes as:

$$Y_t - Y_{t-1} = \beta_2(X_t - X_{t-1}) + v_t \quad \dots (12.17)$$

i.e., $\Delta Y_t = \beta_2 \Delta X_t + v_t$ … (12.18)

where the symbol Δ (read as delta) is the first difference operator. Note that the difference model (12.17) has no intercept. If ρ is not known, then we can estimate ρ by the following two methods.

(i) Durbin Watson Method

From equation (12.6) we see that *d*-statistic and ρ are related. We can this relationship to estimate ρ. The *d*-statistic and ρ are related as:

$$\rho \approx 1 - \frac{d}{2} \quad \dots (12.19)$$

If the value of *d* is known, then $\hat{\rho}$ can be estimated from the *d*-statistic.

((ii) The OLS Residuals (e_t) Method

Here, we consider the first order autoregression scheme as in (12.12), i.e.,

$u_t = \rho u_{t-1} + v_t$. Since u_t is not directly observable, we use its sample counterpart e_t and run the following regression:

$$e_t = \hat{\rho} e_{t-1} + v_t \quad \dots (12.20)$$

Note that $\hat{\rho}$ is an estimator of ρ. In small samples, $\hat{\rho}$ is a biased estimator of ρ. As sample size increases, the bias disappears.

12.6.3 Iterative Procedure

This is also called as the Cochrane-Orcutt iterative procedure. We consider the two variable model with the AR(1) scheme for autocorrelation as discussed earlier. That is, we consider: $Y_t = \beta_1 + \beta_2 X_t + u_t$ where $u_t = \rho u_{t-1} + v_t$ with $-1 \leq \rho \leq 1$. We have taken only one explanatory variable for simplicity but we can have more than one explanatory variable too. The iterative procedure suggested by Cochrane-Orcutt has the following steps:

(i) Estimate the equation $u_t = \rho u_{t-1} + v_t$ by the usual OLS method.

(ii) From the above, obtain the residuals e_t.

(iii) Using the residuals e_t, run the regression $e_t = \hat{\rho}\, e_{t-1} + v_t$ and obtain $\hat{\rho}$.

(iv) Use $\hat{\rho}$ obtained in (iii) above to multiply the equation $u_t = \rho u_{t-1} + v_t$.

(v) Now, obtain the generalized difference equation as:

$Y_t^* = \beta_1^* + \beta_2 X_t^* + e_t$ where, $Y_t^* = Y_t - Y_{t-1}$, $X_t^* = X_t - \rho X_{t-1}$ and

$\beta_1^* = \beta_1(1 - \hat{\rho})$

(vi) We are not sure that $\hat{\rho}$ estimated in (iii) above is the best estimate of ρ. Therefore, we repeat the steps (ii) and (iii) to obtain the new residuals e_t^*.

(vii) Now estimate the regression $e_t^* = \hat{\rho} e_{t-1}^* + w_t$to obtain the new estimate of $\hat{\rho}$.

We thus obtain the second-round estimate of ρ. Since we are not sure if the second round estimate of ρ is the best, we go for the third round estimate and so on. We repeat the same steps again and again. Due to this repetitive steps followed, this procedure, suggested by Cochrane-Orcutt, is called the 'iterative procedure'. We stop the iteration when the successive estimates of ρ differ by a small amount (less than 0.01 or 0.005).

12.7 LAGGED DEPENDENT VARIABLE

The Durbin-Watson method is not applicable when the regression model includes lagged value of the dependent variable as one of the explanatory variables. In such models, the *h*-statistic suggested by Durbin is used to identify the presence of autocorrelation in the regression model. Let us consider the regression model as:

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + v_t \qquad \dots (12.21)$$

In equation (12.21), we have two explanatory variables: X_t and Y_{t-1} with Y_{t-1} as a lagged dependent variable (with one-period lag). For equation (12.21) the *d*-statistic is not applicable to detect autocorrelation. For such models, Durbin suggests replacing the *d*-statistic by the *h*-statistic taken as:

$$h \approx \hat{\rho} = \sqrt{\frac{n}{1 - n\,Var(b_3)}} \qquad \dots (12.22)$$

where, *n* = sample size, $\hat{\rho}$ = the estimator of the autocorrelation coefficient, and

$var(b_3)$ = variance of estimator of β_3, the lagged dependent variable in (12.21).

The null hypothesis is $H_0: \rho = 0$. Durbin has shown that for large samples the *h*-statistic is distributed as h$\sim N(0,1)$. For normal distribution, we know that the critical value at 5 per cent level of significance is 1.96 and at 1 per cent level of significance it is 2.58. Using this information, we can draw inference from equation (12.22) as follows:

(i) If the computed value of h is greater than the critical value of h, we reject H_0. We interpret the result as existence of no autocorrelation.

(ii) If the computed value of h is less than the critical value of h, we do not reject H_0. We interpret the result as existence of autocorrelation.

Check Your Progress 3 [Answer the questions in 50-100 words within the space given]

1) Outline the transformation procedure suggested by Cochrane-Orcutt to resolve the problem of autocorrelation.

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2) State how the iterative procedure of Cochrane-Orcutt is applied in the case of autocorrelation in a dataset. Why is it called iterative procedure?

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3) What is the advantage of using the h-statistic in regression model having autocorrelation problem?

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11.8 LET US SUM UP

The unit has discussed the concept of autocorrelation in regression models. The consequences of the presence of autocorrelation, its detection and techniques that provide remedial measures for such situations have been explained. The unit also discusses the case of autocorrelation in regression models with lagged dependent variables.

11.9 ANSWERS/ HINTS TO CHECK YOUR PORGRESS EXERCISES

Check Your Progress 1

1) Autocorrelation refers to the presence of correlation between the error terms of any two observations. This means if U_i and U_j are the error terms, then *Corr (Ui, Uj)* $\neq 0$ for $i \neq j$. In the CLRM, one of our assumptions is that the *Corr (Ui, Uj) = 0*. This means the two error terms are not correlated. Violation of this assumptions is a situation of autocorrelation.

2) The problem of autocorrelation is more common in time series data. This is because a phenomena affecting the error term in one point of time is more likely to be influencing the error term in the next point of time. This is especially identified as the factor of 'inertia or sluggishness'. Across units of cross section this is less likely. But it cannot be ruled out even in cross section data. In such cases, due to the spatial effect in cross section data, which is more like a demonstration effect, it is distinctly termed as spatial correlation.

3) Inertia or sluggishness, specification error in the model, cobweb phenomenon and data smoothening.

4) The consequences are: (i) least squares estimators are not efficient, (ii) the estimated variances of OLS estimates are biased, (iii) the standard error of true variances are underestimated, (iv) we are more likely to commit an error in deciding on the hypothesis of 'no statistical significance' of a particular estimated coefficient i.e. the decisions based on t and F tests would be unreliable, (v) estimated error variance would be biased and (vi) the value of R^2 would be misleading or unreliable.

Check Your Progress 2

1) Time sequence plotting (graphical method), Durbin-Watson test and Breusch-Godfrey (BG) Test.

2) $\boldsymbol{d} = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$. It is the ratio of the sum of the squared differences in the successive residuals to the residual sum of squares.

3) The regression model includes an intercept term, the X variables are non-stochastic, the error term follows the following mechanism $u_t = \rho u_{t-1} + v_t$, $-1 \leq \rho \leq 1$, and the regression does not contain any lagged values of the dependent variable as one of the explanatory variables.

4) The one drawback of the d-test is that it has two zones of indecision, viz., $d_L < d < d_U$ and ($4 - d_U < d < 4 - d_L$).

5) (i) It can handle non-stochastic regressors as well as the lagged values of Y_t, (ii) it can deal with higher-order autoregressive schemes such as AR(2)… etc. and (iii) it can also handle simple or higher order moving averages.

Check Your Progress 3

1) In this method we lag the regression equation by one period; multiply it by $\boldsymbol{\rho}$; and subtract it from the original regression equation. This gives us a transformed regression model. When estimated by OLS method, the estimators of the transformed model are BLUE.

2) In Sub-Section 12.6.3 we have outlined steps of the Cochrane-Orcutt iterative procedure. You should go through it and answer.

3) The *h*-statistic can be used in regression models having lagged dependent variables as explanatory variables.