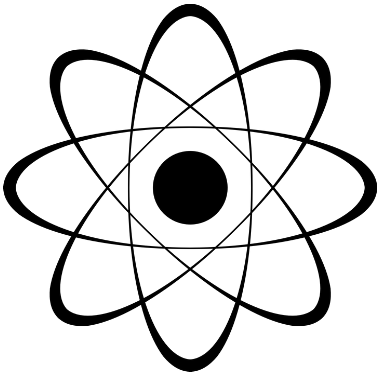


Picture of an atom. The nucleons stay in the nucleus, electrons revolve around it in elliptical orbits.
(Source: https://pixabay.com/vectors/nucleus-physics-atom-protons-35000/

UNIT 14

THE ATOMIC NUCLEUS

Structure

STUDY GUIDE

In this unit, you will learn about the atomic nucleus; its constituents and the nature of the forces that keep the nucleons (protons and neutrons) confined within the nucleus. You will also learn Heisenberg's uncertainty principle, which tells us why negatively charged electrons cannot reside inside the nucleus. You would be able to discuss the stability of the nuclei based on the binding energy curve. This will enable you to estimate the size and density of the nucleus. You will know why some naturally occurring elements are more abundant and stable than others. We have included several examples in the unit to enhance your grasp of various concepts discussed here. Several SAQs and TQs are included in the unit to help you gain the essence of various concepts. You should answer SAQs and work out problems yourself to enhance your conceptual clarity. We are aware that some of the concepts discussed here are known to you. But we advise you to refresh your memory and revise these.

"Imagination is more important than knowledge." ***Albert Einstein***

14.1 INTRODUCTION

In Unit 13, you have learnt about radioactivity exhibited by heavier elements in the form of spontaneous emission of alpha and beta particles and gamma rays to attain stability. From your senior school physics classes, you will recall that existence of atoms was postulated by Greek philosopher Democritus around 450 BC. (Indian scientist Acharya Kanad for the concept of parmanu.) However, Aristotle, an influential Greek philosopher rejected Democritus' idea. But in 1808, John Dalton, a British chemist, revived the brilliant idea of the atom as the smallest indivisible entity of all matter to explain the laws of chemical composition. Through his subsequent researches, Dalton provided evidence for the existence of atoms. As we now know, atoms are too small to be weighed even with most sensitive balance available now or observed even by the most powerful microscope using visible light. (The wavelength of visible light is in the range 400 to 700 μm, which is too large compared to the size of the atom).

Dalton' conjecture was substantiated by later researches of Thomson, Rutherford and Bohr. While Thomson discovered electrons and proposed that these were fundamental constituents of all matter, Rutherford gave the model of atom as consisting of a hard central core –the nucleus--surrounded by electrons. Thereafter, using the ideas of quantum mechanics proposed by Planck, Bohr argued that electrons revolve around the nucleus in well defined orbits. Einstein's theory of Brownian motion by assuming that solute particles were continuously being knocked around by water molecules was experimentally confirmed by Jean Perrin. Using Avogadro's hypothesis, he calculated the size of the atom to be around 10^{-8} cm. These studies opened a very fertile field of research in nuclear physics in the first half of the 20th century. Some of the questions that engaged the leading physicists of that time were:

- How are electrons distributed in an atom?
- What is the nucleus made of?
- How do nucleons stay together in the nucleus?

Answers to these and such other questions led to a pool of brilliant knowledge. In Sec. 14.2, you will learn about constituents of the nucleus and how electrons are distributed in a nucleus. In Sec. 14.3, you will learn that stability of a nucleus is governed by its binding energy per nucleon (BE/A). An important consequence of BE/A is that very light and very heavy elements show tendency for fusion or fission under suitable conditions. (These concepts have been used to generate electricity and you will learn about these in the next unit.) In Sec. 14.4 you will learn about nuclear forces which hold the nucleons (protons and neutrons) together while overcoming Coulomb repulsion between protons.

In the next unit, you will learn about a few applications of nuclear physics.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ list the constituents of the nucleus;
- ❖ explain why electrons cannot residing inside the nucleus;
- ❖ explain the nature of nuclear forces;
- ❖ discuss the stability of nuclei based on the binding energy curve; and
- ❖ calculate the binding energy using Weizsäcker's semi-empirical mass formula.

We begin our journey of the study of the nucleus by discussing what constitutes it.

14.2 CONSTITUENTS OF THE NUCLEUS

Let us estimate the size of the atom.

We know that iron is one of the most abundant elements found on earth. The density of pure iron is $7.874\,\text{gcc}^{-1}$ and its molar mass of 56 g. On the basis of Avogadro's hypothesis, we can say that the number of atoms in 56 g iron is equal to Avogadro's number; 6×10^{23}). To get an estimate of the volume occupied by an iron atom, we will take density of iron as $8\,\text{gcc}^{-1}.$ It means that 6×10^{23} atoms of iron occupy a volume $\frac{56}{8}=7\,\text{cm}^3.$

Therefore, the volume occupied by one iron atom is $\frac{7}{6\times10^{23}}\,\text{cc}$ which can be taken to be approximately as $10^{-23}\,\text{cc}$.

If we assume that iron is packed as hard iron spherical atoms, then its volume will be $\frac{4}{3}\pi r_0^3$ where r_0 is the radius of the iron atom. That is volume of an atom of iron is directly proportional to the third power of its radius. Hence, the order of magnitude of the radius of an iron atom would be $(10^{-23})^{1/3}\sim10^{-8}\,\text{cm}$.

When light of sufficiently high frequency is made to fall on a metal plate, negatively charged particles are emitted due to photo-electric effect. J.J. Thompson identified the negatively charged particles as electrons and measured their charge to mass ratio, $\frac{e}{m}$. He argued that since electrons are constituents of atoms, electrical neutrality of an atom implies that there must be an equal positive charge in the atom. On the basis of these results, Thomson proposed his 'plum-pudding model' of atom. He suggested that the atom consisted of a uniform positive sphere of radius nearly $10^{-10}\,\text{m}.$ with electrons embedded in such a way as to give it a stable configuration. (That is, electrons were embedded in a spherical cloud of positive charges like seeds in a water melon.),

On the basis of his experimental results, Rutherford proposed that the total positive charge of the atom was not distributed uniformly. It was, in fact, concentrated in an extremely small region of size ~ 10^{-14} m inside the atom. Atomic size was found to be roughly ten thousand times the size of the inner region (~ 10^{-10} m) where most of the mass and positive charges resided.

Rutherford thus proposed that

- all the mass and positive charge of an atom is concentrated in a tiny nucleus;
- the radius of the nucleus is of the order of a few fermi (1 fermi = 10^{-15} m) and
- electrons reside outside the nucleus.

In a series of experiments carried out by Chadwick in 1920, it was established that the charge on the nucleus was exactly equal to the total electronic charge of the atom, thus rendering the atom neutral. Each atom can be uniquely characterised by a single number, known as the atomic number. It is denoted by the symbol *Z*. The nucleus thus consists of *Z* positively charged particles called protons and their mass is roughly equal to the mass of atomic hydrogen. You may now ask: Do all nuclei consist of only protons? To answer this question, we consider three forms of hydrogen namely, Hydrogen, Deuterium and Tritium. All three forms of hydrogen have identical chemical properties corresponding to one atomic electron each. Their masses however, differ

$$m_H : m_D : m_T :: 1 : 2 : 3$$

Thus, if the mass of the atom is mainly contributed by the positively charged protons, Hydrogen, Deuterium and Tritium should contain 1, 2 and 3 protons, respectively, in their nuclei. The charge neutrality would then demand that the nucleus must also contain the electrons to neutralize the excess positive charge. Thus, the composition of these three forms of hydrogen should be as follows:

Element	No. of Atomic Electron	Particles in the Nucleus
Hydrogen	1	1 proton
Deuterium	1	2 protons + 1 electron
Tritium	1	3 protons + 2 electrons

But there is a flaw in this reasoning. This brings us to the question: Do electrons reside inside the nucleus? The answer is not affirmative. To know the correct response, go through the following paragraph.

Let us find out whether the electrons stay inside the nucleus or are the constituents of the nucleus. (You have studied about Heisenberg's uncertainty principle in Sec. 6.2, Unit 6 of Block 2.)

If electrons were trapped in the nucleus of size 10^{-14} m, then according to Heisenberg's uncertainty principle, there would be uncertainty in its position $\Delta x \sim 10^{-14}$ m. The corresponding uncertainty in its momentum (according to Heisenberg's Uncertainty Principle ($\Delta x \; \Delta p_x \geq \hbar$))would then be

$$\Delta p_x \sim \frac{\hbar}{\Delta x} = \frac{\hbar c}{\Delta x c} = \frac{1}{c}\frac{1.97\,\text{MeV}}{0.1} \sim 19.7\frac{\text{MeV}}{c}$$

The kinetic energy of the electron would thus to

$$K = \sqrt{p^2c^4 + m_e^2c^4} - m_ec^2 \sim 19\,\text{MeV}$$

Thus, electrons confined inside the nucleus would have an uncertainty in their kinetic energy of the order of 19 MeV where m_ec^2 is the rest mass energy of the electron.

However, the electrons emitted from the nucleus in nuclear beta decay typically had kinetic energy of the order of few MeV. This therefore suggested that electrons could not possibly reside inside the nucleus. You may now ask: If the number of protons inside the nucleus equals the number of atomic electrons (required for charge neutrality), what is the source of missing mass in a nucleus? To know answer to this question, go through the following example.

EXAMPLE 14.1 : MISSING MASS IN THE NUCLEUS

Iron has 26 electrons but its mass is roughly 56 times the mass of the hydrogen atom. Charge neutrality implies that the nucleus should have 26 positively charged protons, which would account for roughly 26 times the hydrogen mass to its overall mass value. Where then has the missing mass (equal to 56 – 26 = 30 proton mass) gone?

SOLUTION ■ This discrepancy in the atomic mass pointed to the possibility of the existence of new particles in the nucleus that contributed to its mass but not to its charge.

Rutherford conjectured the existence of a particle devoid of all charge but slightly heavier than the proton in the nucleus. The particle was named neutron by him. The experimental evidence for the existence of neutrons came in 1932 through the research of Chadwick. It then became clear that all nuclei were composed of protons and neutrons with the exception of hydrogen nucleus. The number of protons and neutrons together define the mass number, denoted by *A*. Neutrons and protons are collectively referred to as nucleons.

Table 13.1: Constituents of the Nucleus

Name	Symbol	Charge	Mass
Proton	*p*	+ve	1.007276 u = 938.28 MeV
Neutron	*n*	0	1.008666 u = 939.57 MeV

14.2.1 Isotopes

We often come across elements whose atoms have the same number of electrons but different number of neutrons. So, their nuclear masses differ. Such atoms are called **Isotopes.** For example, deuterium nucleus has one proton and one neutron and tritium nucleus consists of one proton and two neutrons. Since they have only one electron like hydrogen, these are three different isotopes of hydrogen and are denoted by the symbols ^{1}H, ^{2}H, ^{3}H. Uranium for example has three isotopes ^{233}U, ^{235}U and ^{238}U and so on. You should write out isotopes of other elements known to you.

14.2.2 Nuclear Density

The alpha-particle experiment performed by Geiger and Marsden under the guidance of Rutherford provided the first evidence that nuclei are of extremely small size $(\sim 10^{-15}\,\text{m})$. They bombarded gold foil by alpha particles emitted by radioactive Bismuth. Since then, various experiments have been performed using high energy electrons and neutrons to determine nuclear dimensions. These experiments have revealed that

- nuclei do not have sharp boundaries; and
- the density of nuclear matter is maximum at the centre of the nucleus and decreases gradually to zero as the distance increases.

You would now like to know the order of magnitude of the density of nuclear matter. Let us consider the lightest nucleus of hydrogen of mass $1.673\times10^{-27}\,\text{kg}$ and radius $1.2\times10^{-15}\,\text{m}$. If we assume it to be spherical in shape, the density of nuclear matter can be calculated as follows:

$$d_{\text{H}} = \frac{M_{\text{H}}}{\frac{4\pi}{3}R_{\text{H}}^3} = \frac{1.673\times10^{-27}\,\text{kg}}{\frac{4\pi}{3}(1.2\times10^{-15}\,\text{m})^3} = 2.3\times10^{17}\,\text{kg}\,\text{m}^{-3}$$

This value is extremely high. Recall the densities of water $(=10^3\,\text{kg}\,\text{m}^{-3})$ and mercury $(13.6\times10^3\,\text{kg}\,\text{m}^{-3})$. It means that nuclear matter is extremely densely packed. The mass of our Earth $(=6\times10^{24}\,\text{kg})$, if packed to such high density, would lead to a sphere of radius only ~184 m! You should convince yourself by doing this calculation. Also, calculate the radius of the nuclear sphere whose mass will be equal to the mass of our sun. Your answer should be nearly 10 km!

Next, let us calculate the density of nuclear matter for oxygen. It is reliably known that the radius of oxygen nucleus $R_0 = 3\times10^{-15}\,\text{m}$ and its mass $M_0 = 2.7\times10^{26}\,\text{kg}$. Therefore, the density of oxygen nucleus is given by

$$do = \frac{2.68\times10^{-26}\,\text{kg}}{\frac{4\pi}{3}(3\times10^{-15}\,\text{m})^3} = 2.39\times10^{17}\,\text{kg}\,\text{m}^{-3}$$

Note that the densities of hydrogen and oxygen nuclei are nearly the same. Is it a mere coincidence? To discover the answer, solve the following SAQ.

SAQ 1 – **Density of nuclei**

Calculate the density of carbon and lead nuclei using the following data:

$$M_C = 19.92\times10^{-27}\,\text{kg}, \qquad R_C = 2.7\times10^{-15}\,\text{m}$$

$$M_{\text{Pb}} = 3.4\times10^{-25}\,\text{kg} \quad \text{and} \quad R_{\text{Pb}} = 7\times10^{-15}\,\text{m}$$

On solving SAQ 1, you will come to the conclusion that nuclei of all elements have roughly the same density. This suggests that the

- nucleus is analogous to a drop of liquid, and
- there is an empirical relation between the radius of a nucleus, R, and its mass number A. Indeed, experimental evidences suggests that R and A are connected through the relation $R = 1.12 \times 10^{-15} A^{1/3}\text{m}$.

To get a quick idea about this, let us calculate the radii of carbon and lead nuclei:

$$M_C = M_H A_C = \frac{4\pi}{3} R_C^3 d_C$$

$$R_C = \left[\frac{3}{4\pi}\left(\frac{M_H}{d_C}\right)\right]^{1/3} A_C^{1/3}$$

Similarly, $$R_{\text{Pb}} = \left[\frac{3}{4\pi}\left(\frac{M_H}{d_{\text{Pb}}}\right)\right]^{1/3} A_{\text{Pb}}^{1/3}$$

On substituting the values given and/or calculated in SAQ 1 for carbon and lead, you will get

$$R_C = \left[\frac{3}{4 \times 3.1417}\left(\frac{1.673 \times 10^{-27}\,\text{kg}}{2.42 \times 10^{17}\,\text{kg m}^{-3}}\right)\right]^{1/3} A_C^{1/3}$$

$$= \left(\frac{1.673 \times 0.2387}{2.42}\right)^{1/3} \times 10^{-15} A_C^{1/3}\text{m}$$

$$= \left(\frac{3.993}{2.42}\right)^{1/3} \times 10^{-15} A_C^{1/3}\text{m}$$

$$= (1.6502)^{1/3} \times 10^{-15} A_C^{1/3}\text{m}$$

$$= 1.1181 \times 10^{-15} A_C^{1/3}\text{m}$$

$$= 1.12 \times 10^{-15} A_C^{1/3}\text{m}$$

Similarly, for lead, we have

$$R_{\text{Pb}} = \left[\frac{3}{4 \times 3.1416}\left(\frac{1.673 \times 10^{-27}\,\text{kg}}{2.37 \times 10^{17}\,\text{kg m}^{-3}}\right)\right]^{1/3} A_{\text{Pb}}^{1/3}\text{m}$$

$$= \left(\frac{16.73 \times 0.2387}{2.37}\right)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3}\text{m}$$

$$= \left(\frac{3.993}{2.37}\right)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3}\text{m}$$

$$= (1.685)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3}\text{m}$$

$$= 1.1190 \times 10^{-15} A_{\text{Pb}}^{1/3}\text{m}$$

$$= 1.12 \times 10^{-15} A_{\text{Pb}}^{1/3}\text{m}$$

Now-a-days, atomic masses are expressed in terms of the actual mass of ^{12}C isotope of carbon. The unit of atomic mass, abbreviated as u, is (1/12)th of the actual mass of the ^{12}C. This is equal to 1.66×10^{-27} kg. The energy equivalent of 1u is

$$1\text{u} = (1.66 \times 10^{-27}\,\text{kg}) \times (2.998 \times 10^{8}\,\text{ms}^{-1})^2$$

$$= 14.92 \times 10^{-11}\,\text{J}$$

$$= 931.3 \times 10^{6}\,\text{eV}$$

$$= 931.3\,\text{MeV}$$

since

These calculations show that the relation between the radius of a nucleus, R, and its mass number A is the same, irrespective of the size of the nuclei.

Let us now discuss about binding energy of a nucleus, which determines its stability.

14.3 BINDING ENERGY OF THE NUCLEUS

You now know that the nucleus of deuterium contains one proton and one neutron. The measured (rest) masses of the proton and the neutron are 1.6723×10^{-27} kg and 1.6747×10^{-27} kg, respectively. This means that the total rest mass of a neutron plus a proton is 3.34709×10^{-27} kg. But the rest mass of a deuterium nucleus is 3.34313×10^{-27} kg. This means that measured mass of a deuterium nucleus is (3.34709×10^{-27} kg – 3.34313×10^{-27} kg) = 3.96242×10^{-30} kg less than the measured masses of a neutron and a proton. In fact, it is quite well known now that mass of any nucleus is always less than the sum of the rest masses of its constituent nucleons. This difference is termed the **mass defect**. We denote it by Δm. Mathematically, we can write

$$\Delta m = (Z m_p + N m_n) - (M - Z m_e)$$

$$= Z m_H + N m_n - M \qquad (14.1)$$

where M is the actual mass of the neutral atom containing Z protons and N neutrons. $m_H = (m_p + m_e)$, m_p, m_n and m_e are the masses of the hydrogen atom, the proton, the neutron and the electron respectively. It is often convenient to express the mass defect by its equivalent energy through Einstein's mass-energy equivalence relation (about which you have learnt in Unit 3 of this course):

$$\text{BE} = \Delta m c^2$$

For deuterium, we get

$$\text{BE} = (3.96242 \times 10^{-30}\,\text{kg}) \times (2.998 \times 10^{8}\,\text{ms}^{-1})^2$$

$$= 35.614 \times 10^{-14}\,\text{kgm}^2\text{s}^{-2}$$

$$= 3.5614 \times 10^{-13}\,\text{J}$$

$$= 2.223 \times 10^{6}\,\text{eV} = 2.223\ \text{MeV}$$

since $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$.

This means that we will have to provide at least 2.223 MeV energy to free the constituent nucleons – protons and neutrons – of deuterium nucleus. We can generalise this result to say that **mass defect appears as the energy which binds the nucleons together**. This is essentially used up in doing work against the forces which bind the nucleons together.

If we provide more energy than 2.223 MeV, the extra energy appears as kinetic energy of free nucleons. This result is confirmed by observations of the photo-disintegration of a deuteron. When deuterium is bombarded by gamma ray photons, it breaks up into a proton and a neutron on absorbing a photon of energy at least equal to the binding energy.

$$E_\gamma = (m_p + m_n - m_d)c^2$$

This is shown in Fig. 14.1. Inverting the above argument we can say that when a neutron and a proton combine to form a deuteron, a small mass is found missing. Does this not suggest that the binding energy can be looked upon as a direct measure of nuclear stability? To convince yourself with the answer to this question, you may like you to solve SAQ 2.

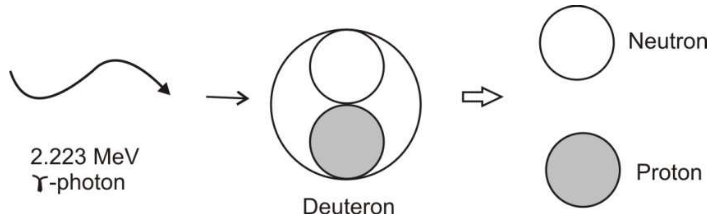


Fig. 14.1: **When deuterium is bombarded by a 2.223 MeV gamma ray photon, it breaks up into a proton and a neutron.**

SAQ 2 – Binding energy of nuclei

Calculate the binding energy of (i) $^4_2$He, (ii) $^{35}_{17}$Cl, (iii) $^{56}_{26}$Fe and (iv) $^{235}_{92}$U. Given $m_n = 1.008665$ u, $M(^1\text{H}) = 1.007825$ u, $M(^4\text{He}) = 4.002604$ u, $M(^{35}\text{Cl}) = 34.96885$ u, $M(^{56}\text{Fe}) = 55.934932$ u and $M(^{235}\text{U}) = 235.043933$ u.

On solving this SAQ you will find that binding energy of a nucleus is an increasing function of the mass number (28.3 MeV for ^{4}He, 298 MeV for ^{35}Cl, 492 MeV for ^{56}Fe and 1784 MeV for ^{235}U. But if we divide these binding energies by the mass numbers of particular nuclei, the binding energies per nucleon are found to be 7.1 MeV, 8.5 MeV, 8.8 MeV and 7.6 MeV. Fig. 14.2 shows an explicit plot of binding energy per nucleon as a function of mass number. You will note that

- binding energy curve shows sharp peaks, particularly for ^{4}He, ^{9}Be, ^{12}C, ^{16}O and ^{20}Ne;
- with the exception of light nuclei ($A \le 20$), the values of binding energy per nucleon fall on a smooth curve;
- binding energy per nucleon increases monotonically with a maximum in the vicinity of *A* = 56 (corresponding to the iron nucleus) with a maximum value of 8.8 MeV; and
- For *A* > 56, BE/*A* decreases steadily and drops to 7.6 MeV for *A* = 238.

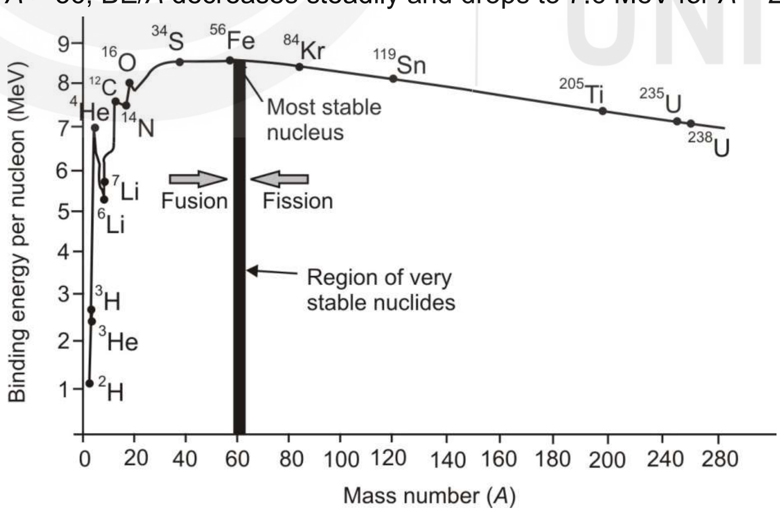


Fig. 14.2: **Binding energy per nucleon as a function of mass number.**

From this discussion, we can conclude that nuclei at either extreme of the periodic table are less stable as compared to the nuclei in the middle. It means that BE/*A*, rather than BE, is a better measure of the stability of a nucleus.

The variation of the binding energy per nucleon with mass number hints at the possibility of tapping the energy of the nucleus. For instance, when two light nuclei fuse to produce a more stable nucleus, energy is released. Such reactions are called **fusion reactions** and are responsible for the release of energy in the Sun. Efforts are now on in France and China to use controlled fusion reaction and create “artificial sun” on the earth. It is hoped that the world would get commercial fusion energy by 2050, which, once achieved, should meet our all future energy needs.

Similarly, when a very heavy nucleus breaks into two parts, the binding energy per nucleon increases leading to liberation of energy. This process is called **nuclear fission.** The amount of energy released in fission is equal to the number of nucleons times the difference in binding energy per nucleon of the reactants and the products. For example, the binding energy per nucleon in ^{235}U is nearly 7.6 MeV, whereas it is about 8.5 MeV for nuclei with mass number around 120. Thus, if a ^{235}U nucleus splits into two nearly equal fragments, there would be a gain in binding energy of the system of 0.9 MeV per nucleon. The total energy released in one fission event would therefore be nearly equal to $235 \times 0.9 \cong 212\,\text{MeV}$. Do you know that the heat of combustion of a carbon atom is only about 4eV. It means that when 1 kg of uranium undergoes fission, we get 2.4 million kWh of power which is huge compared to the power (8kWh) obtained by burning 1kg coal.

14.3.1 Semi-empirical Mass Formula

A semi-empirical formula for calculating the binding energy of nuclei was given by **Weizsäcker** by considering the similarity that exists between forces which make nucleons cling together in a nucleus and the forces that bind molecules in a liquid drop. Following Weizsäcker, we quote the result for the binding energy of a nucleus containing Z protons and *A* nucleons:

$$\text{BE (MeV)} = \alpha A - \beta A^{2/3} - \gamma \frac{(A-2Z)^2}{A} - \delta \frac{Z(Z-1)}{A^{1/3}} \pm \frac{\epsilon}{A^{3/4}} \quad (14.2)$$

where α =15.8, β = 17.8, γ = 23.7, δ = 0.71 and

$$\epsilon = \begin{cases} 34, \text{ for even} - \text{even or odd} - \text{ odd nuclei} \\ 0, \text{otherwise} \end{cases}$$

Before proceeding further, let us understand the meanings of various terms in Eq. (14.2).

The first term in Eq. (14.2) represents the attractive energy of nucleons (volume energy since A is related to the cube root of radius of a nucleus); the second term corrects for the over-estimation due to the weaker binding of nucleons near the surface and is proportional to the surface area (surface effect), the third term is a negative correction due to excess neutrons (asymmetry effect) and the fourth term takes account of the electrostatic energy of protons (Coulomb effect). Since each charged particle in a nucleus repels all the other charged particles, this energy is negative and proportional to the number of pairs of protons, which is equal to $Z(Z-1)/2$. The last term is the spin term. This term is positive for even-even nuclei (i.e., both Z and N even), negative for odd-odd nuclei and zero for odd-even or even-odd nuclei. The relative contributions of the various terms in Weizsäcker’s semi-empirical formula to binding energy per nucleon as a function of mass number are plotted in Fig. 14.3.

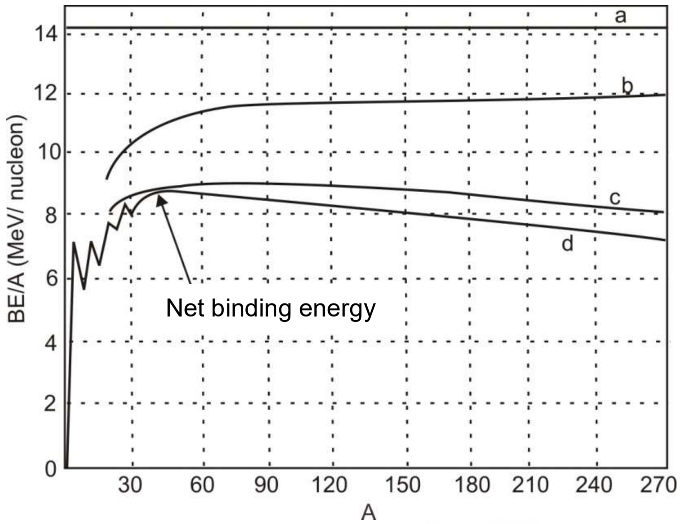


Fig. 14.3: **Plot of binding energy per nucleon as a function of mass number. Curve *a* represents the volume energy. Curve *b* represents the combined effect of volume and surface energies. Curve *c* represents the cumulative effect of the first, second and fourth terms in Weizsäcker's formula. When the contribution of asymmetric and spin terms is added, we get curve *d* for net binding energy per nucleon.**

Now we illustrate how Weizsäcker's formula can be used to calculate BE/*A*.

EXAMPLE 14.2 : WEIZSÄCKER'S SEMI-EMPIRICAL FORMULA

Using Weizsäcker's semi- empirical formula, calculate the binding energy per nucleon for ^{235}U. When it is made to undergo fission, suppose that $^{149}_{60}\text{Nd}$ and $^{85}_{32}\text{Ge}$ are the two fission products. Calculate *BE/*A for these nuclei as well.

SOLUTION ■ In the case of ^{235}U, $A = 235, Z = 92$ and $N = 143$. Also since it is an even-odd nucleus, the contribution of spin term would be zero. Hence, using Eq. (14.2) we get

$$\text{BE (MeV)} = 15.8 \times 235 - 17.8 \times (235)^{2/3} - \frac{23.7 \times (51)^2}{235} - \frac{0.7 \times 92 \times 91}{235^{1/3}}$$

$$= 3713.00 - 677.85 - 262.3 - 963.25 = 1808.6$$

You will note that the Coulomb term (fourth) dominates the surface term (second). This is due to the large number of protons in the nucleus of ^{235}U.

The binding energy per nucleon is $= \dfrac{1808.6}{235} \cong 7.7\,\text{MeV}$.

For $^{149}_{60}\text{Nd}$, we have $A = 149, Z = 60$ and $N = 89$. As before, the contribution of the spin term is zero. Therefore,

$$\text{BE (MeV)} = 15.8 \times 149 - 17.8 \times (149)^{2/3} - \frac{23.7 \times (29)^2}{149} - \frac{0.71 \times 60 \times 59}{(149)^{1/3}}$$

$$= 2354.20 - 500.28 - 133.77 - 474.18$$

$$= 1246.0$$

The binding energy for nucleon is $= \dfrac{1246}{149} \approx 8.4\,\text{MeV}$

What do you observe? We note that for ${}^{149}_{60}\text{Nd}$, the Coulomb term and the surface term are nearly equal. In this case BE per nucleon comes out to be 8.4 MeV showing that fission product nuclei ${}^{149}\text{Nd}$ is more stable than ${}^{235}\text{U}$.

For the case of ${}^{85}_{32}\text{Ge}$, $A = 85, Z = 32$ and $N = 53.$ The contribution of spin term is zero even here. Hence,

$$\text{BE (MeV)} = 15.8 \times 85 - 17.8 \times (85)^{2/3} - \frac{23.7 \times (21)^2}{85} - \frac{0.71 \times 32 \times 31}{(85)^{1/3}}$$

$$= 1343.00 - 344.11 - 122.96 - 160.19 = 715.7$$

Therefore, binding energy per nucleon is 8.4 MeV.

You will note that in this case surface energy is more than the Coulomb energy and, of the two product nuclei, ${}^{85}_{32}\text{Ge}$ is more stable.

In the above example you have seen that stability of nuclei decreases as the difference between the number of neutrons and protons increases. It is now well known that in lighter stable nuclei, the number of neutrons is nearly equal to the number of protons. As we move towards higher *A*, the neutron number increases relative to the proton number and the excess gradually increases with increasing *A*. And only certain combinations of protons and neutrons form stable nuclei. This can be well understood from Fig. 13.1 of Unit 13, where we had plotted the number of neutrons (y-axis) versus the number of protons (x-axis) for stable nuclei (shown by solid circles). We noted that for nuclei with $Z \leq 20$, the stability curve was a straight line with $Z = N$. For $Z > 20$, the stability curve bent towards $N > Z$. This can be understood on the basis of semi-empirical formula by answering the following SAQ.

SAQ 3 – **Semi-empirical formula**

Show that for light nuclei, the fact that $Z \cong N$ is explained by the semi-empirical formula.

It is an observed fact that nuclei like ${}^{58}_{28}\text{Ni}$, ${}^{50}_{22}\text{Ti}$, ${}^{40}_{20}\text{Ca}$, etc. in which either *N* or *Z*, or both, are equal to 2, 8, 20, 28, 50, 82 and 126 have some very special properties, which are markedly different from those of other nuclei:

- these are more abundant in nature; and
- these are more stable than others.

These numbers are called **magic numbers**. These have proved very helpful in speculating the structure of atomic nuclei.

It is instructive to calculate the amount of energy required to remove a neutron from a nucleus. Consider a nucleus of the atom ${}^{A}_{Z}X_N$ where $N = (A - Z)$ is the number of neutrons in the atom which has *Z* atomic electrons and *A* is its mass number. After the removal of one neutron from the nucleus the resulting atom will have one less neutron and change into the isotope ${}^{A-1}_{Z}X_{N-1}$. Thus, the amount of energy required to remove one neutron from the atom would be

$$E_N = \left[m\left({}^{A-1}_{Z}X_{N-1}\right) + m_N - m\left({}^{A}_{Z}X_N\right)\right]c^2 \qquad (14.3)$$

where m_N is the mass of the neutron.

You should now solve an SAQ.

SAQ 4 – **Neutron separation energy**

Calculate the neutron separation energy of ^{17}O and ^{57}Fe. It is given that

$m(^{17}\text{O}) = 16.999132$ u; $m(^{16}\text{O}) = 15.994916$ u

$m(^{57}\text{Fe}) = 56.935398$ u; $m(^{56}\text{Fe}) = 55.934942$ u

Take $m_N = 1.008665$ u; 1u = 931.5 MeV/c^2

14.4 HOW DO NUCLEONS STAY TOGETHER: NUCLEAR FORCE

Once physicists accepted the neutron-proton model of nucleus, an important question arose: How do nucleons stay together? In other words: What is the nature of force that is responsible for the binding of nucleons in a nucleus? Since gravitation and electromagnetic interactions explain most of the observed facts, you may be tempted to identify one of these forces as the likely force. However, the extremely small size of the nucleus, where all the protons and neutrons are closely packed suggests the existence of strong short-range attractive forces to hold them together. Do you know that these attractive forces cannot have electrostatic origin?

This is because electrostatic forces between protons are repulsive and if only these were acting, the nucleons would have been blown apart. Instead, the forces between nucleons are responsible for the large binding energy per nucleon (nearly 8 MeV) in a nucleus. Let us consider the other alternative. The force between nucleons may be gravitational since it is an attractive force between every pair of nucleons. However, it is far too weak to account for the powerful attractive forces between nucleons. If the nucleon – nucleon force is taken to be unity, the gravitational force would be of the order of 10^{-39}. We may, therefore, conclude that the purely attractive forces between nucleons are of a new type with no analogy whatsoever with other known forces in the realm of classical physics. This new attractive force is called **nuclear force**.

Recent experimental evidences suggest that nuclear forces have a charge dependent part. However, it is quite small (< 1%).

The gravitational as well as electrostatic forces obey the inverse square law. The situation in the case of the nucleus is entirely different. All the nucleons are closely packed in the tiny nucleus like a set of marbles in a box. The force that holds the nucleons together must exist between the individual neighbouring nucleons in the nucleus. The nuclear force between nucleons should therefore be a short range force acting over very short distances ($\sim 10^{-15}$ m). The nuclear force is negligible at large distances. It suggests that each nucleon interacts only with its nearest neighbours.

There are two major pieces of evidence for the short range nature of nuclear forces. The first evidence comes from the constancy of the nuclear density irrespective of the number of nucleons present in the nucleus (SAQ 1). The second evidence for the short range nature of nuclear forces follows from the fact that total binding energy is proportional to *A*, a long range force will have the total binding energy proportional to A^2.

We now note that nuclear forces must account for the attractive force between

- a proton and a neutron
- two protons, and
- two neutrons.

Since BE/*A* is roughly constant, irrespective of the mix of neutrons and protons in the nucleus, we are quite justified in considering the force between them as equivalent. That is, we consider **nuclear forces as charge independent**.

If nuclear forces were only attractive in character, nucleons should coalesce under their influence. But we know that the average separation between nucleons is constant, resulting in a nuclear volume proportional to the total number of nucleons. The possible explanation is that nuclear forces exhibit attractive character only so long as nucleons are separated through a certain critical distance. For distances less than this critical value, the character of nuclear forces changes abruptly; attraction changes to repulsion. (You should not confuse this repulsion with electrostatic repulsion.) These qualitative aspects of nuclear forces are shown in Fig. 14.4.

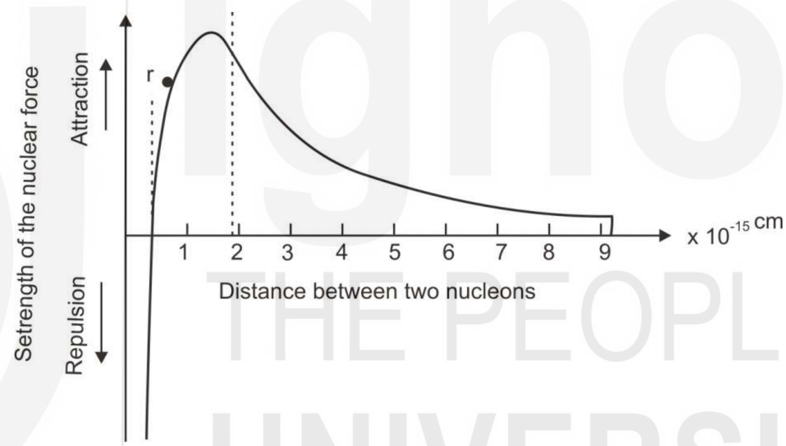
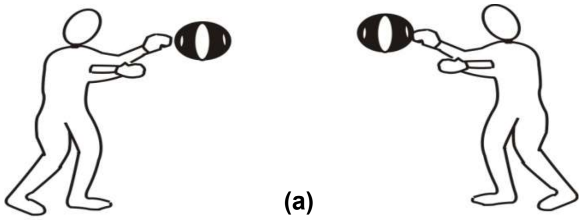


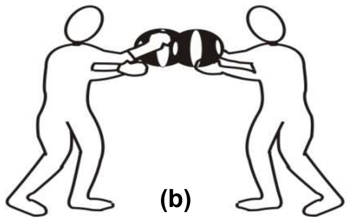
Fig. 14.4: **Typical variation of nuclear forces with distance.**

Let us pause for a while and ask: How do nuclear forces act between nucleons? In 1932, Heisenberg suggested that electrons and positrons shift back and forth between nucleons. A neutron, for instance, might emit an electron and become a proton, while a proton on absorbing the electron would become a neutron. However, theoretical considerations showed that the forces resulting from electron and positron exchange by nucleons would be too small (by a factor of 10^{14}) to be significant. In 1935 Japanese physicist Hideki Yukawa proposed that particles of mass in-between the masses of electrons and nucleons are responsible for nuclear forces. Now these particles are called **pions**. Pions may be charged (π^{+}, π^{-}) or neutral (π^{0}); the word pion is contraction of the original name **pi-meson**.

According to Yukawa's theory, every nucleon continually emits and reabsorbs pions. An emitted pion can also be absorbed by another nucleon. The associated transfer of momentum is equivalent to the action of a force. One of the strengths of Yukawa's theory of nuclear forces is that it can account for their attractive as well as repulsive characters. There is no simple way to demonstrate this aspect formally. But as a rough analogy, let us imagine two boys exchanging volleyballs (Fig. 14.5).



(a)



(b)

Fig. 14.5: **Attractive and repulsive forces can both arise from particle exchange a) Repulsive force due to particle exchange; b) Attractive force due to particle exchange.**

You may now ask: If nucleons constantly emit and absorb pions, why are they not found with other than their usual masses? The answer is provided by Heisenberg's uncertainty principle. We know that the laws of physics refer only to measurable quantities and the accuracy with which certain combinations of measurements can be made is limited by the uncertainty principle. The emission of a pion by a nucleon, which does not change in mass – a clear violation of the law of conservation of energy – is possible if the nucleon absorbs the same or another pion so soon afterward that even in principle it is not possible to measure change in mass. The uncertainty principle does not bar an event in which energy is not conserved for the time less than $\hbar / (2\Delta E)$. This condition enables us to estimate the pion mass. This is illustrated in the following example.

EXAMPLE 14.3 : PION MASS

Assume that a pion travels between nucleons at a speed of $v \sim c$. The emission of a pion of mass m_π, represents a temporary energy discrepancy of $E - mc^2$. Calculate m_π.

SOLUTION ■ Nuclear forces have a maximum range of about 1.7 fm and the time *t* needed for the pion to travel this far is given by

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

Hence $$(m_\pi c^2) \times \frac{r}{c} \sim \hbar$$

so that $$m_\pi \sim \frac{\hbar}{rc}.$$

On substituting the values of $\hbar$, *r* and *c*, we get

$$m_\pi = \frac{1.05 \times 10^{-34}\,\text{J.s}}{(1.7 \times 10^{-15}\,\text{m}) \times (3 \times 10^8\,\text{ms})} = 2.1 \times 10^{-28}\,\text{kg}$$

This is about 230 times the rest mass of the electron. Yukawa's mesons were actually discovered in 1946 in cosmic rays by Powell. The rest mass of charged pions is $273\, m_e$ and that of neutral pions is $264\, m_e$.

Let us now sum up what you have learnt in this unit.

14.4 SUMMARY

Concept	Description
Constituents of the Nucleus	■ The atom consists of a tiny nucleus where roughly all the mass of the atom is concentrated. The nucleus consists of positively charged protons and neutral neutrons. ■ The size of the nucleus is roughly $\sim 1\,\text{fm} = 10^{-15}\,\text{m}$, which is appreciably less than the size of the atom $\sim 10^{-10}\,\text{m}$. ■ Nuclei of all atoms have roughly the same density of the order of $\sim 10^{17}\,\text{kg m}^{-3}$.
Binding Energy	■ The binding energy of the nucleus is equivalent to mass defect, which, in turn, equals the difference between the observed mass of the nucleus and its constituents: $\text{BE} = \Delta mc^2 = (Z\,m_\text{H} + N\,m_\text{N} - M)c^2$
Nature of Nuclear Force	■ Nucleons (protons and neutrons) are held together by strong nuclear force. ■ The nuclear forces are attractive at short range (< 1.7 fm) and charge independent.

14.5 TERMINAL QUESTIONS

1. Calculate the energy released when two ${}^2_1\text{H}$ nuclei are fused together to form a ${}^4_2\text{He}$ nucleus. The binding energy per nucleon of ${}^2\text{H}$ is 1.1 MeV and that of ${}^4\text{He}$ is 7.0 MeV.
2. Calculate the binding energy of a ${}^4\text{He}$ nucleus on the basis of semi-empirical mass formula and compare it with the value obtained on the basis of mass defect (SAQ 2).
3. From the semi-empirical mass formula given by Eq. (14.2), calculate the value of the atomic number (Z) for the most stable nucleus at a given mass number. Calculate Z_0 for A = 56.
4. Calculate the radius of a neutron star of 1.4 Solar mass $(\sim 2.8 \times 10^{30}\,\text{kg})$. In the neutron star, the nucleons are assumed to be squeezed to nuclear density $(\sim 10^{17}\,\text{kg/m}^3)$.
5. Given $m_p = 1.005\,\text{u}$, $m_n = 1.009\,\text{u}$ and $m\left({}^4_2\text{He which is an } \alpha - \text{particle}\right) = 4.003\,\text{u}$, calculate the binding energy of the α-particle.
6. Calculate the total binding energy of the isotopes ${}^3\text{He}$ and ${}^3\text{H}$. Also, calculate the Coulomb energy for ${}^3\text{He}$ using Weizsäcker's formula. How this difference is accounted for?

14.6 SOLUTIONS AND ANSWERS

<u>Self-Assessment Questions</u>

1. $$d_C = \frac{M_c}{\frac{4\pi}{3}R_c^3} = \frac{19.92 \times 10^{-27}\,\text{kg}}{\frac{4}{3} \times 3.1416 \times (2.7 \times 10^{-15}\,\text{m})^3}$$

$$= \frac{19.92}{82.45} \times 10^{18}\,\text{kg m}^{-3}$$

$$= 2.42 \times 10^{17}\,\text{kg m}^{-3}$$

$$d_{\text{Pb}} = \frac{M_{\text{Pb}}}{\frac{4\pi}{3} R_{\text{Pb}}^3} = \frac{3.4 \times 10^{-25}\,\text{kg}}{\frac{4}{3}(7 \times 10^{-15}\,\text{m})^3}$$

$$= \frac{3.4 \times 10^{20}\,\text{kg m}^{-3}}{\frac{4\pi}{3} \times (7)^3}$$

$$= 2.37 \times 10^{17}\,\text{kg m}^{-3}$$

2. From Eq. (14.1) we know that

$$\text{BE} = \Delta m c^2$$

where $\Delta m = Zm_{\text{H}} + Nm_{\text{n}} - M$.

i) For ${}^4_2\text{He}$, we have $Z = 2$ and $N = 2$. On substituting the given values, we get

$$\Delta m(^4\text{He}) = 2 \times (1.007825\text{ u}) + 2 \times (1.008665\text{ u}) - 4.002604\text{ u}$$

$$= 14.03298\text{ u} - 4.002604\text{ u} = 0.030376\text{ u}$$

and

$$\text{BE} = 0.030376 \times (1.66 \times 10^{-27}\,\text{kg}) \times (2.998 \times 10^8\,\text{ms}^{-1})^2$$

$$= 0.030376 \times 14.94 \times 10^{-11}\,\text{J}$$

$$= 0.030376 \times (14.94 \times 10^{-11}\,\text{J})/(1.602 \times 10^{-19}\,\text{JeV}^{-1})$$

$$= 0.030376 \times 9.313 \times 10^8\,\text{eV}$$

$$= 2.829 \times 10^7\,\text{eV}$$

$$= 28.3\,\text{MeV}$$

ii) For ${}^{35}_{17}\text{Cl}$, we have $Z = 17$ and $N = 18$. Therefore,

$$\Delta m(^{35}\text{Cl}) = 17 \times (1.007825\text{ u}) + 18 \times (1.008665\text{ u}) - 34.96885\text{ u}$$

$$= 17.133025\text{ u} + 18.15597\text{ u} - 34.96885\text{ u}$$

$$= 0.320145\text{ u}$$

Since 1u = 931.5 MeV, we find that

$$\text{BE}(^{35}\text{Cl}) = 298.2\,\text{MeV}$$

iii) For ${}^{56}_{26}\text{Fe}, Z = 26$ and $N = 30$. Therefore

$$\Delta m(^{56}\text{Fe}) = 26 \times (1.007825\text{ u}) + 30 \times 1(1.008665\text{ u}) - 55.934932\text{ u}$$

$$= 26.20345\text{ u} + 30.25995\text{ u} - 55.934932\text{ u}$$

$$= 0.528468\text{ u}$$

and

$$\text{BE}(^{56}\text{Fe}) = 492.2\,\text{MeV}$$

iv) For $^{235}U, Z = 92, N = 143$ so that,

$$\Delta m = 92 \times (1.007825\text{ u}) + 143 \times (1.008665\text{ u}) - 235.043933\text{ u}$$

$$= 92.7199\text{ u} + 144.239095\text{ u} - 235.043933\text{ u}$$

$$= 1.915062\text{ u}$$

so that $\text{BE}(^{235}\text{U}) = 1783.8\text{ MeV}$

3. For a given A, the ratio $\frac{N}{Z}$ will be such that the total energy E tends to a minimum. Since $m_p = m_n$, the only terms which we have to consider in discussing this minimum for a particular value of A are $\frac{(N-Z)^2}{A}$ and $\frac{Z^2}{A^{1/3}}$. The first term demands $N = Z$ while the second term demands Z to be as small as possible. This is consistent with the fact that light nuclei have $Z \cong N$.

4. Using the formula given in Eq. (14.3), separation energy of the neutron is given by

$$En(^{17}\text{O}) = (15.994915 + 1.008665 - 16.999132)\,c^2$$

$$= 4.143\text{ MeV}$$

$$En(^{57}\text{Fe}) = (55.934942 + 1.008665 - 56.935398)\;c^2$$

$$= 7.65\text{ MeV}$$

Terminal Questions

1. B.E. of ^4_2He nucleus $= 7.0 \times 4 = 28\text{ MeV}$

   B.E. of ^2_1He nucleus $= 1.1 \times 2 = 2.2\text{ MeV}$

   $\therefore$ Mass of ^4_2He nucleus $= 2m_p + 2m_n - 28.0\text{ MeV}$

   Mass of ^2_1He nucleus $= m_p + m_n - 2.2\text{ MeV}$

   Energy released in the fusion reaction

$$E = 2 \times \text{Mass of } ^2_1\text{He} - \text{mass of } ^4_2\text{He}$$

$$= 2(m_p + m_n - 2.2) - (2m_p + 2m_n - 28.0) = 23.6\text{ MeV}$$

2. For $^4_2\text{He}, A = 4$ and $Z = 2$ and $\varepsilon = 34$

   Hence from Eq. (14.2) we get

$$\text{BE (MeV)} = 15.8 \times 4 - 17.8 \times 4^{2/3} - 0.71 \times \frac{2}{4^{1/3}} + \frac{34}{4^{3/4}}$$

$$= 63.2 - 44.9 - 0.895 + 12.02 = 29.43$$

   This is slightly greater than the value calculated on the basis of mass difference of helium nucleus.

3. For the most stable nucleus at a given mass number A, we have

$$\left(\frac{\mathrm{d\,BE}}{\mathrm{d}Z}\right)_A = 0 \text{ for } Z = Z_0$$

Hence we have from Eq. (14.2)

$$-2\gamma\frac{(A-2Z_0)}{A}(-2)-\frac{\delta}{A^{1/3}}(2Z_o-1)=0$$

$$\Rightarrow \quad 4\gamma(A-2Z_0)-\delta A^{2/3}(2Z_o-1)=0$$

or $$4\gamma A-8\gamma Z_0-2\delta A^{2/3}Z_o+\delta A^{2/3}=0$$

or $$Z_0(8\gamma+2\delta A^{2/3})=4\gamma A+\delta A^{2/3}$$

$$\Rightarrow \quad Z_0=\frac{4\gamma A+\delta A^{2/3}}{8\gamma+2\delta A^{2/3}}$$

$$=\frac{23.7\times 4A+\delta A^{2/3}}{8\times 23.7+2\times 0.71A^{2/3}}$$

$$=\frac{94.8A+\delta A^{2/3}}{1.42\,A^{2/3}+189.6}$$

For

$A = 56$

$$Z_0=\frac{94.8\times 56+0.71\times 56^{2/3}}{1.42\times 56^{2/3}+189.6}=\frac{5319.2}{210.4}$$

$$=25.3\Rightarrow 26$$

4. We know that

$$\frac{4\pi}{3}r^3\rho=2.8\times 10^{30}\,\text{kg}$$

Substituting the value of the density, we get

$$r=\{0.212\times 10^{13}\}^{1/3}\text{m} \quad \approx(6.68)^{1/3}\times 10\,\text{Km}$$

$$\approx 18.8\,\text{Km}$$

5. Using the Eq. of Binding energy, we can write

$$\text{BE}(\alpha)=2m_p+2m_n-m\left({}^4_2\text{He}\right) \qquad \text{(i)}$$

On substituting, the values in Eq. (i), we get

$$=2\times 1.005\,u+2\times 1.009\,u-4.003\,u=0.025\,u$$

or $$=23.28\,\text{MeV}$$

6. B.E. of ${}^3\text{He}=7.718$ MeV

${}^3\text{H}=8.482$

$\therefore$ Difference in B.E. of ${}^3\text{H}$ and ${}^3\text{He}$ is = 0.764 MeV

Electrostatic B.E. from the Weizsacker's mass formula is

$$\text{Electrostatic BE (MeV)} = 0.71\frac{Z(Z-1)}{A^{1/3}}$$

for ^{3}He, $Z = 2$, $A = 3$

$$\therefore \quad \text{Electrostatic BE} = \frac{-0.71\times 2}{3^{1/3}} \approx -0.98\,\text{MeV}$$

As this accounts for the bulk of energy due to Coulomb's repulsion.