UNIT 9 BASIC CONCEPTS OF PROBABILITY

Objectives

After reading this unit, you should be able to:

- appreciate the relevance of probability theory in decision-making
- understand the different approaches to probability
- calculate probabilities in different situations
- revise probability estimate, if added information is available.

Structure

9.1 INTRODUCTION

Uncertainty is a part and parcel of human life. Weather, stockmarket prices, product quality are but some of the areas, where, commenting on the future with certainty becomes impossible. Decision-making in such areas is facilitated through formal and precise expressions for the uncertainties involved. Study of rainfall, spelled out in a form amenable for analysis, may render the decision on water management easy. Intuitively, we see that if there is a high chance of a large quantity of rainfall in the coming year, we may decide to use more water of rainfall for power generation and irrigation this year. We may also take some steps regarding flood control. However, in order to know how much water to release for different purposes, we need to quantify the chances of different quantities of rainfall in the coming year.

Similarly, formal and precise expressions of stockmarket prices and product quality uncertainties, may go a long to help analyse, and facilitate decision on portfolio and sales planning respectively. Probability theory provides us with the ways and means to attain the formal and precise expressions for uncertainties involved in different situations. The objective of this unit is to introduce you to the theory of probability. Accordingly, the basic concepts are first presented, followed by the different approaches to probability measurement that have evolved over time. Finally, in the last two sections, certain important results in quantifying uncertainty which have emerged as a sequel to the theoretical developments in the field, are presented.

Activity A

Mention three events in your life, where you faced total certainty.

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3 ……………………………………………………………………………………

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Activity B

Mention twp major events in your life, where you faced uncertainty in taking decisions. Elaborate as to how you dealt with the uncertainty in each of the cases.

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9.2 BASIC CONCEPTS: EXPERIMENT, SAMPLE SPACE, EVENT

Probability, in common parlance, connotes the chance of occurrence of an event or happening. In order that we are able to measure it, a more formal definition is required. This is achieved through the study of certain basic concepts in probability theory, like experiment, sample space and event. In this section we explore these concepts.

Experiment

The term experiment is used in probability theory in a much broader sense than in physics or chemistry. Any action, whether it is the tossing of a coin, or measurement of a product's dimension to ascertain quality, or the launching of a new product in the market, constitute an experiment in the probability theory terminology.

These experiments have three things in common:

1 There are two or more outcomes of each experiment.

2 It is possible to specify the outcomes in advance.

3 There is uncertainty about the outcomes.

For example, a coin tossing may result in two outcomes, in head or tail, which we know in advance, and we are not sure whether a head or a tail will come up when we toss the coin. Similarly, the product we are measuring may turn out to be undersize or right size or oversize, and we are not certain which way it will be when we measure it. Also, launching a new product involves uncertain outcome of meeting with a success or failure in the market.

Sample Space

The set of all possible outcomes of an experiment is defined as the sample space. Each outcome is thus visualised as a sample point in the sample space. Thus, the set (head, tail) defines the sample space of a coin tossing experiment. Similarly, (success, failure) defines the sample space for the launching experiment. You may note here, that given any experiment, the sample space is fully determined by listing down all the possible outcomes of the experiment.

Event

An event, in probability theory, constitutes one or more possible outcomes of an experiment. Thus, an event can be defined as a subset of the sample space. Unlike the common usage of the term, where an event refers to a particular happening or incident, here, we use an event to refer to a single outcome or a combination of outcomes. Suppose, as a result of a market study experiment of a product, we find that the demand for the product for the next month is uncertain, and may take values from 100, 101, 102... 150. We can obtain different event like:

The event that demand is exactly 100

The event that demand lies between 101 to 120

The event that demand is 101 or 102

In the first case, out of the 51 sample points that constitute the sample space, only one sample point or outcome defines the event, whereas the .number of outcomes used in the second and third case are 20 and 2 respectively.

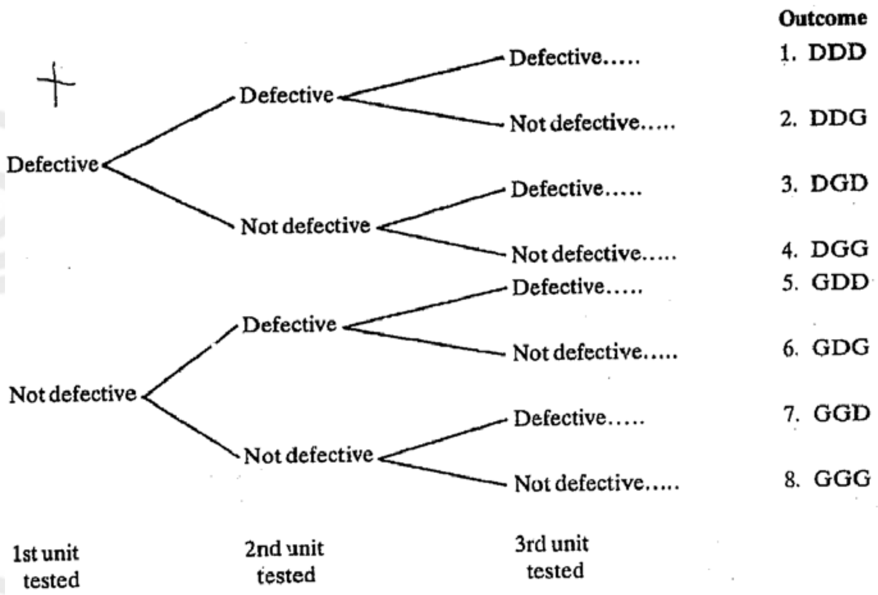
With this background on the above concepts, we are now in a position to formalise the definition of probability of an event. In the next section, we will look at the different approaches to probability that have been developed, and present the axioms for the definition of probability.

Example 1

Consider the experiment of testing three units of a product. *We* are interested in finding the possible outcomes of this test.

Solution

In this experiment, we find that each unit can be either defective or not defective. The test results of the three units may be represented as follows :



The above diagram shows all possible outcome (here 8 in number) of the experiment. Corresponding to each of the two outcomes of the testing of one unit, the second unit may be defective or non-defective, leading to 2 x 2 = 4 outcomes. Corresponding to each of these four outcomes, the third unit may again give two results giving us in total 4 x 2 = 8 possible outcomes of the experiment.

If we denote a defective by D and a non-defective by G, then the sample space(s) can be written down as the list of all possible outcomes of the experiment ;

S = (DDD, DDG, DGD, DGG, GDD, GDG, GGD, GGG)

Example 2

Suppose we are interested in the following Event A in the above experiment: The number of defective are exactly two. How many sample' points does this event correspond to?

Solution

We can see from the sample space that there are three outcomes where D occurs twice, viz, DDG, DGD and GDD, thus the Event A corresponds to 3 sample point.

Activity C

Consider an experiment where four coins are tossed once. List down the possible outcomes of the experiment. In how many outcomes do you find the occurrence of two heads?

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