UNIT 12 DECISION THEORY

Objectives

After reading this unit, you should be able to:

- structure a decision problem involving various alternatives and uncertainties in outcomes
- apply marginal analysis for solving decision problems under uncertainty
- analyse sequential problems using Decision Tree Approach
- appreciate the use of Preference Theory in decision-making under uncertainty
- analyse uncertain situations where probabilities of outcomes are not known.

Structure

12.1 INTRODUCTION

In every sphere of our life we need to take various kinds of decisions. The ubiquity of decision problems, together with the need to make good decisions, have led many people from different time and fields, to analyse the decision-making process. A growing body of literature on Decision Analysis is thus found today. The analysis varies with the nature of the decision problem, so that any classification base for decision problems provides us with a means to segregate the Decision Analysis literature. A necessary condition for the existence of a decision problem is the presence of alternative ways of action. Each action leads to a consequence through a possible set of outcome, the information on which might be known or unknown. One of the several ways of classifying decision problems has been based on this knowledge about the information on outcomes. Broadly, two classifications result:

a) The information on outcomes are deterministic and are known with certainty, and
b) The information on outcomes are probabilistic, with the probabilities known or unknown.

The former may be classified as Decision Making under certainty, while the latter is called Decision Making under uncertainty. The theory that has resulted from analysing decision problems in uncertain situations is commonly referred to as Decision Theory. With our background in the Probability Theory, we are in a position to undertake **a** study of Decision Theory in this unit. The objective of this unit is to study certain methods for solving decision problems under uncertainty. The methods are consequent to certain key issues of such problems. Accordingly, in the next section we discuss the issues and in subsequent sections we present the different methods for resolving them.

12.2 CERTAIN KEY ISSUES IN DECISION THEORY

Different issues arise while analysing decision problems under uncertain conditions of outcomes. Firstly, decisions we take can be viewed either as independent decisions, or as decisions figuring in the whole sequence of decisions that are taken over a period of time. Thus, depending on the planning horizon under consideration, as also the nature of decisions, we have either a single stage decision problem, or a sequential decision problem. In real life, the decision maker provides the common thread, and perhaps all his decisions, past, present and future, can be considered to be sequential. The problem becomes combinatorial, and hence difficult to solve. Fortunately, valid assumptions in most of the cases help to reduce the number of stages, and make the problem tractable. In Unit 10, we have seen a method of handling a single stage decision problem. The problem was essentially to find the number of newspaper copies the newspaper man should stock in the face of uncertain demand, such that, the expected profit is maximised. A critical examination of the method tells us that the calculation becomes tedious as the number of values the demand is taking increases. You may try the method with a discrete distribution of demand, where demand can take values from 31 to 50. Obviously a separate method is called for. We will be presenting Marginal Analysis for solving such single stage problems. For sequential decision problems, the Decision Tree Approach is helpful and will be dealt with in a later section. The second issue arises in terms of selecting a criterion for deciding on the above situations. Recall as to how we have used `Expected Profit' as a criterion for our decision. In both the Marginal Analysis and the Decision Tree Approach, we will be using the same criterion. However, this criterion suffers from two problems. Expected Profit or Expected Monetary Value (EMV), as it is more commonly known, does not take into account the decision maker's attitude towards risk. Preference Theory provides us with the remedy in this context by enabling us to incorporate risk in the same set up. The other problem with Expected Monetary Value is that it can be applied only when the probabilities of outcomes are known. For problems, where the probabilities are unknown, one way out is to assign equal probabilities to the outcomes, and then use EMV for decision-making. However this is not always rational, and as we will find, other criteria are available for deciding on such situations.

For the purpose of this unit, we will be discussing the issues as raised above. This will be achieved through a study of the following:

1 Marginal Analysis for single stage decision problems.

2 Decision Tree Approach for sequential decision problems.

3 Preference Theory.

4 Other approaches for problems where probabilities are unknown.

In the subsequent sections we take up the above in the order presented.

Activity A

Suppose you have the option of investing either in Project A or in Project B. The outcomes of both the projects are uncertain. If you invest in Project A, there is a 99' chance of making Rs. 20,000 profit, and a 1% chance of losing Rs. 1,00,000. If project B is choosen, there is a 50-50 chance of making a profit of Rs. 6,000 or Rs. 18,000. Which project will you choose and why?

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

Activity B

Suppose in Exercise 1, you have calculated the expected payoff (EMV) for both the projects as follows.

EMV_A = 99 x 20,000 - .01 x 1,00,000 = Rs. 18,000.

EMVB = .5 X6,000- .5 x 18,000 = Rs. 12,000.

You have thus found that by investing in Project A, you can expect more money, so you have chosen A. Your friend, when given the same option, chooses B, arguing that he would not like to go bankrupt (losing 1 lakh) by choosing A. How do you reconcile these two arguments?

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

12.3 MARGINAL ANALYSIS

In Unit 10, we have seen how expected value can be used while deciding on one alternative from among several alternative courses of actions, each of which is characterised by a set of uncertain outcomes. It is easy to see that the computations become tedious as the number of values, the random variable can take, increases. Consider the example of the newspaper man discussed in section 10.4. Instead of six values of the demand that we have assumed there, if the demand could take, say, twenty values, with different chances of occurrence of each Value, the computation would become very tedious. In such cases, marginal analysis is very helpful. In this section, we explain the concept behind this analysis.

Consider Example 1 in section 10.4 with the following change. Let us assume that the newspaper man has found from the past data that the demand can take values ranging from 31, 32... to 50. For easy representation, let us assume that each of these values has got an equal chance of occurrence, viz. , $\frac{1}{20}$. The problem is to decide on the number of copies to be ordered.

Marginal Analysis proceeds by examining whether ordering an additional unit is worthwhile or not. Thus, we will order X copies, provided ordering the X^{th} copy is worthwhile but ordering the $(X+1)^{th}$ copy is not. To find out whether ordering X copies is worthwhile, we note the following. Ordering of the X^{th} copy may meet with two consequences, depending on the occurrences of two events:

A The copy can be sold.

B The copy cannot be sold.

The X^{th} copy can be sold only if the demand exceeds or equals X, whereas, the copy cannot be sold if the demand turns out to be less than X. Also, if event A occurs, we will make a profit of 50 p. on the extra copy, and if even B occurs, there will be a loss of 30 p. As this profit and loss pertains to the additional or marginal unit, these are referred to as marginal profit or loss and the resulting analysis is called marginal analysis.

Using the following notations

K_1 = Marginal profit = 50p.

K_2 = Marginal loss = 30p.

P(A) = Probability (Demand $\geq$ X) = 1-Probability (Demand $\leq$ X - 1).

P(B) = Probability (Demand < X) = Probability (Demand $\leq$ X - 1).

We can write down the expected marginal profit and expected marginal loss as :

Expected Marginal Profit = K_1 P(A)

Expected Marginal Loss = K_2 P(B)

Ordering the X^{th} copy is worthwhile only if the expected profit due to it is more than the expected loss, so that

$K_1 P(A) \geq K_2 P(B)$

Now, if F(D) denotes the c.d.f. of demand, then by definition, Probability

Demand $\leq (X-1) \pm F(X-1)$

Hence, $K_1 [1-F(X-1)] \geq K_2 F(X-1)$

or; $K_1 \_ K_1 F(X-1) - K_2 F(X-1) \geq 0$

or; $F(X-1) \leq \frac{K_1}{K_1 + K_2}$ .... (CONDITION 1)

Thus, if condition 1 holds good, it is worthwhile to order the X^{th} copy.

If the optimal decision is to order X copies, then ordering the $(X+1)^{th}$ copy will not be worthwhile, i.e. the expected marginal profit due to the $(X+1)^{th}$ copy should be less than the expected loss.

Proceeding with the analysis in the same way as above, we have :

Expected Marginal Profit = K_1 Probability (Demand $\geq$ X + 1)

= $K_1 [1 – F(X)]$

Expected Marginal Loss = K2 F(X)

$\therefore$ For the $(X+1)^{th}$ copy : $K_1 [1-F(X)] \leq K_2 F(X)$

From conditions (1) and (2) and the definition of Fractile, it is clear that X will be the $\left(\frac{K_1}{K_1+K_2}\right)^{th}$ the fractile of the Demand distribution.

Thus, for our problem, given the above result, all that we have to do is to calculate $K = \frac{K_1}{K_1+K_2}$ and find the K^{th} fractile of the distribution, which will give us the required answer.

In our problem :

$K = \frac{.5}{.5 + .3} = .625$ and the $.625^{th}$ fractile is 43.

$\therefore$ The optimal decision is to order 13 copies.

We can verify quickly that in the problem given in section 10.4, the $.625^{th}$ fractile of the demand distribution is 33. So the optimal decision there is to order 33, which is the answer that we have obtained there.

The above shows how marginal analysis helps us in arriving at the optimal decision with very little computation. This is especially useful when the random variable of interest takes a large number of values. Though we have demonstrated this for a discrete demand distribution the same logic can be shown to be applicable for continuous distributions also. Instead of the distribution we have taken, if we would have assumed that demand is normal with a specific μ and σ, then also the same K^{th} fractile of $N(\mu, \sigma)$ would have given us the optimal decision.

Activity C

The demand for a particular perishable item is known to be N (50, 6). The cost of understocking (K_1), and the cost of overstocking (K_2) per unit is known to be Rs. 20 and Re. 1 respectively. How much of the item should be stocked to minimise the cost due to understocking and overstocking?

(Note that understocking implies stocking less than what is demanded, the loss being in terms of contribution, while overstocking implies stocking more than what is demanded, and hence, there is the cost of not being able to sell. These are K_1 and K_2 respectively as discussed in the text.)

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

12.4 DECISION TREE APPROACH

In the earlier section we nave seen a single stage decision problem. Quite often the decision maker has to take decisions in a sequence, the decisions coming later in the sequence being dependent on those coming earlier. The sequence is either built-in, or it is possible to engineer such a sequence for a better decision. For example, consider the periodic production decision for a certain item with uncertain demand (say, refrigerator); for each period, a decision on the number of units to be produced is to be taken, given the uncertainties in demand during different periods. Thus, we will have a number of decisions for each period, with intervening uncertainties in outcomes for each decision between any two periods. In such cases, the sequence is built-in.

In contrast to the above, we find situations, where the time-frame of decisions are such, that before going for the final decision, it is possible to go for a method for generating extra information that will facilitate the final decision, For example, before deciding on marketing a product nationally, one can decide on Test Marketing. Similarly, in a production situation, where a machine produces an unknown percentage of defectives, one may have an option to buy a special attachment that helps to produce a known low fraction of defectives. The trade-off then, is between not buying the attachment and thereby risking a high percentage of defectives, of buying the attachment at a cost, to safeguard against the risk. An infinite sequence of decisions can be engineered in this case by allowing sampling from the current process, to ascertain the percentage of defectives. Thus, at each stage we can have two alternatives :

a) buying, and

b) not buying and sampling.

This can go on till we decide to stop sampling due to some reason (e.g. sampling cost becomes prohibitive).

The Decision Tree Approach provides us with a useful way to analyse such sequential decision problems. We illustrate this approach through an example. The oil drilling example has been a favourite of many authors. We have taken the following example from Management Decision Science by Berry et al., with some modifications.

Example I

Consider the decision of drilling for oil in a particular region, confronting our decision maker. The chances of getting oil in the region as per the geologist's report is known to be 0.6. To start with, the decision maker has got Rs. 1.5 lakh. The consequences of drilling and getting oil and that of drilling and not getting oil, in terms of cash left after decision, are known to be Rs. 5 lakh and Rs. 40,000 respectively. The decision maker has got an option to undertake a seismic test that will increase his knowledge about the oil content of the region. The test will cost him Rs. 5,000; however, the benefit in having the test is that, if oil is actually there the test would predict it correctly for 90% of the time; and if there is actually no oil, that would be predicted correctly for 70% of the time. What should we do and why?

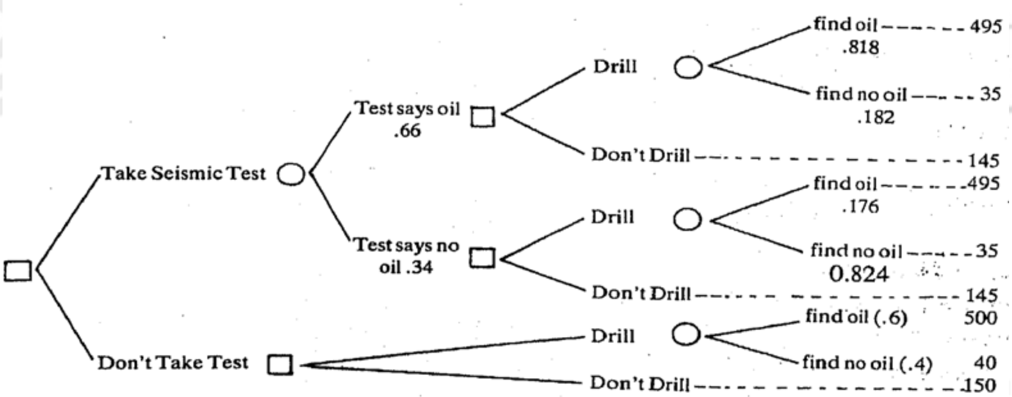
The first step is to structure the decision problem. In Decision Tree Approach a square "□" is used to denote an action or a decision point, and a circle "○" is used to, illustrate the point of uncertainty. First the alternatives of courses of action are shown as emanating from the decision point and then corresponding to each decision, the possible outcomes are shown emanating from the uncertainty point. The probability and consequence for each outcome are listed by the side of the outcome. The resulting diagram is called a Decision Tree. For our example, we have to start with two possible actions:

1 Take the Seismic Test

2 Do not take the Seismic Test

If the test is taken, the test may say that there will be oil, or it may say that there will not be any oil. These outcomes are uncertain as the test is not a perfect test. Once the test outcomes are known, the decision maker has again to decide on whether to drill or not. The outcomes corresponding to each decision are once again known here. Similarly, If it is decided that the test is not to be taken, one has to still decide on whether to drill or not.

The Decision Tree, thus, can be drawn as follows:



The sequences shown beside each outcome are in thousand rupees.

The second step is to write down the probabilities corresponding to each outcome. If the test is not taken, the chances of finding oil is given directly by the geologist's report as 0.6. Therefore, the chances of not getting oil = 1-.6 = .4. These can then be written corresponding to each of the outcomes with consequences of 500 and 40 thousand. However, once the test is taken, the chances of the test saying positive (presence of oil) or negative (no oil) is dependent on the predictive capability of the test, and has to be calculated. Similarly, the probability of finding oil given that test has yielded positive results is expected to be more than 0.6. These and related probabilities are to be calculated. also. The probability calculations can be done by using Bayes' Theorem discussed in section 9.5.

Using the same notations, we find two mutually exclusive and collectively exhaustive events A and B as follows :

A : find oil

B : find no oil

The other events defined in the context of the same experiment are :

C : Test says oil is there (positive results).

D : Test says no oil is there (negative results).

The data given to us are

P(A) = Probability of finding oil = 0.6

P(B) = Probability of not finding oil = 0.4

P(C/A) = Probability test predicts correctly when oil is actually there = 0.9

P(D/A) = Probability test predicts incorrectly when oil is actually there = 0.1

P(D/B) = Probability test predicts correctly when actually oil is not there = 0.7

P(C/B) = Probability test predicts incorrectly when actually no oil is there = 0.3

We are interested in finding

P(C) = Probability that test says oil is there.

P(D) = Probability that test says no oil is there.

P(A/C) = Probability of finding oil, given positive test results.

P(A/D) = Probability of finding oil, given negative test results.

P(B/C) = Probability of finding oil, given positive test results.

P(B/D) = Probability of finding oil, given negative test results.

We have Bayes' Theorem:

$$P(A/C) = \frac{P(C/A) \times P(A)}{P(C/A)\,P(A) + P(C/B)\,P(B)} = \frac{0.9 \times 0.6}{.9 \times .6 + .3 \times .4} = .818$$

$$P(B/C) = \frac{P(C/B) \times P(B)}{P(C/B)\,P(B) + P(C/A)\,P(A)} = \frac{0.3 \times .4}{.3 \times .4 + .9 \times .6} = .182$$

$$P(A/D) = \frac{P(D/A) \times P(A)}{P(D/A)\,P(A) + P(D/B)\,P(B)} = \frac{.1 \times .6}{.1 \times .6 + .7 \times .4} = .176$$

$$P(B/D) = \frac{P(D/B) \times P(B)}{P(D/A)\,P(A) + P(D/B)\,P(B)} = \frac{.7 \times .4}{.1 \times .6 + .7 \times .4} = .824$$

We also know that,

P(C) = P(C/A) P(A) + P(C/B) P(B) = .9 x .6 + .3 x .4 = .66

P(D) = P(D/A) P(A) + P(D/B) P(B) = .1 x .6 + .7 x .4 = .34

[Check P(C) + P(D) = 1, P(A/C) + P(B/C) = 1, P(A/D) + P(B/D) = 1]

These probabilities are incorporated in the decision tree diagram. The final step consists of finding the Expected Monetary Value (EMV) for the decisions. We start from the Northeast corner of the diagram and "fold back" the tree as follows :

The extreme Northeast decision is "to drill" with the outcomes of finding oil. or not finding oil with chances of occurrence of .818 and .182. The respective contributions are Rs, 4,95,000 and Rs. 35,000.

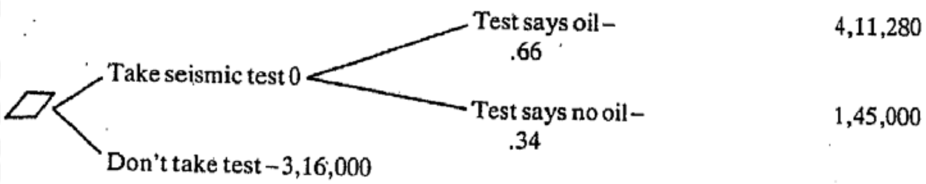
$\therefore$ EMV of decision to drill = 4,95,000 X .818 + 35,000 x .182

= Rs. 4,11,280

This being greater than the payoff due to not drilling (1,45,000), we can say that once the test says oil, it is better to go for drilling, and the corresponding expected payoff in that case is Rs. 4,11,280.

Similarly, when the test says no oil, we find that "not drilling" is a better option than "drilling", as the expected payoff in the former is more (Rs. 1,45,000) vis-à-vis the latter (= .176 x 495 + .824 x 35 = 115960).

The earlier diagram is thus reduced as shown:



If test is not taken, the expected payoff of drilling is:

500 x .6 + 40 x .4 = 3,16,000

This being greater than not drilling (1,50,000) it is better to go for drilling if the test has not been taken. This is shown in the diagram. We now calculate the EMV of taking a seismic test :

.66 x 4,11,280 + .34 x1,45,000 = 3,20,745

Therefore, as this payoff is more than what one can expect if the test is not taken, it is better to take the test.

Hence, the decision is to "Take the Test". If the test result says no oil then one should not drill, and if the test result is positive one should drill. This decision will maximise the EMV.

Activity D

ABC Company is a small time manufacturer of L.P. records. The record business is almost a monopoly of another Calcutta Based company (XYZ), and ABC's ability to. survive so far may be attributed to their able and experienced Managing Director Mr. A. As all the topmost artists are under the contract of XYZ, ABC's strategy has been to get hold of new faces for recording. Mr. A's intuition in this respect has proved useful. He has been actively participating in recruiting new faces, and he believes that apriori 70% of his recruits stand the chance of being successful nationally. Once a new face is chosen, a tape is cut and an initial production of 5,000 records is undertaken for test marketing. It has been found that when the recruit is actually a success nationally, test marketing would have predicted the outcome 90% of the time, and when the recruit is actually a failure nationally, the outcome would have been predicted 70% of the time. Based on test marketing results, the decision to go for national marketing is taken up. National marketing involves a production of 50,000 records. The artist is paid a sum of 5,000 once a tape in cut. The variable cost per record for production run of 5,000 and 50,000 works out to Rs. 13 per record and Rs. 10 per record respectively and the selling price is Rs. 40 per record.

Mr. A is thinking of entering the ghazal market, and has currently recruited a ghazal singer, He feels that the prediction capability of test marketing will be on the lower side for ghazals: His estimate is that the test marketing would predict a success, when it is actually a success for only 70% of the time (as against 90% earlier), and in case of failure, it would predict correctly only 60% of the time (as against 70% earlier). Given the low prediction capability, he is wondering whether it is worthwhile to go for test marketing at all.

Can you help him in his decision? You may assume that a success in case of test or National marketing would imply an ability to sell 5,000 and 50,000 records respectively, whereas a failure in both cases would amount to zero sales, for all practical purposes.

12.5 PREFERENCE THEORY

So far, while deciding on an action, we have used the criterion of maximising the EMV or expected payoff. This does not take into account the decision maker's attitude towards risk. If a company is financially weak, it may decide not to use the EMV maximising action, if there is even a small chance of going bankrupt following that action. Preference Theory helps us in such situations by providing a systematic way of measuring the consequences on a preference scale, that reflects the decision maker's attitude towards risk. The objective of this section is to illustrate how Preference Theory can be used for decision-making.

The procedure consists of eliciting information from the decision maker (d.m.), on his `certainly equivalents' (CE) corresponding to each alternative; CE of an alternative being the amount he is ready to exchange for the uncertain consequences of the particular alternative. For example, consider any alternative of investing in a project, the possible outcomes of which are (a) net loss of Rs. 1,00,000 with probability 0.1, and (b) net gain of Rs. 20,000 with probability 0.9. Now, if the d.m. is risk averse, he might not like even the small odds of losing 1 lakh, and he might be content in having an alternative paying him a certain amount of Rs. 5,000 as against the above (EMV of above Rs. 8,000). You can imagine that this investment gamble is the exclusive right of a class of people, and our d.m. is one among them. Thus, if this exclusive right is allowed to be sold to other people, the d .m. is ready to sell it for Rs. 5,000. The difference between the EMV and the CE is defined as the risk premium. Here, CE is Rs. 5,000; hence the risk premium is Rs. 3,000.

As the number of alternatives increase, it becomes difficult to collect preference information in this way. The Preference curve, which is a plot of the monetary value (X - axis) and the preference (Y- axis) is then obtained as follows. First, the best and the worst consequences corresponding to any decision are identified. The preference values of 1 and 0 are then given corresponding to the best and worst consequences respectively, giving us two points in the Preference curve. The step for obtaining the subsequent points are given below :

Let Ro = Consequence corresponding to worst decision.

P(Ro) = Preference corresponding to Ro = O.

R_l = Consequence corresponding to the best decision.

$P(R_l)$ = Preference corresponding to R_l = 1.

Step 1 We find the d.m 's CE of a 50-50 chance of getting Rs. R_0 or Rs. R_l. Suppose, he gives the value Rs. (CE_1).

Step 2 We find the preference corresponding to CE_1 i.e. $P(CE_1)$.

Preference of an alternative is defined as the mathematical expectation of preferences corresponding to the consequences of the alternative. A preference P(x) assigned to a consequence x implies that the d.m. is indifferent

to having an amount x for certain or having uncertain consequences of (a) [

1-p(x)] of Rs . Ro and (b) P(x) of achieving Rs. R_I.

$\therefore$ $P(CE_1) = .5 \times 0 + 0.5 \times 1 = .5$

Step 3 Now, we ask the d.m., as to what certain amount would make him indifferent to uncertain consequences of Rs. (CE_1) with probability 0.5 and Rs. R_1 with probability 0.5. Say, he says Rs. (CE_2).

Step 4 We find $P(CE_2) = 0.5\ P(CE_1) + 0.5\ P(R_1) = .5 \times .5 + .5 \times 1 = .75$

Step 5 We continue till sufficient values of P(x) corresponding to different x are generated, and the curve of P(x) vs x can be drawn.

Once the preference curve is drawn, the preferences corresponding to each consequence of the problem can be obtained. In the same Decision Tree, the consequence can now be replaced by the preferences and the criterion of maximising expected preference be used for arriving at the decision. We now illustrate the above through an example.

Example 2

Let us take Example i of the earlier section. Suppose the decision maker is not a player of long run average (expected value). We want to get his preference curve for the problem, and arrive at the decision that maximises his expected preference.

Solution

We obtain the Preference curve of the d.m. as follows :

Step 1 From the Decision Tree of the earlier section, we see the worst consequence Rs. 35.000

the best consequence = Rs. 5,00,000

Question to d.m. : Suppose you have got a 50-50 chance of getting Rs, 35,000 or Rs. 5,00.000; for what certain amount will you exchange it?

Answer : Suppose he says Rs. 1,00,000 i.e. CE_1 = Rs. 1,00,000.

Step 2

Question to d.m.: Suppose you have a 50-50 chance of getting Rs. 1 lakh or Rs. 5 lakh, for what certain amount will you exchange it?

Answer : CE_2 = Rs. 2 lakh.

Step 3

Question to d.m.: What is your CE for a 50-50 chance of getting Rs. 2 lakh or Rs. 5 lakh.

Answer : CE_3 = Rs. 2.5 lakh.

Step 4. Continue questioning to obtain CE values till sufficient points, are there to draw a graph.

Step 5 Calculate P_1, P_2, P3 .... the preference corresponding to CE_1, CE_2,

CE_3 ....

$P1 = 0x.5+1x.5=.5$

$P_2 = .5 \times .5 + 1 \times .5 = .75$

$P_3 = .75 \times .5 + 1 \times .5 = .875$ etc.

Step 6. Draw the graph of P vs CE and look up the P values corresponding to the relevant consequences of the Decision Tree. Let us say, we get the preference values as .03, .61, .63, .99 corresponding to the consequences of Rs. 40,000, Rs. 1,45,000, Rs**.** 1,50,000 and Rs. 4,95,000 respectively.

Step 7 We calculate the expected Preferences.

Expected Preference for Drilling, given that the test says oil

=.818 x.99+ 182x-0=.809

This is greater than the preference of not drilling, given that test says oil.

$\therefore$ If test says oil, it is better to drill and expected preference in that case is .809.

Similarly, if test says no oil, expected preference of drilling (.174) is less than not drilling (.61). Hence if test says no oil, it is better not to drill and expected preference then is .61.

Expected Preference of taking test = .66 x .809 + .34 x .61 = .741. The Expected preference of not taking the test is given by :

$$.6 \times 1 + .03 \times .4 = .612.$$

Hence decision to take test will maximise his expected preference, i.e., in this case the decision is same as EMV maximising action. Though this need not always be true.

Activity E

Draw the Preference Curve for a decision maker who believes in maximising EMV. Consider another decision maker who is risk averse. Will the Preference Curve of the latter always be below that of the former? Justify your answer.

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

12.6 OTHER APPROACHES

In the foregoing sections, we have assumed that the probabilities associated with the outcomes are known. In practice, we find situations where it is not possible to make any probability assessment. The EMV and preference criteria fail in such cases. The objective of this concluding section is to discuss some criteria that can be used under such circumstances.

Criteria when probability are not known.

a) **Criterion of Pessimism :** At the name suggests, the decision-making is based on pessimism, viz, the assumption that whatever alternative is chosen, the worst payoff corresponding to each alternative is actually going to occur. A rational criterion for decision-making in such a case is to maximise the minimum payoff.

b) **Criterion of Optimism :** A variant of (a), here, over and above the maximum of the nninumum payoff (say, M_I), the maximum of the maximum payoff (say, M_2) is determined. Choosing M_M would mean complete optimism (the opposite of choosing M). It is suggested that the d.m. find the$_y$ maximum and minimum payoff for each alternative and then weigh them by his coefficient of optimism to arrive at the expected payoff for each alternative. The alternative with maximum expected payoff can then be chosen. Coefficient of Optimism lies between 0 and 1. It gives us the degree by which the maximum payoff is favoured by the d.m. vis-a-vis the minimum payoff.

c) **Criterion of Regret :** The criteria stems from the fact that a regret inbuilt-in in the decision-making, as the final decision on an alternative and the actual outcome after the decision has been taken, may not match, A regret of zero occurs when it matches. The regret can be measured as follows Consider our d.m. having two alternative investment proposals, the outcome corresponding to each proposal will be a failure or

Success depending on whether there is an economic depression or not. The consequences are as follows :

Outcome / Alt.	Depression	No Depression
1	-10	40
2	-6	20

Thus, if alternative 1 is chosen, and a depression actually occurs, then there is a cause for regret, as choosing 2 would have meant a loss of only 6 (vis-a-vis 10), thus regret = 10 - 6 - 4. Similarly, if there is no depression actually, and alt. 2 has been chosen, then a regret of 40-20 = occurs. Choosing alternative1 and later finding no depression would mean zero regret. Thus, the regret matrix is found:

	D	ND
1	4	0
2	0	20

Now, a pessimistic stand is taken and the criterion of minimising maximum regret is used for decision. For each alternative, the maximum regret is found, and finally the alternative with minimum value of maximum regret is chosen. Thus our d.m. would have chosen alternative 1.

d) **Subjectivists' Criterion :** The outcomes are assumed to be equally probable in this case, and EMV is used for decision. This is known as the subjectivists' stand.

The above four criteria are the best-known ones. Selection of the final criterion is purely subjective, as the obvious by now. However, each provides us with certain rationale and the d.m. can choose any,. depending on his own inclination.

Activity F

Consider the following problem where the decision maker has three alternative courses of action. Corresponding to each action there are possible outcomes, the probabilities of occurrence of which are unknown. The monetary payoff in each case is given in the matrix below :

Outcomes / Actions	0_1	0_2	0_3	0_4
A_1	10	15	25	20
A_2	30	20	45	15
A_3	25	40	55	10

For example, if the decision maker chooses A_1, and the outcome 0_1 occurs, he will get Rs. 10.

What will be the decision if the decision maker follows the criterion of pessimism? Will this decision change if he adopts the criterion of minimising the regret?

……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………
……………………………………………………………………………………

12.7 SUMMARY

Decision Theory provides us with the framework and methods for analysing decision problems under uncertainty. A decision problem under uncertainty is characterised by different alternative courses of action and uncertain outcomes corresponding to each action. The problems can involve a single stage or a multi-stage decision process. Marginal Analysis is helpful in solving single stage problems, whereas the Decision Tree Approach is useful for solving multi-stage problems. In this unit we have examined how these methods can be applied to solve decision problems. While using these methods, we have used the criterion of maximising the Expected Monetary Value (EMV). Thus, EMV basically assumes that the decision maker is risk neutral. Preference Theory helps in incorporating the preference of the decision maker in the Decision Tree framework. We have seen how instead of maximising the EMV, we can maximise the expected preference, and thereby consider the decision maker's attitude towards risk. In the final section of this unit we have examined certain other criteria that are helpful in taking decisions, when the probabilities of occurrence of the outcomes are not known.

12.8 FURTHER READINGS

Raiffa, H., 1970. *Decision Analysis,* Addison-Wesley.

Schlairfer; R.,1969. *Analysis of Decisions under Uncertainty,* McGraw-Hill.

Schlairfer, R., 1959. *Probability and Statistics for Business Decision,* McGraw-Hill (Ch. 38)

Berry, W.L. et al., 1980. *Management Decision Sciences,* R.D. Irwin, Inc.: Homewood. (Ch. 5)

Miller, D.W. and M.K. Starr, 1978. *Executive Decisions and Operations Research,* Prentice-Hall: Englewood-Cliffs. (Chs. 1, 4, 5 & 6).