UNIT 2 FUNCTIONS AND PROGRESSIONS

Objectives

After studying this unit, you should be able to understand and appreciate:

- the need to **identify** or **define** the relationships that exists among business variables
- how to **define** functional relationships
- the **various types** of functional relationships
- the **use** of graph to depict functional relationships
- the **manageria**l applicability and use of functional relationships in diverse fields
- the **progressions** and their applications.

Structure

2.1 INTRODUCTION

For decision problems which use mathematical tools, the first requirement is to identify or formally define all significant interactions or relationships among primary factors (also called variables) relevant to the problem. These relationships usually are stated in the form of an equation (or set of equations) or inequations. Such type of simplified mathematical relationships help the decision-maker in understanding (any) complex management problems. For example, the decision-maker knows that demand of an item is not only related to price of that item but also to the price of the substitutes. Thus if he can define specific mathematical relationship (also called model) that exists, then the demand of the item in the near future can be forecasted. The main objective of this unit is to study mathematical relationships (or functions) in the context of managerial problems.

2.2 DEFINITIONS

Variable

A variable is something whose magnitude can vary or which can assume various values. The variables used in applied mathematics include: sale, price, profit, cost, etc. Since magnitude of variables can vary, therefore these are represented by symbols (such as x, y, z etc.) instead of a specific number. In applied mathematics a variable is represented by the first letter of its name, for example p for price or profit; q for quantity, c for cost; s for saving or sales; d for demand and so forth. When we write $x=5$, the variable takes specific value.

Variables can be classified in a number of ways. For example, a variable can be **discrete** (suspect to counting, e.g. 2 houses, 3 machines etc.), or **continuous** (suspect to measurement, e.g. temperature, height. etc.).

Constant and Parameter

A quantity that remains fixed in the context of a given problem or situation is called a **constant.**

An absolute (or numerical) constant such as $\sqrt{2}$, π, e, etc. retains the same value in all problems whereas an arbitrary (or parametric) constant or **parameter** retains the same value throughout any particular problem but may assume different values in different problems, such as wage rates of different category of labourers in an industrial unit.

The absolute or numerical value of a constant 'b' is denoted by $|b|$ and means the magnitude of `b' regardless of its algebraic sign. Thus $|b| = |-b|$.

Functions

We come across situations in which two or more variables are related to each other. For example, demand (D) of a commodity is related to its price (p).It can be mathematically expressed as

$$D = f(p) \qquad (2\text{-}1)$$

This relationship is read as "demand is function of price" or simply "f of p". It does not mean D equals f times p. This mathematical relationship has two variables, D and p. These are called variables because they can take on different numerical values.

Let us now consider a mathematical relationship that contains three variables. Assume that the demand (D) of a commodity is related to the price (p) per unit of the commodity, and the level of advertising expenditure (A). Then the general relationship among these variables can be expressed as

$$D = f(p, A) \qquad (2\text{-}2)$$

The functional notations of the type (2-1) and (2-2) are meant to give a general idea that certain variables are, somehow, related. However for making managerial decisions, we need a specific and explicit, not a general and implicit relationship among selected variables. For example, for the purpose of finding the value of demand (D), we make the general relationship (2-2) more specific as shown in (2-3).

$$D = 4 + 3p - 2pA + 2A^2 \qquad (2\text{-}3)$$

Now for any given values of p and A, the value of D can be calculated using the relationship (2-3). This means that the value of D **depends** on the values of p and A. Hence D is called the **dependent variable** and p and A are called independent variables. In this case, it may be noted that we have established a **rule of correspondence** between the dependent variable and independent variable(s). That is, as soon as values are assigned to the independent variable(s), the corresponding **unique** value for the dependent variable is determined by the given specific relationship. That is why a **function is sometimes defined as a rule of correspondence between variables.** The set of values given to independent variable is called the **domain** of the function while the corresponding set of values of the dependent variable is called the **range** of the function. Other examples of functional relationships are as follows:

i) The distance (d) covered is a function of time (T) and speed (s), i.e. d = f (T, s).

ii) Sales volume (v) of the commodity is a function of price (p), i.e. V = f (p).

iii) Total inventory cost (T) is a function of order quantity (Q), i.e. T = f (Q).

iv) The volume of the sphere (v) is a function of its radius (r), i.e. V = f (r) or $v = \frac{4}{3}\pi r^3$

v) The extension (y) of a spring is proportional to the weight (m) (Hooke's law), i.e. $y \propto m$ or y = km.

vi) The net present value (y) of an investment is a function of net cash flows. (C,) in different time periods, project's initial cash outlay (B), firm's cost of capital (P) and the life of the project (N), i.e. $y = f(C_t, B, P, N)$.

It is important to note that every mathematical relationship may not be a function. For example, consider the following relationship:

$$y = \sqrt{x}$$

It is not a function because corresponding to any value of x, the value of y is not unique. For example, when x = 4, y = +2 and - 2.

The dimension of a function is determined by the number of independent variables For example:

D = f (p) is a single-variable (or one-dimensional) function

D = f (p, A) is a two-variable (or two-dimensional) function

Y = f (C_t, B, P, N) is a multi-variable (or multi-dimensional) function.

In order to understand the nature of mathematical relationship (also called model) between independent variable(s) and dependent variable we must be familiar with such terms as parameter, constants and variables. The Example-1 will illustrate the meaning of these terms.

Example 1

Suppose an industrial worker gets Rs. *25* per day. If he works for 26 days in a particular month, then his total wage for this month is 25 X 26= Rs. 650. During some other month he may have worked a total of only 25 days, then he would have earned Rs. 625. Thus, the total wages of the worker, assuming no overtime, can always be calculated as follows:

Total wages = 25 X number of days worked

If we let,

T = total wages

D = number of days worked

then,

T = 25 D.

This represents the relationship between total wages and number of days worked. In general, the above relationship can also be written as:

T = KD

where K is a constant for particular class of worker(s), to be assigned or determined in a specific situation. Since the value of K can vary for a specific situation, problem or context therefore it is called a **parameter,** whereas constants such as pi (denoted by π) which has approximate value of 3.1416 remains same from one problem context to another are called **absolute** constants. Quantities such as T and D which can assume various values in a given problem are called **variables.**

Activity A

1. Find the domain and range of each of the following functions

a) $y = \dfrac{1}{x - 1}$

b) $y = \sqrt{x};\ y \leq 0$

c) $y = \sqrt{4 - x};\ y \geq 0$

2. Let 4p + 6q = 60 be an equation containing variables p (price) and q (quantity). Identify the meaningful domain and range for the given function when price is considered as independent variable.

2.3 TYPES OF FUNCTION

In this Section some different types of functions are introduced which are particularly useful in calculus.

1 Linear Functions

A linear function is one in which the power of independent variable is 1, the general expression of linear function having only one independent variable is:

$$Y = f(x) = a + bx$$

Where a and b are given real numbers and x is an independent variable taking all numerical values in an interval.

A function with only one independent variable is also called single variable unction. Further, a single-variable function can be linear and non-linear. For example,

$$y = 3 + 2x, \text{ (linear single-variable function)}$$

And

$$y = 2 + 3x - 5x^2 + x^2, \text{ (non-linear single-variable function)}$$

A linear function with one variable can always be graphed in a two dimensional plane (or space). This graph can always be plotted by giving different values to x and calculating corresponding values of y. The graph of such functions is always a straight line.

Example 2

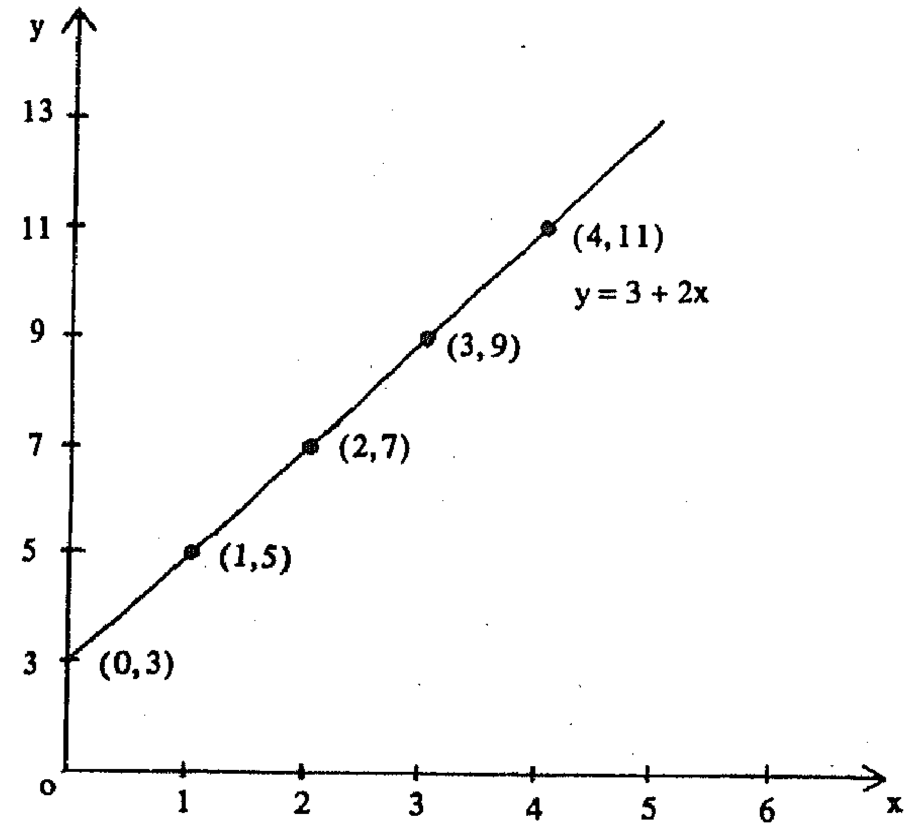
Plot the graph of the function, $y = 3 + 2x$

For plotting the graph of the given function, assign various values to x and then calculate the corresponding values of y as shown in the table below:

x	0	1	2	3	4	5	…..
y	3	5	7	9	11	13	…..

The graph of the given function is shown in Figure 1.

Figure I



A function with more than one independent variable is defined, in general, form, as:

$$y = f(x,, x_2, \ldots\ldots\ldots\ldots.., x_n) = a_o + a_1x_1 + a_2x_2 + \ldots + a_nx_n$$

where a_0, a_l, a_2,....., a_n are given real numbers and x_1, x_2, ... , x_n are independent variables taking all numerical value in the given intervals. Such functions are also called multivariable functions. A multivariable function can be linear and non-linear, for example,

$$y = 2 + 3x_1 + 5x_2 \text{ (linear multi-variable function)}$$

and

$$y = 3 + 4x_1 + 15x_1x_2 + 10x_2^2 \text{ (non-linear multivariable function)}$$

Multivariable functions may not be graphed easily because these require three-dimensional plane or more dimensional plane for plotting the graph.

In general, a function with n variables will require (n + 1) dimensional plane for plotting its graph.

2 Polynomial Functions:

A function of the form

$$y = f(x) = a_1x^n + a_2x^{n-1} + \ldots\ldots.+ a_nx + a_{n+1} \qquad (2 - 4)$$

where a_i' s (i = 1 , 2, ... , n + 1) are real numbers, $a_1 \neq 0$ and n is a positive integer is called **a polynomial of degree n.**

a) If n = 1, then the polynomial function is of degree 1 and is called a linear function. That is, for n = 1, function (2-4) can be written as:

$$y = a^1x^1 + a_2x^\circ \ (a_1 \neq 0)$$

This is usually written as

$$y = a + bx \qquad (\because\ x^0 = 1)$$

where 'a' and 'b' symbolise a_2 and a_1 respectively.

b) If n = 2, then the polynomial function is of degree 2 and is called a quadratic function. That is, for n = 2, function (2-4) can be written as:

$$y = a_1x^2 + a_2x^1 + a_3 \ (a_1 \neq 0)$$

This is usually written as:

$$y = ax^2 + bx + c$$

where

$$a_1 = a,\ a_2 = b \text{ and } a_3 = c$$

3 Absolute Value Functions

The functional relationship expressed by

$$y = |x|$$

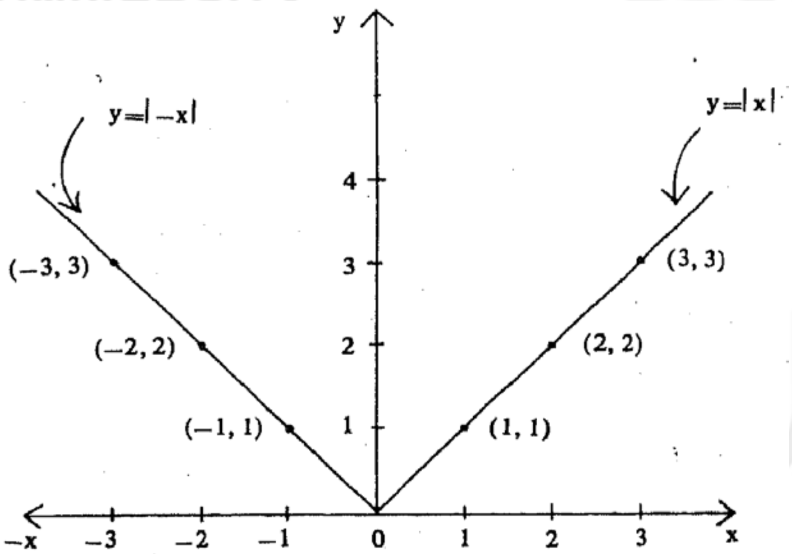
is known as an absolute value function, where $|x|$ is known as magnitude (or absolute value) of x. By absolute value we mean that whether x is positive or negative, its absolute value remains positive. For example $|7| = 7$ and $|-6| = 6$.

Plotting of the graph of the function $y = |x|$, assigning various values to x and then calculating the corresponding values of y, is shown in the table below:

x	….	-3	-2	-1	0	1	2	3	……..
y	…..	3	2	1	0	1	2	3	……..

The graph of the given function is shown in Figure II.

Figure II



4 **Inverse Function**

Take the function y = f(x). Then the value of y, can be uniquely determined forgiven values of x as per the functional relationship. Sometimes, it is required to consider x as a function of y, so that for given values of y, the value of x can be uniquely determined as per the functional relationship. This is called the inverse function and is also denoted by $x = f^{-1}(y)$. For example consider the linear function:

$$y = ax + b$$

Expressing this in terms of x, we get

$$X = \frac{y - b}{a}$$

$$= \frac{y}{a} - \frac{b}{a} = cy + d$$

where $c = \frac{1}{a}$, and $d = \frac{-b}{a}$

This is also a linear function and is denoted by $x=f^{-1}(y)$

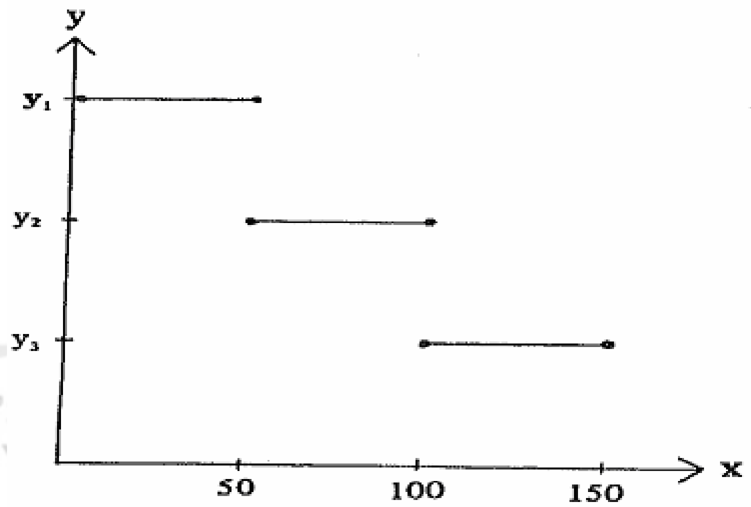
5 Step Function

For different values of an independent variable x in an interval, the dependent variable y = f(x) takes a constant value, but takes different values in different intervals. In such cases the given function y = f(x) is called a step function. For example

$$y = f(x) = \begin{cases} y_1, \text{if } 0 \le x < 50 \\ y_2, \text{if } 50 \le x < 100 \\ y_3, \text{if } 100 \le x < 150 \end{cases}$$

The shape of the graph of this function looks as shown in Figure III. for $y_3 < y_2 < y_1$,

Figure III



6 Algebraic and Transcendental Functions

Functions can also be classified with respect to the mathematical operations (addition, subtraction, multiplication, division, powers and roots) involved in the functional relationship between dependent variable and independent variable(s). When only finite number of terms are involved in a functional relationship and variables are affected only by the mathematical operations, then the function is called an **algebraic function,** otherwise transcendental function. The following functions are algebraic functions of x.

i) $y = 2x^3 + 5x^Z - 3x + 9$

ii) $y = \sqrt{x} + \frac{1}{x^2}$

iii) $y = x^3 - \frac{1}{\sqrt{x}} + 2$

The sub-classes of transcendental functions are follows:

a) Exponential Function

If the independent variable in any functional relationship appears as an exponent (or power), then that functional relationship is called exponential function, such as

i) $y = a^x,\ a \neq 1$

ii) $y = ka^x,\ a \neq 1$

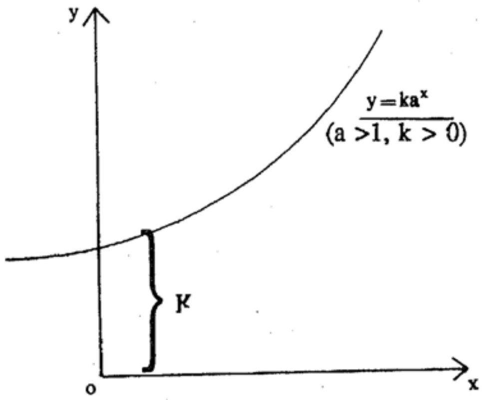
iii) $y = ka^{bX},\ a \neq 1$

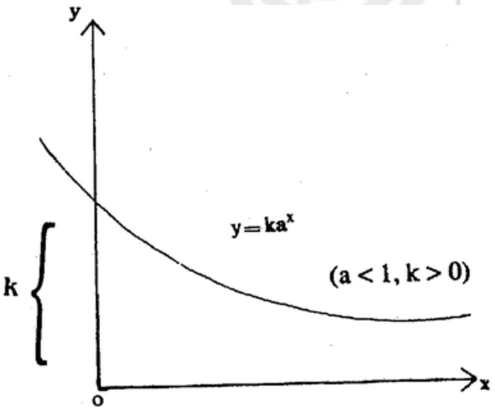
iv) $y = ke^x$

where a, b, e and k are constants with `a' taking only a positive value.

Such functions are useful for describing sharp increase or decrease in the value of dependent variable. For example, the exponential function $y = ka^x$ curve rises to the right for $a > 1$, $k > 0$ and falls to the right for $a < 1$, $k > 0$ as shown in the Figure IV(a) and (b).

Figure IV(a) **Figure IV(b)**





b) Logarithmic Functions

A logarithmic function is expressed as

$$y = \log_a x$$

where $a \neq 1$ and >0 is the base. It is read as `y' is the log to the base a of x . This can also be written as

$$x = a^y$$

Thus from an exponential function $y = a^x$, we may construct the logarithmic function $x = \log_a y$ by interchanging the variables. This shows that the inverse of an exponential function is a **logarithmic function.**

The two most widely used bases for logarithms are `10' and `e' $\cong$ 2.7182).

i) **Common logarithm:** It is the logarithm to the base 10 of a number x. It is written as $\log_{10} x$. If $y = \log_{10} x$, then $x=10^Y$.

ii) **Natural logarithm:** It is the logarithm to the base `e' of a number x. It is written as $\log_e x$ or ln x. When no base is mentioned, it will be understood that the base is e.

Some important properties of the logarithmic function $y = \log_e x$ are as follows:

i) $\log 1 = 0$

ii) $\log e = 1$

iii) $\log (xy) = \log x + \log y$

iv) $\log \left(\frac{x}{y}\right) = \log x - \log y$

v) $\log (x^n) = n \log x$

vi) $\log_e 10 = \frac{1}{\log_{10} e}$

vii) $\log_e a = (\log_e 10)(\log_{10} a) = \frac{\log_{10} a}{\log_{10} e}$

viii) logarithm of zero and negative number is not defined.

Activity B

1 Draw the graph of the following functions

a) $y = 3x - 5$

b) $y = x^2$

c) $y = \log_2 x$

2 The data of machine operating cost (c) and the age (t) of the machine art shown in the following table:

t (years)	:	1	2	3	4	5
c (in '000's)	:	5	8	13	20	29

i) Express operating cost as a function of the machine age

ii) Sketch the graph of the function derived in (i).

2.4 SOLUTION OF FUNCTIONS

The value(s) of x at which the given function f(x) becomes equal to zero are called the roots (or zeros) of the function f(x). For the linear function

$$y = ax + b$$

the roots are given by

$$ax + b = 0$$

$$\text{or} \qquad x = -\frac{b}{a}$$

Thus if $x = -\frac{b}{a}$ is substituted in the given linear function y = ax + b then it becomes equal to zero.

In the case of quadratic function

$$y = ax^2 + bx + c,$$

we have to solve the equation $ax^2 + bx + c = 0;\ a \neq 0$ to find the roots of y.

The general value of x for which the given quadratic function will become zero is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, in general, there are two values of x for which y becomes zero. One value is

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and other value is

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

It is very important to note that the number of roots of the given function are always equal to the highest power of the independent variable.

Particular Cases:

The expression $b^2 - 4ac$ in the above formula is known as discriminant which determines the nature of the roots as discussed below:

i) If $b^2 - 4ac > 0$, then the two roots are real and unequal.

ii) If $b^2 - 4ac = 0$ or $b^2 = 4ac$, then the two roots are equal and are equal to $-\frac{b}{2a}$

iii) If $b^2 - 4ac < 0$, then the two roots are imaginary (not-real) because of the square root of a negative number.

The roots of a polynomial of the form:

y = (x-a)(x-b)(x-c)(x-d) .. .

are a, b, c, d, ...

Activity C

Given that f(x) = (x - 4)(x + 3); then find

a) f(4), f(-1), f(-3)

b) Roots of the function

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2.5 BUSINESS APPLICATIONS

We often talk of supply and demand functions; cost functions; profit functions; revenue functions; production functions; utility, functions; etc. in applied mathematics. In this section, a few examples are given by constructing such functions and obtaining their solutions:

Example *3 (Linear Functions)*

A company sells x units of an item each day at the rate of Rs. 50 per unit. The cost of manufacturing and selling these units is Rs. 35 per unit plus a fixed daily overhead cost of Rs. 1000. Determine the profit function. How would you interpret the situation if the company manufactures and sells 400 units of the items a day.

Solution:

The total revenue received by the company per day is given by:

Total revenue (R) = (price per unit) X (number of items sells)

= 50.x

The total cost of manufactured items per day is given by:

Total cost (c) = (Variable cost per unit) X (number of items manufactured) + (fixed daily overhead cost)

= 35.x + 1000

Thus, Total profit (p) = (Total revenue) - (Total cost)

= 50.x - (35.x + 1000) =15.x -1000

If 400 units of the item are manufactured and sold, then the profit is given 'by:

P = 15 X 400 – 1000

= - 400

The negative profit indicates loss. Thus if the company manufactures and sells 400 units of the item, it would incur a loss of Rs. 400 per day.

Example *4 (Quadratic Functions)*

Let the market supply function of an item be q =160 + 8p, where q denotes the quantity supplied and p denotes the market price. The unit cost of production is Rs. 4. It is felt that the total profit should be Rs. 500. What market has to be fixed for the item so as to achieve this profit?

Solution:

Total profit function can be constructed as follows:

Total profit (P) = Total revenue - Total cost

= (Price per unit X Quantity supplied) - (Cost per unit X Quantity supplied)

= p.q – c.q

= (p – c).q

Given that c = Rs. 4 and q = 160 + 8p. Then total profit function becomes

$$P = (p - 4)(160 + 8p)$$

$$= 8p^2 + 128p - 640$$

If P = 500, then we have

$$500 = 8p^2 + 128p - 640$$

or $$8p^2 + 128p - 1140 = 0$$

$$\because\ p = \frac{-128 \pm \sqrt{(128)^2 - 4 \times 8 \times (-1140)}}{2 \times 8}$$

$$= \frac{-128 \pm 229.92}{16}$$

$$= 6.36 \text{ or } -22.37$$

Since negative price has no economic meaning, therefore the required price per unit should he Rs. 6.37.

Activitiy D

a) Consider the quadratic equation $2x^2 - 8x + c=.-O$. For what value of c, the equation has

   i) real roots,

   ii) equal roots, and

   iii) imaginary roots?

b) A newsboy buys papers for p_1 paise per paper and sells them at a price of p_2 paise per paper ($p_2>p_1$). The unsold papers at the end of the day are bought by a wastepaper dealer for p_3 paise per paper ($p_3 <p_1$).

   i) Construct the profit function of the newsboy.

   ii) Construct the opportunity loss function of the newsboy.

2.6 SEQUENCE AND SERIES

Sequence

If for every positive integer n, there corresponds a number a_n such that a_n is related to n by some rule, then the terms a, , a_2 , ... a_n ... are said to form a sequence.

A sequence is denoted by bracketing its nth term, i.e. (a_n) or $\{a_n\}$. Example of a few sequences are:

i) If $a_n = n^2$, then sequence $\{a_n\}$ is 1, 4, 9, 16, ..., n^2, .. .

ii) If $a_n = 1/n$, then sequence $\{a_n\}$ is 1,1/2,1/3,1/4 . . .1/n .. .

iii) If $a_n = \frac{n^2}{n+1}$, then sequence $\{a_n\}$ is 1/2,4/3,9/4, ... $n^2/n + 1$, ...

The concept of sequence is very useful in finance. Some of the major areas where it plays a vital role are: `instalment buying'; `simple and compound interest problems'; 'annuities and their present values', mortgage payments and so on.

Series

A series is formed by connecting the terms of a sequences with plus or minus sign. Thus if a_n is the nth term of a sequence, then

$$a_1 + a_2 + ... + a_n$$

is a series of n terms.

2.7 ARITHMETIC PROGRESSION (AP)

A progression is a sequence whose successive terms indicate the growth or progress of some characteristics. An arithmetic progression is a sequence whose term increases or decreases by a constant number called **common difference** of an A.P. and is denoted by d. In other words, each term of the arithmetic progression after the first is obtained by adding a constant d to the preceding term. The standard form of an A.P. is written as

$$a, a + d, a + 2d, a + 3d,...$$

where `a' is called the first term. Thus the corresponding standard form of an arithmetic series becomes

$$a + (a + d) + (a + 2d) + (a + 3d) +...$$

Example 5

Suppose we invest Rs. 100 at a simple interest of 15% per annum for 5 years. The amount at the end of each year is given by

115, 130, 145, 160, 175

This forms an arithmetic progression

The nth Term of an A.P.

The nth term of an A.P. is also called the general term of the standard A.P. It is given by

$$T_n = a + (n - 1) d; \qquad n =1,2,3,.....$$

Sum of the First n terms of an A.P.

Consider the first n terms of an A.P.

$$a, a + d, a + 2d, a + 3d,..., a + (n - 1) d$$

The sum, S_n of these terms is given by

$$s_n = a + (a + d) + (a + 2d) + (a + 3d) + ... + a + (n - 1)d$$

$$= (a + a + ... + a) + d \{1 + 2 + 3 + ... + (n - 1)\}$$

$$= n.a + d \{\frac{n(n-1)}{2}\}$$ (using formula for the sum of first (n - 1) natural numbers)

$$= \frac{n}{2}\{2a+(n-1)d\}$$

Example 6

Suppose Mr. X repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will be take to clear his loan?

Solution

Since Mr. X increases the monthly payment by a constant amount, Rs. 15 every month, therefore d = 15 and first month instalment is, a = Rs. 20. This forms an A.P. Now if the entire amount be paid in n monthly instalments, then we have

$$s_n = \frac{n}{2}\{2a + (n - 1)d\}$$

$$\text{or} \quad 3250 = \frac{n}{2}\{2 \times 20 + (n - 1)15\}$$

$$6500 = n \{25 + 15n\}$$

$$15n^2 + 25n - 6500 = 0$$

This is a quadratic equation in n. Thus to find the values of n which satisfy this equation, we shall apply the following formula as discussed before.

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-25 \pm \sqrt{(25)^2 - 4 \times 15 \times (-6500)}}{2 \times 15}$$

$$= \frac{-25 \pm 625}{30} = 20 \text{ or } -21.66$$

The value, n = - 21.66 is meaningless as n is positive integer. Hence Mr. X will pay the entire amount in 20 months.

Activity E

1 Find the 15th term of an A.P. whose first term is 12 and common difference is 2.

2 A firm produces 1500 TV sets during its first year. The total production of the firm at the end of' the 15th year is 8300 TV sets, then
   a) estimate by how many units, production has increased each year.
   b) based on estimate of the annual increment in production, forecast the amount of production for the 10th year.

2.8 GEOMETRIC PROGRESSION (GP)

A geometric progression (GP) is a sequence whose each terms increases or decreases by a constant ratio called **common ratio** of G.P. and is denoted by r. In other words, each term of G.P. is obtained after the first by multiplying the preceding term by a constant r. The standard form of a G.P. is written as

$$a, ar, ar^2, ...$$

where `a' is called the first term. Thus the corresponding geometric series in standard form becomes

$$a + ar + ar^2 + ...$$

Example 7

Suppose we invest Rs. 100 at a compound interest of 12% per annum for three years. The amount at the end of each year is calculated as follows:

i) Interest at the end of first year = $100 \times \frac{12}{100}$ = Rs. 12

Amount at the end of first year = Principal + Interest

$= 100 + 100\ (12/100)$

$= 100\ (1 + \frac{12}{100})$

This shows that the principal of Rs. 100 becomes Rs. $100\ (1 + \frac{12}{100})$ at the end of first year.

ii) Amount at the end of second year =

(Principal at the beginning of second year) $\{1 + \frac{12}{100}\}$

$$= 100\ \{1 + \frac{12}{100}\}\ \{1 + \frac{12}{100}\}$$

$$= 100\ \{1 + \frac{12}{100}\}^2$$

iii) Amount at the end of Third year = $100\ \{1 + \frac{12}{100}\}^2 \{1 + \frac{12}{100}\}$

$$= 100\ \{1 + \frac{12}{100}\}^3$$

Thus, the progression giving the amount at the end of each year is

$$100\{1 + \frac{12}{100}\};\ 100\{1 + \frac{12}{100}\}^2;\ 100\{1 + \frac{12}{100}\}^3;\ ....$$

This is a G.P. with common ratio $r = (1 + \frac{12}{100})$

In general, if P is the principal and i is the compound interest rate per annum, then the amount at the end of first year becomes $p\ (1 + \frac{i}{100})$ Also the amount at the end successive years forms a G.P.

$$P\ (1 + \frac{i}{100})\ ;\ P\ (1 + \frac{i}{100})^2;\ ...$$

with $r = (1 + \frac{i}{100})$

The nth Term of G.P.

The nth term of G.P. is also called the general term of the standard G.P. It is given by

$$T_n = ar^{n-1},\ n = 1, 2, 3, ...$$

It may be noted here that the power of r is oneless than the index of T_n, which denotes the rank of this term in the progression.

Sum of the First n Terms *in G.P.*

Consider the first n terms of the standard form of G.P.

$$a, ar, ar^2, \ldots, ar^{n-1}$$

The sum, Sn of these terms is given by

$$S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1} \qquad (2\text{-}4)$$

Multiplying both sides by r, we get

$$rS_n = ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n \qquad (2\text{-}5)$$

Subtracting (2.5) from (2.4), we have

$$S_n - rS_n = a - ar^n$$

$$S_n\ (1 - r) = a\ (1 - r^n)$$

$$\text{or} \quad S_n = \frac{a(1 - r^n)}{(1 - r)};\ r \neq 1 \text{ and } <1$$

Changing the of the numerator and denominator, we have

$$S_n = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1 \text{ and } >1$$

a) If r= 1, G.P. becomes a, a, a, ... so that S_n in this ease is S_n = n.a.

b) If number of terms in a G.P. are infinite, then

$$S_n = \frac{a}{1 - r}, \ r<1$$

For $r \geq 1$, the sum tends to infinity

Example 8

A car is purchased for Rs. 80,000. Depreciation is calculated at 5% per annum for the first 3 years and 10% per annum for the, next 3 years. Find the money value of the car after a period of 6 years.

Solution:

i) Depreciation for the first year = 80,000 X $\frac{5}{100}$. Thus the depreciated value of the car at the end of first year is:

$$= (80,000 - 80,000 \times \frac{5}{100})$$

$$= 80,000(1 - \frac{5}{100})$$

ii) Depreciation for the second year

= (Depreciated value at the end of first year) X (Rate of depreciation for second year)

= 80,000 (1 - 5/100) (5/100)

Thus the depreciated value at the end of the second year is

= (Depreciated value after first year) - (Depreciation for second year)

$$= 80,000 \left(1 - \frac{5}{100}\right) - 80,000 \left(1 - \frac{5}{100}\right) \left(\frac{5}{100}\right)$$

$$= 80,000 \left(1 - \frac{5}{100}\right) \left(1 - \frac{5}{100}\right)$$

$$= 80,000 \left(1 - \frac{5}{100}\right)^2$$

Calculating in the same way, the depreciated value at the end of three years is

iii) Depreciation for the fourth year

$$= 80,000 \left(1 - \frac{5}{100}\right)^3 \left(\frac{10}{100}\right)$$

Thus the depreciated value at the end of the fourth year is

= (Depreciated value after three year) X (Depreciation for fourth year)

$$= 80,000 \left(1 - \frac{5}{100}\right)^3 - 80,000 \left(1 - \frac{5}{100}\right)^3 \left(\frac{10}{100}\right)$$

$$= 80,000 \left(1 - \frac{5}{100}\right)^3 \left(1 - \frac{10}{100}\right)$$

Calculating in the same way, the depreciated value at the end of six years becomes

$$= 80,000 \left(1 - \frac{5}{100}\right)^3 \left(1 - \frac{10}{100}\right)^3$$

= Rs. 49,980.24

Activity F

1 Determine the common ratio of the G.P.

49, 7, 1, 1/7, 1/49, ...

a) Find the sum to first 20 terms of G.P.

b) Find the sum to infinity of the terms of G.P.

2 The population of a country in 1985 was 50 crore.

Calculate the population in the year 2000 if the compounded annual rate of increase is (a) 1% (b) 2%.

2.9 SUMMARY

The objective of this unit is to provide you exposure to functional relationship among decision variables. We started with the mathematical concept of function and defined terms such as constant, parameter, independent and dependent variable. Various examples of functional relationships are mentioned to see the concept in broad perspective. Various types of functions which are normally used in managerial decision-making are enumerated along with suitable examples, their graphs and solution procedure. Finally, the applications of functional relationships are demonstrated through several examples.

Attention is then directed to defining the Arithmetic and Geometric Progressions and subsequently to their applications.

2.10 KEY WORDS

Arithmetic Progression (A.P.): An A.P. is a sequence whose terms increases or decreases by a constant number.

Algebraic and Transcendental Function: When only finite number of terms are involved in a functional relationship and variables are affected only by the mathematical operations, then functions are called algebraic function, otherwise transcendental function.

Constant: A quantity that remains fixed in the context of a given problem or situation.

Exponential Function: If the independent variable in any functional relationship appears an exponent (or power), then such functional relationship is called exponential function,

Function: It is the rule of correspondence between dependent variable and independent variable(s) so that for every assigned value to the independent variable, the corresponding unique value for the dependent variable is determined.

Geometric Progression (G.P.): A G.P. is a sequence whose terms increases or decreases by a constant ratio.

Linear Function: A function whose graph is a straight line is called a linear function.

Logarithmic Function: The inverse of exponential function is called a logarithmic function.

Parameter: A quantity that retains the same value throughout any particular problem but may assume different values in different problem.

Polynomial Function: A function of degree n is called a polynomial function of degree n.

Series: A series is formed by connecting the terms of a sequence with plus or minus sign.

Sequence: If for any positive integer n, there corresponds a number a_n such that a_n is related to n by some rule, then the terms a_1, a_2, ... a_n, are said to form a sequence.

Step Function: If for values of an independent variable, the dependent variable takes a constant value in different intervals then the function is called step function.

Variable: A quantity that can assume various values.

2.11 FURTHER READINGS

Childress, R.L., 1974. *Mathematics for Managerial Decision,* Prentice Hall Inc.: Englewood-Cliffs.

Dean, B.V., Sassieni, M.W.; and Gupta, S.K., 1978. *Mathematics for Modern Management,* Wiley Eastern: New Delhi.

Draper, J.E.; and J.S. Klingrnan, 1972. *Mathematical Analysis: Business and Economic Applications,* Harper and Row Publishers: New York.

Raghavachari, M., 1985. *Mathematics for Management: An Introduction;*

Tata McGraw-Hill Pub. Comp. Ltd.: New Delhi.