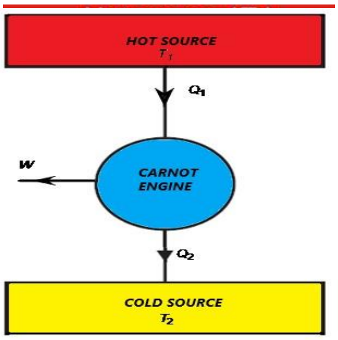

Carnot engine led to the industrialisation of Europe.

UNIT 8

CARNOT CYCLE

Structure

STUDY GUIDE

In the previous units of this course, you have learnt about the zeroth law and the first law of thermodynamics. As you now know, these laws facilitated introduction of the concepts of temperature and internal energy, respectively. In this unit, you will learn about conversion of heat into work, Carnot cycle and the second law of thermodynamics.

In TQ 9 of Unit 7, you have obtained expression for the work done in Carnot cycle depicted on an indicator diagram. We will extend this result to calculate the efficiency of Carnot engine and show that no engine can be more efficient than a Carnot engine. As mentioned earlier, thermodynamics is a phenomenological science and its laws need no proof. In fact, the second law of thermodynamics has been stated in two different but equivalent forms by Kelvin-Planck and Clausius. You will learn these equivalent statements.

The mathematics used here is rather simple and basically, we will use the results derived in Unit 7. You are, therefore, advised to master that unit before reading this unit. Moreover, if you work out SAQs and TQs given in this unit on your own, you will appreciate the subject matter better.

"Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world."

Albert Einstein

8.1 INTRODUCTION

We now know that the first law of thermodynamics is a statement of conservation of energy for thermodynamic processes. But it does not give us information about the direction of flow of heat. For instance, it is a common experience that heat flows from a hotter body to a colder body spontaneously but it cannot flow by itself from a colder body to a hotter body. However, the first law of thermodynamics does not rule out this possibility. Similarly, it is a common experience that it is possible to completely convert work into heat via friction, say. But the first law of thermodynamics puts no definite limitation on conversion of heat into work, though engineering experience refrains us from achieving 100% conversion. If this were not true, we could convert virtually unlimited heat of the environment into work and energy crisis would not have been such an issue for present day civilisation. We can similarly consider many natural processes where energy is conserved but those never happen. This suggests that besides the first law, we must have some other fundamental principle which satisfactorily explains these facts of experience. This principle is known as the *second law of thermodynamics*. In fact, the second law goes far beyond conversion of heat into work.

In Sec. 8.2, we begin our discussion by considering convertibility of heat into work using a heat engine. For simplicity, we confine ourselves to the framework of reversible Carnot cycle. We derive an expression for the efficiency of a Carnot engine in Sec. 8.3. You will learn that the direction of operation of Carnot cycle determines whether a device acts as a heat engine or a refrigerator. It is for such reasons that Carnot cycle is the most important reversible cycle of great practical utility. You will also learn that Carnot engine has maximum efficiency but it is a theoretical idealisation.

It may be mentioned here that contributions of Carnot facilitated industrial revolution in Europe. As we now know, the work of Carnot led Clausius, Thomson (later Lord Kelvin) and Planck, among others, to study convertibility of heat into work. These studies led them to sum up generalisations of experiences in different statements of the second law. However, the two most well-known statements of the second law are due to Kelvin-Planck and Clausius. These statements are discussed in detail and their equivalence has also been established in Sec. 8.4. We show that if one statement is not obeyed, the other one is also violated. In Sec. 8.5, we have discussed Carnot theorem.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ derive the expression for the efficiency of a Carnot engine;
- ❖ explain the physics of the working of a heat engine and a refrigerator;
- ❖ state Kelvin-Planck and Clausius statements of the second law of thermodynamics, discuss their implications and prove their equivalence; and
- ❖ establish Carnot theorem.

8.2 HEAT ENGINES: CONVERSION OF HEAT INTO WORK

Let us begin our discussion by stating the preliminaries of heat engines which convert heat into work.

Basic Terminology

We know that heat flows spontaneously from a hotter to a colder body. If we intercept this flow with a machine, some of it can be converted into work. *A machine that can convert heat into work is known as heat engine*. To be a useful device, a heat engine must operate continuously; absorb heat at a higher temperature and reject it at a lower temperature. That is to say, **a heat engine operates between two heat reservoirs** (Fig. 8.1). Moreover, the processes which take place inside an engine must not cause permanent changes. This means that **an engine has to operate in a cycle.**

The material used in the operation of an engine is called the *working substance*. The working substance can be solid, liquid or gas. In a steam engine, the working substance is steam (water). Other familiar working substances for automobile engines are petrol, diesel and CNG. In a refrigerator, the most widely used working substances used to be chlorofluorocarbon compounds. But these have now been phased out as these deplete the ozone layer present in stratosphere.

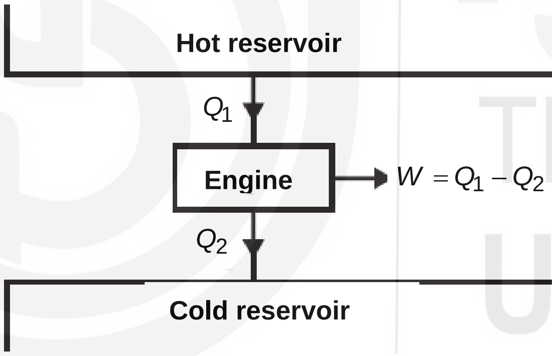

Fig. 8.1: **Schematics of operation of a heat engine.**

Efficiency

The efficiency of an engine is defined as the ratio of the net work done to the heat absorbed during one complete cycle. It is usually denoted by the symbol η (pronounced as eta):

$$\eta = \frac{\text{Useful work done}}{\text{Heat absorbed}} \qquad (8.1)$$

After one complete cycle, the engine returns to its original state. Therefore, there will be no change in its internal energy, i.e. $\Delta U = 0$. Using the first law, we can write

$$\Delta U = Q_1 - Q_2 - W = 0$$

or

$$W = Q_1 - Q_2 \qquad (8.2)$$

Nicolas Leonard Sadi Carnot (1796-1832) was a French physicist and engineer. With his pioneering work on heat engines, he successfully proposed an engine based on reversible thermodynamic processes, which offered maximum possible efficiency. Unfortunately, his work was not appreciated during his life time. Clausius and Kelvin used his ideas to propose the second law of thermodynamics.

where Q_1 is the heat absorbed from the source, Q_2 is the heat rejected to the sink and W is the work done during one cycle (Fig. 8.1). Note that in a real engine, heat is rejected to the surroundings in the form of hot exhaust gases or steam and, therefore, Q_2 contributes to *thermal pollution* of our environment.

From Eqs. (8.1) and (8.2), we can write

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \qquad (8.3)$$

This result shows that efficiency of a heat engine will always be less than one. Nevertheless, it is desirable to design an engine with maximum efficiency. To know how high η can be, we have to consider the conditions in which an engine operates and the role of the working substance. You will learn about these in the following sections. However, it may suffice to say here that Carnot was the first researcher who recognised that for maximum efficiency, a heat engine should be (thermodynamically) reversible. That is, all stages of operation should be carried out infinitely slowly so that there are no dissipative losses due to friction or turbulence, leading to wastage of energy. (In practice, however, there are always some losses.) It may be mentioned here that any heat engine operating in a Carnot cycle is called a *Carnot engine* and the working substance exchanges heat with heat reservoirs. We will discuss it in some detail now. But before that let us now summarise what you have learnt in this section.

Recap

CONVERSION OF HEAT INTO WORK

- A machine repsonsible for conversion of heat into work is called a heat engine.
- In a steam engine (power-plant or an automobile) we burn fuel for generating heat which, in turn, makes the engine do work through the motion of a piston (turbine).
- The difference in the heat generated and the amount utilised to do work is released to surroundings and is one of the causes of thermal pollution of our environment.
- The ratio of work done and heat absorbed characterises the efficiency of a machine which converts heat into work.

8.3 THE CARNOT CYCLE

The Carnot cycle consists of four stages. These are schematically depicted in Fig. 8.2. Suppose that T_1 and T_2 are temperatures of the heat reservoirs such that $T_1 > T_2$. The working substance, say a gas, is contained within a cylinder fitted with a frictionless piston. To simulate the working of a real engine, we consider the following reversible sequence:

i) isothermal expnasion, ii) adiabatic expansion, iii) isothermal compression, and iv) adiabatic compression.

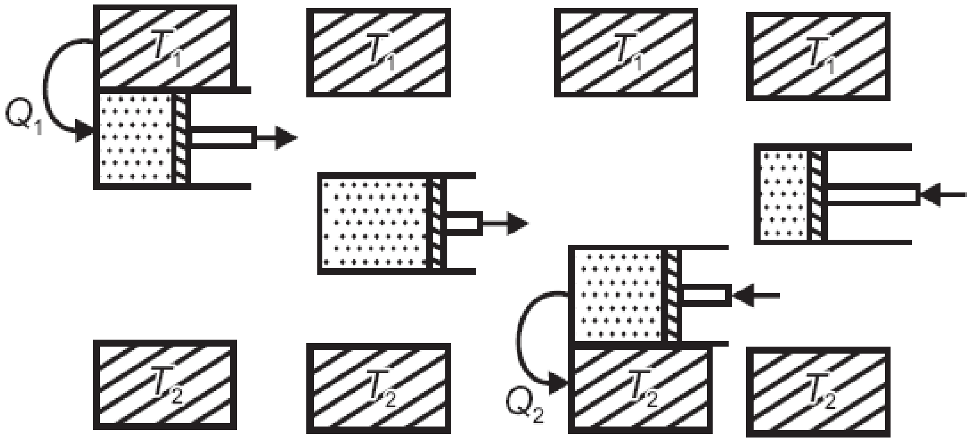

Fig. 8.2: **Four stages in the Carnot cycle.**

To realise the processes stated above in an engine, it is operated as follows:

1. We place the cylinder in thermal contact with the hot reservoir and let the gas undergo *reversible isothermal expansion*. Suppose heat Q_1 flows from the reservoir into the gas in this process. We have indicated this change as A to B on the p-V diagram in Fig. 8.3. (It was similarly depicted in Fig. 7.8 in TQ 9 in Unit 7). Note that the process is reversible so that the temperature of the working substance continues to be equal to the temperature of the reservoir during heat transfer.

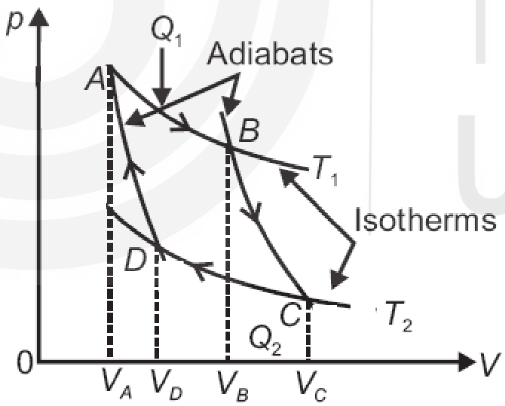

Fig. 8.3: **Carnot cycle on indicator diagram.**

2. Next, the gas is thermally isolated and allowed to undergo *reversible adiabatic expansion*. The temperature falls from T_1 to T_2, the temperature of the cold reservoir. Do you know, why the temperature drops? It is because work is done by the gas at the cost of its internal energy. This change is indicated as B to C on the p-V diagram.

3. On attaining the state defined by C, the working substance is at relatively low pressure and to use it in a cycle, it has to be restored to its initial state. Therefore, the gas is compressed in two stages: First isothermally and then adiabatically. This is done by placing the cylinder in thermal contact with the cold reservoir at lower temperature T_2 and compressing the gas

isothermally and reversibly. Suppose heat Q_2 is given up by the gas to the cold reservoir. This change is indicated as *C* to *D*.

4. Next, the gas is thermally isolated and compressed under reversible adiabatic conditions till its original state is restored.

8.3.1 Efficiency of a Carnot Engine

While answering TQ 9 of Unit 7, you have obtained expression for the work done by the gas in a Carnot engine. We just quote the result here:

$$W = nRT_1 \ln (V_B / V_A) - nRT_2 \ln \frac{V_C}{V_D} \qquad (8.4)$$

To simplify this expression, we note that *B* and *C* (in Fig. 8.3) lie on the same adiabatic curve. Then, using Eq. (7.21), we can write

$$T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$$

or

$$\frac{T_1}{T_2} = \left(\frac{V_C}{V_B}\right)^{\gamma-1} \qquad (8.5)$$

Similarly, for states *D* and *A* we can write

$$\frac{T_1}{T_2} = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \qquad (8.6)$$

To simplify Eq. (8.7) we use the algebraic theorem that if powers are positive and equal, the bases are also equal.

On comparing Eqs. (8.5) and (8.6), we get

$$\left(\frac{V_C}{V_B}\right)^{\gamma-1} = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \qquad (8.7)$$

We can rewrite it as

$$\left(\frac{V_C}{V_B}\right) = \left(\frac{V_D}{V_A}\right)$$

or

$$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

Using this result in Eq. (8.4), we get

$$W = nR(T_1 - T_2)\ln (V_B / V_A) \qquad (8.8)$$

You have learnt in TQ 9 in Unit 7 that using the first law of thermodynamics for an isothermal process, we can write

$$W_1 = Q_1$$

since $\Delta U = 0$

Work done by the gas on the piston is given by

$$W_1 = \int_A^B p\,dV$$

$$W_1 = Q_1 = nRT_1 \int_{V_A}^{V_B} \frac{dV}{V}$$

$$\therefore \quad Q_1 = nRT_1 \ln (V_B / V_A)$$

On substituting this expression for *W* in Eq. (8.4), we can express the efficiency of a Carnot engine in terms of the temperatures within which it operates:

$$\eta = \frac{W}{Q_1} = \frac{nR}{nRT_1} \frac{(T_1 - T_2)\ln (V_B / V_A)}{\ln (V_B / V_A)}$$

$$= \frac{(T_1 - T_2)}{T_1} = 1 - \frac{T_2}{T_1} \qquad (8.9)$$

We can draw the following conclusions from this result:

1. Efficiency of a Carnot engine depends on the temperature difference between the source and the sink; greater the difference, higher will be efficiency. In practice, the temperature of the sink is limited by the surroundings and the only way to increase η is to raise temperature of the source, T_1. It means that heat is more useful when it is supplied at a higher temperature. This explains why saturated steam at high pressure is a more efficient working substance.

2. Efficiency of a Carnot engine is less than one. This is a fundamental limitation imposed on the convertibility of heat into work by the second law of thermodynamics. (We know that most of the electricity is generated in large fossil fuel (coal, oil, gas) or nuclear power plants. These are basically heat engines (where energy is released in chemical or nuclear reactions).

   The working substance, water, gets heated in a boiler and converted into steam at high pressure. It is made to expand adiabatically in a turbine, which is coupled to a generator and converts mechanical energy into electrical energy. The maximum efficiency of a power plant is about 50%. (This is also true of diesel and petrol engines.) It means that only half of the heat generated (fuel used) in a plant is converted into useful work. In fact, a substantial amount of our expensive fuel ends up as waste heat; it is released in the environment and causes thermal pollution, which is responsible for various ecological problems. It is, therefore, desirable to design maximum efficiency engines.

3. If the source and the sink are at the same temperature, the efficiency will be zero. It means that we cannot operate an engine (and convert heat into work) if there is no temperature difference. To understand this, consider the following situation:

   You take a motor boat to sea and run out of fuel. (If you are lucky, you may be rescued by another boat.) The first law of thermodynamics permits you safe return as the ocean has a vast amount of energy. But the second law tells us that this energy cannot be converted into useful work because ocean surface is at an almost uniform temperature.

4. The efficiency of a Carnot engine is independent of the nature of the working substance. You may expect that real engines will also be independent of the working substance and ask: Why are we then so concerned about a particular fuel? The answer to this question lies in their availability, economics, tenchonological feasibility and environmental factors. That is to say, thermodynamic considerations alone do not decide between various fuels and methods of harnessing energy sources.

5. On comparing the expressions of efficiency given in Eqs. (8.3) and (8.9), we can correlate the ratios of heat absorbed and heat rejected to temperature of the source and temperature of the sink:

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}.$$

Before proceeding further, answer the following SAQ.

SAQ 1 – Carnot engine

Can efficiency of a Carnot engine be increased more effectively by increasing T_1 or lowering T_2? Explain your answer.

You should now go through the following solved example.

EXAMPLE 8.1: EFFICIENCY CALCULATION

The cluster of nuclear power plants at Tarapur produces 540 MW electric power. In the reactor core, energy is released (as heat) at the rate of 1600 MW. Steam produced in the reactor enters the turbine at a temperature of 560K and leaves it at 350K. Calculate the efficiency of the power plant.

SOLUTION ■ The thermodynamic efficiency is given by Eq. (8.9):

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{350\text{K}}{560\text{K}} = 0.375$$

That is, the system is only 37.5% efficient.

The actual efficiency of a power plant is defined as the ratio of the electric power output to the thermal power produced:

$$\eta = \frac{540\,\text{MW}}{1600\,\text{MW}} = 0.337$$

The waste heat of 1060 MW is normally discharged in a river (like Ganges, Mahanadi) or sea. This is a huge amount of energy and harmful for aquatic life.

To reduce problems arising out of this, the designers of the power plant at Narora (UP) made use of cooling towers where expandig steam is made to cool by releasing heat to the atmosphere (air) rather than to water. This nevertheless causes thermal pollution in the troposphere.

Diesel engines used in vehicles constitute another example of heat engines. A typical automobile engine operates at about 800K and releases exhaust gases to the environment at about 300K.

The maximum possible efficiency is then

$$\eta = 1 - \frac{300\,\text{K}}{800\,\text{K}} = 0.63$$

In practice, the actual efficiency is much lower (~ 40%) and emanating hot gases are responsible for thermal pollution of our environment.

You may now like to answer an SAQ.

SAQ 2 – Efficiency of a Carnot engine

a) In the tropics, the temperatures at the surface of the ocean and at a depth of 300m are 25°C and 5°C, respectively. Will you recommend to tap this energy? Discuss.

b) A Carnot engine is made to work between ice point (273K) and nitrogen temperature (77K). Calculate its efficiency. Is it possible to attain this figure in actual practice?

Now go through the following example.

EXAMPLE 8.2: EFFICIENCY OF A CARNOT ENGINE

A Carnot engine has an efficiency of 60% when its sink temperature is at 27°C. Calculate the change in the source temperature for increasing its efficiency to 70%.

SOLUTION ■ Let the initial temperature of the source be T_1. The temperature of the sink, $T_2 = 27°C = 300K$.

Using Eq. (8.9) for a Carnot engine, we can write

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1} = 0.6$$

On rearranging terms and solving for T_1, we get

$$T_1 = 750 \text{ K}$$

Let the temperature of the source be raised to $(750 + T)$K for efficiency to become 70%. Thus, we can write

$$\eta = 1 - \frac{300}{(750 + T)} = 0.7$$

On solving this, we get, $T = 250$K

Hence, the temperature of the source should be raised by 250 K.

SAQ 3 – Efficiency of a Carnot engine

An engine has an efficiency of 40%. Its efficiency is to be raised to 45%. By how much must the temperature of the source be increased if heat is released to atmosphere at 27°C?

You will now agree that the beauty of Carnot cycle lies in the fact that all its stages are completely reversible. So if you invert the sequence of processes occuring in a heat engine, you will obtain a refrigerator. Do you know that an air conditioner is also a refrigerator designed to cool a room? (The first modern electrical air conditioner was designed by Willis Carrier in 1902 in Buffalo, New York.) Let us now understand the physics of this device.

8.3.2 Carnot Cycle as Refrigerator

Most of us now use refrigerators in our homes to keep various food items fresh so that they do not get stale and lose taste. This is done by keeping these cool. Have you ever thought: How is cooling achieved in a refrigerator? The most beautiful aspect of a Carnot engine is that we can run the whole system backward so that the sequence of events and their functions are reversed. Thus, Carnot cycle working in the reverse direction will act as an ideal *refrigerator*, in which heat is extracted from the reservoir at lower temperature and transferred to the reservoir at higher temperature. Therefore, in a sense, a refrigerator is also a heat engine.

Let us re-examine Fig. 8.3 again. If the directions of the arrows are reversed, the cycle $ABCDA$ becomes $ADCBA$. Since each process is reversible, the cycle is also reversible. Therefore, magnitudes of heat taken, heat rejected and the work done remain the same, except that their signs are reversed. It means that heat Q_2 is absorbed by the working substance from the lower temperature reservoir and heat Q_1 is rejected to the reservoir at higher temperature. And the work W represents the work done on the system (Fig. 8.4). In a domestic refrigerator, heat is pumped out of its interior, which is at a temperature lower than the surroundings and work is done by the motor driving the refrigerator. Thermodynamically, a refrigerator makes heat to flow from a lower temperature to a higher temperature, i.e., in a direction it does not spontaneously go. You can feel it by putting your hand near the coils, body, of the refrigerator. (You should not however touch the coils.)

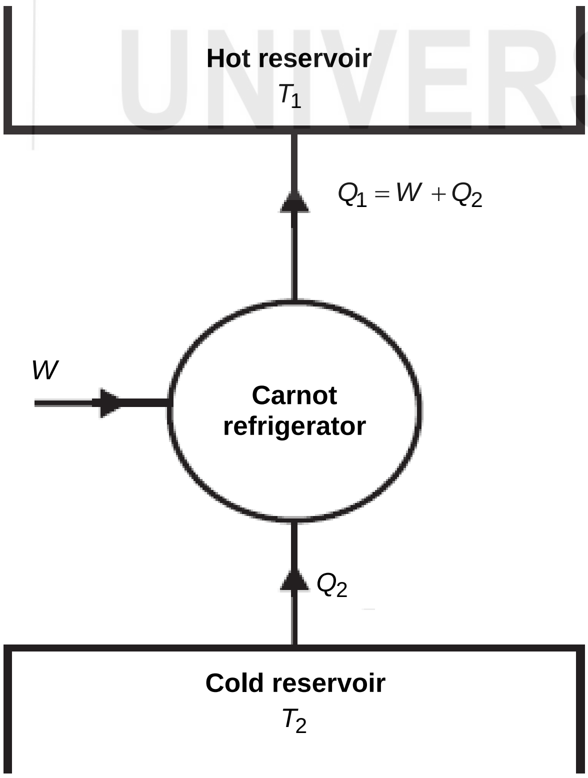

Fig. 8.4: **A Carnot refrigerator.**

The ability of a refrigerator is rated in terms of the *coefficient of performance* or *figure of merit*. We denote it by the symbol ω and define it as

$$\omega = \frac{\text{heat extracted at low temperature from the object to be refrigerated}}{\text{work input}}$$

Mathematically, we write

$$\omega = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \qquad (8.10)$$

where Q_2 is heat absorbed at the low temperature (cooler body) and Q_1 is heat rejected at higher temperature (kitchen environment).

In terms of temperatures of the eatables kept inside the refrigerator and the kitchen environment, we can express the coefficient of performance as

$$\omega = \frac{T_2}{T_1 - T_2} \qquad (8.11)$$

where T_1 and T_2, respectively denote the temperatures of the kitchen environment and the eatables kept inside the refrigerator.

On comparing the expressions for ω given by Eqs. (8.10) and (8.11), we can write

$$W = Q_2 / \omega = Q_2\left(\frac{T_1 - T_2}{T_2}\right) \qquad (8.12)$$

We can derive following conclusions from Eqs. (8.11) and (8.12):

i) ω is directly proportional to T_2 . That is, the coefficient of performance will be small when T_2 is low. In fact, ω approaches zero as $T_2 \to 0$. This means that more work will have to be done or more energy will be used up by the refrigerator for transferring the same amount of heat as T_2 decreases. If $T_2 = 0$, infinite amount of work will be required to produce cooling. This essentially implies that it is not possible to attain absolute zero mechanically.

ii) ω is inversely proportional to $T_1 - T_2$, i.e., lesser the difference between the hot and cold bodies, greater will be the coefficient of performance. As $(T_1 - T_2)$ approaches zero, ω approaches infinity. This means that a refrigerator will be most effective when eatables/chemicals placed inside it are close to the temperature of surroundings. So to conserve energy, it is advisable to put eatables in a kitchen refrigerator while they have cooled to room temperature. (You may have seen your maid/mother/sister allowing boiled milk to cool down to room tempearture before putting it in the refrigerator. If they are not doing so, advise them accordingly.)

iii) Unlike the efficiency of a heat engine, the coefficient of performance of a refrigerator can be *greater than unity*. That is, the amount of heat removed from the refrigerated space can be greater than the work input. (In fact, one of the reasons for expressing the efficiency of a refrigerator by another nomenclature – the coefficient of performance – is the intention to avoid confusion of having thermal efficiencies greater than unity.) To give you an idea about the figure, let us consider that freezer in a refrigerator or cold storage is maintained at -10°C and the room temperature is 30°C. You can readily convince yourself that the value of coefficient of performance in this case will be $\omega = \frac{263}{40} = 6.58$.

You should now go through the following examples to grasp the ideas discussed in this section.

EXAMPLE 8.3: COEFFICIENT OF PERFORMANCE

A typical home freezer operates between –18°C and 30°C. Calculate the maximum value of ω of this refrigerator. With this ω, how much electrical energy would be required to freeze 0.5 kg of water, initially at 0°C. Given latent heat of fusion = 334 kJ kg^{-1}.

SOLUTION ■ The coefficient of performance is given by Eq. (8.11) as

$$\omega = \frac{T_2}{T_1 - T_2} = \frac{255\text{K}}{303\text{K} - 255\text{K}} = \frac{255\text{K}}{48\text{K}} = 5.3$$

To produce 0.5 kg of ice, you have to extract heat from water. It is given by

$$Q = mL$$

where L is latent heat of fusion. Hence,

$$Q_2 = (0.5\text{ kg}) \times (334\text{ kJ kg}^{-1}) = 167\text{ kJ}$$

Using Eq. (8.10), you can write: $W = \frac{Q_2}{\omega} = \frac{167\text{kJ}}{5.3} = 31.5\text{kJ}$

In actual practice, ω would be lower and the corresponding work input would be higher because a real engine is not completely reversible.

EXAMPLE 8.4: COEFFICIENT OF PERFORMANCE

A domestic refrigerator is driven by a 1000 W electric motor, which operates at an efficiency of 60%. If the refrigerator can be treated as a reversible heat engine operating between –10°C and 20°C, calculate the time required by it to freeze 10 kg of water which is at 0°C. Neglect heat losses. Take latent heat of fusion of ice as 334 kJ kg^{-1}.

SOLUTION ■ We know that work done by a refrigerator is given by

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

Here $T_1 = 20\ ^{\circ}\text{C} = 293\text{ K}$, and $T_2 = -10^{\circ}\text{C} = 263\text{ K}$. Since the refrigerator is being driven by a motor of 1000 W with 60% efficiency, we get

$$W = 1000 \times \frac{60}{100} = 600\text{ W} = 600\text{Js}^{-1}$$

Hence, $Q_2 = W \times \dfrac{T_2}{T_1 - T_2} = 600 \times \dfrac{263K}{293K - 263K} = 5260\,\text{Js}^{-1}$.

But the heat required to freeze 10 kg of water $= mL = 10 \times 334$ KJ. Hence the time required to extract 334X10^4 J of heat

$$t = \frac{334 \times 10^4\,\text{J}}{5260\text{Js}^{-1}} = 635\text{s} = 10\,\text{min}\ 35\text{s}$$

In a household refrigerator, Freon-12 is used as the working substance. It has a boiling point of -29° C. (Freon-12 is a gas at room temperature.) Freon gas in a tube is made to expand suddenly from high pressure to low pressure. In this process it cools and a vapour-liquid mixture is obtained. This cold fluid, circulated through expansion coil around the region to be cooled, absorbs heat from the eatables kept inside the refrigerator and the entire liquid in the mixture changes into vapour. The vapour is compressed and work is done by the compressor on the vapour. The temperature as well as pressure of the vapour rise. The compressed vapour rejects heat to the surrounding medium such as the kitchen air and condenses through a set of tubes (called condenser and located at the back of the refrigerator).

It has been observed that CFCs adversely affect the life protecting layer of ozone in our atmosphere. So, there is now growing emphasis on phasing out CFCs. In India, non-CFC refrigerators are available.

An air-conditioner is also a refrigerator and the refrigerated space is a room rather than the food compartment. A window air-conditioning unit produces cooling by discharging heat of the air in the room outside. (When you travel by an aeroplane, sit in an air-conditioned room/office for long hours, it is advisable to drink water every half-hourly to avoid dehydration due to loss of body heat in the form of perspiration.) The same unit can also be used as heat pump by installing it backward. Now-a-days, systems fitted with controls so as to operate them as air-conditioners in summer and as heat pump in winter are available in the market.

We hope that now you appreciate the importance of Carnot's work on convertibility of heat into work. In fact, Carnot's genius lay in his imagination that a heat engine is the most efficient machine when it is operated in a reversible cycle. Historically, the work of Carnot led to the formulation of the second law of thermodynamics, which is a generalization of certain experiences and observations about the direction of transfer of thermal energy. This law has been stated in two different ways: (i) by Kelvin and Planck and (ii) by Clausius. We now discuss these in turn.

8.4 THE SECOND LAW OF THERMODYNAMICS

Kelvin and Planck confined themselves to the working of a heat engine and summarised the fact that it converts only a part of heat into work; the rest is rejected to a sink at a lower temperature. Let us now learn about it.

8.4.1 The Kelvin-Planck and Clausius Statements

Kelvin-Planck statement of the second law of thermodynamics is as follows:

> *No process is possible whose sole result is complete conversion of heat into work.*

This statement implies that one cannot devise a machine which just absorbs heat from a reservoir and produces 100% work. That is, we need two reservoirs for exchange of heat and running an engine in a cycle.

There are other processes in which energy is conserved but they do not occur. For example, it is a fact of experience that heat does not flow on its own from a body at a lower temperature to a body at higher temperature. That is, spontaneous heat flow is unidirectional and is a fact of experience. It is contained in the Clausius statement of the second law of thermodynamics, which is as follows:

> *No process is possible whose sole result is the transfer of heat from a body at a lower temperature to a body at a higher temperature.*

Note that Clausius statement is relevant for the working of a refrigerator. An important implication of this statement is that it is not possible to transfer heat from a cold body to a hot body without some change somewhere, including the working substance/surroundings of the system.

Note that the two statements of the second law apparently seem different or unconnected but they are equivalent. In fact, each statement implies the other. If one statement is untrue, will the other statement necessarily be untrue? Indeed, it is so and the truth of either form is both a necessary and sufficient condition for the truth of the other. We now discuss the equivalence of Kelvin-Planck and Clausius' statements.

8.4.2 Equivalence of Kelvin-Planck and Clausius Statements

The equivalence of these statements implies that if one statement is untrue, the other statement is necessarily untrue.

1. Let us suppose that the Clausius statement of the second law is violated by a hypothetical refrigerator *A*. Suppose that it transfers Q_2 units of heat in each cycle from a cold reservoir at temperature T_2 to a hot reservoir at temperature T_1 without expenditure of any work (Fig. 8.5a). Let us now assume that a heat engine working between the same heat reservoirs draws an amount of heat Q_1 from the hot reservoir and rejects heat Q_2 to the low temperature reservoir and performs work $W_{net} = Q_1 - Q_2$ in one cycle. Further, suppose that the heat engine operates at such a rate that it completes one cycle in the same period as does the refrigerator.

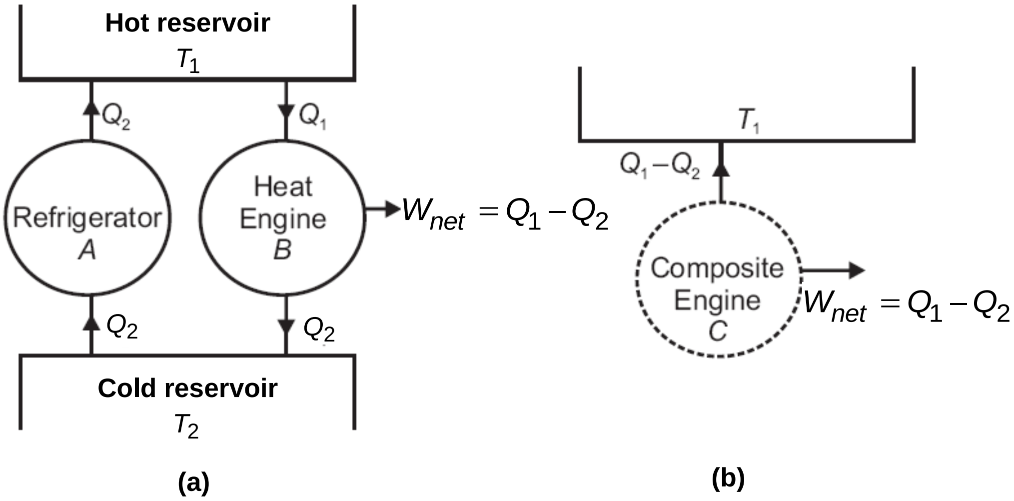

Fig. 8.5: **a) Refrigerator *A* supposedly violates the Clausius statement of the second law, whereas *B* does not violate the law; b) the composite engine violates the Kelvin-Planck statement.**

Now suppose that a composite engine is formed by considering the refrigerator and the heat engine to act together (Fig. 8.5b).

Since the heat drawn by the heat engine Q_2 is equal to the heat rejected by the refrigerator, the need for the hot reservoir will be eliminated completely, if heat Q_1 were fed to the heat engine by the hotter reservoir. That is, even though the composite engine exchanges heat with only one reservoir at a fixed temperature, there is net work output in each cycle.

Such a composite engine obviously violates Kelvin-Planck statement, which implies that no engine can run with just one reservoir.

2. To prove that if Kelvin-Planck statement is violated, the Clausius statement is also violated, let us consider a hypothetical heat engine which extracts heat Q_1 from the hot reservoir, converts it completely into work and rejects no heat to the low temperature reservoir (Fig. 8.6a).

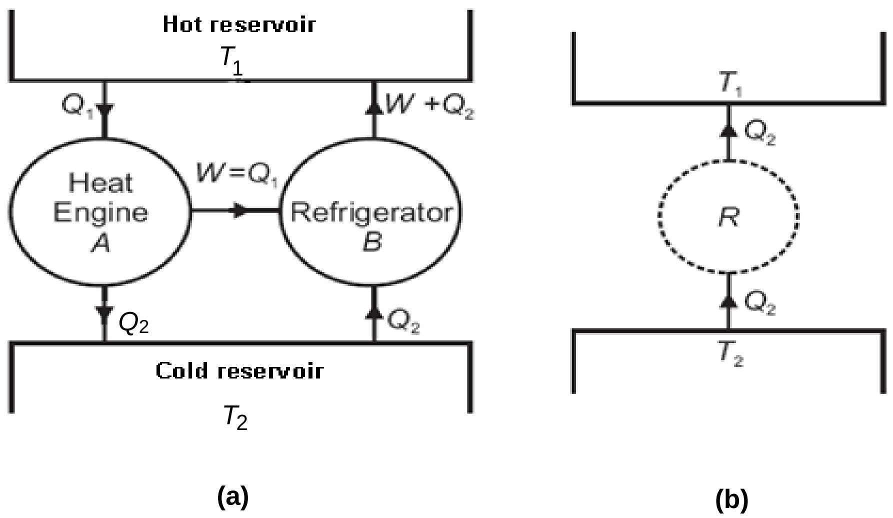

Fig. 8.6: **a) Heat engine *A* violates Kelvin-Planck statement of the second law, whereas refrigerator *B* does not violate the law; b) the composite engine violates Clausius' statement.**

Suppose that the work performed by the heat engine is used to drive a refrigerator operating between reservoirs at temperatures T_1 and T_2. Further, suppose that the refrigerator absorbs heat Q_2 from the low temperature reservoir and rejects heat $W + Q_2$ to the hot reservoir per cycle. As before, we also assume that the refrigerator completes one cycle in the same period as does the heat engine.

You will agree that the refrigerator by itself does not violate any law but when it is made to form a composite engine with a heat engine (Fig. 8.6b), the net result of operation of the composite system will be to transfer heat Q_2 from the low temperature reservoir to the higher temperature reservoir **without any external work**. This obviously violates Clausius statement of the second law. You can now conclude that both the statements of the second law are equivalent.

Having analysed the operation of Carnot cycle, we can do two things: a) show that no real engine can be more efficient than the Carnot engine, i.e., prove *Carnot theorem*, and b) introduce the concept of *thermodynamic temperature*.

Let us now discuss Carnot theorem.

8.5 CARNOT THEOREM

Carnot theorem states that *of all heat engines working between the same (constant) temperatures, the reversible Carnot engine has the maximum efficiency.* Let us consider an irreversible engine (*I*) and a reversible engine (*R*) operating between the same reservoirs which are at temperatures T_1 and T_2. Suppose that the irreversible engine is more efficient than a reversible engine. That is, we assume that

$$\eta_I > \eta_R$$

And Carnot theorem demands that this assumption is to be proved wrong. So, if the assumption is valid, then we must have

$$\frac{W}{Q_1} > \frac{W}{Q_1'} \qquad (8.13)$$

where Q_1 is heat absorbed by the irreversible engine and $W = Q_1 - Q_2 = Q_1' - Q_2'$. This implies that $Q_1' > Q_1$. That is, heat absorbed by the reversible engine is more than that absorbed by an irreversible engine.

We now couple these engines and regard the system of combined engines to be a single device. Now suppose that the work produced by the irreversible engine is used to drive the reversible engine backwards so that it acts as a (Carnot) refrigerator, as shown in (Fig. 8.7). Thus, the overall effect of the combined engine is to transfer a net amount of heat $(Q_1 - W) - (Q_1' - W) = Q_1 - Q_1'$ from the cold reservoir to the hot reservoir on its own. That is, the combined engine acts as a self-acting device, which requires no external input. But this is forbidden by the Clausius statement of the second law. Therefore, the assumption that $\eta_I > \eta_R$ is not valid, i.e., an irreversible engine cannot have efficiency greater than that of the reversible engine i.e., $\eta_I < \eta_R$. In fact, the efficiency of a Carnot engine is maximum.

We should expect it physically because friction and heat losses in an irreversible engine will make it less efficient.

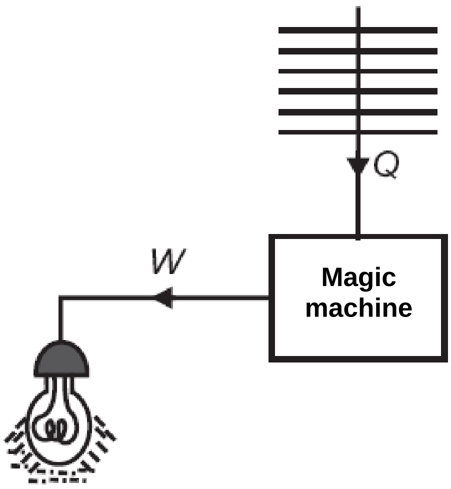

Fig. 8.7: **Illustrating the proof of Carnot's theorem.**

A corollary of Carnot theorem is: *All reversible engines operating between the same temperature limits have the same efficiency*.

It may be remarked here that Carnot engine is an ideal device because all losses due to conduction, radiation or friction are assumed to be absent. However, in real appliances, some useful energy is always dissipated. Yet, a study of this idealized engine helps us to understand the working of a real engine. You should now go through the following example.

EXAMPLE 8.5: Carnot theorem

For a reversible engine, show that $\sum \frac{Q}{T} = 0$.

SOLUTION ■ According to Carnot theorem: $\eta_I \le \eta_R$

where η_R denotes the efficiency of a Carnot engine and η_I is the efficiency of any other engine operating between the same temperature limits. In terms of heats exchanged, we can write

$$1 - \frac{Q_2'}{Q_1'} \le 1 - \frac{Q_2}{Q_1} \quad \text{or} \quad \frac{Q_2'}{Q_1'} \ge \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Hence, $$\frac{Q_1'}{T_1} - \frac{Q_2'}{T_2} \le 0$$

Therefore, for any cycle in which heat exchange takes place with two reservoirs only, the algebraic sum

$$\sum \frac{Q}{T} \le 0.$$

Note that the equality sign holds for a reversible cycle, whereas the inequality sign holds for an irreversible cycle.

We now sum up what you have learnt in this unit.

8.6 SUMMARY

Concept	Description
Efficiency of a Carnot engine	■ The efficiency of Carnot's engine is maximum. The efficiency of a heat engine is given by: $$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$
Second law of thermodynamics	■ Two equivalent statements of second law of thermodynamics are due to Kelvin-Planck, and Clausius.
	■ The Kelvin-Planck statement governs the working of a heat engine. It states that *it is impossible to construct an engine, no matter how ideal, which, working in a cycle, will transform the entire heat into*
	■ The Clausius statement of second law governs the working of a refrigerator. It states that *it is impossible to make a refrigerator operate in a cycle so that its sole effect is transfer of heat from a cooler body to a hotter body.*
Coefficient of performance	■ The coefficient of performance of a refrigerator is given by: $$\omega = \frac{Q_2}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

8.7 TERMINAL QUESTIONS

1. A reversible engine works between two temperatures and the difference of two temperatures is 110°C. If it absorbs 756 J of heat from the source and gives 536 J to the sink, calculate the temperature of source and sink.

2. A Carnot engine whose sink is at 300 K has an efficiency of 40 percent. (i) Determine the source temperature. (ii) Obtain the increment in the temperature of source to increase the efficiency by 25 percent of original efficiency?

3. A Carnot engine has efficiency 25%. It operates between reservoirs of constant temperature with temperature difference of 80 K. Calculate the temperature of the low-temperature reservoir in Celsius.

4. The efficiency of a Carnot's engine at particular source and sink temperatures is $\frac{1}{2}$. When the sink temperature is reduced by 100°C, the engine efficiency becomes $\frac{2}{3}$. Calculate the new source temperature.

5. An ideal Carnot engine, whose efficiency is 40%, receives heat at 500 K. If its efficiency is 50%, calculate the intake temperature for the same exhaust temperature.

8.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. The efficiency of a Carnot engine can be increased more effectively by increasing the temperature of source. Increasing the temperature of sink will have opposite effect.

2. a) $$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{278}{298} = \frac{20}{298} = 0.07 = 7\%$$

   This is a highly inefficient system and it is not advisable to tap this source of energy.

   b) $$\eta = 1 - \frac{77}{273} = \frac{196}{273} = 0.72 = 72\%$$

   It is too high to be attainable in practice.

3. From Eq. (8.9) we recall that

$$\eta = 1 - \frac{T_2}{T_1}.$$

Here $T_2 = 273 + 27 = 300$ K and $\eta = 0.40$.

Using this data, we can easily calculate the temperature of the source:

$$0.40 = 1 - \frac{300}{T_1}$$

so that $$T_1 = \frac{300}{0.60} = 500\,\text{K} \quad \text{(i)}$$

For the increased efficiency, we can write

$$T_1' = \frac{300}{0.55} = 545.5 \text{ K} \quad \text{(ii)}$$

Hence the temperature of the source should be raised by

$$\Delta T = T_1' - T_1 = (545.5 - 500)\,\text{K} = 45.5 \text{ K}$$

Terminal Questions

1. Let the temperature of the source and sink be T_1 and T_2, respectively. It is given that

$$T_1 - T_2 = 110°\text{C} = 110\text{K}$$

(As temperature differences in Celsius and Kelvin scales are the same.)

Using Eq. (8.9), the efficiency of the heat engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \left(\text{as } \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$$

or $$\eta = \frac{T_1 - T_2}{T_1}$$

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} = \frac{756}{536} \qquad \Rightarrow \qquad \frac{T_2}{T_1} = \frac{536}{756}$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{536}{756} = \frac{220}{756}$$

or $$\frac{110}{T_1} = \frac{220}{756} \qquad (\text{since } T_1 - T_2 = 110\,\text{K})$$

On solving the above equation, we get

$T_{1_{source}} = 378\,\text{K}$ and

$T_{2\,sink} = 268\,\text{K}$ $(\because T_1 - T_2 = 110\,\text{K})$

2. i) Using Eq. (8.9), we can write

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the temperature of the source and T_2 that of the sink. On inserting the values in the above expression, we get

$$\frac{40}{100} = 1 - \frac{300}{T_1}$$

Hence, $\frac{2}{5} = 1 - \frac{300}{T_1}$

or $2T_1 = 5T_1 - 1500$

$\therefore$ $T_1 = 500\,\text{K}$

Hence, the source temperature is 500 K.

ii) 25% of original efficiency = 10%. Therefore, we can write

$$\frac{40 + 10}{100} = 1 - \frac{300}{T_1 + x}$$

where x is the increment in temperature. On solving, we get

$$\frac{1}{2} = \frac{T_1 + x - 300}{T_1 + x}$$

or $T_1 + x = 2T_1 + 2x - 600$

or $500 + x = 1000 + 2x - 600$

or $500 + x = 400 + 2x$

$\therefore$ $x = 100$ K

Hence, the temperature of source should be raised by 100 K.

3. We use the expression of efficiency,

$$\eta = 1 - \frac{T_L}{T_H},$$

where T_L and T_H are the temperatures of reservoirs in Kelvin.

$$\therefore \quad \frac{1}{4} = 1 - \frac{T_L}{T_H} \qquad \text{(i)}$$

$$\text{or} \quad T_H = \frac{4}{3} T_L \qquad \text{(ii)}$$

Also, it is given that $T_H - T_L = 80$

On substituting the value of T_H from Eq. (i), we can write

$$\frac{4}{3} T_L - T_L = 80\,\text{K}$$

$$T_L\left(\frac{4}{3} - 1\right) = 80\,\text{K}$$

$$\text{or} \quad T_L\left(\frac{4-3}{3}\right) = 80\,\text{K}$$

$$\text{or} \quad T_L = 240\,\text{K}$$

In Celsius, $T_L = (240 - 273)°\text{C} = -33°\text{C}$

4. The expression for efficiency is given by Eq. (8.9) as

$$\eta = 1 - \frac{T_2}{T_1} \qquad \text{(i)}$$

Substituting the value of $\eta = \frac{1}{2}$ in Eq. (i), we can write

$$\eta = \frac{1}{2}$$

$$\therefore \quad 1 - \frac{(T_2 - 100)}{T_1} = \frac{2}{3} \qquad \text{(ii)}$$

On solving Eq. (i), we get

$$\frac{T_2}{T_1} = \frac{1}{2} \qquad \text{(iii)}$$

Similarly, solving Eq. (ii), we get

$$\frac{(T_2 - 100)}{T_1} = \frac{1}{3} \qquad \text{(iv)}$$

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{T_2}{(T_2 - 100)} = \frac{3}{2}$$

or $$2T_2 = 3T_2 - 300$$

or $$T_2 = 300 \text{ K}$$

$$T_1 = 600 \text{ K.}$$

Hence, the new source temperature will be 600 K.

5. We can write the expression of efficiency using Eq. (8.9) as

$$\eta = 1 - \frac{T_2}{T_1}$$

On substituting the values, we can write

$$0.4 = 1 - \frac{T_2}{500}$$

On solving, we get

$$T_2 = 300 \text{ K}$$

Now, $$0.5 = 1 - \frac{300}{T_1}$$

$\therefore$ $$T_1 = 600 \text{ K}$$