Differential form of Ampere's law

- Differential form of ampere's law is

$$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}}$$

where $\vec{\mathbf{J}}$ is the current density at a given point.

Magnetic vector potential

- In terms of magnetic vector potential $\vec{\mathbf{A}}$, the magnetic field is given as

$$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$$

because diverge of a curl is zero and divergence of $\vec{\mathbf{B}}$ is equal to zero.

- In terms of magnetic vector potential $\vec{\mathbf{A}}$, Ampere's law is written as

$$\nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

13.6 TERMINAL QUESTIONS

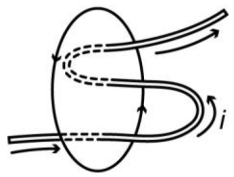

Fig. 13.11: **Diagram for TQ 2.**

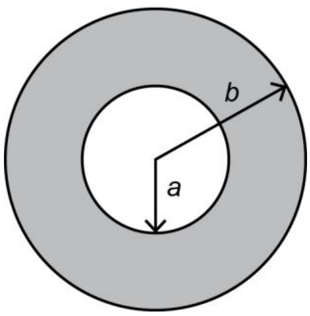

Fig. 13.12: **Diagram for TQ 4.**

1. Five very long, straight, insulated wires are closely bound together to form a small cable. Currents carried by the wires are $i_1 = 20$ A, $i_2 = -6$ A, $i_3 = 12$ A, $i_4 = -7$ A and $i_5 = 18$ A (negative currents are opposite in direction to the positive). Calculate the magnitude of $\vec{\mathbf{B}}$ at a distance of 10 cm from the cable.
2. Consider the surface bounded by the closed path shown in Fig. 13.11 with the value of *i* equal to 15 A. What is the net current passing through the surface? Calculate the value of the line integral of $\vec{\mathbf{B}}$ for this closed path.
3. A long, straight wire of diameter 4 mm carries a uniformly distributed 10 A current. At what distance from the axis of the wire the magnitude of $\vec{\mathbf{B}}$ will be maximum? Justify your answer.
4. A long, hollow conducting cylinder carries a current *i* which is uniformly distributed over the cross-section as shown in Fig. 13.12. Determine the value of magnetic field at at point a distance *r* from the axis of the cylinder for i) $r \le a$, ii) $a < r \le b$, and iii) $b \le r$.
5. A long solenoid with 900 turns per meter has a 2.6 A current. i) What is the magnitude of the magnetic field at the centre of the solenoid? ii) If the length of the solenoid is 300 mm, how many turns of wire are on the solenoid?
6. A toroid has 600 turns and a current of 200 mA is flowing in it. If the inner and outer diameters of the toroid is 80 mm and 95 mm respectively, calculate the maximum and minimum values of the magnetic field in the toroid.
7. A 15 cm long solenoid having diameter 1.5 cm carrier a current 1.5 A and the value of the magnetic field at its centre is 0.04T. If the wire used to wind the solenoid has diameter 0.6 mm, determine the number of layers in the winding and total length of the wire used.

13.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. The line integral $\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}}$ in the Ampere's law depends on the net current encircled by the Amperian loop. The Amperian loops 1 and 3 in Fig. 13.2

encircles current i. Thus, for these loops, $\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_0 i$. And for the Amperian loop 2 in Fig. 13.2, $\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = 0$ because net current is zero.

2. The variation of B with distance T from the axis of the wire is shown in Fig. 13.13.

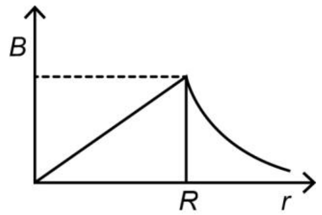

Fig. 13.13: **Diagram for answer to SAQ 2.**

3. The magnetic field inside a solenoid is given by

$$B = \mu_0 ni$$

We can answer all the questions on the basis of above relation:

i) The field will be doubled because we have made $n = 2n$.
ii) The field will be doubled because we have made $i = 2i$.
iii) The field will be halved because we have made $n = n/2$.
iv) The field remains unchanged because we have kept n unchanged.
v) The field remains unchanged as it is independent of the diameter of the solenoid.

4. The magnetic field within the toroid is given as $B = \dfrac{\mu_0 ni}{2\pi r}$

We have, $N = 6000, i = 10\text{ A}, r = 20\text{ cm} = 0.2\text{ m}, \mu_0 = 4\pi \times 10^{-7}\text{ T mA}^{-1}$

So, $B = \dfrac{(2 \times 10^{-7}\text{ T mA}^{-1}) \times (6000) \times (10\text{ A})}{(0.2\text{ m})} = 0.06\text{ T}$

Terminal Questions

1. Let us consider an Amperian loop of radius 10 cm having centre at the axis of the cable comprising five current carrying wires. Then, the magnitude of magnetic field at a distance of 10 cm from the cable is given by $B = \dfrac{\mu_0 i_{encircled}}{2\pi r}$

$i_{encircled} = i_1 + i_2 + i_3 + i_4 + i_5 = 20\text{ A} - 6\text{ A} + 12\text{ A} - 7\text{ A} + 18\text{ A} = 37\text{ A}$

So, $B = [(2 \times 10^{-7}\text{ T mA}^{-1}) \times (37\text{ A})]/(0.1\text{m}) = 7.4 \times 10^{-5}\text{ T}$

2. Note from Fig. 13.11 that the current crosses the surface thrice; thus, the net current passing through the surface is $i = 15$ A. The line integral of $\vec{\mathbf{B}}$ for this closed path is given by Ampere's law as

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{l}} = \mu_0 i_{encircled} = \mu_0(15\text{ A}) = (4\pi \times 10^{-7}\text{ T mA}^{-1}) \times (15\text{ A})$$

$$= 1.88 \times 10^{-5}\text{ Tm}$$

3. The magnitude of the magnetic field due to current carrying wire is given by Eq. (13.4): $B = (\mu_0 i)/(2\pi r)$.

Thus, as the distance r from the axis increases, B decreases. However, from Sec. 13.3 (Example 13.1) you know that inside the wire, where distance $r < R$, the radius of the wire, B is given as

$$B = (\mu_0 ir)/(2\pi R^2)$$

This expression shows that inside the wire, B increases as r increases. And, the value of B reaches its maximum value when $r = R$. Thus, B is maximum at (4 mm/2) = 2mm.

4. i) If we take an Amperian loop of radius a around the axis of the hollow conducting cylinder, the current encircled by the loop is zero. So, $B = 0$.

   ii) For an Amperian loop having radius $r > a$ and $\leq b$, the situation is similar to the one discussed in Example 13.2. So, you can show that the magnitude of B will be

   $$B = \left(\mu_0 i(r^2 - a^2)\right)/\left(2\pi r(b^2 - a^2)\right)$$

   iii) In this case, any Amperian loop having radius $r \geq b$ will encircled the total current i. So, $B = (\mu_0 i)/(2\pi r)$

5. i) The magnitude of $\vec{\mathbf{B}}$ at the centre of a solenoid is given by Eq. (13.8): $B = \mu_0 ni$. We have, i = 2.6 A and n = 900. So,

   $$B = \mu_0 ni = (4\pi \times 10^{-7}\,\text{T m A}^{-1}) \times (900) \times (2.6\text{ A}) = 2.9 \times 10^{-3}\,\text{T}$$

   ii) Since 1 m length has 900 turns, 300 mm will contain 270 turns.

6. From Eq. (13.9), we know that the magnetic field due to a toroid is given as $B = (\mu_0 Ni)/(2\pi r)$ where r is the radial distance from the axis of the toroid. We also know that the value of B is zero inside the inner edge as well as outside the outer edge of the toroid. Further, from the above (1/r) dependence of B, we note that the value of B will be maximum just inside the toroid for which r = 80 mm. And, B will be minimum just inside the outer edge for which r = 95 mm. So,

   $$(B)_{max} = [(2 \times 10^{-7}\,\text{T m A}^{-1}) \times (600) \times (0.2\text{ A})]/[0.08\text{ m}] = 0.3\text{ mT}$$

   $$(B)_{min} = [(2 \times 10^{-7}\,\text{T m A}^{-1}) \times (600) \times (0.2\text{ A})]/[0.095\text{ m}] = 0.25\text{ mT}$$

7. The magnetic field due to a solenoid is given by Eq. (13.8): $B = \mu_0 ni$

   We have, $B = 0.04\text{ T}, i = 1.5\text{ A}$. So,

   $$n = B/(\mu_0 i) = (0.04\text{ T})/[(4\pi \times 10^{-7}\,\text{T m A}^{-1}) \times (1.5\text{ A})] = 2.1 \times 10^4$$

   So, the number of turns per meter is 2.1×10^4.

   The length of the solenoid is 15 cm. So, it will contain 3.15×10^3 turns. Since the length of the solenoid is 15 cm and diameter of the wire is 0.6 mm, in one layer, there would be (0.15 m)/ $(0.6 \times 10^{-3}\text{m}) = 2.5 \times 10^2$ turns. So, number of layers is equal to $(3.15 \times 10^3)/(2.5 \times 10^2) = 12.6$ layers.

   Further, the circumference of the solenoid is $2\pi r = 2 \times 3.14 \times 0.0075\text{ m}$ $= 0.047\text{ m}$. So, in one turn, 0.047 m length of wire is used. So, total length of the wire used is approximately $(0.047\text{ m}) \times (3.15 \times 10^3) = 148\text{ m}$. (Note that we have neglected the gradual increase in the circumference layer after layer).