Particle accelerators utilise very high potential differences to produce high energy charged particles used in *atom smashing* experiments for studying nuclear structure. This is a picture of the Large Hadron Collider located at CERN, near Geneva.

(Picture source: Wikimedia Commons)

UNIT 9

ELECTRIC POTENTIAL OF CONTINUOUS CHARGE DISTRIBUTIONS

Structure

STUDY GUIDE

In this unit, we will continue our discussion on electric potential begun in the previous unit. You will learn how to determine the electric potential of continuous charge distributions such as charged wire, spherical shell and non-conducting solid sphere. While studying this unit, you should focus on how to calculate the total charge for a given continuous charge distribution. The mathematical tools used for these calculations are similar to those you have learnt in Block 1 of this course. However, you will do better if you revise Units 3 and 4 of Block 1 on vector integral calculus and school integral calculus. Further, you should also focus on how the value of electric field can be calculated at a point using the expression for potential at that point due to a given continuous charge distribution. To help you understand and practice the method of determining electric potential better, we have given several examples, SAQs and TQs. Try to solve them yourself to check your understanding of the concepts and methods discussed in the unit.

"I have not failed. I've just found 10,000 ways that won't work."

Thomas A. Edison

9.1 INTRODUCTION

In the previous units of this block, you have learnt how to determine the electric field $\vec{E}$ and electric potential V due to a point charge and a system of discrete charges. You have learnt how to calculate potential by evaluating the line integral of $\vec{E}$. You have also learnt how to calculate $\vec{E}$ from potential V by taking its gradient. In this unit, we shall extend these ideas to determine electric potential of continuous charge distributions.

You know that the electrical appliances we use in our homes work on a potential difference of 220 V. Apart from these appliances, the concept of potential difference plays an important role in the design and manufacturing of high voltage sources used by physicists to do interesting experiments. For example, if a charged particle is allowed to fall through a potential difference, it accelerates and its kinetic energy increases. The machines called *particle accelerators* have been designed on this basic principle to produce high energy charged particles used in *atom smashing* experiments for studying nuclear structure. In electrical appliances and machines, the desired potential difference is created by charging objects of appropriate geometry. Therefore, it is important to study electric potential of continuous charge distributions.

We begin the discussion by determining the electric potential of three types of continuous charge distributions, namely, line charge, spherical shell and non-conducting solid sphere (Sec. 9.2). In Sec. 9.3, you will learn about equipotential surface which is a useful concept because it is characterised by the fact that no net work is done in moving a charge from one point to other on this surface. In Sec. 9.4, you will learn how to calculate electrostatic potential energy of a system of discrete charges as well as continuous charge distributions if electric potential is known.

In the next unit (Block 3), you will study the macroscopic properties of the dielectrics kept in an electric field. The understanding of the concepts of electric field and potential studied in this block will help you appreciate the properties of dielectrics better.

Expected Learning Outcomes

After studying this unit, you should be able to:

- obtain the expression for electric potential of a line charge;
- determine the electric potential of a uniformly charged spherical shell;
- derive the expression of electric potential of uniformly charged non-conducting sphere;
- explain the concept of equipotential surface; and
- calculate the electrostatic potential energy for a given charge distribution.

9.2 ELECTRIC POTENTIAL OF CONTINUOUS CHARGE DISTRIBUTIONS

In the previous unit, you have learnt how to determine the electric potential of a point charge at a given point. You have also learnt how to use the superposition principle to obtain the expression for electric potential of multiple discrete charges.

Now, suppose that we need to determine the electric potential of a charged object such as a metal rod or a solid sphere. **In general, for determining the electric potential of a continuous charge distribution, we first calculate the potential due to a small element of the charge distribution and then integrate this expression over appropriate limits to include the effect of total charge in it.** We now determine the electric potential of three types of continuous charge distributions: line charge, uniformly charged spherical shell and uniformly charged non-conducting sphere.

NOTE

Can we consider a charged object as a collection or distribution of discrete charges and use the method described in the previous unit for determining its electric potential? No, we cannot. This is so because for such uniformly charged objects, we can only know the total charge on them. There is no way to ascertain the position of individual charges because the charge is uniformly distributed all over the object. A uniformly charged object is called continuous charge distribution because the separation between individual charges on such objects is very, very small.

9.2.1 Line Charge

In Unit 7 of this block, you have learnt how to determine the electric field at a point near an **infinitely long charged wire** (or **a line charge**). It is given by:

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}} \tag{9.1}$$

where λ is the charge per unit length on the wire or the **linear charge density**, r is the perpendicular distance of the point from the wire, ε_0 is the permittivity of free space, and $\hat{\mathbf{r}}$ is the unit vector along the direction of increasing r from the line charge (Fig. 9.1).

The question now is: What is the potential of this wire at a point a situated at a perpendicular distance of r_a from the wire? From Eq. (8.15) of Unit 8, you can write

$$V = -\int_{\infty}^{r_a} \vec{\mathbf{E}}.d\vec{\mathbf{l}} \tag{9.2}$$

Let us evaluate the line integral in Eq. (9.2) by first moving a unit positive charge from a finite distance r_b instead of infinity, to point a at distance r_a and then let r_b go to infinity. Here r_b is the distance of point b from the wire (see Fig. 9.1). This integral then gives us the difference in potentials between points a and b, i.e.

$$V_a - V_b = -\int_{r_b}^{r_a} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{r_a}^{r_b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{r_a}^{r_b} \vec{\mathbf{E}}.d\vec{\mathbf{r}} \tag{9.3}$$

because for the path a to b, $d\vec{\mathbf{l}}$ is parallel to $d\vec{\mathbf{r}}$. Inserting the expression for $\vec{\mathbf{E}}$ from Eq. (9.1) we get

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{\hat{\mathbf{r}}.d\vec{\mathbf{r}}}{r}$$

Since $\hat{\mathbf{r}}$ and $d\vec{\mathbf{r}}$ are in the same direction, we have

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{dr}{r}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln r\right]_{r_a}^{r_b} = -\frac{\lambda \ln r_a}{2\pi\varepsilon_0} + \frac{\lambda \ln r_b}{2\pi\varepsilon_0} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_a}{r_b}\right) \tag{9.4}$$

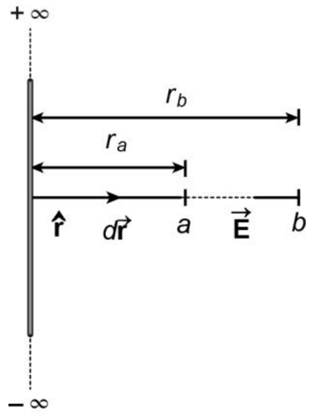

Fig. 9.1: **Electric potential at a point *a* due to an infinitely long charged wire.**

As you have learnt in the previous unit, we take a point at infinity with zero potential as our reference point and calculate the potential at a given point with respect to infinity. In the instant case, if we consider point *b* to be located at infinity, i.e. $r_b = \infty$ and take the potential V_b equal to zero, then the RHS of Eq. (9.4) tells us that the potential V_a at point *a* will be infinite. This is expected also because the infinitely long line charge having uniform charge distribution means an infinite amount of charge. Therefore, the sum of finite contributions from each part or element of an infinite line charge leads to an infinite potential.

Thus, to have a physically meaningful expression for potential at a point finite distance away from the line charge, we cannot take infinity with zero potential as our reference point. However, the inability to have a reference point with zero potential does not cause any problem because in practical situations, we are interested in difference in potential between two points rather than its absolute value at a given point. Thus, Eq. (9.4) which gives the potential difference between points *a* and *b* (Fig. 9.1) with both r_a and r_b having finite values meets our requirement.

Note that an infinite line charge contain infinite amount of charge. Thus, we cannot calculate *V* at a point for such a continuous charge distribution by taking total charge into consideration. That method will not work as it will give infinite potential everywhere. That is why we have used the relation between *V* and $\vec{\mathbf{E}}$ and the expression for $\vec{\mathbf{E}}$ for an infinite line charge to obtain a physically meaningful expression for *V*.

Further, to check whether we can obtain the value of electric field at a point, say *a*, using Eq. (9.4), let us assume that point *b* located at a finite distance r_b is the reference point with zero potential. This implies that r_b is fixed and $V_b = 0.$ Hence, the second term on the RHS of Eq. (9.4) is constant. Thus, we can write Eq. (9.4) as

$$V_a = -\frac{\lambda \ln r_a}{2\pi\varepsilon_0} + \text{const} \qquad (9.5)$$

You may recall that electric field $\vec{\mathbf{E}}$ and electric potential *V* are related by Eq. (8.20):

$$\vec{\mathbf{E}} = -\vec{\nabla} V = -\hat{\mathbf{r}}\frac{dV_a}{dr} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}} \qquad (9.6)$$

Note that Eq. (9.6) is the same as Eq. (9.1).

To concretise the ideas discussed above, go through the following example.

EXAMPLE 9.1 : ELECTRIC POTENTIAL OF A LINE CHARGE

An infinite line charge has linear charge density $\lambda = 2.0\,\mu\text{C}\,\text{m}^{-1}$. Calculate the electric potential at a point on a line perpendicular to the line charge, at a distance of 3.0 m from the line charge. Assume that the electric potential of the line charge is zero at the perpendicular distance of 4.0 m.

SOLUTION ■ From Eq. (9.4), note that the potential difference between two points *a* and *b* due to an infinite line charge is given as

$$V_a - V_b = -\frac{\lambda}{2\pi\varepsilon_0}\ln(r_a / r_b) \qquad \text{(i)}$$

From the problem, we have $r_a = 3.0\,\text{m}$ and $r_b = 4.0\,\text{m}$. It is also given that $V_b = 0$ at $r_b = 4.0\,\text{m}$. Substituting these values and $\lambda = 2.0\times10^{-6}\,\text{C m}^{-1}$ in Eq. (i), we get

$$V_a - 0 = -\frac{2.0\times10^{-6}\,\text{C m}^{-1}}{2\times3.14\times(8.85\times10^{-12}\,\text{F m}^{-1})}\ln\left(\frac{3.0\,\text{m}}{4.0\,\text{m}}\right)$$

or $$V_a = 10.93\times10^{3}\,\text{V}$$

Before proceeding further, solve an SAQ.

SAQ 1 - Electric potential of a line charge

The linear charge density of an infinite line charge is $3.0\times10^{-6}\,\text{C m}^{-1}$. Assuming that the electric potential at a perpendicular distance of 5.0 m from the wire is zero, calculate the potential at the perpendicular distance of 6.0 m.

Now, let us discuss how to determine the electric potential of a uniformly charged spherical shell at a given point.

9.2.2 Uniformly Charged Spherical Shell

You know that a spherical shell is a hollow sphere. For determining the electric potential of a uniformly charged spherical shell, there are two regions of interest: one at a point inside the spherical shell and the other at a point outside.

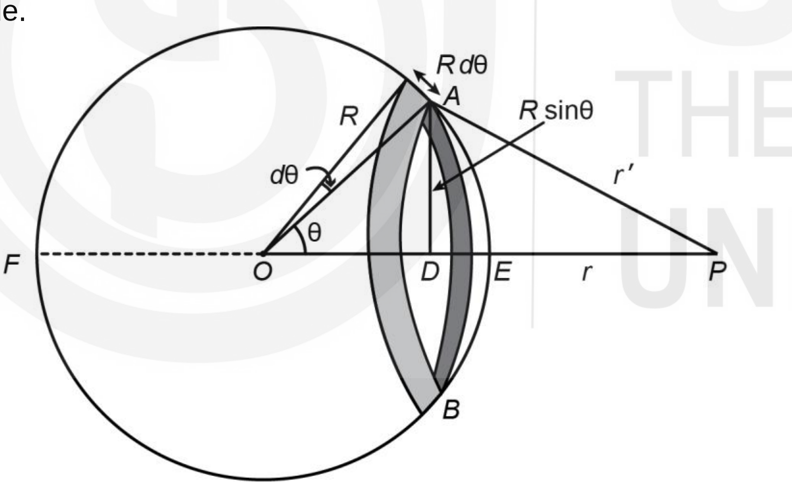

Fig. 9.2: **A uniformly charged spherical shell of radius *R* and point *P* is an external point.**

Study Fig. 9.2, which shows a uniformly charged spherical shell of radius *R*. To obtain an expression for the potential at an external point *P*, we first identify a suitable element of charged shell. The charged surface of the shell can be considered as a collection of a large number of thin rings such as the ring *AB*. The orientation of these rings are so selected that the axis of the rings is along *OP*, the line joining the centre *O* of the shell with the point *P*.

Now, let the ring *AB* be contained between the directions θ and $\theta + d\theta$ with respect to the axis *OP*. Let it be of infinitesimal width so that every point on it is at the same distance, say r', from *P*. The angular width of the ring is $d\theta$, its width is $Rd\theta$ and its radius is $R\sin\theta$. The circumference of the ring is $2\pi R\sin\theta$ and hence, its area is given by

$$dA = (2\pi R \sin\theta) R\, d\theta = 2\pi R^2 \sin\theta d\theta \qquad (9.7)$$

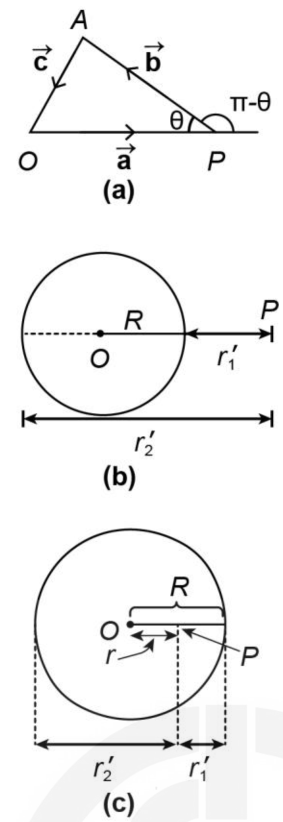

Fig. 9.3: **Diagrams for calculating electric potential of charged spherical shell.**

What is the charge on the ring? If the total charge on the shell is Q, then charge per unit area, $\sigma = Q/(4\pi R^2)$. Thus, using Eq. (9.7) we can write the charge on the ring

$$Q_{ring} = \frac{Q}{4\pi R^2} \times (2\pi R^2 \sin\theta)\, d\theta = \frac{Q}{2} \sin\theta d\theta \qquad (9.8)$$

We shall now determine the electric potential at point P due to the ring AB. The ring is made up of a large number of point charges each having charge equal to, say δQ. So, the electric potential for one such point charge is $\frac{1}{4\pi\varepsilon_0}\frac{\delta Q}{r'}$. So, the electric potential due to the ring will be

$$dV_{ring} = \sum\left(\frac{1}{4\pi\varepsilon_0}\frac{\delta Q}{r'}\right) = \frac{1}{4\pi\varepsilon_0 r'}\sum \delta Q = \frac{1}{4\pi\varepsilon_0}\frac{Q_{ring}}{r'}$$

So, on using Eq. (9.8), we get

$$dV_{ring} = \left(\frac{1}{4\pi\varepsilon_0 r'}\right)\left(\frac{Q}{2}\sin\theta d\theta\right) \qquad (9.9)$$

As we mentioned above, the shell can be imagined to be made of rings like AB having a common axis OP. Since electric potential is a scalar quantity, we shall integrate Eq. (9.9) to get the electric potential V of the shell.

Note that on the RHS of Eq. (9.9), we have two variables θ and r'. It will be convenient if we can express it in terms of a single variable. For this, we shall consider the relation between r', r and R. To do so, refer to Fig. 9.3a. From triangle OAP, we have (see Margin Remark):

$$r'^2 = r^2 + R^2 - 2rR\cos\theta$$

For triangle OAP (Fig. 9.3a):

$$\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$$

$$\Rightarrow \vec{\mathbf{a}} + \vec{\mathbf{b}} = -\vec{\mathbf{c}}$$

$$\Rightarrow (\vec{\mathbf{a}} + \vec{\mathbf{b}}).(\vec{\mathbf{a}} + \vec{\mathbf{b}}) = c^2$$

$$\Rightarrow a^2 + b^2 + 2\vec{\mathbf{a}}.\vec{\mathbf{b}} = c^2$$

or

$$a^2 + b^2 + 2ab\cos(\pi - \theta) = c^2$$

or

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

On differentiating with respect to θ, we get

$$2r'\frac{dr'}{d\theta} = 2rR\sin\theta$$

or

$$\frac{dr'}{rR} = \frac{\sin\theta d\theta}{r'} \qquad (9.10)$$

Substituting Eq. (9.10) in Eq. (9.9), we get

$$dV_{ring} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2}\right)\left(\frac{dr'}{rR}\right) \qquad (9.11)$$

To obtain the electric potential due to the entire shell, we need to integrate Eq. (9.11) over appropriate limits of integration to include the contribution of every ring of the shell:

$$V = \int dV_{ring} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2rR}\int_{r_1'}^{r_2'} dr' \qquad (9.12)$$

where r_1' and r_2' are, respectively, the minimum and maximum values of r'. To write the values of r_1' and r_2' in terms of r and R, we consider the two cases – point P outside the shell and point P inside the shell – separately :

a) *Point* P *outside the shell*

In this case, as shown in Fig. 9.3b, the values of r_1' and r_2' are

$$r_1' = r - R \qquad \text{and} \qquad r_2' = r + R$$

So, Eq. (9.12) becomes

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2rR}\int_{(r-R)}^{(r+R)} dr' = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2rR}\right)[2R] = \frac{Q}{4\pi\varepsilon_0 r} \qquad (9.13)$$

Eq. (9.13) gives the electric potential due to a uniformly charged spherical shell at a point outside the shell. Note that Eq. (9.13) is same as Eq. (8.14) which is for the electric potential of a point charge at a point at distance r.

> **Thus, we may conclude that, for an external point, the uniformly charged spherical shell behaves as a point charge located at the centre of the shell.**

b) *Point* P *inside the shell*

Refer to Fig. 9.3c which depicts the point P inside the shell. From the figure, you may note that for $r < R$

$$r_1' = R - r \qquad \text{and} \qquad r_2' = R + r$$

Substituting these values of r_1' and r_2' as limits of integration in Eq. (9.12), we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2rR}\int_{(R-r)}^{(R+r)} dr' = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2rR}\right)[2r] = \frac{Q}{4\pi\varepsilon_0 R} \qquad (9.14)$$

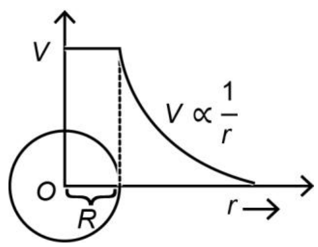

Fig. 9.4: **The variation of electric potential due to a spherical shell shell with distance from its centre.**

From Eq. (9.14), which gives electric potential at an internal point P, we note that the electric potential is independent of r, the distance of point P from the centre O of the shell. This means that the electric potential at every point inside the shell is same and its value is equal to its value at the surface. If we plot the variation of potential for a spherical shell with distance from its centre, we obtain a curve as shown in Fig. 9.4.

On the basis of Eq. (9.14) and Fig. 9.4, can you guess what will the value of electric field inside the uniformly charged spherical shell be? Note from Fig. 9.4 that the electric potential is constant everywhere inside the shell. So, if we move a test charge from one point to another inside the shell, no work is to be done because both the points are at the same potential. This is possible only if the value of the electric field inside the shell is zero. Thus, we conclude that:

Don't forget

> **The value of electric field inside a uniformly charged spherical shell is zero.**

Now, before proceeding further, solve an SAQ.

SAQ 2 **– Electric potential of a uniformly charged spherical shell**

The radius and surface charge density of a uniformly charged spherical shell are 20 cm and $3.0\,\mu\text{C m}^{-2}$, respectively. Calculate the electric potential at a distance (a) 40 cm and (b) 15 cm from the centre of the shell.

9.2.3 Uniformly Charged Non-conducting Sphere

Let ρ be the volume charge density (charge per unit volume) of a uniformly charged non-conducting sphere. Let the radius of the sphere be R (see Fig. 9.5). As in the case of spherical shell, here also we have two regions of interest for determining electric potential: one at a point outside the sphere and the other at a point inside it.

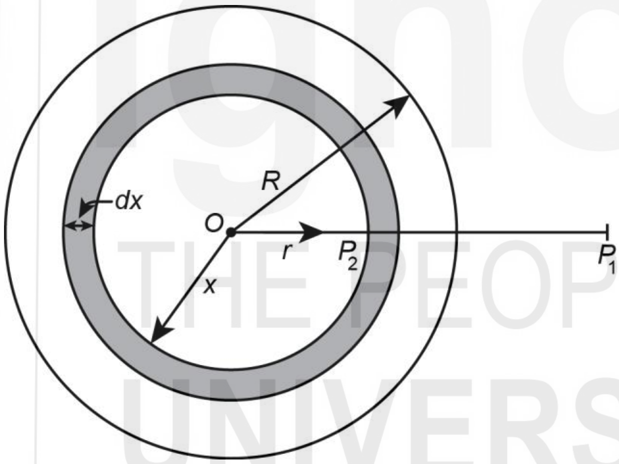

Fig. 9.5: **A uniformly charged non-conducting sphere of radius R with point P_1 outside the sphere and point P_2 inside the sphere.**

a) Electric potential at a point outside the sphere

For points outside the non-conducting sphere, such as P_1, located at distance r from the centre O of the sphere, the whole charge spread throughout the volume of the sphere behaves like a point charge located at its centre O. This fact can easily be deduced on the basis of the derivation of the potential at an external point due to a spherical shell discussed in the previous section. We can divide the non-conducting sphere into a large number of thin concentric shells as shown in Fig. 9.5. For each of these shells, the charge can be regarded as concentrated at the centre O for points outside the shell. Thus, for a point outside the sphere, such as P_1 in Fig. 9.5, the whole charge of the sphere can be regarded as a point charge located at its centre O. Hence, for points outside the sphere, the expression for electric potential due to a non-conducting charged sphere will be the same as for a uniformly charged spherical shell [(Eq. (9.13)]:

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{9.15}$$

where $Q\left(= (4\pi/3)R^3\rho\right)$ is the total charge on the sphere and r is the distance of external point P_1 from the centre O of the sphere. **You must, however, note that the expression for the total charge, Q is different for a uniformly charged non-conducting solid sphere from that for the uniformly charged spherical shell** (it is spread in a volume, whereas for a shell, it is spread on its surface)**.**

b) Electric potential at a point inside the sphere

Let point P_2 be an internal point at a distance r from the centre O such that $r < R$ (see Fig. 9.5). If we divide the sphere into a large number of thin concentric shells with centre O, then for shells with radii $\leq r$, point P_2 is outside and for shells which have radii between r and R, point P_2 is inside. For shells with radii less than or equal to r, potential V_1 at P_2 can be written as if point P_2 is an external point and hence it is given by Eq. (9.15):

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r} = \frac{4\pi}{3} \times \frac{r^3\rho}{4\pi\varepsilon_0 r} = \frac{\rho r^2}{3\varepsilon_0} \tag{9.16}$$

To evaluate the contribution to electric potential by the shells for which P_2 is inside the sphere, let us consider a shell of radius x and thickness dx as shown in Fig. 9.5. For this shell, the total charge Q_2 is equal to volume times charge density, i.e. $Q_2 = 4\pi x^2 dx\rho$. This charge contributes a constant electric potential dV_2 at any internal point and is given by (see Eq. (9.14)):

$$dV_2 = \frac{4\pi x^2\, dx\, \rho}{4\pi\varepsilon_0\, x} = \frac{\rho x\, dx}{\varepsilon_0} \tag{9.17}$$

For adding the contributions from all such shells for which P_2 is an internal point, we integrate Eq. (9.17) for x varying from r to R. This gives the electric potential V_2 at P_2 due to shells for which point P_2 is internal as

$$V_2 = \int_r^R dV_2 = \frac{\rho}{\varepsilon_0}\int_r^R x\, dx = \frac{\rho}{\varepsilon_0}\left(\frac{R^2 - r^2}{2}\right) \tag{9.18}$$

Thus, adding Eqs. (9.16) and (9.18), we can write the electric potential V of the non-conducting sphere at an internal point P_2 as:

$$V = V_1 + V_2 = \frac{\rho}{3\varepsilon_0} r^2 + \frac{\rho}{\varepsilon_0}\left(\frac{R^2 - r^2}{2}\right)$$

$$= \frac{\rho}{3\varepsilon_0}\left(\frac{3R^2 - r^2}{2}\right) = \frac{4\pi R^3\rho}{3 \times 4\pi\varepsilon_0}\left(\frac{3R^2 - r^2}{2R^3}\right) = \frac{Q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3} \tag{9.19}$$

where $Q[= (4/3)\pi R^3\rho]$ is the total charge on the uniformly charged non-conducting sphere.

To fix these ideas, you may like to go through the following example, which is for conducting sphere.

EXAMPLE 9.2 : ELECTRIC POTENTIAL DUE TO CHARGED CONDUCTING SPHERE

Two charged spherical conductors of radius $r_1 = 8.0\,\text{cm}$ and $r_2 = 2.0\,\text{cm}$ are separated by a distance much larger than 10 cm. These spheres are connected by a conducting wire and a total of 60 nC charge is placed on one of the spheres. (a) Calculate the charge on each sphere. b) Calculate the electric potential of each sphere at a point on their surfaces.

SOLUTION ■ Since the charged conducting sphere is connected through a conducting wire to the uncharged sphere, the 60 nC charge will redistribute between the two sphere in such a manner so that both sphere have same electric potential. Let the final charge be q_1 (on the larger sphere) and q_2 on the smaller sphere.

(a) From the conservation of charge, we have

$$q_1 + q_2 = 60\,\text{nC} \qquad \text{(i)}$$

Further, since the electric potential of both spheres are equal, we can write,

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \Rightarrow q_2 = \frac{r_2}{r_1}q_1 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we can write

$$q_1 + \frac{r_2}{r_1}q_1 = 60\,\text{nC}$$

$$\Rightarrow \quad q_1 = \left(\frac{r_1}{r_1 + r_2}\right) \times 60\,\text{nC} = \frac{80\,\text{cm}}{10\,\text{cm}} \times 60\,\text{nC} = 48\,\text{nC}$$

So, $\quad q_2 = (60\,\text{nC} - 48\,\text{nC}) = 12\,\text{nC}$

Note that the spheres in Example 9.2 are conducting spheres.

b) Using the values of q_1 and q_2, we can write the potential V_1 and V_2 of the two spheres at a point on their surfaces as

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{48\,\text{nC}}{(8.0\times10^{-2}\,\text{m})} = \frac{(9\times10^{9}\,\text{Nm}^2\text{C}^{-2})\times(48\times10^{-9}\,\text{C})}{(8.0\times10^{-2}\,\text{m})} = 5.4\,\text{kV}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{12\,\text{nC}}{(2.0\times10^{-2}\,\text{m})} = \frac{(9\times10^{9}\,\text{Nm}^2\text{C}^{-2})\times(12\times10^{-9}\,\text{C})}{(2.0\times10^{-2}\,\text{m})} = 5.4\,\text{kV}$$

Now, you may like to solve an SAQ.

SAQ 3 - Electric potential of a charged conducting sphere

An isolated solid sphere of aluminium having radius 7.0 cm is at a potential of 500 V. Calculate the number of electrons which have been removed from the sphere to raise it to this potential.

Let us now learn about the concept of equipotential surface.

9.3 EQUIPOTENTIAL SURFACES

To understand the concept of equipotential surface, recall that electric potential of a point charge Q, at a point at distance r is given as:

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)$$

From the above equation, we note that electric potential depends only on *r.* Now, you know that the locus of points having the same value of r is the surface of a sphere of radius r with the point charge as its centre. For a different value of *r,* we get a different surface of the sphere (see Fig. 9.6a). On any such surface, the value of electric potential will be the same everywhere because r is same for all points of this surface. Such a surface is called an **equipotential surface. Formally, we define equipotential surface as the locus of all points having the same electric potential.** Further, the geometry of Fig. 9.6a suggests that the electric field lines due to the point charge Q located at the centre of the concentric spheres are everywhere perpendicular to the equipotential surfaces. The consequences of this fact are very important which we shall discuss shortly.

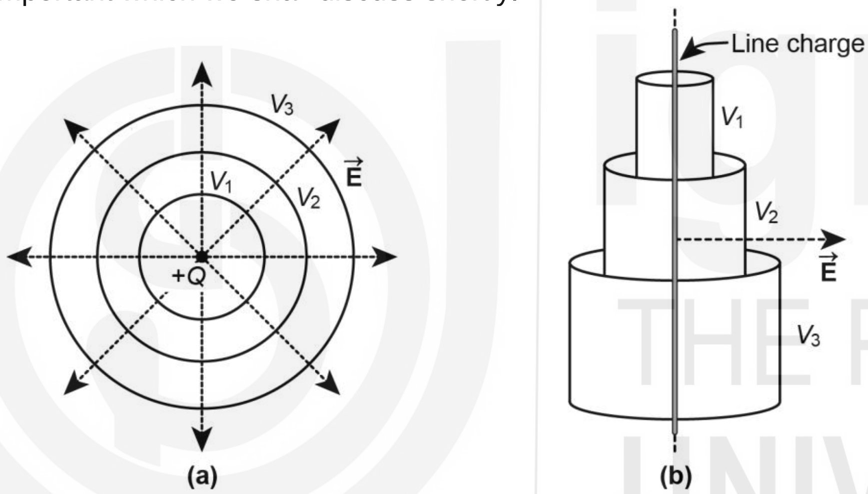

Fig. 9.6: a) Equipotential surfaces of a point charge $+Q$; the electric field lines are radial (dashed). Solid circles are intersections of equipotential surfaces on the plane of paper; b) The equipotential surfaces (cylindrical surfaces) of a uniform infinite line charge.

Can you guess the nature of equipotential surfaces for a uniform infinite line charge? From Eq. (9.5), you may note that for a uniform infinite line charge, the electric potential is same at all points equidistant from the line charge. Therefore, for such a charge distribution, equipotential surfaces are cylindrical with the line charge as the axis of the cylinder (Fig. 9.6b).

Yet another example of an equipotential surface is a conducting surface. **An ideal conducting surface must be an equipotential surface.** Can you guess why it is so? This is because if there were any potential difference between two points on the conducting surface, charges would move from higher to lower electric potential (or *vice-versa*) until the electric potential everywhere became equal. You will see later in Unit 11 that this property of conductors helps us determine the electric field and potential in the space between the plates of a capacitor easily.

Since an equipotential surface is a surface having **constant electric potential**, the potential difference between any two points on it is zero. This

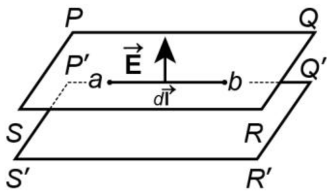

Fig. 9.7: **Direction of electric field vector $\vec{\mathbf{E}}$ relative to equipotential surfaces.** *PQRS* **and** *P′Q′R′S′* **are part of equipotential surfaces.**

implies that the work done in moving a unit charge from one point to another on such a surface is also zero. Thus, if *a* and *b* are two points on a equipotential surface, we can write

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} = 0 \tag{9.20}$$

where V_a and V_b are potential at points *a* and *b*, respectively. You will agree that Eq. (9.20) will hold only when the electric field $\vec{\mathbf{E}}$ and the small displacement vector $d\vec{\mathbf{l}}$ are perpendicular to each other. Since $d\vec{\mathbf{l}}$ is an infinitesimal displacement on the equipotential surface, $\vec{\mathbf{E}}$ has to be perpendicular at all points on such a surface (see Fig. 9.7). It is for this reason that we have drawn the electric field lines as perpendicular to the equipotential surfaces in Fig. 9.6.

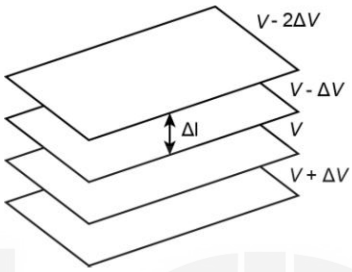

Fig. 9.8: **Separation between equipotential surfaces for arbitrary distribution of charges. Portions of four equipotential surfaces are shown.**

For an arbitrary charge distribution, the equipotential surfaces may look like the ones drawn in Fig. 9.8. **By convention, the equipotential surfaces are drawn such that there is a constant difference of potential, say** ΔV, **between the adjacent surfaces as shown in Fig. 9.8.**

Further, you may note in Fig. 9.9 which depicts the equipotential surfaces for an arbitrary charge distribution, that the equipotential surfaces may or may not be parallel to each other. They are relatively closer where the magnitude of $\vec{\mathbf{E}}$ is large, and are relatively far apart where the magnitude of $\vec{\mathbf{E}}$ is small. It is so because the difference in potential, ΔV between any two given equipotential surfaces is constant and we know that

$$\Delta V = Ed = E\Delta l \tag{9.21}$$

Thus, for constant ΔV, if Δl decreases, *E* must increase.

The magnitude of electric field is greater in the region where equipotentials are closer to each other.

So, we have seen that the sketch of equipotential surfaces gives us a fairly good idea about the magnitude of electric field in that region. You have also learnt that the electric field is directed perpendicular to an equipotential surface. Can we also draw some inference about the sense of the direction of electric field on the basis of equipotentials? Yes, we can. To find out, refer again to Fig. 9.9. Note that, on the left hand side of the figure, equipotential surfaces are closer to each other as compared to the right hand side. Now, you may recall from Unit 8 that the relation between $\vec{\mathbf{E}}$ and electric potential is given by:

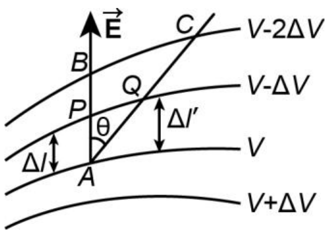

Fig. 9.9: **Direction of electric field $\vec{\mathbf{E}}$ from equipotential surfaces.**

$$\vec{\mathbf{E}} = -\vec{\nabla} V \tag{9.22}$$

The negative sign in Eq. (9.22), along with the fact that the electric field $\vec{\mathbf{E}}$ is always perpendicular to equipotential surfaces, implies that $\vec{\mathbf{E}}$ always points in the direction of decreasing *V*. To understand this better, let us consider two probable directions *APB* and *AQC* for the electric field $\vec{\mathbf{E}}$ (Fig. 9.9). Let the separation between two adjacent equipotentials along *APB* and *AQC* be Δl

and $\Delta l'$ respectively. Since ΔV is constant and the geometry of the figure suggests that $\Delta l' > \Delta l$, we have

$$\frac{\Delta V}{\Delta l} > \frac{\Delta V}{\Delta l'}$$

This implies that $\vec{\mathbf{E}}$ is directed along $\Delta\vec{\mathbf{l}}$, that is, along *APB* because the decrease in *V* is fastest along this line. **Thus, we conclude that the electric field $\vec{\mathbf{E}}$ is always along the direction of maximum (or the steepest) decrease of potential, *V*.**

Thus, a sketch of the equipotential surfaces gives us a visual picture of both the direction and the magnitude of $\vec{\mathbf{E}}$ in a region of space containing a single charge, a group of charges, or a charge distribution of some particular form (or shape).

On the basis of the above discussion, we can summarise the properties of equipotential surfaces as follow:

Recap

- **The electric field is perpendicular to equipotential surface.**
- **The electric field is directed along the maximum (steepest) decrease of potential. That is, it points from surface at higher electric potential to lower electric potential.**
- **No work is done in moving a charge between any two points on an equipotential surface.**
- **The tangential component of electric field along an equipotential surface is zero. If it were not so, a finite work would be required to be done in moving a charge along the surface.**

So far, we have described the electrostatic field in terms of electric field vector, potential and equipotential surfaces. In the next section we shall discuss the electrostatic energy associated with discrete and continuous charge distributions. But, before studying the next section, you may like to try an SAQ.

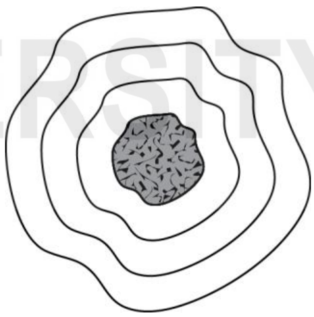

Fig. 9.10: **Diagram for SAQ 4b.**

SAQ 4 – **Equipotential surfaces**

a) Suppose you are given a sketch of electric field lines due to a group of charges and asked to draw the equipotential surfaces. List the various points you will keep in mind while drawing equipotential surfaces.

b) The equipotential surfaces for a charged solid metal object are shown in Fig. 9.10. Draw the electric field lines.

9.4 ELECTROSTATIC POTENTIAL ENERGY

In the previous unit, you have learnt about the electrostatic potential energy of charge *q* in the field of another charge *Q*. We now extend this discussion to discrete and continuous charge distributions.

Let us first consider two charged particles q_1 and q_2 very far apart from one another as shown in Fig. 9.11a. Now, if we bring these two particles slowly towards each other to a distance between them be r_{21}, then how much work is done in this process? Recall that the work done will be the same whether we move q_2 and keep q_1 fixed or *vice-versa*. The work done is the integral of the product of the force between the charges and displacement in the direction of force.

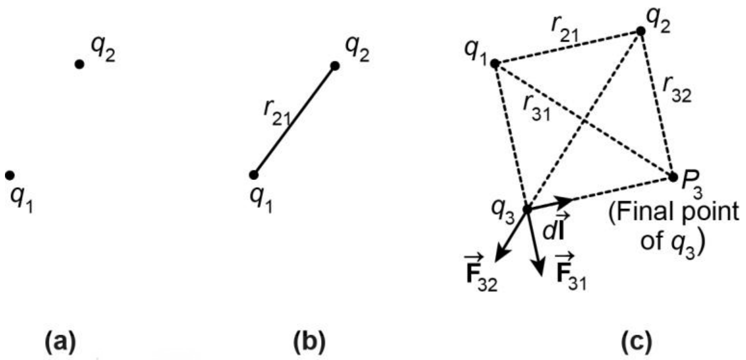

Fig. 9.11: **a) Two charged particles q_1 and q_2 at a very large distance from each other; b) the two charges at a separation r_{21} from each other; c) three charges q_1, q_2 and q_3 are brought near one another.**

The work done in bringing the charges q_1 and q_2 separated by a large distance to a separation r_{21} from each other is:

$$W_1 = \int \vec{\mathbf{F}}_{21}.d\vec{\mathbf{l}} = \int_{r=\infty}^{r_{21}} \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 (-dr)}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}} \qquad (9.23a)$$

Note that we have taken distance as (– dr) because r is changing from $\propto$ to r_{21}. We have taken q_1 and q_2 to be positive; so the charges must be pushed together and the displacement is opposite to the direction of Coulomb force.

You know from Unit 8 that the work done in moving a charge from infinity to a finite distance r in a field due to another charge is independent of the path we take. With this understanding, let us now bring a third charge q_3 from infinity (that is, from very large distance from charges q_1 and q_2) and bring it to a position such that its distance from q_1 is r_{31} and from q_2, r_{32} (Fig. 9.11c). So, the work done in moving charge q_3 to this position is

$$\int \vec{\mathbf{F}}.d\vec{\mathbf{l}} = \int (\vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{32}).d\vec{\mathbf{l}} = -\int \vec{\mathbf{F}}_{31}.d\vec{\mathbf{r}} - \int \vec{\mathbf{F}}_{32}.d\vec{\mathbf{r}} \qquad (9.23b)$$

Eq. (9.23b) is written due to the fact that the work done to bring q_3 to point P_3 is the sum of the work needed when q_1 alone is present and the work needed when q_2 alone is present. So,

$$W_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{31}} + \frac{q_2 q_3}{r_{32}}\right)$$

So, the total work done in assembling this arrangement of three charges q_1, q_2 and q_3 is

$$W = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{21}} + \frac{q_1q_3}{r_{31}} + \frac{q_2q_3}{r_{32}}\right) \quad (9.24)$$

The work done given by Eq. (9.24) is defined as electrostatic potential energy of the system.

We can now generalise the result contained in Eq. (9.24) to any number of charges. If we have N different charges in any configuration in space, the electrostatic potential energy of the system can be written as sum over all pairs. So, for a system of N charges $q_1, q_2, q_3, ..., q_N$, the electrostatic potential energy can be written as

$$P.E. = \frac{1}{2}\sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}\frac{q_j q_k}{4\pi\varepsilon_0 r_{jk}} \quad (9.25)$$

Note that the double summation notation. $\sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}$ implies that when we take $j = 1$, we need to sum over all values of k except 1; that is, we sum over $k = 2, 3, ..., N$; then we take $j = 1$ and sum over $k = 1, 3, 4, ..., N$ (leaving $k = 2$), and so on. So, we find that the double summation includes every pair twice and the factor of (1/2) has been included in Eq. (9.25) to correct this double counting.

In terms of electric potential V_j at the position of charge q_j, Eq. (9.25) may be written as

$$\boxed{P.E. = \frac{1}{2}\sum_{j=1}^{N} q_j V_j} \quad \text{where} \quad V_j = \sum_{\substack{k=1 \\ j\neq k}}^{N}\frac{q_k}{4\pi\varepsilon_0 r_{jk}} \quad (9.26)$$

Eq. (9.26) implies that, for calculating the electrostatic potential energy for a group of point charges, one may consider each charge by turn, and the corresponding potential at its position due to all other charges except the one under consideration.

Continuous Charge Distribution

Since most of the charged real physical systems such as the plates of parallel plate capacitor are described as continuous charge distributions, you may like to know how to determine their electrostatic or electrical potential energy. To learn the method, take a simple example of adding point charges gradually, in steps, on an isolated conductor. In such a situation, the work done can be calculated as follows.

Let the charge on a conductor at a given time be q. Then, the potential V of this charged conductor is proportional to q. Thus, the work done δW in adding an additional charge δq on q (isolated conductor) is

$$\delta W = V\delta q$$

Further, we can write V as $V = kq$ where k is the constant of proportionality. Hence

$$\delta W = kq\,\delta q$$

As we go on adding more and more charges to this conductor, the total work done is the electrical potential energy of the charged body. The total work done can be calculated by integration (equivalent to summation). Thus, if Q is the final charge on the isolated conductor, then its electrical potential energy can be expressed as:

$$P.E. = \int_0^Q \delta W = \int_0^Q k\, q\, \delta q = k\left[\frac{q^2}{2}\right]_0^Q = k\frac{Q^2}{2} = \frac{Q}{2}V_f \qquad (9.27)$$

where $(V_f = kQ)$ is the final electric potential of the charged isolated conductor.

Eq. (9.27) gives the electrical potential energy of a charged conductor. We can write this expression in terms of charge density. For example, if in an infinitesimal volume $d\tau$, we assemble point charges such that the volume charge density is ρ and the electric potential is V then Eq. (9.27) for electrical potential energy can be written as

$$P.E. = \frac{1}{2}\int_{volume} \rho V d\tau \qquad (9.28)$$

Note that $\rho d\tau$ in Eq. (9.28) gives charge in the volume element $d\tau$ and when we integrate it over volume, we get Q, total charge on the conductor.

Similarly, for a charge distribution on a surface, if σ is the charge per unit area, then Eq. (9.28) takes the form

$$P.E. = \frac{1}{2}\int_{surface} \sigma V\, dS \qquad (9.29)$$

where dS is the element of surface area. And for a line charge distribution, if λ is the charge per unit length, then Eq. (9.27) for potential energy becomes

$$P.E. = \frac{1}{2}\int_{line} \lambda V\, dl \qquad (9.30)$$

where dl is line element.

Now, let us work out an example on electrical potential energy.

EXAMPLE 9.3 : ELECTRICAL POTENTIAL ENERGY

Three charges are arranged as shown in Fig. 9.12. Calculate the electrical potential energy of the system. Assume $q = 1.0\times10^{-5}\,\text{C}$, and $d = 0.10\,\text{m}$.

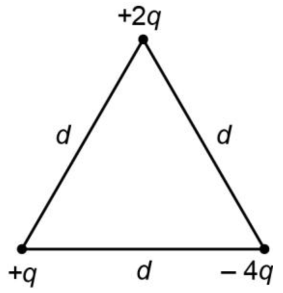

Fig. 9.12: **Diagram for Example 9.3.**

SOLUTION ■ The total electrical potential energy ($P.E.$) of the system is the algebraic sum of the electrical potential energies of all pair of charges, viz.,

$$P.\,E. = \frac{1}{4\pi\varepsilon_0}\left[\frac{(+q)\times(-4q)}{d} + \frac{(+q)\times(+2q)}{d} + \frac{(-4q)\times(+2q)}{d}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-10q^2}{d}\right] = \frac{-(9.0\times10^9\,\text{N m}^2\,\text{C}^{-2})\times(10)\times(1.0\times10^{-5}\,\text{C})^2}{0.10\,\text{m}}$$

$$= -\,90\,\text{J}$$

***SAQ 5* - Electrical potential energy**

With the help of a suitable diagram, estimate the number of terms that will contribute to the electrical potential energy for a system of five point charges.

We now sum up what you have learnt in this unit.

9.5 SUMMARY

Concept	Description
Potential due to infinite line charge	■ The potential difference between two points *a* and *b* on the line perpendicular to infinite line charge at distance r_a and r_b, respectively is: $$V_a - V_b = -\frac{\lambda \ln r_a}{2\pi\varepsilon_0} + \frac{\lambda \ln r_b}{2\pi\varepsilon_0}$$ If we assume that point *b* is at finite distance r_b and it is the reference point having zero electric potential (that is, $V_b = 0$), then $$V_a = -\frac{\lambda \ln r_a}{2\pi\varepsilon_0} + \text{const}$$
Potential due to uniformly charged spherical shell	■ At a point distant *r* from the centre and outside the spherical shell of radius *R*: $$V = \frac{Q}{4\pi\varepsilon_0 r}$$ where $Q = 4\pi R^2 \sigma$. At a point inside the shell: $$V = \frac{Q}{4\pi\varepsilon_0 R}$$
Potential due to uniformly charged non-conducting sphere	■ At an external point at distance *r* from the centre of the sphere: $$V = \frac{Q}{4\pi\varepsilon_0 r}$$ where $Q = (4\pi/3)R^3\rho$. At an internal point: $$V = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2)$$ where *R* is the radius of the sphere and *r* (< *R*) is the distance of the internal point from the centre.
Equipotential surface	■ Equipotential surfaces are surfaces on which the potential at each point is same. The electric field $\vec{\mathbf{E}}$ is always directed perpendicular to an equipotential surface. It is always along the direction of the fastest decrease of the electric potential. No work is done in moving a charge between any two points on an equipotential surface. Equipotential surfaces are closer to each other in regions of strong electric field and are relatively far apart in regions of weak electric field.

Electrical potential energy

- The electrical potential energy is the energy stored in a system of charges. It is equal to the amount of work done in assembling the system together by bringing the charges from infinity.

  The electrical potential energy for a group of N discrete point charges is given as:

  $$P.E. = \frac{1}{2}\sum_{j=1}^{N} q_j V_j$$

  where V_j is the potential at the position of charge q_j due to all the charges except the charge q_j.

  The electrical potential energy of a charged conductor is

  $$P.E. = \frac{1}{2}\int_{volume} \rho V d\tau$$

  where ρ is volume charge density.

  The electrical potential energy of a charge distribution on a surface is

  $$P.E. = (1/2)\int_{surface} \sigma V dS$$

- The electrical potential energy of a line charge is

  $$P.E. = (1/2)\int_{line} \lambda V dl$$

9.6 TERMINAL QUESTIONS

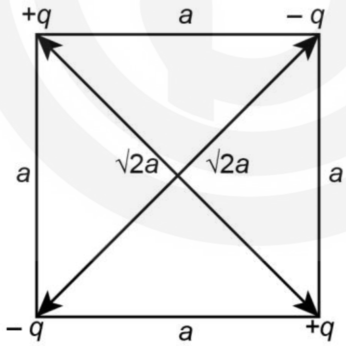

Fig. 9.13: **Diagram for TQ 4.**

1. If electric field $\vec{\mathbf{E}}$ equals zero at a given point, must V (electric potential) equal zero at that point? Give one example to justify your answer.

2. An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} Cm^{-2}. How far apart are the equipotential surfaces whose potentials differ by 5.0 V?

3. A uniformly charged sphere has electric potential of 375 V on its surface. At a radial distance of 25 cm from the surface of the sphere, the electric potential is 125 V. Calculate the radius and charge on the sphere.

4. Derive an expression for the work required to put the four charges together as indicated in Fig. 9.13.

5. Calculate the gain or loss of electrical potential energy when a droplet of radius R carrying a charge Q splits into two equal sized droplets of charge $Q/2$ and radius r. Assume that the droplets are repelled to a large distance compared to r because of electrostatic repulsion.

6. There are two charged conducting spheres of radii a and b. Suppose that they are connected by a conducting wire. What will happen? Using the result from this arrangement, explain why charge density on sharp and pointed ends of a conductor is higher than on its flatter portions.

7. Devise an arrangement of three point charges, separated by finite distances, that has zero potential energy.

9.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. We know that the potential difference between two points P and Q due to an infinite line charge is given as [Eq. (9.4)]:

$$V_P - V_Q = -\frac{\lambda}{2\pi\varepsilon_0}\ln(r_P / r_Q)$$

As per the problem, let point Q be at a distance of 5.0 m from the line charge where potential $V_Q = 0$. So, the potential at point P located at 6.0 m can be written as

$$V_P = -\frac{2\lambda}{4\pi\varepsilon_0}\ln(r_P / r_Q) = -2\times(3.0\times10^{-16}\,\text{C m}^{-1})\times(9\times10^9\,\text{N m}^2\text{C}^{-2})\times\ln\left(\frac{6.0\,\text{m}}{5.0\,\text{m}}\right)$$

$$= -9.8\times10^3\,\text{V}$$

2. a) The point located at 40 cm from the centre of the shell where potential is to be calculated is an external point because radius is 20 cm. So, we can write

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{(9\times10^9\,\text{Nm}^2\,C^{-2})\times 4\pi\times(0.20\,\text{m})^2\times(3.0\times10^{-6}\,\text{C m}^{-2})}{(0.40\,\text{m})}$$

$$= 8.4\times10^4\,\text{V}$$

b) The point located at 15 cm from the centre is an internal point. For any such point, potential has a constant value given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{(9\times10^9\,\text{N m}^2\text{C}^{-2})\times 4\pi\times(0.20\,\text{m})^2\times(3.0\times10^{-6}\,\text{C m}^{-2})}{(0.20\,\text{m})}$$

$$= 6.7\times10^4\,\text{V}$$

3. Let Q be the charge on the aluminium sphere and n number of electrons have been removed to raise it to potential of 500 V. So, $Q = ne$, where e is electronic charge. So, $n = (Q/e)$. Further, the potential of the sphere is given as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{ne}{r}$$

So, $$n = \frac{(4\pi\varepsilon_0)Vr}{e} = \frac{(500\,\text{V})\times(.07\,\text{m})}{(9\times10^9\,\text{N m}^2C^{-2})\times(1.6\times10^{-19}\,\text{C})} = 2.4\times10^8$$

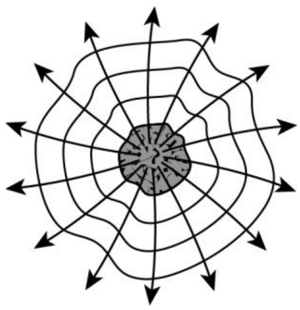

Fig. 9.14: **Diagram for answer to SAQ 4b.**

4. a) i) Equipotentials are always perpendicular to the electric field lines.

ii) Separation between the equipotentials depends on the strength of the electric field.

b) Electric field lines for the charged metal object are shown in Fig. 9.14.

5. The diagram for a system of five charges is shown in Fig. 9.15. Since each pair of charge has a potential energy and there are 10 pairs between 5 point charges, 10 terms would be contributing to the potential energy of 5 charges.

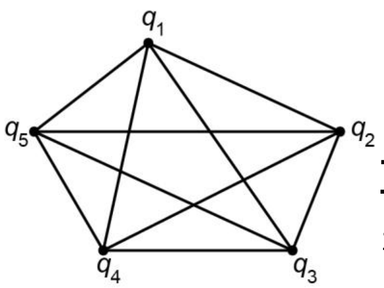

Fig. 9.15: **Diagram for answer to SAQ 5**.

(Rule: If there are n charges, the number of terms (pairs) contributing to the potential energy is $\frac{n(n-1)}{2}$).

Terminal Questions

1. We know that the electric field is related to potential as $\vec{\mathbf{E}} = -(dV/dx)$. Thus, if $|\vec{\mathbf{E}}| = 0$, electric potential has to be a constant. It is not necessary that V be equal to zero when $|\vec{\mathbf{E}}| = 0.$ Consider, for example two identical charges separated by a distance $2a$. At the mid-point between the charges,

$$|\vec{\mathbf{E}}| = 0, \text{ but } V = \frac{1}{2\pi\varepsilon_0}\frac{q}{a}$$

2. The magnitude of electric field near an infinite charged sheet is given by (see Unit 6):

$$|\vec{\mathbf{E}}| = \frac{\sigma}{2\varepsilon_0}$$

where σ is the surface charge density. Therefore, for the problem under consideration,

$$|\vec{\mathbf{E}}| = \frac{1.0 \times 10^{-7}\ \text{Cm}^{-2}}{2 \times 8.9 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\text{m}^{-2}} = 5.6 \times 10^3\ \text{NC}^{-1}$$

The spacing Δl between the equipotential surface is given by $\Delta l = (\Delta V/E)$ where ΔV is the potential difference between the adjacent surfaces. With $\Delta V = 5.0\,\text{V}$, we have

$$\Delta l = \frac{5.0\,\text{V}}{5.6 \times 10^3\ \text{NC}^{-1}} = 0.89 \times 10^{-3}\text{m} = 0.89\ \text{mm}$$

3. The potential of a charged sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \quad \Rightarrow \quad 375\ \text{V} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \qquad \text{(i)}$$

Also, as per the problem

$$125\ \text{V} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(4 + 0.25\ \text{m})} \qquad \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{375\ \text{V}}{125\ \text{V}} = \frac{(r + 0.25\ \text{m})}{r}$$

$$3r - r = 0.25\ \text{m} \quad \Rightarrow \quad r = 0.13\ \text{m}$$

And, total charge on the sphere is

$$Q = V(4\pi\varepsilon_0)r = \frac{375\ \text{V} \times 0.13\ \text{m}}{9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}} = 5.2 \times 10^{-9}\,\text{C}$$

4. The work required to assemble four charges together as shown in Fig. 9.13 is equal to the electric potential energy of the system. The electrical potential energy of the system may be obtained by considering the charges in pairs:

$$= \frac{1}{4\pi\,\varepsilon_0}\left[\frac{(q)(-q)}{a} + \frac{(-q)\,(q)}{a} + \frac{(-q)(q)}{a} + \frac{(-q)(q)}{a}\right] + \left[\frac{(-q)(-q)}{\sqrt{2}\,a} + \frac{(q)(q)}{\sqrt{2}\,a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-4q^2}{a} + \frac{2q^2}{\sqrt{2}\,a}\right] = \frac{1}{4\pi\varepsilon_0}(-4 + \sqrt{2})\frac{q^2}{a}$$

5. Total volume of 2 droplets after splitting $= 2 \times (4\pi/3)\, r^3$. Volume of the original droplet $= (4\pi/3)\, R^3$. Since volumes have to be equal, we have

$$2 \times (4\pi/3)\, r^3 = (4\pi/3) R^3 \quad \Rightarrow r = (1/2)^{1/3} R \qquad \text{(i)}$$

Electrical potential energy (P.E.) of the original droplet with charge Q is

$$P.E. = \frac{1}{2} QV = \frac{1}{2} Q \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \text{(ii)}$$

Total electrical potential energy of 2 droplets after splitting is

$$(P.E.)_{\text{split}} = 2 \times \frac{1}{2}\,\frac{Q}{2}\,\frac{Q/2}{4\pi\varepsilon_0 r} = \frac{Q^2/2}{8\pi\varepsilon_0 r}$$

Using Eq. (i), we have

$$(P.E.)_{\text{split}} = \frac{Q^2}{8\pi\varepsilon_0 R}\left(\frac{1}{2}\right)^{2/3} \qquad \text{(iii)}$$

Thus, the loss in electrical potential energy after splitting is

$$\frac{Q^2}{8\pi\varepsilon_0 R}\left[1 - \frac{1}{(2)^{2/3}}\right]$$

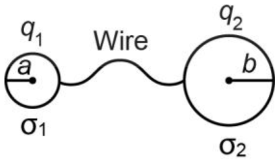

Fig. 9.16: **Diagram for answer to TQ 6.**

6. When two charged conducting spheres are connected by a wire as shown in Fig. 9.16, the charges redistribute themselves till both spheres are at the same potential, i.e.,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

where q_1 and q_2 are charges on spheres of radii a and b respectively. This gives

$$\frac{q_1}{q_2} = \frac{a}{b} \qquad \text{(i)}$$

The surface charge densities σ_1 and σ_2 on these spheres are:

$$\sigma_1 = \frac{q_1}{4\pi a^2} \quad \text{and} \quad \sigma_2 = \frac{q_2}{4\pi b^2}$$

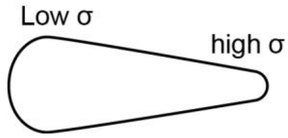

Fig. 9.17: **Diagram for the answer of TQ 6.**

Thus, we can write

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2} \times \frac{b^2}{a^2} \qquad \text{(ii)}$$

Combining Eqs. (i) and (ii), we get

$$\frac{\sigma_1}{\sigma_2} = \frac{a}{b} \times \frac{b^2}{a^2} = \frac{b}{a}$$

Thus, we see that the surface charge densities of conducting spheres are inversely proportional to their radii. For sharp and pointed ends, the radii are small, resulting in high surface charge densities. For flatter ends, the radii are larger which result in smaller surface charge densities. See Fig. 9.17.

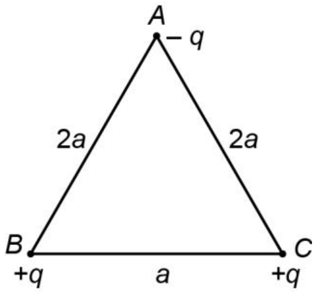

Fig. 9.18: **Diagram for the answer to TQ 7.**

7. If we devise an arrangement as shown in Fig. 9.18, the electrical potential energy (P.E.) turns out to be zero because the P.E. of the arrangement is:

$$P.E. = \frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)\,q}{2a} + \frac{(-q)\,(q)}{2a} + \frac{(q)\,(q)}{a}\right] = 0$$