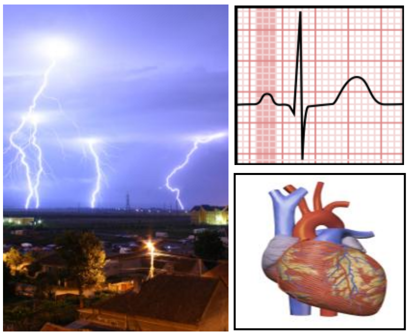

Electric potential and potential differences abound in nature ranging from several hundred million volts in a typical lightning bolt to about 90 mV in heart cell membranes. (Picture source: Wikimedia Commons)

UNIT 8

ELECTRIC POTENTIAL

Structure

8.1 Introduction
- Expected Learning Outcomes

8.2 Work done in Moving a Charge
- Line Integral of Electric Field
- Electrostatic Potential Energy

8.3 Electric Potential due to Point Charges
- Electric Potential due to a Point Charge
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STUDY GUIDE

In this unit, you will study electric potential which is a concept closely related to electrostatic force and electric field. It is a very useful concept for studying the behaviour of charged objects in an electric field. You know that the electrostatic force and electric field are vector quantities. The electric potential, however, is a scalar quantity. Since electric potential is a scalar quantity, the calculation of electric potential at a point in space due to a charge or a system of charges is much easier than that of an electric field – a vector quantity. To understand the contents of this unit better, you should refresh vector algebra given in Block 1 and the concepts of conservative force and potential energy from Block 2 of the 1st semester course entitled Mechanics (BPHCT-131). You should also revise the vector calculus given in Block 1 of this course. In particular, you should refresh the concept of gradient of a scalar field, integration of a vector function, line integral of scalar and vector fields discussed in Block 1 of this course. We advise you to work through the steps of mathematical derivations as you study the unit. You should also try to solve SAQs and TQs yourself to check your understanding of the concepts discussed in the unit.

> ***"You must be ready to give up even the most attractive ideas when experiment shows them to be wrong."***
>
> ***Alessandro Volta***

8.1 INTRODUCTION

In Unit 5 of this block, you have learnt Coulomb's law which enables us to calculate the electrostatic force between any two charges. You have also learnt the concept of electric field which makes the computation of electrostatic force far easier and convenient than using Coulomb's law. In Units 6 and 7, you have learnt how to calculate electric field directly or by using Gauss's law.

In most problems in electrostatics, our aim is to calculate the electric field. Since electric field is a vector quantity, its determination requires calculation of each of its scalar components. Many a time, to make this calculation easier, we first calculate a scalar quantity known as the electric potential *V*, from which electric field can be determined using a simple relation. Since electric potential is a scalar quantity, its calculation in most cases is not as difficult as the calculation of electric field.

The concept of electric potential is also important because it is closely linked to the work done by the electrostatic force due to charged particles and their potential energies. To explain the concept of electric potential, we draw analogy from mechanics (Unit 10, BPHCT-131). In that unit, you studied gravitational potential energy, which arises from the work done in moving an object from one point to another against gravitational force. You have learnt in Unit 5 of this course that the gravitational force between charges is very small (compared to the electrostatic force). So, the gravitational potential energy of a charge is negligible. In the same way, when a charge is moved from one point to another against electrostatic force (or field), work needs to be done which is stored as electrostatic potential energy of the charge. And, the electric potential at a point in an electric field is defined as electrostatic potential energy per unit charge at that point.

We begin this unit by determining the work done in moving a charge from one point to another in an electric field in Sec. 8.2. In doing so, you will learn how to calculate the line integral of electric field $\vec{\mathbf{E}}$. In Sec. 8.3, we shall define a scalar quantity called **electric potential** in terms of the line integral of electric field and calculate its value at a point due to an isolated charge as well as due to a system of charges. In Sec. 8.4, you will learn how to calculate electric field at a point if the value of electric potential at that point is known.

You have learnt the concept of electric dipole in Unit 5. You know that it is a unique configuration of two charges which is of immense practical utility in physics. Therefore, in Sec. 8.5 of this unit, we shall explain how to determine electric potential due to electric dipole at a given point. In Sec. 8.6, we discuss the effect of electric field on an electric dipole and explain the conditions under which electrostatic potential energy can be stored in an electric dipole.

In the next unit, you will learn how to calculate electric potential due to continuous charge distributions.

Expected Learning Outcomes

After studying this unit, you should be able to:

- calculate the work done in moving a charge from one point to another in an electric field;
- define electric potential as line integral of an electric field;
- determine the electric potential at a point due to a single charge and a system of charges;
- establish the relation between electric potential and electric field;
- calculate electric field at a point knowing the electric potential;
- determine the electric potential due to an electric dipole at a given point; and
- determine the torque experienced by an electric dipole in a uniform electric field.

8.2 WORK DONE IN MOVING A CHARGE

The concept of electric potential is closely linked to (a) the work done by electrostatic force in moving a charge from one point to another in an electric field, and (b) the relation between work done and potential energy. For the gravitational force, you have learnt how to determine the work done in moving an object from one point to another in Example 9.8 of Unit 9 of the course on Mechanics (BPHCT-131). You have also learnt in Unit 10 that the gravitational force is a conservative force which enables us to define the gravitational potential energy. On similar lines, we shall determine the work done by the electrostatic force in moving a charge from one point to another in an electric field. We shall also show that the electrostatic force is conservative and thereby define electrostatic potential energy and electric potential.

From Sec. 5.3 of Unit 5, you know that a single charge, say Q, *sets up* an electric field in the region surrounding it. The electric field $\vec{\mathbf{E}}$ due to the charge at a point is defined as the electrostatic force experienced by a unit positive test charge placed at that point. If, instead of a unit positive charge, we place a charge q at that point, then electrostatic force $\vec{\mathbf{F}}$ experienced by the charge q in the electric field $\vec{\mathbf{E}}$ is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} \qquad (8.1a)$$

where the electric field $\vec{\mathbf{E}}$ is given by Eq. (5.6a) of Unit 5:

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad (8.1b)$$

where $\hat{\mathbf{r}}$ is a unit vector in the radial direction away from charge Q.

Now, let us suppose that the charge q is moving from point a to b along an arbitrary path as shown in Fig. 8.1.

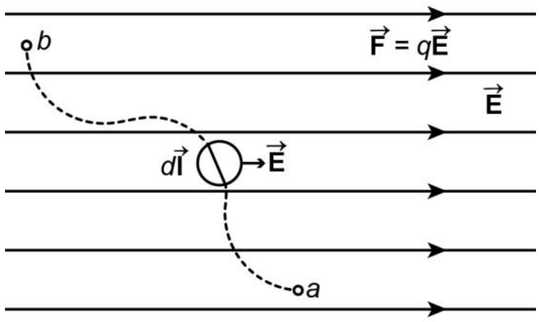


Fig. 8.1: **The charge** q **moves from point** a **to point** b **along an arbitrary path in electric field** $\vec{\mathbf{E}}$ **of charge** Q **(not shown in the figure).**

From Sec. 3.3 of Unit 3, you may recall that the work W, done in moving the charge q from point a to b is given by the line integral [Eq. (3.18b)]:

$$W = \int_a^b \vec{\mathbf{F}}.d\vec{\mathbf{l}} = q\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} \qquad (8.2)$$

Now, what happens if instead of the charge q, we move only a unit positive charge between a and b? You can see that in this case, the work W', done is obtained simply by dividing W by q, i.e.,

$$W' = \frac{W}{q} = \int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} \qquad (8.3)$$

We will be solving the line integrals of Eqs. (8.2) and (8.3) and obtain W' for a given charge. We will thus arrive at some interesting results. But, before proceeding further, we would like you to solve an SAQ.

SAQ 1 – **Work done in moving a charge**

Calculate the work done in moving a unit positive charge through a distance l in a uniform electric field parallel to the field direction.

Let us now evaluate the line integral of the electric field.

8.2.1 Line Integral of Electric Field

Let us consider the electric field due to a charge Q as shown in Fig. 8.2a. Let there be two points a and b at distances r_a and r_b from the charge Q as shown in Fig. 8.2a. Let us determine the line integral given by Eq. (8.3) for an arbitrary path between points a and b. Note that the path from a to b is a continuous curve. Let us evaluate Eq. (8.3), i.e., the work done in moving a unit positive charge from a to b. Suppose the unit charge moves from a to a' (which is an arc of a circle) and then from a' to b as shown in Fig. 8.2a. Then we can write [(recall Eq. (3.33), Unit 3, Block 1 of this course]:

$$W' = \int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_a^{a'} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + \int_{a'}^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} \qquad (8.4)$$

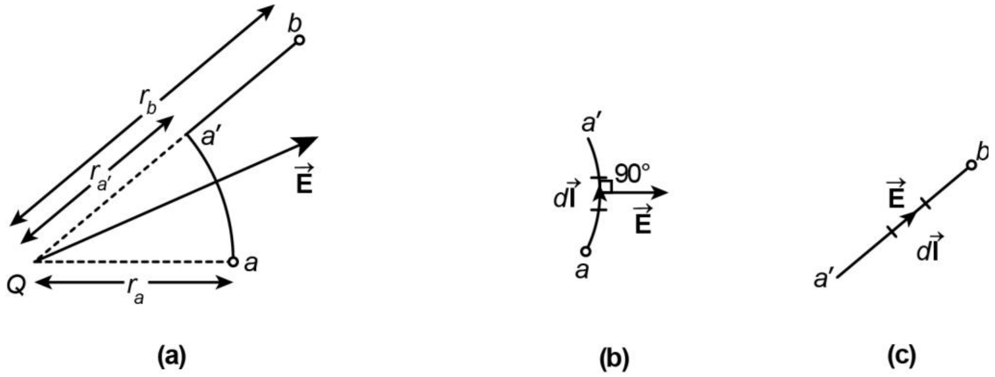


Fig. 8.2: **Work done in moving a unit positive charge from point** *a* **to point** *b* **along the path shown as continuous curve.**

The dot or scalar product of two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is defined as

$$\vec{\mathbf{a}}.\vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

where θ is the angle between vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.

The first line integral on the right-hand side of Eq. (8.4) represents the work done in moving the unit charge from *a* to a' along the arc of a circle of radius say, r_a. The second integral represents the work done in moving the same charge from a' to *b* along a straight line. The integrand $\vec{\mathbf{E}}.d\vec{\mathbf{l}}$ of the first line integral is equal to zero as $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$ are perpendicular to each other (see Fig. 8.2b). The integrand $\vec{\mathbf{E}}.d\vec{\mathbf{l}}$ of the second line integral in Eq. (8.4) is equal to $|\vec{\mathbf{E}}||d\vec{\mathbf{l}}|$ as both $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$ are parallel to each other along the path $a'b$ (see Fig. 8.2c). Can you tell why it is so? This is because, in the case of first integral, θ is 90° and hence $\cos\theta = 0$ and in the second integral, θ is zero and hence $\cos\theta = 1$ (see the margin remark).

Note that when we integrate over *r*, the limits of integration are from $r_{a'}$ to r_b.

Let us now determine the second integral of Eq. (8.4). Using Eq. (8.1b) for $\vec{\mathbf{E}}$, replacing $d\vec{\mathbf{l}}$ by $d\vec{\mathbf{r}}$ (since the path from a' to *b* is radial), and writing $d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ (where $\hat{\mathbf{r}}$ is a unit vector in the radial direction away from charge *Q*), we can write Eq. (8.4) as:

$$W' = 0 + \int_{a'}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{r_{a'}}^{r_b} \vec{\mathbf{E}}.d\vec{\mathbf{r}} = \int_{r_{a'}}^{r_b} \frac{Q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}.\hat{\mathbf{r}}}{r^2}(dr) = \int_{r_a}^{r_b} \frac{Q}{4\pi\varepsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

since $r_{a'} = r_a$.

Therefore, Eq. (8.4) becomes

$$W' = \int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \tag{8.5}$$

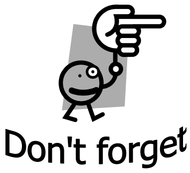
for the path shown in Fig. 8.3a between points *a* and *b*.

Now, you may recall from Sec. 3.4.1 of Unit 3, Block 1 of this course that a scalar potential can be associated with a conservative vector field. Since our aim here is to define an electric potential associated with electric field, we should establish that it is a conservative vector field. To do so, we examine if the electric field of a charge is conservative.

One of the characteristics of a conservative force is that the work done by this force in moving a particle from one point to another is independent of the path chosen to move the particle between the two points. The converse of this statement is also true: if the work done by a force in moving a particle from one point to another is independent of the path, the force is a conservative force.

This characteristic is exhibited by gravitational force (for a particle) as well as by electrostatic force (for a charged particle).

From Eq. (8.5), we note that the work done in moving a unit positive charge between any two points in the electric field of charge *Q* depends only on the distance of those points from charge *Q* and is independent of the path we choose to move the unit charge from one point to the other. So, **the line integral of the electric field is independent of the path**. Therefore, the electric field is a conservative vector field.



The electric field of a stationary charge is conservative.

8.2.2 Electrostatic Potential Energy

Recall from Sec. 10.3 of Unit 10, Block 2 of the Semester 1 course entitled Mechanics (BPHCT-131) that gravitational force is conservative. **You have learnt that we can define potential energy of an object moving under the influence of a conservative force.** For example, you have learnt that the change in gravitational potential energy, ΔU in moving an object from point *a* to point *b* is equal to the negative of the work done by the gravitational force in moving it from point *a* to *b*, that is,

$$(\Delta U)_{ba} = -W_{ab}$$

Now, you have learnt in Sec. 8.2.1 of Unit 8 that the electric field is a conservative vector field. Thus, we can say that the electrostatic force is a conservative force. So, we can also define electrostatic potential energy in the same way as we defined gravitational potential energy.

Thus, we can say that the change in electrostatic potential energy of a charge *q* in moving it from point *a to b* in an electric field of a charge *Q* is equal to the negative of the work done by the electrostatic force in moving the charge from point *a* to point *b*. If U_a and U_b are the initial and final electrostatic potential energy, respectively, of charge *q*, then we can write

$$\Delta U = (U_b - U_a) = -W_{ab} \tag{8.6}$$

where, W_{ab} is the work done by the electrostatic force in moving the positive charge *q* from point *a* to point *b* in the electric field $\vec{\mathbf{E}}$ due to charge *Q*. Now, from Eqs. (8.3) and (8.5), we can write:

$$W_{ab} = qW' = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \tag{8.7}$$

where W' is the work done in moving a unit positive charge from point *a* to point *b*. So, from Eqs. (8.6) and (8.7), we can write

$$\Delta U = (U_b - U_a) = -\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) \tag{8.8}$$

Eq. (8.8) gives the change in electrostatic potential energy of a positive charge *q* when it is moved from point *a* to *b* in the electric field due to charge *Q*. To fix these ideas, you may like to go through the following example.

EXAMPLE 8.1 : ELECTROSTATIC POTENTIAL ENERGY

The magnitude of a uniform electric field $\vec{\mathbf{E}}$ along the positive *x*-axis is 120 NC^{-1}. Calculate the change in electrostatic potential energy of a proton moving along a path parallel but opposite to the direction of $\vec{\mathbf{E}}$ through a distance 25 m.

SOLUTION ■ We know that the work done by a constant force $\vec{\mathbf{F}}$ in moving a particle through a displacement $\vec{\mathbf{d}}$ is given as

$$W = \vec{\mathbf{F}}.\vec{\mathbf{d}}$$

In the instant case, the electrostatic force on charge *q* due to an electric field $\vec{\mathbf{E}}$ is $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. Thus, the work done on the proton is

$$W = q\vec{\mathbf{E}}.\vec{\mathbf{d}} = qEd\cos\theta$$

where θ is the angle between $\vec{\mathbf{E}}$ and $\vec{\mathbf{d}}$. Now, the displacement of proton is parallel and opposite to the direction of $\vec{\mathbf{E}}$, i.e. $\theta = 180°$. Thus, we have

$$W = qEd\cos 180° = (1.6\times 10^{-19}\,\text{C})\times(120\,\text{NC}^{-1})\times(25\,\text{m})\times\cos 180°$$

$$= -4.8\times 10^{-16}\,\text{J}$$

If U_i and U_f are the initial and final electrostatic potential energy of the proton, we can write

$$\Delta U = U_f - U_i = -W = 4.8\times 10^{-16}\,\text{J}$$

So, we discover that the electrostatic potential energy of proton increases (as $U_i < U_f$) when it moves opposite to the direction of electric field.

Before proceeding further, you should answer an SAQ.

SAQ 2 - Electrostatic potential energy

In a region, the uniform electric field is $200\,\hat{\mathbf{i}}\,\text{NC}^{-1}$. Calculate the work done in moving i) an electron, ii) a proton through a distance 30 m along the field direction. What will be the change in electrostatic potential energy of these charged particles?

With this understanding of work done by electrostatic force in moving a charge and the related concept of electrostatic potential energy, you can learn about electric potential.

8.3 ELECTRIC POTENTIAL DUE TO POINT CHARGES

In Unit 5 of this Block, you have learnt that the electric field, defined as electrostatic force per unit charge, is a very useful concept for determining the forces experienced by a charge or a group of charges of any sign and magnitude. **Now, let us ask ourselves: Can we define a simpler concept which enables us to determine the electrostatic force and electric field**

due to a charge or a system of charges? The answer is, yes, we can. **The electric potential** is such a concept. Let us elaborate it with the help of the relation between work done and electrostatic potential energy discussed in Sec. 8.2.2.

8.3.1 Electric Potential due to a Point Charge

Let us first define electric potential. **The electric potential is defined as electrostatic potential energy** per unit charge, that is,

$$V = \frac{U}{q} \tag{8.9}$$

where V is electric potential at a given point in the electric field and U is the electrostatic potential energy of charge q at that point. You know that the difference in electrostatic potential energy of charge q, when it is moved from point a to b in the electric field of charge Q is given by Eq. (8.8). Thus, on the basis of the definition of electric potential given above, we can write the difference in electric potential between points a and b as

$$\Delta V = \frac{\Delta U}{q}$$

or $$V_b - V_a = \frac{U_b - U_a}{q} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right] \tag{8.10}$$

You know that ΔU is related to the work done by the electrostatic force in moving charge q from point a to b through Eq. (8.6). Also, the work done per unit charge is related to the electric field by Eq. (8.3). Thus, on the basis of Eqs. (8.6) and (8.3), we can write Eq. (8.10) in terms of the line integral of electric field $\vec{\mathbf{E}}$ as

$$V_b - V_a = -\frac{W_{ab}}{q} = -\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}} \tag{8.11}$$

Further, from Eq. (8.10) we note that the difference in electric potential is a difference between two numbers (or scalars): $\left(\frac{Q}{4\pi\varepsilon_0 r_b}\right)$ and $\left(\frac{Q}{4\pi\varepsilon_0 r_a}\right)$.

Let us now see what happens if we assume that initial point a is at infinity (that is, $r_a = \infty$) and the electric potential at infinity is zero, that is, $V_a = 0$. Then, we can write Eq. (8.10) as

$$V_b = \frac{Q}{4\pi\varepsilon_0 r_b} \tag{8.12}$$

Eq. (8.12) gives the **electric potential** at point b at a distance r_b from a point charge Q. Further, for the condition that point a is located at infinity, i.e., $r_a = \infty$ and $V_a = 0$, Eq. (8.11), which defines electric potential at point b at a distance r_b in terms of line integral of $\vec{\mathbf{E}}$, reduces to

$$V_b = -\int_\infty^{r_b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} \tag{8.13}$$

Note that Eqs. (8.12) and (8.13) are equivalent definitions of electric potential. Eq. (8.12) signifies that electric potential is a scalar quantity. Eq. (8.13) helps us understand what we mean when we say that the electric potential at a point in an electric field has some finite value. The RHS of Eq. (8.13) tells us that the electric potential at any point *b* at a distance r_b is the work done in bringing a unit positive charge from infinity up to that point (see Fig. 8.3). The SI unit for electric potential is the joule / coulomb (JC^{-1}). This combination occurs so often that a special unit, the volt (abbreviation V named after Alessandro Volta), is used to represent the unit of electric potential.

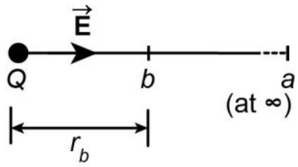


Fig. 8.3: **Work done in moving a unit positive charge from point *a* (at infinity) to point *b* in the electric field of charge *Q*.**

ELECTRIC POTENTIAL DUE TO A POINT CHARGE

Electric potential associated with electric field $\vec{\mathbf{E}}$ due to a charge *Q* at a point at distance *r* from it is defined as

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{8.14}$$

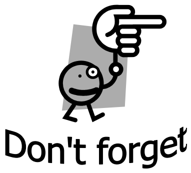
Electric potential *V* associated with the electric field $\vec{\mathbf{E}}$ due to a point charge *Q* at a distance *r* from it is given in terms of line integral as

$$V = -\int_{\infty}^{r} \vec{\mathbf{E}}.d\vec{\mathbf{l}} \tag{8.15}$$

From Eq. (8.14), we note that in the electric field of a positive charge *Q*, the potential at a point at distance *r* is positive; while for a negative charge, it is negative. Now, let us pause for a moment and ask ourselves: What is the physical meaning of this statement?

Note that in the electric field due to a positive charge, **work is done on** the unit positive test charge to move it from infinity to the given point **against the repulsive force** between the positive charge and the test charge. This **work done by an external agent** increases the electrostatic potential energy of the system and hence electric potential due to a positive charge at some finite distance is positive. On the other hand, in the electric field due to a negative charge, **the work is done by the electric field** in bringing the unit positive charge from infinity and the electrostatic potential energy of the system decreases. Therefore, the electric potential due to a negative charge at some finite distance is negative.

It is, therefore, clear that, **when work is done against the force field** (in this case **electric field**), **potential energy of the system increases**. This can be easily understood by considering an example in the case of gravitational field. When a body of finite mass is raised to a height against the force of gravity acting downwards, then the potential energy of the body increases. Here, work is done against gravity. And when work is done by the force of gravity as in case of free fall of a body, the potential energy decreases. The difference in potential energy gets converted into kinetic energy of the freely falling object.



A positive point charge produces positive electric potential and a negative point charge produces negative electric potential.

So far, you have learnt the concept of work done in moving a charge in an electric field, electrostatic potential energy and electric potential and how these concepts are related to each other.

Now, to concretise these ideas, you should go through the following example.

EXAMPLE 8.2 : ELECTRIC POTENTIAL DUE TO A POINT CHARGE

A particle of charge 5.0 μC is located on the *x*-axis at the point $x = 6.0\,\text{cm}$. Calculate the electric potential due to this charge at the origin, $x = 0$. Also calculate the work done in moving a charge $-6.0\,\mu\text{C}$ from infinity to the origin keeping the first charge fixed.

SOLUTION ■ From Eq. (8.14), we write the electric potential as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Substituting the values of $Q = 5.0\times10^{-6}\,\text{C}$, $r = 6.0\times10^{-2}\,\text{m}$ and $(1/4\pi\varepsilon_0) = 9.0\times10^{9}\,\text{Nm}^2\text{C}^{-2}$ we have

$$V = (9.0\times10^{9}\,\text{Nm}^2\text{C}^{-2})\times\frac{5.0\times10^{-6}\,\text{C}}{6.0\times10^{-2}\,\text{m}} = 7.48\times10^{5}\,\text{V}$$

Further, to calculate the work done in moving the charge $-6.0\,\mu\text{C}$ from infinity to the origin, we use Eq. (8.11) with the understanding that potential at infinity is zero:

$$V = W/q \Rightarrow W = qV = (-6.0\times10^{-6}\,\text{C})\times(7.48\times10^{5}\,\text{V}) = -\,4.48\,\text{J}$$

In electrostatics, we associate three quantities with a static electric charge. The magnitude of the electrostatic force on a test charge *q* at a distance *r* from the point charge *Q* is given as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

The magnitude of the electric field at a point distance *r* is given as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

The electric potential at distance *r* from the point charge *Q* is given as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Note the nature of dependence of these quantities on the distance *r*, point charge *Q* and test charge *q*.

Potential Difference and Zero Potential

The way we have defined the electric potential at a point by Eq. (8.14) may give you an impression that it is an absolute quantity. It is, however, not true because we have **arbitrarily chosen a reference point at infinity and assumed that the electric potential at infinity is zero.** The more fundamental quantity is the **potential difference** as given by Eq. (8.10) and Eq. (8.11) which refers to the change in electrostatic potential energy or the work done when a unit positive charge is moved from one point to another in an electric field. To determine the potential difference between any two points in an electric field, we do not need any reference point.

Potential difference is a very important concept in the field of electrostatics and current electricity. Its knowledge helps us in determining the exact value of the current which flows between any two points in an electric circuit, provided the (electric) resistance between the two points is known.

Though potential difference is a more fundamental concept than absolute potential, it is of immense practical importance to define a

reference point where the value of potential can be taken to be zero. Such a reference point with zero potential enables us to assign an absolute value of electric potential to a point in electric field. We did that by choosing the reference point at infinity with zero potential and defined electric potential at a point by Eq. (8.14).

You should, however, remember that the choice of the reference point with zero potential is arbitrary and it is done in such a manner which makes the mathematical treatment of the problem simpler. For example, in most of the problems involving electric potential in electrical circuits, the potential of the Earth is taken as reference point with zero potential. This choice of reference potential is guided by the fact that the potential of the Earth remains constant even if it gains or loses electricity. This choice of reference with zero potential for electric situations is similar to our choice of sea level as reference point for describing the height of a place or a mountain on the Earth.

Before studying further, try to solve the following SAQ.

***SAQ 3 -* Calculating electric potential, potential difference and work done**

a) Refer to Fig. 8.4 which shows two points *X* and *Y* located at distances 8 m and 12 m, respectively, from a point charge $+7\mu C$. (i) Calculate the electric potential at points *X* and *Y* and the potential difference between points *X* and *Y*. (ii) Suppose that the point charge $+7\mu C$ is replaced by a point charge $-7\mu C$. Calculate the electric potential at points *X* and *Y* and the potential difference between *X* and *Y*. (iii) If the point charge $+7\mu C$ is fixed at its position, calculate the work done in moving a charge $+3\mu C$ from infinity to the point *X*.

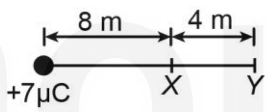


Fig. 8.4: Diagram for SAQ 3a.

b) The radius of a gold nucleus is 6.6×10^{-15} m and the atomic number, *Z* of gold is 79. Assuming that the nucleus acts as a point charge, and electronic charge $e = 1.6\times10^{-19}$ C, calculate the electric potential at the surface of a gold nucleus.

From Eq. (8.14), you know how to determine electric potential due to an isolated charge at a point located at distance *r* from the charge. Now, suppose that we have many discrete changes located at different points in space. How do we determine electric potential at some given point due to this system of discrete charges? You will learn it now.

8.3.2 Electric Potential due to a System of Discrete Charges

From Unit 5 of this course, you know that electric field obeys superposition principle which enables us to calculate $\vec{\mathbf{E}}$ at a given point due to a system of discrete charges. The superposition principle for electric fields implies that (a) the electric field at a given point due to any one charge of the system is unaffected by the presence of the remaining charges, and (b) the net value of $\vec{\mathbf{E}}$ at a given point is the **vector sum** of the fields due to individual charges of the system, at that point.

You may, therefore, ask: **Can we use the superposition principle to determine the value of electric potential due to a system of charges?** The answer is: Yes, we can. Since electric potential is a scalar quantity, its value at a given point is the **algebraic sum** of the electric potential due to individual charges of the system. Thus, using the superposition principle for electric potential is much simpler than using it for $\vec{\mathbf{E}}$ because, in case of $\vec{\mathbf{E}}$, we have to deal with vector sum of the fields due to individual charges.

Suppose we have a system of charges $q_1, q_2, ..., q_N$ located at distances $r_1, r_2, ..., r_N$, respectively, from the point *P*. So, according to the superposition principle, the potential at point *P* can be written as the algebraic sum of the potential at *P* due to $q_1, q_2, ..., q_N$:

$$V_P = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + ... + \frac{q_N}{4\pi\varepsilon_0 r_N}$$

Note that here each individual charge is acting as if the other charges are not present. The above expression may be written in a summation form as:

$$V_P = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i} \qquad (8.16)$$

As a caution, you may keep in mind that the sum given in Eq. (8.16) is an algebraic sum and not a vector sum as the potential at a point is a scalar quantity. To get a feel for the value of potential due to a system of discrete charges, go through the following example.

EXAMPLE 8.3 : ELECTRIC POTENTIAL DUE TO MANY DISCRETE CHARGES

Three point charges are placed on the *x*-axis: $2\mu C$ at $x = 20$ cm, $-3\mu C$ at $x = 30$ cm, $-4\mu C$ at $x = 40$ cm. Calculate the electric potential at $x = 0$.

SOLUTION ■ To calculate the electric potential at a point due to many discrete charges, we use Eq. (8.16):

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{3} \frac{q_i}{r_i}$$

On substituting the numerical values of q_i and r_i, we get

$$V = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2} \times \left[\frac{2 \times 10^{-6} \text{ C}}{0.20 \text{ m}} - \frac{3 \times 10^{-6} \text{ C}}{0.30 \text{ m}} - \frac{4 \times 10^{-6} \text{ C}}{0.40 \text{ m}} \right]$$

$$= 9 \times 10^9 \text{ Nm}^2\text{C}^{-2} \times [10^{-5} \text{ m}^{-1} - 10^{-5} \text{ m}^{-1} - 10^{-5} \text{ m}^{-1}]$$

or $$V = -9 \times 10^4 \text{ Nm C}^{-1} = -9 \times 10^4 \text{ V}$$

Note that each of the three charges are placed at different points on the same line (x-axis). But, the electric potential at a given point $(x = 0)$ on the same line due to one charge is not affected by the presence of the other two charges.

Before proceeding further, answer an SAQ.

SAQ 4 - Electric potential due to many charges

Two point charges $+q$ and $-2q$ are placed along a straight line at a distance of 9 m from each other. Determine the distance of a point, from the charge $+q$, between the two charges where the electric potential is zero.

On the basis of the discussion so far, you have learnt that the electric field $\vec{\mathbf{E}}$ at a point in space gives us the magnitude and direction of electrostatic force and electric potential gives the work done by the electrostatic force in moving a unit positive charge from one point to another. So, if we have a relation which enables us to compute electric field at a point if the potential at that point is known, solving problems of electrostatics becomes far easier. It is far easier to use the concept of electric potential since it is a scalar. You will agree that working with vectors is more complicated than working with scalars. Let us now learn the relation between electric field and electric potential.

8.4 RELATION BETWEEN ELECTRIC FIELD AND ELECTRIC POTENTIAL

You know from Eq. (8.11) that the difference in electric potential, $V_{ba} (= V_b - V_a)$ between two points b and a in the electric field $\vec{\mathbf{E}}$ of charge Q is equal to the negative of the line integral of $\vec{\mathbf{E}}$ between the same two points:

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}}$$

If the separation $d\vec{\mathbf{l}}$ between the two points a and b is small, we can write the potential difference dV between any two points as

$$dV = -\vec{\mathbf{E}}.d\vec{\mathbf{l}} \tag{8.17}$$

or $$dV = -E\cos\theta \left| d\vec{\mathbf{l}} \right|$$

or $$-E\cos\theta = \frac{dV}{\left| d\vec{\mathbf{l}} \right|} \tag{8.18}$$

The presence of cos θ term in Eq. (8.18) indicates that the electric field is not a simple derivative of the potential function V; rather, it is some special kind of derivative of the potential. We call it directional derivative about which you studied in Unit 1, Block 1 of this course.

As you have studied in Sec. 1.3, Unit 1, Block 1 of this course, the rates of change of scalar fields such as temperature and potential in different directions can be expressed by using the gradient operator. From Eq. (1.8), you know that the difference *df* in the value of a scalar function *f* between two points separated by $d\vec{\mathbf{r}}$ is given as

$$df = (\vec{\nabla} V).d\vec{\mathbf{r}}$$

Since electric potential is a scalar function, we can use the above general relation and write the electric potential difference between two points separated by $d\vec{\mathbf{l}}$ as

$$dV = (\vec{\nabla} V).d\vec{\mathbf{l}} \tag{8.19}$$

So, comparing Eqs. (8.17) and (8.19), we can write

$$\vec{\mathbf{E}} = -\vec{\nabla} V = -\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right) \tag{8.20}$$

The components of $\vec{\mathbf{E}}$ along *x, y* and *z* directions are

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \tag{8.21}$$

Thus, we find that the electric field $\vec{\mathbf{E}}$ is the negative of the gradient of the electric potential *V at any point*.

Eq. (8.20) or Eq. (8.21) enables us to calculate the electric field at a point if we know the value of electric potential at that point. To understand this method, go through the following example.

EXAMPLE 8.4 : ELECTRIC FIELD FROM ELECTRIC POTENTIAL

The electric potential at a point is given by the relation $V = Ax + By - Cz$ where *A*, *B* and *C* are constants. Determine the electric field $\vec{\mathbf{E}}$ at that point.

SOLUTION ■ From Eq. (8.20), we have

$$\vec{\mathbf{E}} = -\vec{\nabla} V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

Substituting the value of *V*, we get

$$\vec{\mathbf{E}} = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)(Ax + By - Cz)$$

$$\therefore \quad \vec{\mathbf{E}} = -[A\hat{\mathbf{i}} + B\hat{\mathbf{j}} - C\hat{\mathbf{k}}]$$

Now apply this method yourself to solve SAQ 5.

SAQ 5 - Electric field from electric potential

The electric potential at any point is given by $V = x(y^2 - 4x^2)$. Calculate the electric field $\vec{\mathbf{E}}$ at that point.

In Unit 5 of this block, you have learnt how to calculate electric field due to multiple discrete charges and, especially the electric dipole. In the following section, you will learn how to determine the electric potential due to an electric dipole.

8.5 POTENTIAL DUE TO AN ELECTRIC DIPOLE

In Unit 5, you have learnt about the electric dipole. You know that it is a pair of equal and opposite charges, $\pm q$, separated by some distance, $2a$. **Then $2\vec{\mathbf{a}}$ is a vector along the axis of the dipole, drawn *from* the negative *to* the positive charge (Fig. 8.5).**

Let us now determine the electric potential due to a dipole. We shall use polar coordinates for mathematical convenience. Refer to Fig. 8.5 which shows point *P* at a distance *r* from the midpoint *C* of the dipole *AB*. The line joining *P* and *C* makes an angle θ with the dipole axis. So, the polar coordinates of point *P* are *r* and θ with the origin at *C*, the midpoint of dipole. We now determine the electric potential at *P* due to the two charges $-q$ and $+q$ of the dipole.

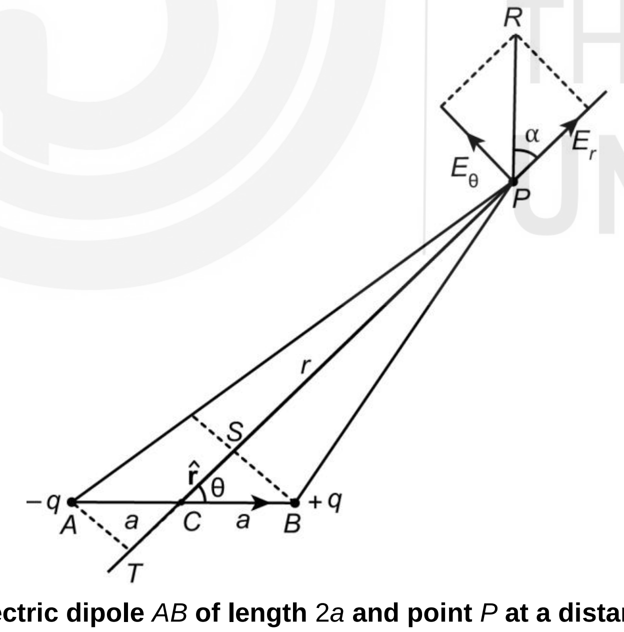


Fig. 8.5: **An electric dipole** *AB* **of length** $2a$ **and point** *P* **at a distance** *r* **from the mid-point** *C* **of the dipole.**

Study Fig. 8.5. Note that the distances of point *P* from $-q$ and $+q$ are *AP* and *BP*, respectively. Also note we have drawn perpendiculars from *B* to *S* and *A* to *T*. Thus, under the condition that point *P* is far away from the dipole so that $r \gg 2a$, you can see from the figure that

$$BP = SP = PC - CS = r - a\cos\theta$$

and

$$AP = TP = TC + CP = r + a\cos\theta$$

Thus, using the superposition principle [Eq. (8.16)], we can write the potential at P due to charges q and $-q$ of the dipole as:

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right] = \frac{2q\,a\cos\theta}{4\pi\varepsilon_0(r^2 - a^2\cos^2\theta)} \quad (8.22)$$

Now, let us suppose that $\vec{\mathbf{r}}$ is a vector from C to P and the unit vector along $\vec{\mathbf{r}}$ is $\hat{\mathbf{r}}$. Also, you know [Eq. (5.11)] that the dipole moment, $\vec{\mathbf{p}} = 2q\,\vec{\mathbf{a}}$. Since $\vec{\mathbf{p}}.\hat{\mathbf{r}} = 2q\,\vec{\mathbf{a}}.\hat{\mathbf{r}} = 2qa\cos\theta$, we can write Eq. (8.22) for V as

$$V = \frac{\vec{\mathbf{p}}.\hat{\mathbf{r}}}{4\pi\varepsilon_0(r^2 - a^2\cos^2\theta)} \quad (8.23)$$

When point P is far away from the dipole, r^2 is large compared to $a^2\cos^2\theta$. So, we can neglect $a^2\cos^2\theta$ in the denominator in comparison to r^2, and write Eq. (8.23) as

$$V = \frac{\vec{\mathbf{p}}.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \quad (8.24)$$

Eq. (8.24) gives the general expression for the electric potential due to dipole at a distance r from its mid point.

On the basis of Eq. (8.24), you can conclude that:

- **The electric potential due to dipole varies with r as $1/r^2$ whereas the potential due to point charge varies as $1/r$. The comparison of these variations shows that the potential decreases more rapidly with r for a dipole than for a point charge.**
- **The electric potential due to dipole is zero for all points which lie on the perpendicular bisector of the dipole axis because, for any such point, $\theta = 90°$ and $\cos\theta = 0$. Hence, no work is done in moving a test charge along the perpendicular bisector.**

We will now determine the electric field of a dipole from its electric potential. But before studying further, you may like to solve an SAQ.

SAQ 6 - Electric potential due to an electric dipole

A straight line from the centre of an electric dipole and along the axis of the dipole first passes through point P_1 and then through point P_2. The distances of points P_1 and P_2 from the centre of the dipole are 40 cm and 60 cm, respectively. The dipole length is much smaller than 40 cm. If the potential at point P_1 is 60 V, calculate the potential at point P_2.

To determine the electric field from electric potential, we will use the relation given by Eq. (8.20). However, since we have used polar coordinates to specify the location of point P, we must use the expression for the del operator in

Eq. (8.20) in polar coordinates. In polar coordinates, the operator $\vec{\nabla}$ is given as

$$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta}$$

Thus, Eq. (8.20) can be expressed in polar coordinates as

$$\vec{\mathbf{E}} = -\vec{\nabla}V = -\left[\hat{\mathbf{r}}\frac{\partial}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial}{\partial \theta}\right]V$$

Now, substituting the value of V from Eq. (8.24), we can write

$$\vec{\mathbf{E}} = -\left[\hat{\mathbf{r}}\frac{\partial}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial}{\partial \theta}\right]\left[\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right] = -\left[\hat{\mathbf{r}}\frac{\partial}{\partial r}\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial}{\partial \theta}\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\left[\hat{\mathbf{r}}(2\cos\theta) + \hat{\boldsymbol{\theta}}\sin\theta\right] \quad (8.25)$$

From Eq. (8.25), we can write the radial (E_r) and tangential (E_θ) components of electric field $\vec{\mathbf{E}}$ at point P (see Fig. 8.5) as

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \quad (8.26)$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \quad (8.27)$$

The radial and tangential components of the electric field $\vec{\mathbf{E}}$ at point P are shown in Fig. 8.5. From the figure, note that the resultant electric field $\vec{\mathbf{E}}$ at point P is directed along PR and it makes an angle α with the (extended) line CP, i.e. the direction of the radial component E_r.

Thus, the magnitude of the electric field is given as

$$|\vec{\mathbf{E}}| = \sqrt{E_r^2 + E_\theta^2} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1} \quad (8.28)$$

To determine the direction of the resultant field $\vec{\mathbf{E}}$, we make use of Eqs. (8.26) and (8.27) and note from the geometry of Fig. 8.5:

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{\sin\theta}{2\cos\theta} = \frac{1}{2}\tan\theta \quad (8.29)$$

The advantage of using polar coordinates for obtaining expressions for potential and hence electric field at a point due to dipole can be understood on the basis of Eqs. (8.26) and (8.27). Refer to Fig. 8.5. If we take $\theta = 0$, then the point P will shift to a point along the axis of the dipole. For any such point, Eqs. (8.26) and (8.27) show that only the radial component will be present; the tangential component, E_θ will be zero because of the $\sin\theta$ term. So, the magnitude of the electric field due to the dipole at a point along its axis can be written as

$$|\vec{\mathbf{E}}| = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

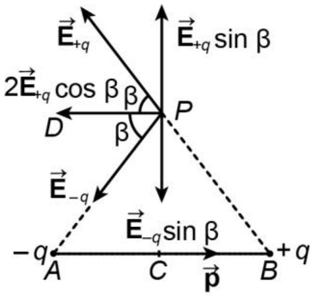


Fig. 8.6: **Direction of electric field at a point on the perpendicular bisector of dipole.**

And, Eq. (8.29) indicates that direction of the electric field will be along the axis of the dipole because, for $\theta = 0$, $\alpha = 0$ and α is the angle between the resultant electric field and the dipole axis. Thus, the electric field due to dipole at a point along its axis at a distance *r* from the mid point of dipole, such that *r* >> *a*, is given as

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{\mathbf{p}}}{r^3} \qquad (8.30)$$

Eq. (8.30) is the same as Eq. (i) of Example 5.4, Unit 5 obtained for electric field due to dipole at a point along its axis.

For $\theta = \pi/2$, point *P* will be a point on the perpendicular bisector of the dipole axis (Fig. 8.6). In this case, the radial component of electric field will be zero as $\cos\theta = 0$ in Eq. (8.26). Thus, the magnitude of the electric field at such a point will have contribution only from the tangential compound, E_θ. Thus, we can write Eq. (8.28):

$$|\vec{\mathbf{E}}| = \sqrt{E_\theta^{\,2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \qquad (8.31)$$

We cannot use Eq. (8.29) for determining the direction of $\vec{\mathbf{E}}$ at a point on the bisector of the dipole because, $\tan\theta = \tan(\pi/2)$ is not defined. We can, however, make use of the fact that the value of potential at every point on the bisector is zero [see Eq. (8.24)]. This means that no work is done in moving a charge along the bisector of a dipole. Further, the work done in moving a unit charge by distance $d\vec{\mathbf{l}}$ is given as $\vec{\mathbf{E}}.d\vec{\mathbf{l}}$. Thus, $\vec{\mathbf{E}}.d\vec{\mathbf{l}} = 0$ implies that field $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\mathbf{l}}$, the direction of the perpendicular bisector. Now, to determine whether $\vec{\mathbf{E}}$ is along or opposite to $\vec{\mathbf{p}}$, refer to Fig. 8.6 which shows the electric field due to the dipole at point *P*. The components $E_{+q}\sin\beta$ and $E_{-q}\sin\beta$ of $\vec{\mathbf{E}}_{+q}$ and $\vec{\mathbf{E}}_{-q}$ respectively will cancel each other. However, the component $E_{+q}\cos\beta$ and $E_{-q}\cos\beta$ will add up along *PD*, a direction perpendicular to the bisector and opposite to the direction of dipole moment $\vec{\mathbf{p}}$. Thus, the electric field due to the dipole at any point on its perpendicular bisector is anti-parallel to $\vec{\mathbf{p}}$. Thus, we can write

$$\vec{\mathbf{E}} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{\mathbf{p}}}{r^3} \qquad (8.32)$$

Eq. (8.32) is same as Eq. (i) of Example 5.5, Unit 5 obtained by computing electric fields due to dipole at a point on its bisector.

We mentioned in the beginning of this section that understanding the behaviour of an electric dipole under the influence of an external electric field is very useful in analysing the effect of electric field on dielectric materials.

So, let us now study the effect of electric field on a dipole.

8.6 DIPOLE IN AN ELECTRIC FIELD

Let us consider a dipole of length 2*a* in a uniform external electric field $\vec{\mathbf{E}}$ as shown in Fig. 8.7. A uniform electric field means that its magnitude and direction are the same everywhere. Let the dipole moment vector $\vec{\mathbf{p}}\,(=2q\vec{\mathbf{a}})$ makes an angle θ with the electric field, $\vec{\mathbf{E}}$.

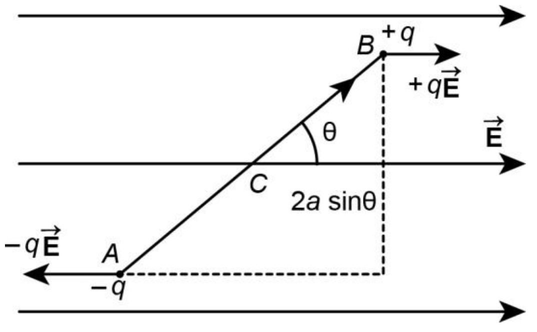


Fig. 8.7: **Torque experienced by a dipole placed in a uniform electric field $\vec{\mathbf{E}}$.**

Due to the external electric field $\vec{\mathbf{E}}$, the charge $+q$ of the dipole experiences a force $\vec{\mathbf{F}}_+ = q\vec{\mathbf{E}}$ while the charge $-q$ experiences an equal and opposite force $\vec{\mathbf{F}}_- = -q\vec{\mathbf{E}}$. Since the field is uniform, the net force $\vec{\mathbf{F}}$ on the dipole is zero, i.e.,

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_+ + \vec{\mathbf{F}}_- = q\vec{\mathbf{E}} - q\vec{\mathbf{E}} = \vec{\mathbf{0}} \tag{8.33}$$

As the net force on the dipole is zero, the centre of mass of the dipole is not accelerated, that is, there is no effect on its translational motion.

You may, therefore, ask: Does it mean that the external electric field has no effect on the dipole? No, it is not so. The dipole still experiences a **turning effect** due to the torque about its centre of mass *C*. This turning effect arises because the two equal and opposite forces, which cancel each other as free vectors, are acting at different points. That is, the forces experienced by charges $+q$ and $-q$ of the dipole do not have same line of action and hence they provide a turning effect.

From Fig. 8.7, note that the centre of mass C of the dipole is at a distance *a* from each charge of the dipole. Thus, we can write the magnitude of net torque $\vec{\boldsymbol{\tau}}$ as

$$\tau = qEa\sin\theta + qEa\sin\theta = 2qaE\sin\theta = pE\sin\theta$$

The above expression can be written in vector form as

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}} \tag{8.34}$$

You know that the unit of torque is Newton metre (N m). The direction of the torque is obtained from right-hand rule (refer Sec. 12.3, Unit 12 of 1st semester course BPHCT-131) and is along $-\vec{\mathbf{k}}$ if the electric field $\vec{\mathbf{E}}$ and dipole are in the *xy*-plane.

Under the action of the torque, the dipole tend to align itself along the field direction with dipole moment vector $\vec{\mathbf{p}}$ parallel to $\vec{\mathbf{E}}$ vector. So, when $\vec{\mathbf{p}}$ is aligned along $\vec{\mathbf{E}}$, the torque on the dipole is zero because for $\theta = 0°, \sin\theta = 0$. **The system (that is, the dipole) is in stable equilibrium when $\vec{\mathbf{p}}$ is aligned with $\vec{\mathbf{E}}$.**

From Fig. 8.7, we note that the torque acting on the dipole tends to align it along $\vec{\mathbf{E}}$. So, the rotation of the dipole is in the clockwise direction.

Potential Energy of an Electric Dipole

Now, let us ask ourselves: What will happen to the potential energy of the dipole if it is rotated from its stable position? Whenever the dipole is rotated from its stable configuration ($\vec{\mathbf{p}}$ parallel to $\vec{\mathbf{E}}$) external work must be done. This external work is stored as potential energy of the dipole.

To obtain an expression for the potential energy of a dipole we need to calculate the work done by the electric field to rotate the dipole from some initial value of θ to final value of θ. The work done, in terms of torque and angular displacement $d\theta$ is

$$dW = -\tau\, d\theta$$

$$= -pE \sin\theta\, d\theta \qquad (8.35)$$

The negative sign in Eq. (8.35) indicates that the torque opposes any increase in θ. Thus, the work done by $\vec{\mathbf{E}}$ to rotate the dipole from an angle θ_0 to θ is

$$W = \int_{\theta_0}^{\theta} dW$$

$$= \int_{\theta_0}^{\theta} (-pE \sin\theta)\, d\theta$$

$$= pE(\cos\theta - \cos\theta_0) \qquad (8.36)$$

The change in potential energy ΔU of the dipole is the negative of the work done by the electric field. Thus, we have

$$\Delta U = U_f - U_i = -W = -pE(\cos\theta - \cos\theta_0) \qquad (8.37)$$

Note that $U_i = -pE\cos\theta_0$ is the potential energy at the initial or reference orientation of the dipole. As in the case of point charge for which we define potential energy to be zero at infinity, we need to define the orientation of dipole with respect to $\vec{\mathbf{E}}$ for which we can consider its potential energy to be zero. It turns out that when the dipole is aligned perpendicular to $\vec{\mathbf{E}}$, that is, when $\theta = \pi/2$ in Fig. 8.7, potential energy of the dipole can be taken to be zero.

Thus, the initial potential energy $U_i = 0$. So, we can write Eq. (8.37) as

$$U = -pE\cos\theta$$

or

$$U = -\vec{\mathbf{p}}.\vec{\mathbf{E}} \qquad (8.38)$$

Eq. (8.38) gives the potential energy of a dipole in a uniform electric field. It shows that the potential energy is minimum (most negative) when the dipole is aligned along the field direction (i.e., $\theta = 0°$), and is maximum (most positive) when it is aligned opposite to the field direction (i.e., $\theta = 180°$).

Let us now sum up what we have learnt in this unit.

8.7 SUMMARY

Concept	Description
Work done and line integral	■ The work W' done by the electric field $\vec{\mathbf{E}}$ in moving a unit positive charge from point *a* to *b*, is equal to the line integral of $\vec{\mathbf{E}}$: $$W' = \int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}}$$
Path independence	■ The work done, that is, the line integral of $\vec{\mathbf{E}}$, in moving a unit positive charge from one point to another in an electric field is independent of the path between the two points.
Electrostatic potential energy	■ The difference in electrostatic potential energy of a charge between two points *a* and *b* in an electric field is equal to the negative of the work done by the field in moving the charge from *a* to *b*: $$\Delta U = U_b - U_a = -W'_{ab}$$
Electric potential as line integral	■ The negative of the work W' done by the electric field in carrying a unit positive charge from infinity to some point at distance *r* from the charge giving rise to the field is defined as the electric potential *V* at that point: $$V = -W' = -\int_{-\infty}^{r} \vec{\mathrm{E}}.d\vec{\mathbf{l}}$$
Electric potential	■ The electric potential *V* at a point at a distance *r* from a point charge *Q* is given as: $$V = \frac{Q}{4\pi\varepsilon_0 r}$$
Relation between V and $\vec{\mathbf{E}}$	■ The electric field $\vec{\mathbf{E}}$ at a point is the negative gradient of the electric potential *V* at that point: $$\vec{\mathbf{E}} = -\vec{\nabla}V$$
Electric potential due to dipole	■ The electric potential at any point *P*, at a distance *r* from the midpoint of the dipole, on a line which makes an angle θ with the axis of the dipole is given by: $$V = \frac{\vec{\mathbf{p}}.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$ where $\hat{\mathbf{r}}$ is a unit vector from the centre of dipole to the point *P* where potential is to be determined and $\vec{\mathbf{p}}(=2q\vec{\mathbf{a}})$ is the dipole moment vector.
Torque on a dipole in electric field	■ Electric dipole in a uniform electric field experiences a turning effect. The torque $\vec{\boldsymbol{\tau}}$ experienced by the dipole is given by: $$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$
Electrostatic potential energy of the dipole	■ The electrostatic potential energy of an electric dipole in an electric field is given by $U = -\vec{\mathbf{p}}.\vec{\mathbf{E}}$. Its value is minimum when dipole moment vector $\vec{\mathbf{p}}$ is parallel to electric field $\vec{\mathbf{E}}$ and maximum when dipole moment vector $\vec{\mathbf{p}}$ is anti-parallel to $\vec{\mathbf{E}}$.

8.8 TERMINAL QUESTIONS

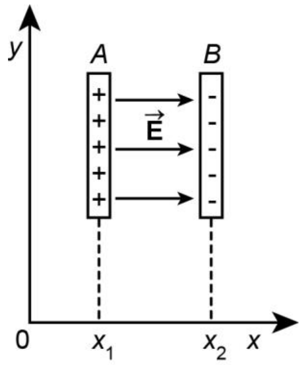


Fig. 8.8: **Diagram for TQ 2.**

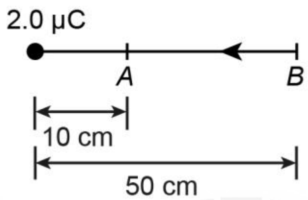


Fig. 8.9: **Diagram for TQ 3.**

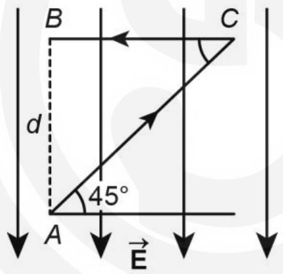


Fig. 8.10: **Diagram for TQ 4.**

1. Show that the line integral of the electric field $\vec{\mathbf{E}}$ over a closed path is equal to zero.

2. Show that, in a pair of oppositely charged plane parallel plates, the electric field $\vec{\mathbf{E}}$ is equal to the potential difference between the plates divided by their separation. You may assume that the electric field is confined between the plates as shown in Fig. 8.8.

3. Calculate the electric potential at two points *A* and *B* at distances of 10 cm and 50 cm from a charge $2.0\,\mu\text{C}$ as shown in Fig. 8.9. Also calculate the work done in bringing a charge of $0.05\,\mu\text{C}$ from point *B to A.*

4. Calculate the potential difference between points *A* and *B* assuming that a test charge q_0 is moved without acceleration from *A* to *B* along the path shown in Fig. 8.10.

5. Mark the following statements as True or False:

   a) If the electric field is zero in some region of space, the electric potential must also be zero in that region.

   b) If the electric potential is zero at a point, the electric field must also be zero at that point.

   c) The value of potential can be chosen to be zero at any convenient point.

   d) Electric field at a point is negative of the gradient of electric potential at that point.

   e) The electric field and potential due to an electric dipole decrease much faster with distance as compared to a point charge.

6. A uniform electric field of $3\times10^3\,\text{NC}^{-1}$ is in the positive *x*-direction. A positive point charge 2 μC is released from rest at the origin.

   a) Calculate the potential difference *V* (5 m) – *V* (0).

   b) What is the change in electrostatic potential energy of the charge when it is moved from $x = 0$ to $x = 5\,\text{m}$?

   c) Calculate the kinetic energy of the charge when it is at $x = 5\,\text{m}$.

   d) Calculate the value of the potential *V*(*x*) if electric potential is chosen to be zero at i) $x = 0$ and ii) $x = 1\,\text{m}$.

7. A uniform electric field is in the negative *x*-direction. Two points *a* and *b* are at $x = 3\,\text{m}$ and $x = 7\,\text{m}$, respectively.

   a) Is the potential difference $V_b - V_a$ positive or negative?

   b) If the value of the potential difference of $(V_b - V_a)$ is $10^4\,\text{V}$, calculate the magnitude of the electric field.

8. How much work needs to be done to transport an electron from the positive terminal of a 12 V battery to its negative terminal?

8.9 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. Let the electric field be $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$ be the element of path length. Since both $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$ are parallel, the angle θ between the two vectors is zero. Thus, using Eq. (8.3), we write the work done in moving a unit positive charge as

$$W' = \int \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int E(\cos\theta)dl = \int E(\cos 0^\circ)dl = \int Edl = El$$

2. The work done by a constant force $\vec{\mathbf{F}}$ in moving a particle through displacement $\vec{\mathbf{l}}$ is $W = \vec{\mathbf{F}}.\vec{\mathbf{l}} = Fl\cos\theta$ where θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{l}}$.

   i) As per the problem, the electron is moving along the direction of $\vec{\mathbf{E}}$. So, $\theta = 0°$. Thus, the work done is

$$W = (qE)l\cos 0°$$

$$= (-1.6\times 10^{-19}\,\text{C})\times(200\,\text{NC}^{-1})\times(30\,\text{m}) = -9.6\times 10^{-16}\,\text{J}$$

   The change in electrostatic potential energy of the electron is

$$\Delta U = U_f - U_i = -W = 9.6\times 10^{-16}\,\text{J}$$

   Thus, the electrostatic potential energy of electron increases as it moves along the direction of the electric field.

   ii) As per the problem, the proton is moving along $\vec{\mathbf{E}}$. So, $\theta = 0°$. Thus, we have

$$W = qEl\cos 0°$$

$$= (1.6\times 10^{-19}\,\text{C})\times(200\,\text{NC}^{-1})\times(30\,\text{m}) = 9.6\times 10^{-16}\,\text{J}$$

   So, the change in electrostatic potential energy of proton is

$$\Delta U = U_f - U_i = -W = -9.6\times 10^{-16}\,\text{J}$$

   Thus, we find that the electrostatic potential energy of proton decreases as it moves along the direction of the electric field.

3. a) From Eq. (8.14), we have the electric potential due to a point charge

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r}$$

   i) So, the electric potential at point X is

$$V_X = [(9\times 10^9\,\text{Nm}^2\text{C}^{-2})\times(7\times 10^{-6}\,\text{C})]/(8\,\text{m})$$

$$= 7.87\times 10^3\,\text{V} = 8\times 10^3\,\text{V}$$

   up to one significant digit. And the electric potential at point Y is

$$V_Y = [(9\times 10^9\,\text{Nm}^2\text{C}^{-2})\times(7\times 10^{-6}\,\text{C})]/(12\,\text{m})$$

$$= 5.25\times 10^3\,\text{V} = 5\times 10^3\,\text{V}$$

   Thus, the potential difference between the points X and Y is

$$V_X - V_Y = (7.87\times 10^3\,\text{V} - 5.25\times 10^3\,\text{V}) = 2.62\times 10^3\,\text{V} = 3\times 10^3\,\text{V}$$

ii) When the point charge $+7\,\mu C$ is replaced by $-7\,\mu C$, we have electric potential at point X

$$V_X = [(9\times10^9\,Nm^2C^{-2})\times(-7\times10^{-6}\,C)]/(8\,m) = -7.87\times10^3\,V$$

$$V_Y = [(9\times10^9\,Nm^2C^{-2})\times(-7\times10^{-6}\,C)]/(12\,m) = -5.25\times10^3\,V$$

Thus, the potential difference between the points X and Y is

$$V_X - V_Y = -7.87\times10^3\,V - (-5.25\times10^3\,V) = -2.62\times10^3\,V = -3\times10^3\,V$$

iii) The work done in moving the charge $+3\,\mu C$ from infinity to point X is

$$W = qV_X = (3\times10^{-6}C)\times(7.87\times10^3\,V) = 2.36\times10^{-2}\,J = 2\times10^{-2}\,J$$

b) Charge on the nucleus $Q = Ze = 79\times1.6\times10^{-19}\,C$ and $r = 6.6\times10^{-15}$ m

Thus, from Eq. (8.14), we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{(9\times10^9\,Nm^2C^{-2})\times(79\times1.6\times10^{-19}\,C)}{6.6\times10^{-15}\,m}$$

$$= 1.7\times10^7\,Nm\,C^{-1} = 1.7\times10^7\,V \quad (\because\ NmC^{-1} = JC^{-1})$$

4. Let the point P be at a distance x from the point charge $+q$ and the electric potential at P due to the two charges be zero (Fig. 8.11). The electric potential at point P is

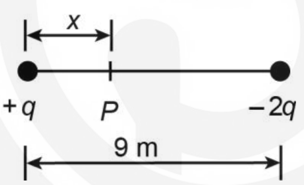


Fig. 8.11: **Diagram for answer to SAQ 4.**

$$V = \frac{1}{4\pi\varepsilon_0 r}\left[\frac{q}{x} + \frac{(-2q)}{(9-x)}\right]$$

Since $V = 0$ at P, we have

$$\frac{q}{x} = \frac{2q}{(9-x)} \Rightarrow q(9-x) = 2qx \Rightarrow 9q = 3qx \Rightarrow x = 3\,m$$

5. The relation between $\vec{\mathbf{E}}$ and V is:

$$\vec{\mathbf{E}} = -\left[\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right]$$

As per the problem, $V = x(y^2 - 4x^2)$

Thus,

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial x}[xy^2 - 4x^3] = y^2 - 12x^2; \quad \frac{\partial V}{\partial y} = \frac{\partial V}{\partial y}[xy^2 - 4x^3] = 2xy$$

$$\frac{\partial V}{\partial z} = \frac{\partial V}{\partial z}[xy^2 - 4x^3] = 0$$

So, $\quad \vec{E} = -[\hat{\mathbf{i}}(y^2 - 12x^2) + \hat{\mathbf{j}}(2xy) + \hat{\mathbf{k}}(0)] = (12x^2 - y^2)\hat{\mathbf{i}} - 2xy\,\hat{\mathbf{j}}$

6. The electric potential due to an electric dipole is given by Eq. (8.24):

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

So, for point P_1 we have

$$(V)_{P_1} = \frac{p\cos\theta}{4\pi\varepsilon_0 (0.40\,\text{m})^2} \Rightarrow \frac{p\cos\theta}{4\pi\varepsilon_0} = (60\,\text{V}) \times (0.40\,\text{m})^2 \quad \text{(i)}$$

For point P_2, we can write using Eq. (i)

$$(V)_{P_2} = \frac{p\cos\theta}{4\pi\varepsilon_0 \times (0.60\,\text{m})^2} = \frac{(60\,\text{V}) \times (0.40\,\text{m})^2}{(0.60\,\text{m})^2} = 27\,\text{V}$$

Terminal Questions

1. Let us consider a closed path starting from and ending at a as shown in Fig. 8.12. Let b be some point on this closed path. A unit positive charge can be moved between points a and b through two paths: L and L'. If V_a and V_b are potentials at a and b, respectively, we can write

$$-\int_{a \text{ along } L}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = V_b - V_a \quad \text{(i)}$$

also

$$-\int_{a \text{ along } L'}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = V_b - V_a \quad \text{(ii)}$$

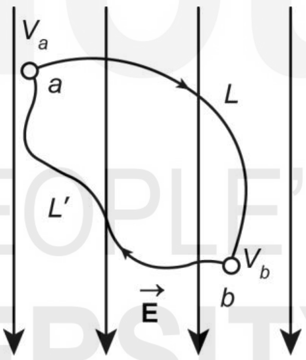


Fig. 8.12: **Diagram for answer to TQ 1.**

Now, by changing the limits of integration, we can write Eq. (ii) as:

$$-\int_{a \text{ along } L'}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{b \text{ along } L'}^{a} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = V_a - V_b \quad \text{(iii)}$$

Adding Eqs. (i) and (ii) and making use of Eq. (iii), we can write

$$-\int_{a \text{ along } L}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} - \int_{a \text{ along } L'}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = -\int_{a \text{ along } L}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} + \int_{b \text{ along } L'}^{a} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = V_b - V_a + V_a - V_b = 0$$

That is, along a closed path, the line integral of the electric field is equal to zero.

Alternative method: We can also use the fact that the line integral of electric field is independent of the path. Thus, we can write

$$\int_{a \text{ along } L}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_{a \text{ along } L'}^{b} \vec{\mathbf{E}}.d\vec{\mathbf{l}}$$

or

$$\underset{\text{along } L}{\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}}} - \underset{\text{along } L'}{\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}}} = 0 \Rightarrow \underset{\text{along } L}{\int_a^b \vec{\mathbf{E}}.d\vec{\mathbf{l}}} + \underset{\text{along } L'}{\int_b^a \vec{\mathbf{E}}.d\vec{\mathbf{l}}} = 0$$

Note that $(L + L')$ implies a closed path between points *a* and *b* in Fig. 8.12.

2. Let *A* and *B* be two oppositely charged plates separated by a distance *d* (Fig. 8.8). Let $\vec{\mathbf{E}}$ be the uniform electric field between the two plates. Then, the potential difference between the two plates can be written as [Eq. (8.11)]:

$$-\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{l}} = (V_B - V_A)$$

where V_A and V_B are the potentials at the plates *A* and *B* respectively.

In the present case, writing $\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{l}}$ *as* $\int_{x_1}^{x_2} \vec{\mathbf{E}}.\hat{\mathbf{i}}\,dx$, and noting that both $\vec{\mathbf{E}}$ and $\hat{\mathbf{i}}\,dx$ are parallel, we can write

$$V_B - V_A = -\int_{x_1}^{x_2} \vec{\mathbf{E}}.\hat{\mathbf{i}}\,dx = -E\,[x]_{x_1}^{x_2} = -E\,(x_2 - x_1) = -E\,d$$

That is, the magnitude of the electric field between two oppositely charged parallel plates is equal to the difference of potential between them divided by their separation.

3. The electric potential *V* at a point distant *r* from a charge *Q* is given by Eq. (8.14):

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

As per the problem, $Q = 2.0\,\mu\text{C} = 2.0 \times 10^{-6}\,\text{C}, 1/(4\pi\,\varepsilon_0) = 9 \times 10^9\,\text{Nm}^2\text{C}^{-2}$ and $r = 0.10\,\text{m}$ and $0.50\,\text{m}$. Substituting these values, we get

$$V_A = (9 \times 10^9\,\text{Nm}^2\,\text{C}^{-2}) \times \frac{2.0 \times 10^{-6}\,\text{C}}{0.10\,\text{m}} = 1.8 \times 10^5\,\text{V}$$

$$V_B = (9 \times 10^9\,\text{Nm}^2\,\text{C}^{-2}) \frac{2.0 \times 10^{-6}\,\text{C}}{0.50\,\text{m}} = 0.36 \times 10^5\,\text{V}$$

Work done in moving charge 0.5 μC from point *B* to *A* is $W = q\,(V_A - V_B)$, where $q = 0.05 \times 10^{-6}\,\text{C}$,

$\therefore \quad W = (0.05 \times 10^{-6}\,\text{C})\,(1.8 \times 10^5 - 0.36 \times 10^5)\,\text{V} = 7.2 \times 10^{-3}\,\text{J}$

4. We can write

$$V_B - V_A = (V_B - V_C) + (V_C - V_A) = -\int_C^B \vec{\mathbf{E}}.d\vec{\mathbf{l}} - \int_A^C \vec{\mathbf{E}}.d\vec{\mathbf{l}}$$

For path C to B, $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}}$ are perpendicular to each other. Therefore,

$$\int_C^B \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \left|\vec{\mathbf{E}}\right| \, d\vec{\mathbf{l}} \cos 90° = 0$$

For path A to C, the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{l}} = 135°$. Thus,

$$\int_A^C \vec{\mathbf{E}}.d\vec{\mathbf{l}} = \int_A^C Edl \cos 135°$$

$$= -\frac{E}{\sqrt{2}} \int_A^C dl = -\frac{E}{\sqrt{2}}(AC) = -\frac{E}{\sqrt{2}}\sqrt{2}\, d = -Ed$$

since $AC = d/\cos 45° = \sqrt{2}\, d$. Thus, we have

$$V_B - V_A = -\int_A^C \vec{\mathbf{E}}.d\vec{\mathbf{l}} = Ed$$

You may note that this is also the value obtained via the direct path from A to B (shown in Fig. 8.10 by dotted lines.)

5. a) False

   b) False (See Eq. (8.24) and (8.32) for any point on the perpendicular bisector of an electric dipole.)

   c) True

   d) True

   e) True

6. a) $V(5\,\text{m}) - V(0) = -\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{l}} = -\int_0^{5\text{m}} Edl = -(3 \times 10^3\,\text{NC}^{-1}) \times (5\,\text{m}) = -15 \times 10^3\,\text{V}$

   b) The difference in electrostatic potential energy and potential difference is related by

   $$\Delta U = q\Delta V = (2 \times 10^{-6}\,\text{C}) \times (-15 \times 10^3\,\text{V}) = -3.0 \times 10^{-2}\,\text{J}$$

   c) From the conservation of energy, we know that

   $$\Delta U + \Delta K = 0$$

   where ΔU is the change in potential energy and ΔK is the change in kinetic energy. So,

   $$[K(5\,\text{m}) - K(0)] + \Delta U = 0 \Rightarrow K(5\,\text{m}) = -\Delta U = 3.0 \times 10^{-2}\,\text{J}$$

   d) We know that for uniform electric field, $V = Ed$. So

   $$V(x) - V(0) = -E_x(x - x_0) = -(3 \times 10^3\,\text{NC}^{-1})(x - x_0)$$

   i) for $V(0) = 0$

   $$V(x) = -(3 \times 10^3\,\text{NC}^{-1})\,x$$

ii) for $V(1\text{m}) = 0$

$$V(x) - 0 = -(3 \times 10^3 \text{ NC}^{-1}) \times (x - 1)$$

or $$V(x) = 3 \times 10^3 \text{ V} - (3 \times 10^3 \text{ Vm}^{-1})\, x$$

7. Note that the electric field is along negative *x*-direction. So, the relation between potential and electric field can be written as

$$E_x = \frac{dV}{dx}$$

So, the value of potential will be higher for larger value of *x*. So, $(V_B - V_A)$ is positive.

Further, to determine the magnitude of E_x for $V_B - V_A = 10^4$ V, we can write

$$E_x = \frac{\Delta V}{\Delta x} = \frac{V_B - V_A}{(7\text{ m} - 4\text{ m})} = \frac{10^4 \text{ V}}{4\text{ m}} = 2.5 \times 10^3 \text{ Vm}^{-1}$$

8. In going from the positive terminal of a battery to the negative terminal, the electron (a negatively charged particle) moves from a point at a higher potential to a point at a lower potential. Thus, if *A* and *B* are, respectively, the positive and negative terminals of the battery, we have

$$V_B - V_A = -12 \text{ V}$$

Thus, the work done in moving an electron from the positive to the negative terminal is

$$W = q(V_B - V_A) = (-1.6 \times 10^{-19} \text{ C}) \times (-12 \text{ V}) = 1.92 \times 10^{-18} \text{ J}$$