**How can sitting inside a closed conducting surface such as a car prevent you from being struck by lightning? Find the answer in this unit!**

UNIT 7

# APPLICATIONS OF GAUSS'S LAW

## Structure



## STUDY GUIDE

In Unit 6, you have studied the concept of electric flux and Gauss's law. You have learnt how to apply Gauss's law to discrete point charges and continuous charge distributions that are spherically symmetric such as a uniformly charged sphere and thin spherical shell. In this unit, you will learn applications of Gauss's law to some more continuous charge distributions having cylindrical and planar symmetry such as a uniform infinite line charge and a plane sheet of charge. You will determine the electric fields due to a uniformly charged infinite wire, a uniform cylindrical charge distribution and an infinite sheet of charge. You will also learn of its application to an isolated charged conductor.

You will learn how to choose appropriate Gaussian surfaces to solve the surface integrals involved in each case. Revise the divergence theorem that you have learnt in Unit 4. You should also revise the methods of solving surface and volume integrals to be able to master the concepts of this unit. Try to solve the Examples, SAQs and Terminal questions given in this unit on your own.

***"It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment."***

***Carl F. Gauss***

## 7.1 INTRODUCTION

In Unit 6, you have learnt the concept of electric flux and studied Gauss's law. You have also learnt how to apply Gauss's law to obtain the electric flux and electric field due to discrete charges. You have applied the law to continuous charge distributions that are spherically symmetric and uniformly charged such as a uniformly charged sphere and thin spherical shell. You have learnt that the Gaussian surface for such charge distributions is spherical and concentric with them. It also passes through the point at which the electric field is to be determined.

In this unit, you will first learn how to apply Gauss's law to charge distributions having cylindrical symmetry such as uniform line charge and uniformly charged cylinder (Sec. 7.2). You will begin by learning the concept of cylindrical symmetry. Then we will explain why Gauss's law is useful for determining the electric fields due to cylindrically symmetric charge distributions. With this understanding, you can learn how to apply Gauss's law to determine the electric fields due to an infinite uniform line charge and infinite uniformly charged cylinder.

In Sec. 7.3, you will learn how to apply Gauss's law to calculate the electric field due to an infinite uniformly charged sheet that possesses planar symmetry. Once again, we will explain what planar symmetry is and how Gauss's law is useful for determining the electric fields due to charge distributions having planar symmetry.

The applications of Gauss's law described in Secs. 7.2 and 7.3 find use in computing the capacitance of coaxial capacitors and parallel plate capacitors as you will learn in Unit 11 of the next block. As you may know, such capacitors are very commonly used around us, for example, in electronic appliances like the TV and computers, and power storage systems, etc. Finally, in Sec. 7.4, we apply Gauss's law to an isolated charged solid conductor and a conductor with a cavity. This too has many interesting applications in real life. One of these is shown in the picture on the first page of this unit.

In the next two units, we introduce the concept of electric potential and its relation with the electric field. You will learn another way of calculating electric fields and electrostatic forces using the concept of electric potential.

### Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ apply Gauss's law to determine the electric field due to cylindrically symmetric charge distributions such as a uniform infinite line charge and an infinite uniformly charged cylinder;
- ❖ determine the electric field due to an infinite uniform plane sheet of charge using Gauss's law; and
- ❖ use Gauss's law to explain why the electric field inside an isolated charged conductor is zero and why the charge on it is distributed entirely on its surface.

# 7.2 ELECTRIC FIELD DUE TO CYLINDRICALLY SYMMETRIC CHARGE DISTRIBUTIONS

We have said in the introduction of this unit that in this section we will determine the electric field due to charge distributions that possess cylindrical symmetry such as a line charge and a charged cylinder. You may like to ask:

- **What is cylindrical symmetry**?
- **How is Gauss's law useful for a charge distribution that possesses cylindrical symmetry**?

Let us begin our discussion by answering these two questions.

## 7.2.1 Gauss's Law and Cylindrically Symmetric Charge Distributions

Let us answer the first question and define **cylindrical symmetry**.

A charge distribution (or any object) is said to possess **cylindrical symmetry** if it remains **unchanged** (or is **invariant**) when it is

- moved along its axis (*AB* in Fig. 7.1a and *CD* in Fig. 7.1b), that is, the line running through its core (translational symmetry);
- rotated around its axis (rotational symmetry);
- rotated by 180° around any axis perpendicular to its axis, (*PQ* in Fig. 7.1a and *RS* in Fig. 7.1b), (180° rotational symmetry);
- reflected across any plane passing through its axis (reflection symmetry);
- reflected across any plane perpendicular to its axis (reflection symmetry).

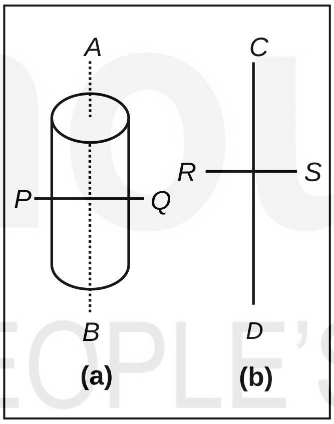



**Fig. 7.1**: **a) The axis *AB* (dotted line) of a cylinder; b) for a line or a wire, the axis *CD* lies on the line/wire itself.**

Try to apply the above transformations to any cylindrical object around you such as a can or a water pipe. Verify that it possesses cylindrical symmetry before studying further. An infinite line or wire (like the axis of an infinite cylinder) *also* possesses cylindrical symmetry (Fig. 7.1b).

Let us now answer the second question and explain **how Gauss's law is useful** for determining the electric field due to a cylindrically symmetric charge distribution.

While studying Secs. 6.4 to 6.6 of Unit 6, you would have noted that due to the choice of the spherical Gaussian surface enclosing the charge distribution, the calculations became very simple for two reasons:

- the electric field was directed parallel to the area vector for a surface element on the Gaussian surface so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS$; and
- the magnitude $E$ of the electric field was the same at all points on the Gaussian surface so that it could be treated as constant and taken out of the surface integral.

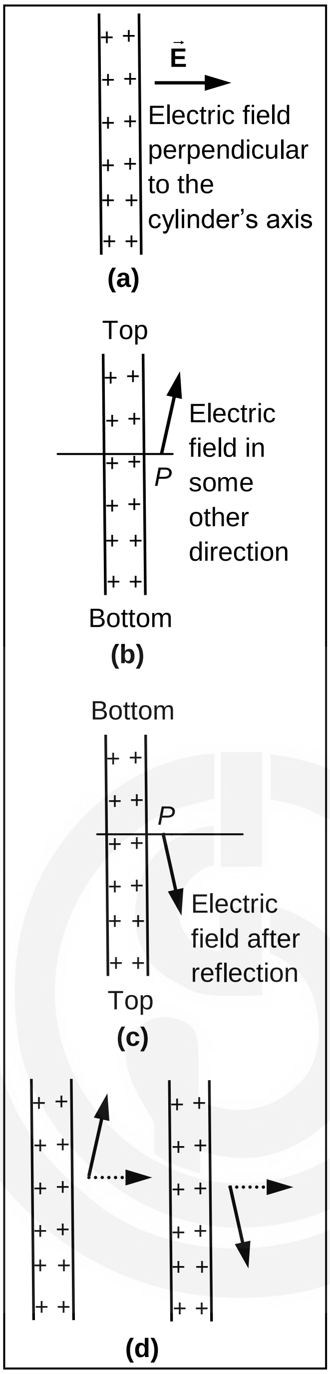



**Fig. 7.2**: **a) The direction of electric field $\vec{E}$ due to a section of an infinite charged cylinder is perpendicular to its axis; b) electric field in some other direction; c) reflected electric field; d) the directions of electric fields are different at the same point for the same charge distribution, which is not possible. So, $\vec{E}$ can only be in the direction shown by dotted arrows.**

Let us now ask: What is the **direction of the electric field** at any point due to a cylindrical charge distribution?

Consider Fig. 7.2a showing a small section of an infinite cylinder carrying positive charge (it could also be a charged infinite wire). The direction of the electric field of the charge distribution is perpendicular to its axis, and the electric field is directed radially outward from the axis for positively charged cylinder (Fig. 7.2a). For a negative cylindrical charge distribution, it will be directed radially inward and perpendicular to its axis. You may ask: **Why**?

To answer this question, suppose that the electric field due to the cylindrical charge distribution at some point *P* is directed in some other direction as shown in Fig. 7.2b. Note that we have arbitrarily labelled one end of this section of the cylinder as 'top' and the other one as 'bottom' just to show what happens when it is reflected.

Now let us reflect this cylindrical charge distribution about a horizontal line perpendicular to its axis and passing through *P*. So the 'top' of the section is now its 'bottom' and the 'bottom', its 'top' (Fig. 7.2c).

What is the direction of the electric field after the cylindrical charge distribution is reflected? After reflection, the direction of the electric field becomes as shown in Fig. 7.2c because the electric field is also reflected in the same manner.

Now compare Figs. 7.2b and 7.2c by putting them alongside each other as in Fig. 7.2d. What do you find? You can see that the charge distribution remains the same after reflection but the electric fields are different. (The labels 'top' and 'bottom' were only for our convenience. Otherwise, we cannot tell the difference.)

This is a contradiction: **How can there be different electric fields at the same point for the same charge distribution**? If the charge distribution remains unchanged, the electric field also has to be the same. If it is not so, there must be some mistake.

Now we ask: **What is the direction of the electric field that will not lead to such a contradiction**? From Fig. 7.2d, you can see that if the electric field (shown by dotted arrows) were in the direction perpendicular to the cylindrical axis, it would remain the same under this symmetry operation.

You can check it for all other symmetry operations on the cylinder. This is how we conclude from symmetry considerations that **the direction of the electric field due to a cylindrical charge distribution at a point can only be perpendicular to its axis**.

**It points outward, for a positive charge distribution and inward, for a negative charge distribution**.

Now you may like to know: **What does the magnitude of the electric field of a charge distribution having cylindrical symmetry depend on**?

The answer is: It depends only on the perpendicular distance, say *r*, of the point from the cylinder's axis. **Why is it so**?

Suppose the **magnitude of the electric field due to the cylindrical charge distribution at any point varied with the angular coordinates of the point**. Then it would have different values at different points, say, $P$ and $Q$ situated at the **same** perpendicular distance from the axis (i.e., on the dotted cylindrical surface of the same radius $r$ in Fig. 7.3). But this is a contradiction. This is because due to cylindrical symmetry, the charged cylinder will look the same from all points on the cylindrical surface of radius $r$ (Fig. 7.3). So,

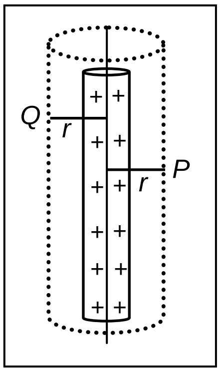



**Fig. 7.3**: **The magnitude of the electric field due to a cylindrical charge distribution at any point depends on its perpendicular distance *r* from the axis of symmetry. If it were not so, the magnitude of the electric field would be different at different points, (e.g., *P* and *Q*) on the same surface for the same charge distribution, which is incorrect.**

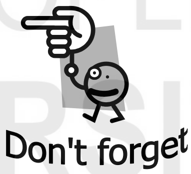
The **magnitude of the electric field cannot have different values at different points on a given cylindrical surface for the same cylindrical charge distribution**.

Therefore, at any point, it will depend only on the perpendicular distance of the point from the axis of the cylindrical charge distribution.

To conclude, due to cylindrical symmetry, the magnitude of the electric field due to a cylindrical charge distribution at any point depends only on the perpendicular distance of the point from the cylindrical axis. So, all points on the cylindrical surface of a given radius are equivalent as far as the magnitude of the electric field of any cylindrical charge distribution is concerned: it could be a line charge, charged wire or charged solid/hollow cylinder.

Then we can treat the magnitude of the electric field of such systems at a given cylindrical surface as constant and take it out of the surface integral. You will appreciate this point better in the next section.

To sum up, you must always remember the following for **any charge distribution having cylindrical symmetry**:




- **The electric field due to a charge distribution having cylindrical symmetry is directed perpendicular to its axis of symmetry.**
- **The magnitude of the electric field at any point depends only on its perpendicular distance from the axis of symmetry.**

So now can you quickly say what kind of Gaussian surface we should choose for a cylindrically symmetric charge distribution such as a line charge? The Gaussian surface should indeed be **cylindrical**. Why so?

As you have learnt just now, for a cylindrical Gaussian surface coaxial with the cylindrical charge distribution (charged line or cylinder), the electric field is normal to the surface at all points on it. You know that for any area element centred at a point on the Gaussian surface, the area vector $d\vec{\mathbf{S}}$ is directed normal to the surface (Fig. 7.4). So, the electric field $\vec{\mathbf{E}}$ at any point due to the cylindrically symmetric charge distribution will be parallel to the area vector $d\vec{\mathbf{S}}$ and, therefore

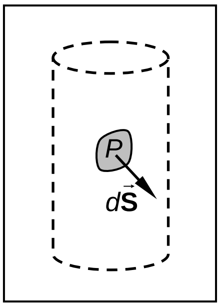



**Fig. 7.4**: **Area vector $d\vec{\mathbf{S}}$ for an element of area centred at any point *P* on a cylindrical Gaussian surface is normal to the surface.**

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS \qquad (7.1)$$

You have also learnt that the magnitude of the electric field at any point is the same everywhere on the cylindrical Gaussian surface passing through that point. So we can treat it as constant for that surface and take it out of the surface integral.

With this understanding of cylindrical symmetry of charge distributions, we can apply Gauss's law to a uniform infinite line charge.

## 7.2.2 Infinite Uniform Line Charge

Recall that you have calculated the electric field for an infinite line charge using Coulomb's law in Example 5.7 of Unit 5. You have learnt how to solve the lengthy integral involved in the calculation. Let us now apply Gauss's law to a similar problem. Consider an infinitely long wire carrying uniform linear charge density $\lambda$. Let us determine the electric field at a distance $r$ from the wire using Gauss's law.

Before studying further, you may like to quickly verify that the infinite line charge distribution has cylindrical symmetry by carrying out the symmetry operations on a wire. Let us now draw a Gaussian surface, i.e., the surface of a right circular cylinder of radius $r$ and length $L$ coaxial with the wire (Fig. 7.5).

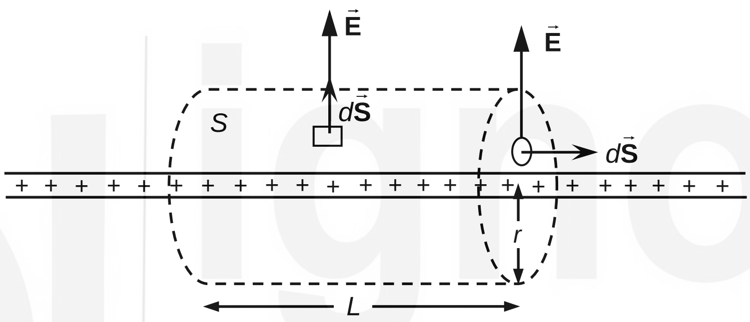



**Fig. 7.5**: **Applying Gauss's law to an infinite uniformly charged wire carrying positive charge. The Gaussian surface is cylindrical having length *L* and radius *r*. It encloses a section of the charged wire.**

What is the **magnitude** of the electric field at any point on the cylindrical Gaussian surface? You have learnt in Sec. 7.2.1 that due to cylindrical symmetry, it would be the same everywhere on the surface of the cylinder as it depends only on the perpendicular distance of the point from the wire's axis. As you can see in Fig. 7.5, this distance is just the radius ($r$) of the cylinder. So, it is the same for all points on the cylindrical surface of radius $r$ and can be treated as constant for that particular surface.

For a cylindrical surface, the direction of the electric field is normal to the surface at all points as shown in Fig. 7.5. For positively charged wire, the electric field is directed radially outwards from the wire's axis. If the charge on the wire were negative, the electric field would point inwards towards the wire's axis. Thus, $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are parallel to each other for each area element on the curved part of the cylinder's surface:

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS \qquad (7.2a)$$

The electric flux at all points through both circular ends of the cylinder is zero because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are perpendicular to each other on these ends (Fig. 7.5). Therefore, the product $\vec{\mathbf{E}}.d\vec{\mathbf{S}}$ is finite only for the curved part of the cylindrical surface.

Thus, from Gauss's law, we have

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oiint_S E\,dS = E\oiint_S dS \quad dS = E\,2\pi rL = \frac{Q_{encl}}{\varepsilon_0} \qquad (7.2b)$$

where we have taken $E$ out of the integral as it is constant on this Gaussian surface $S$. In Eq. (7.2b), we have also used the result that the area of the curved surface of a cylinder of radius $r$ and length $L$ is $2\pi rL$. So from Eq. (7.2b), we have

$$E\,2\pi rL = \frac{Q_{encl}}{\varepsilon_0}$$

or

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0\, r\, L} \qquad (7.2c)$$

For the uniform line charge density $\lambda$, the charge enclosed by the cylinder of length $L$ is given by

$$Q_{encl} = \int_0^L \lambda\, dl' = \lambda \int_0^L dl' = \lambda L, \text{ since } \lambda \text{ is constant} \qquad (7.2d)$$

Substituting Eq. (7.2d) in Eq. (7.2c), we get

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0\, r\, L} = \frac{\lambda L}{2\pi\varepsilon_0\, r\, L}$$

or

$$E = \frac{\lambda}{2\pi\varepsilon_0\, r} \qquad (7.3)$$

Note that the electric field in Eq. (7.3) does not depend on the length of the cylindrical Gaussian surface.

**The electric field is directed perpendicular to the line charge or charged wire.** This is the same result as the one we got in Example 5.7 after a very lengthy calculation! So, you see that for a symmetrical distribution of charges, the calculation of electric field becomes quite simple if we use Gauss's law.

You should, however, note that **Gauss's law is always true**, no matter what the distribution of charges. **But it is very useful for symmetric charge distributions** since its application makes the calculation much simpler.

In applying Gauss's law, the choice of the Gaussian surface is very important for simplifying calculations. This is especially true for symmetric charge distributions as you have learnt in Unit 6. You will appreciate this point time and again in this unit.

You may like to know: **Why do charge distributions have to be symmetric for Gauss's law to be applied to determine electric fields**?

Recall what you have learnt so far and you will be able to arrive at the answer: The symmetry of the distribution helps us determine the surfaces over which the magnitude of the electric field is constant (i.e., the distance $r$ is constant). Also we know the direction of the electric field for a given type of symmetry.

Then the trick is to **choose the Gaussian surface to be the surface over which the magnitude of the electric field is constant**. Also, **the direction of the electric field should be parallel/perpendicular to the area vector at all points on the surface**.

You must note that this is true for all applications of Gauss's law that you have studied so far, such as the charged sphere and the spherical shell in Unit 6 and the infinite charged wire in this section. For example, in this unit, for the infinite line charge, you have seen that the magnitude of the electric field is the same at all points of the curved part of the cylindrical surface as its radius is constant. The direction of $\vec{\mathbf{E}}$ is normal to the curved part of the surface and therefore, in the same direction as the area element $d\vec{\mathbf{S}}$. Since $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\mathbf{S}}$ on the cylinder's ends, for all points on the circular ends of the cylinder, $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0$. This has made the calculation of electric field quite simple. Of course, it is also simple because the line charge density is uniform, i.e., constant.

Suppose, we had chosen some other shape for the Gaussian surface, then Gauss's law would still apply but $\vec{\mathbf{E}}$ may not have been in the same direction as $d\vec{\mathbf{S}}$ and its magnitude may not have been constant over the surface. Then we could not have taken $E$ out of the integral. That would have made the calculation difficult. So, **symmetry is important for such applications of Gauss's law**. You must have appreciated this point by now having studied charge distributions possessing spherical and cylindrical symmetry. We end this section with an SAQ for you.

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### *SAQ 1* – Applying Gauss's law to line charge

The electric field due to an infinite line charge has magnitude $9.0 \times 10^3$ $\text{NC}^{-1}$ at a distance of 1.0 m. Calculate the linear charge density.

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Let us now determine the electric field due to an infinite uniformly charged cylinder using the same symmetry considerations as for the wire at points both outside and inside the cylinder. Such calculations of electric fields for a cylindrical charge distribution are required for determining the capacitance of capacitors having cylindrical geometry.

## 7.2.3 Uniformly Charged Infinite Cylinder

Consider an infinitely long charged solid cylinder of radius $R$, which has uniform volume charge density $\rho$. Let us determine the electric field due to this charge distribution at a point outside the cylinder.

We use Gauss's law to obtain the electric field for the uniformly charged infinite cylinder at a point $P$ lying outside it at a distance $r$ from its axis.

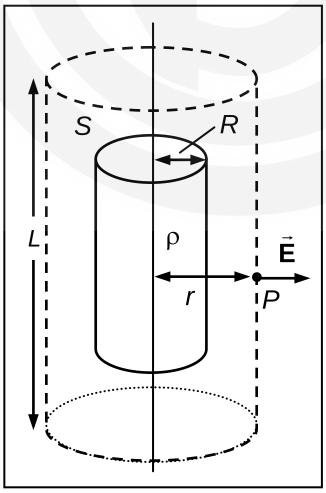



**Fig. 7.6**: **Electric field at a point $P$ lying outside a uniformly charged infinite cylinder. The cylindrical Gaussian surface is of length $L$ and radius $r > R$.**

Study Fig. 7.6 showing a section of the infinite cylinder by a solid line. You can verify that the charge distribution is cylindrically symmetric. For a point $P$ outside the cylinder, we draw a cylindrical Gaussian surface of length $L$ and radius $r$, passing through $P$. Recall that we have drawn a similar surface for the infinitely long wire in Fig. 7.5. We now follow the same steps and argument as in Sec. 7.2.1 to determine the electric field due to a uniformly charged infinitely long cylinder for $r \geq R$.

Once again we note that for the curved part of the cylindrical Gaussian surface, the direction of the electric field is normal to the surface at all points. Also, the electric field is directed radially outwards from the positively charged cylinder's axis. Therefore, $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are parallel to each other for each area element on the curved part of the Gaussian surface and $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS.$ As in Sec. 7.2.2, you can see that the electric flux through both circular ends of the cylindrical Gaussian surface is zero because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are perpendicular to each other at all points on these ends. Therefore, from Gauss's law, we have

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oiint_S E\,dS = E\oiint_S dS \quad dS = E\,2\pi rL = \frac{Q_{encl}}{\varepsilon_0} \qquad (7.4a)$$

Here, since $E$ is the same on all points of the Gaussian surface $S$, we have taken it to be constant for the surface and have taken it out of the integral. In Eq. (7.4a), we have also used the result that the total surface area of a cylinder of radius $r$ and length $L$ is $2\pi rL.$ So from Eq. (7.4a), we have

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0\, r\, L} \quad \text{for } r \geq R \qquad (7.4b)$$

We now have to determine $Q_{encl}$ in Eq. (7.4b), which is the net charge enclosed by the cylindrical Gaussian surface, given that $\rho$ is constant. It is just the charge on the cylinder of length $L$ and radius $R$ (because the charge distribution of the infinite cylinder is zero beyond its radius $R$). By definition, it is given by the following volume integral:

$$Q_{encl} = \iiint_V \rho\, dV \qquad (7.5a)$$

Since $\rho$ is uniform (constant), we can take it out of the integral and write

$$Q_{encl} = \rho\iiint_V dV = \rho\,\pi R^2 L \qquad (7.5b)$$

where the volume integral is just the volume of the cylinder of length $L$ and radius $R$. Therefore,

$$E = \frac{\rho\,\pi R^2 L}{2\pi\varepsilon_0\, r\, L} \quad \text{for } r \geq R \qquad (7.5c)$$

or

$$\vec{\mathbf{E}} = \frac{\rho R^2}{2\varepsilon_0\, r}\hat{\mathbf{r}} \quad \text{for } r \geq R \qquad (7.6)$$

where $\hat{\mathbf{r}}$ is the unit vector in the radial direction pointing outward from the cylinder's axis. Notice from Eq. (7.6) that the electric field of a cylindrical charge distribution at points lying outside it decreases as the distance from the axis increases.

Let us now ask: **What is the electric field of an infinite uniformly charged cylinder at a point inside it**?

You will learn the answer in Example 7.1.

## EXAMPLE 7.1 : ELECTRIC FIELD INSIDE A CYLINDER

An infinitely long uniformly charged cylinder of radius $R$ has positive volume charge density $\rho$. Determine the electric field at a point inside the cylinder.

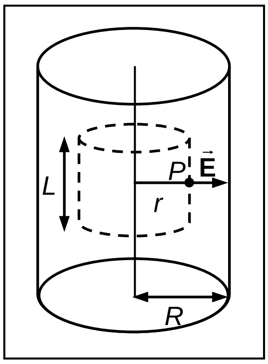



**Fig. 7.7**: **Electric field inside an infinite uniformly charged cylinder. The Gaussian surface is a cylindrical surface of length $L$ and radius $r < R$.**

**SOLUTION ■** We use Gauss's law to obtain the electric field at a point $P$ inside the cylinder at a distance $r$ from its axis.

Since the charge distribution is cylindrically symmetric, we draw a **cylindrical** Gaussian surface of length $L$ and radius $r$ passing through $P$ (Fig. 7.7). For any point inside the cylinder, $r < R$ and the Gaussian surface lies inside the cylinder. From symmetry considerations that you have learnt in Sec. 7.2.1 for cylindrical charge distributions, you know that the electric flux has contribution only from the curved surface of the Gaussian cylinder and not its ends. Hence, from Gauss's law, we have

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,2\pi r\,L = \frac{Q_{encl}}{\varepsilon_0} \qquad \text{for } r < R \qquad \text{(i)}$$

The charge enclosed by this Gaussian surface is

$$Q_{encl} = \rho \iiint_V dV$$

where the volume is just the volume of the cylinder of length $L$ and radius $r$. Therefore,

$$Q_{encl} = \rho\pi r^2 L$$

and from Eq. (i), $$E\,2\pi r\,L = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho\pi r^2 L}{\varepsilon_0}$$

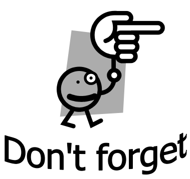
$$\therefore \qquad E = \frac{\rho r}{2\varepsilon_0} \qquad \text{for } r < R$$

and $$\vec{\mathbf{E}} = \frac{\rho r}{2\varepsilon_0}\hat{\mathbf{r}} \qquad \text{for } r < R \qquad (7.7)$$

where $\hat{\mathbf{r}}$ is the unit vector in the radial direction pointing outward from the cylinder's axis.

So, inside the cylindrical charge distribution, the electric field increases linearly with an increase in the distance from the axis.

For an infinite uniformly charged cylinder, always remember the following:




- **The electric field of an infinite uniformly charged cylinder at points outside it decreases with an increase in distance from its axis.**
- **The electric field inside an infinite uniformly charged cylinder increases linearly with an increase in distance from its axis.**

In the next section, we will apply Gauss's law to a charge distribution having planar symmetry. Examples of such charge distributions are uniform two-dimensional sheets of charge, thin plate carrying charge or uniform slabs of charge as well as combinations of such sheets or slabs like the ones used in parallel plate capacitors. But before you study further, try an SAQ to revise what you have learnt in this section.

**SAQ 2 – Applying Gauss's law to a solid charged cylinder**

A long non-conducting solid cylinder of radius 0.60 m carries a uniform volume charge density $+4.8\mu\text{Cm}^{-3}$. Calculate the magnitude of the electric field at a distance of (a) 0.40 m and (b) 1.0 m from the axis of the cylinder.

## 7.3 ELECTRIC FIELD DUE TO AN INFINITE UNIFORMLY CHARGED PLANE SHEET

In this section, we apply Gauss's law to an infinite uniformly charged plane sheet carrying a constant surface charge density $\sigma$. A large plastic sheet uniformly charged on one side is an example of a non-conducting sheet of charge. An aluminium foil is an example of a conducting sheet.

**What kind of symmetry does an infinite sheet (planar charge distribution) possess**? It remains the same if it is

- translated parallel to itself,
- rotated about any axis perpendicular to its plane, and
- reflected about any axis lying in its plane or perpendicular to its plane.

It follows from the symmetry considerations for a sheet of charge that the electric field due to it is everywhere perpendicular to the plane of the sheet. It is directed outward from the sheet, if positively charged and inward, if negatively charged. You may like to know: **Why is the electric field perpendicular to the plane everywhere**?

To answer this question, we follow the same line of argument as we have done for all symmetric charge distributions so far.

Refer to Fig. 7.8a, which shows the side view of a small section of the infinite sheet of charge. Suppose that the electric field of the sheet at some point *P* were directed in some other direction as shown in Fig. 7.8a. Note that we have arbitrarily labelled one end of this section of the infinite sheet as 'top' and the other one as 'bottom' just to show what happens when the sheet is reflected.

Let us now reflect this sheet of charge about a horizontal line perpendicular to its plane and passing through *P*. So the 'top' of the sheet is now the 'bottom' of the sheet and the 'bottom', its 'top' (Fig. 7.8b). What is the direction of the electric field after the sheet is reflected? After reflection, the direction of the electric field at *P* becomes as shown in Fig. 7.8b because the electric field is also reflected in the same manner.

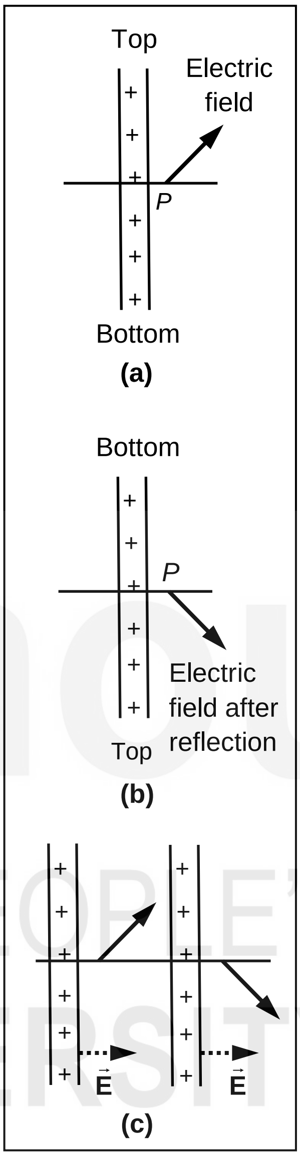



**Fig. 7.8**: **a) and b) If the electric field at any point due to a sheet of charge is not perpendicular to its plane everywhere, it will have different values at the same point for the same charge distribution under reflection, which is a contradiction; c) this contradiction does not exist if the electric field (shown by the dotted arrows) at any point due to the sheet of charge is perpendicular to its plane everywhere.**

But you can compare Figs. 7.8a and b by putting them alongside each other as in Fig. 7.8c. What do you find? You can see that the charge distribution remains the same after reflection. As before, we have labelled 'top' and 'bottom' on the sheet for our convenience. Otherwise, we cannot tell the difference. In this case, we find again that the electric fields before and after reflection are different at the same point. This is a contradiction: **How can there be different electric fields at a given point for the same charge distribution**? **If the charge distribution remains unchanged, the electric field at the point *P* cannot be different; it has to be the same**. Since it is not so, the direction of the electric field in Fig. 7.8a is incorrect.

Again we ask: **What is the direction of the electric field that does not lead to such a contradiction**? From Fig. 7.8c, you can see that the electric field remains the same under reflection only if it is directed perpendicular to the sheet of charge. It is shown by dotted arrows in Fig. 7.8c. You can verify that this is indeed the electric field direction for all other symmetry operations on the sheet. This is how we conclude that from symmetry of the sheet of charge, the direction of the electric field can only be perpendicular to its plane.

Let us now determine the electric field due to the infinite uniformly charged sheet at a distance *r* from it. Let its surface charge density be $\sigma$. Here we assume that the thickness of the sheet is much less than *r*. Now to use Gauss's law meaningfully, we need to choose a Gaussian surface that exploits the fact that the electric field is directed normal to the charged sheet. **What is that Gaussian surface**? We choose a closed cylindrical Gaussian surface perpendicular to the sheet with each end of the cylinder located at an equal distance (*r*) from the sheet. So, the length of the Gaussian cylindrical surface is 2*r* (see Fig. 7.9a). Such a Gaussian surface is also called the **Gaussian 'pillbox'**. In Fig. 7.9b, we show the side view of the sheet and the pillbox. Let the area of cross-section of the Gaussian pillbox (i.e., the area of its ends) be *S*.

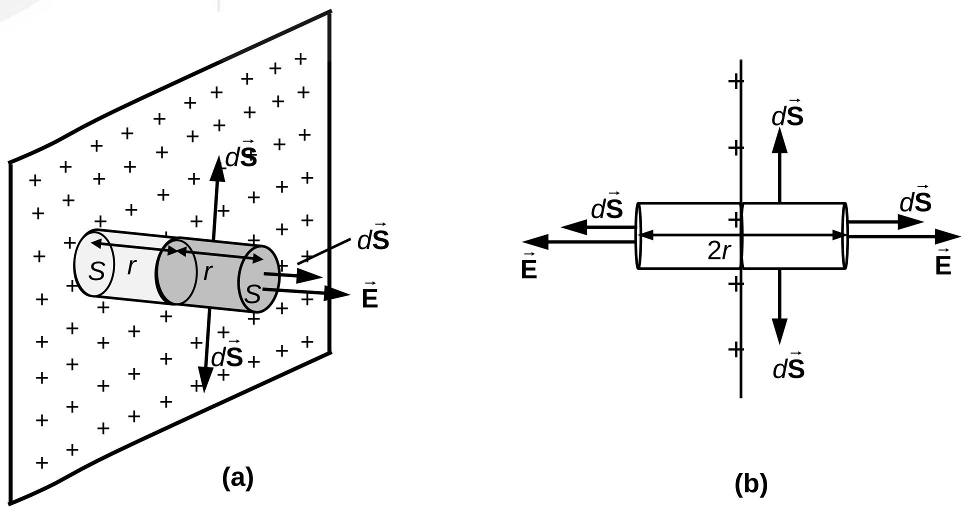



**Fig. 7.9**: **a) A sheet of positive charge and the Gaussian pillbox for which the electric field $\vec{E}$ and area vector $d\vec{S}$ are parallel at the ends and perpendicular to each other on the curved part of the surface; b) the sheet in its side view showing the electric field vectors and area vectors for the pillbox.**

Since the charge is positive, the electric field is directed away from the sheet and is perpendicular to the sheet. This means that for the curved part of the cylindrical Gaussian surface, the electric field vector is perpendicular to the area vector at all points (see Fig. 7.9b). Thus,

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0 \quad \text{for all points on the curved part of the cylindrical surface}$$

The electric field vectors point in an outward direction from the two ends of the Gaussian pillbox, i.e., in the same direction as the area vectors for the ends. So, the contribution to the electric flux is only from the ends of the Gaussian pillbox and

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS \quad \text{for all points on one end of the cylindrical surface}$$

Since there are two ends on the Gaussian pillbox, we need to consider the surfaces of both ends while applying Gauss's law and divide the surface integral into three parts corresponding to the two ends and the curved part. Then Gauss's law gives us

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \iint_{\substack{Curved\\part}} \vec{\mathbf{E}}.d\vec{\mathbf{S}} + \iint_{Both\,ends} \vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0 + ES + ES = \frac{Q_{encl}}{\varepsilon_0} \qquad (7.8a)$$

or

$$E = \frac{Q_{encl}}{2\varepsilon_0 S} \qquad (7.8b)$$

Now, we need to express the charge on the sheet enclosed by the Gaussian cylinder in terms of the uniform surface charge density $\sigma$. This is just the charge enclosed by the area of the sheet equal to the cylinder's cross-section, i.e., the area $S$. Since $\sigma$ is uniform (i.e., constant), it is equal to the ratio of the charge on a given surface to its area. Therefore, for the charge $Q_{encl}$ enclosed by the area $S$, it is

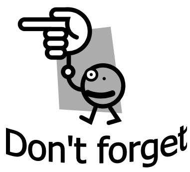
$$\sigma = \frac{Q_{encl}}{S} \quad \Rightarrow \quad Q_{encl} = \sigma S \qquad (7.9)$$

Substituting the value of $Q_{encl}$ from Eq. (7.9) in Eq. (7.8b), we get

$$E = \frac{\sigma}{2\varepsilon_0} \qquad (7.10)$$

**where the direction of the electric field is perpendicular to the sheet. Eq. (7.10) holds for both non-conducting and conducting sheets of charge provided the layer of charge on the sheet is very thin (or its thickness is very small compared to the distance at which the electric field is being calculated)**. It also holds for very large sheets of charge at points far from the edges of the sheet and at distances much larger than the thickness of the sheet or the layer of charge on the sheet. Eq. (7.10) tells us that

> **The electric field due to an infinite (or very large) uniformly charged sheet has the same value at all points lying outside it and points in a direction perpendicular to the sheet.**



Let us apply Gauss's law to two infinite or large sheets of charge in the following example.

## EXAMPLE 7.2 : TWO INFINITE SHEETS OF CHARGE

Two thin infinite non-conducting charged sheets are kept parallel to each other as shown in Fig. 7.10a. The surface charge density of the negatively charged left sheet is $\sigma_1$ and that of the right sheet carrying a positive charge is $\sigma_2$. Determine the net electric field in the region (1) to the left of the sheets, (2) between the sheets and (3) to the right of the sheets.

**SOLUTION ■** We apply Gauss's law to both sheets using the result obtained for an infinite uniformly charged sheet. We use the fact that the charges are fixed and obtain the electric field due to each sheet as if it were isolated. Then we apply the principle of superposition to obtain the net electric field.

Remember that from Eq. (7.10), the magnitude of the electric field at any point does not depend on the distance of the point from the sheet. It depends only on the surface charge density. The directions of the electric fields depend on the sign of the charge carried by them. The magnitudes of the electric field due to the negatively and positively charged sheets having surface charge densities $\sigma_1$ and $\sigma_2$, respectively, are given by

$$E_- = \frac{\sigma_1}{2\varepsilon_0} \quad \text{and} \quad E_+ = \frac{\sigma_2}{2\varepsilon_0}$$

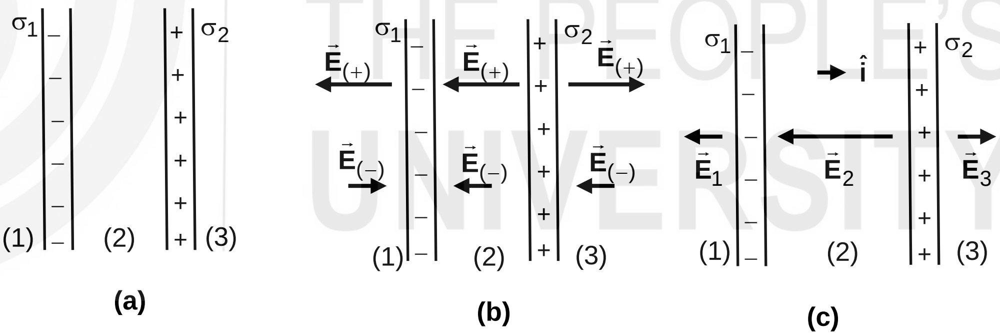



**Fig. 7.10**: **Diagram for Example 7.2.**

Fig. 7.10b shows the directions of the electric fields in each region. Note that the electric field due to the positively charged sheet points away from it in each of the three regions. The electric field due to the negatively charged sheet points towards it in each region. Let us denote the unit vector to the right of the sheets by $\hat{\mathbf{i}}$ (Fig. 7.10c). Then the resultant electric field in each of these regions (Fig. 7.10c) is given by

a) Region (1): $\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = (E_+)(-\hat{\mathbf{i}}) + (E_-)\hat{\mathbf{i}} = \dfrac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)\hat{\mathbf{i}}$

b) Region (2): $\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = (E_+ + E_-)(-\hat{\mathbf{i}}) = -\dfrac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)\hat{\mathbf{i}}$

c) Region (3): $\vec{\mathbf{E}}_3 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = (E_+)\hat{\mathbf{i}} + (E_-)(-\hat{\mathbf{i}}) = \dfrac{1}{2\varepsilon_0}(\sigma_2 - \sigma_1)\hat{\mathbf{i}}$

You will realise the importance of these calculations when you determine the electric fields of parallel plate capacitors in the next block and learn how useful capacitors are in our daily lives. You may now like to attempt an SAQ.

## *SAQ 3* – Uniformly charged thin sheets

Suppose in Example 7.2, the surface charge density of the negatively charged sheet is $\sigma_1 = 9.0 \times 10^{-9}\,\text{Cm}^{-2}$ and that of the positively charged sheet is $\sigma_2 = 6.0 \times 10^{-9}\,\text{Cm}^{-2}$. Determine the magnitudes and directions of the electric fields in the three regions. What would the net electric fields in the three regions be if the two sheets were interchanged?

While studying Unit 6 and Unit 7 so far, you must have realised that the symmetry of the charge distribution plays an important role in applications of Gauss's law. As you have learnt, the calculation of the surface integral in Gauss's law is greatly simplified for symmetric charge distributions. You have learnt about three kinds of symmetry for which application of Gauss's law is particularly useful. These are: **spherical symmetry**, **cylindrical symmetry** and **planar symmetry**. Let us revise the method of applying Gauss's law for each one of these.

Recap

## APPLICATIONS OF GAUSS'S LAW

1. For a **spherically symmetric charge distribution, you should draw a concentric Gaussian sphere**. This means that the centre of the Gaussian sphere should be on the point charge or the centre of the charged sphere or spherical shell. Also the point on which the electric field is to be determined should be on the surface of the Gaussian sphere. Then the electric field is normal to the Gaussian spherical surface, $\vec{\mathbf{E}} \parallel d\vec{\mathbf{S}}$ so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS$ and $E$ is constant on the surface.

2. For a **cylindrically symmetric charge distribution, you should draw a coaxial cylindrical Gaussian surface.** This means that the axis of the cylindrical Gaussian surface should be the same as that of the charge distribution (charged wire or charged cylinder). Also the point on which the electric field is to be determined should lie on the Gaussian surface. Then the electric field is normal to the curved part of the Gaussian cylindrical surface ($\vec{\mathbf{E}} \parallel d\vec{\mathbf{S}}$ so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS$) and parallel to the flat ends of the Gaussian cylinder ($\vec{\mathbf{E}} \perp d\vec{\mathbf{S}}$ so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0$). Also, $E$ is constant on the Gaussian surface.

3. For a **planar charge distribution, you should draw a Gaussian pillbox with its axis perpendicular to the plane of the charge distribution.** Then the electric field is perpendicular to the curved surface of the Gaussian pill box ($\vec{\mathbf{E}} \perp d\vec{\mathbf{S}}$ so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0$) and parallel to the flat ends of the Gaussian pill box ($\vec{\mathbf{E}} \parallel d\vec{\mathbf{S}}$ so that $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS$). Also $E$ is constant on the Gaussian surface.

So far, we have applied Gauss's law to non-conducting charged distributions. Does the law give different results for charged conductors? In the last section of this unit, we will apply Gauss's law to isolated charged conductors. This application of Gauss's law is quite important in our daily lives. This is especially so when we are caught in a thunderstorm. It will help you understand what you should do when you are travelling in a vehicle and are caught in a thunderstorm accompanied by lightning.

## 7.4 CHARGED ISOLATED CONDUCTOR

We can use Gauss's law to verify the following property of charged isolated conductors:

> "**If any excess unbalanced, static charges are placed on a conductor, they must reside on the surface of the conductor**. **The excess amount of charge moves to the surface of the conductor. When the charges stop moving, none of the charges will remain within the body of the conductor**."

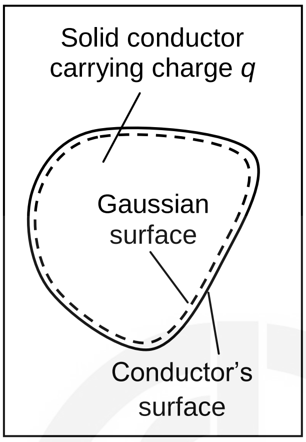



**Fig. 7.11**: **An isolated charged solid metallic conductor carrying excess charge *q* and the Gaussian surface just inside it**.

Let us use Gauss's law to explain how this is possible.

Consider the cross-section of an insulated solid metallic conductor such as the one shown in Fig. 7.11 carrying an excess charge *q*. We choose the Gaussian surface to lie just inside the actual surface of the conductor. The dashed line in Fig. 7.11 shows the Gaussian surface.

**Once the excess charge stops moving, the electric field inside the charged conductor must become zero**. **Why is this so**? We can see why this is so without a formal calculation. Suppose that this were not true and that there was an electric field inside conductor. Then a force would be exerted by the electric field on the charges inside the conductor that are always present in it and are free to move (e.g., electrons in this case).

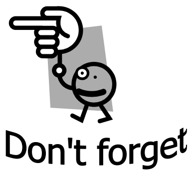
Thus, internal currents would be set up and would always exist within a conductor because charge would flow from one point to another under the action of this force. But no such perpetual currents are observed in any isolated charged conductor. So, the only conclusion is that the internal electric field of an isolated charged conductor is zero. Its interior is always free of electric fields.

For the time when the conductor is being charged, internal electric fields do exist inside it. But once the charging stops and the conductor is isolated, the excess charge is quickly distributed in a way that the **net electric field is zero everywhere inside the conductor**.

Now since the electric field is zero inside the conductor, it must be zero for all points on the Gaussian surface because that surface, though close to the surface of the conductor, lies **inside** it. This means that the electric flux through the Gaussian surface is zero. Then according to Gauss's law, the net charge enclosed by the Gaussian surface is also zero. So, if the excess charge is not inside the Gaussian surface, it must lie outside it. This means that it must lie on the actual surface of the isolated conductor.

So, always remember that

**The net electric field is zero everywhere inside the conductor. If a net charge does reside on an isolated conducting body/object, it can be distributed only over the surface layer of that conductor**.




You may like to know: **What is the electric field at any point lying outside a conductor carrying a net charge on its outer surface**?

The results for **the electric fields for all <u>conducting</u> symmetric charge distributions at a point outside the conductor will be the same as the results obtained for the corresponding non-conducting charge distributions**. So, the electric fields due to various symmetric **conducting** and **non-conducting charge distributions** (**at any point lying outside them**) are same and are given in Table 7.1.

**Table 7.1: Electric fields due to conducting and non-conducting charge distributions at points lying outside them.**

| Conducting and non-conducting charge distribution | Electric field at a point lying outside the charge distribution |
|---|---|
| **Uniform spherical charge distribution of radius *R* carrying net positive charge *Q*** | $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{\mathbf{r}} \quad r \geq R$ |
| **Uniformly charged thin spherical shell of radius *R* carrying net charge *Q*** | $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{\mathbf{r}} \quad r \geq R$ |
| **Infinite line of charge having uniform line charge density $\lambda$** | $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ directed perpendicular to the line charge |
| **Infinite cylindrical charge distribution of radius *R* having uniform volume charge density $\rho$** | $\vec{\mathbf{E}} = \dfrac{\rho R^2}{2\varepsilon_0 r}\hat{\mathbf{r}} \quad r \geq R$ |
| **Infinite thin sheet of charge having uniform surface charge density $\sigma$** | $E = \dfrac{\sigma}{2\varepsilon_0}$ directed perpendicular to the sheet |


## *SAQ 4* – Charged isolated conductor

An isolated conducting sphere of radius 1.0 m carries a uniform surface charge density $2.7\,\mu\text{C m}^{-2}$. What is the net charge on the sphere? Calculate the net electric flux leaving the surface of the sphere. What is the electric field due to the conductor at a point 3.0 m from its centre?

We now consider an example for determining the electric field due to two concentric conductors in different regions around them. Such problems are useful in determining the electric fields due to various geometries in capacitors.

## *EXAMPLE 7.3* : CONCENTRIC SPHERE AND SHELL

A solid conducting sphere is concentric with a thin conducting spherical shell as shown in Fig. 7.12a. The sphere of radius $r_1$ carries charge $Q_1$ and the spherical shell of radius $r_2$ carries charge $Q_2$ with $r_1 < r_2$. Determine the electric fields at a distance $r$ from the centre of the sphere for (a) $r < r_1$, (b) $r_1 < r < r_2$ and (c) $r > r_2$. d) What will happen if the sphere and the shell are connected with a wire? e) What will the electric fields be for $r < r_2$ and $r > r_2$ after this?

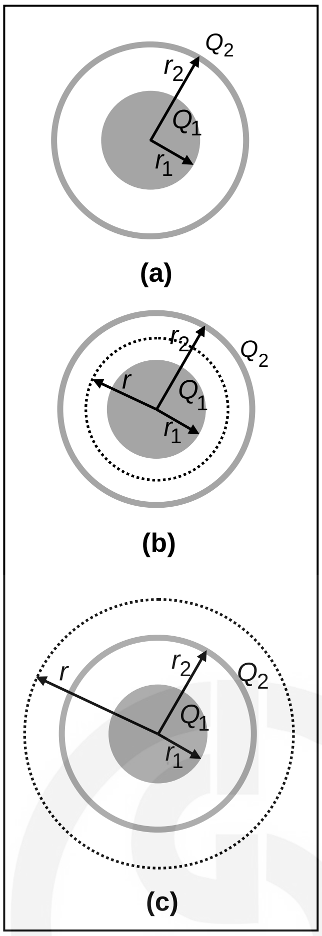



**Fig. 7.12**: **Diagram for Example 7.3.**

**SOLUTION ■** We apply Gauss's law to both conducting sphere and conducting shell using the result obtained for a conductor in this section.

Remember that the electric field at any point inside a conductor is zero.

a) The points corresponding to $r < r_1$ lie **inside** the conducting sphere. Therefore, the electric field at all such points is zero.

b) For the points $r_1 < r < r_2$, we draw a spherical Gaussian surface of radius $r$ at any point between the conducting sphere and the conducting shell (see Fig. 7.12b). The net charge enclosed by it is just the charge on the conducting sphere, i.e., $Q_1$. Therefore, from Eq. (6.22), the electric field is given as

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\hat{\mathbf{r}} \qquad \text{for} \quad r_1 < r < r_2$$

c) For the points $r > r_2$, we draw a spherical Gaussian surface of radius $r\ (> r_2)$ lying outside the conducting shell (Fig. 7.12c). The surface encloses a net charge $(Q_1 + Q_2)$. Therefore, from Eq. (6.22), the electric field is given as

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q_1 + Q_2)}{r^2}\hat{\mathbf{r}} \qquad \text{for} \quad r > r_2$$

Remember, you have to take the algebraic sum of charges to determine the net charge. So, while solving problems, always take into account the signs of the charges.

d) When the conducting sphere and the conducting shell are connected with a wire, charges flow in the system until equilibrium is reached. At equilibrium, there is no charge inside both the conductors and the system behaves like a single conductor. So there is no charge on either the inner sphere or the inner surface of the shell. The net charge $(Q_1 + Q_2)$ resides on the outer surface of the spherical shell.

e) The electric field for $r < r_2$ will be zero since the point lies inside a conductor.

If we draw a spherical Gaussian surface for $r > r_2$, it encloses the net charge $(Q_1 + Q_2)$. Therefore, we have

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q_1 + Q_2)}{r^2}\hat{\mathbf{r}} \qquad \text{for} \quad r > r_2$$

You may quickly like to apply the results of Example 7.3 for practice. Attempt the following SAQ.

## *SAQ 5* – Charged conductors

Suppose in Example 7.3, $Q_1 = Q$ and $Q_2 = -2Q$. What will the electric fields be for (a) $r < r_1$, (b) $r_1 < r < r_2$ and (c) $r > r_2$, if all other parameters are the same?

Now suppose we create a cavity inside the conductor. Will the results for charged isolated conductors still hold? We explain what happens in this case in the following example.

## *EXAMPLE 7.4* : AN ISOLATED CONDUCTOR WITH A CAVITY

A cavity is created inside an isolated conductor. Explain why any excess charge placed on the conductor will reside on its outer surface.

**SOLUTION** ■ We use Gauss's law to give the explanation.

Consider Fig. 7.13, which shows an isolated conductor with a cavity inside it. Now, you have learnt that there are no unbalanced charges inside the solid conductor. Therefore, we can assume reasonably that when we scoop out some of the material, leaving a hollow cavity, we do not change the charge distribution or the electric fields that existed in the solid conductor.

Once again, we draw the Gaussian surface so that it is **inside the conductor** and surrounds the cavity wall very close to it as shown in Fig. 7.13. Since the net electric field inside the conductor is zero $(\vec{\mathbf{E}}_{net} = \vec{\mathbf{0}})$, the electric flux through this surface must also be zero. Therefore, from Gauss's law, this surface cannot enclose any net charge. Thus, we can say that there is no net charge on the cavity wall. All excess charge remains on the outer surface of the isolated conductor.

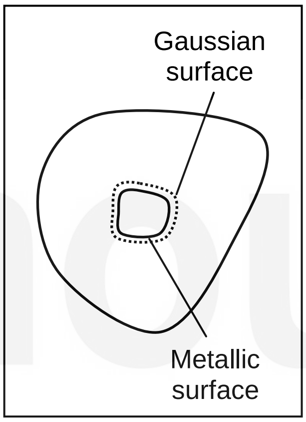



**Fig. 7.13: An isolated charged conductor having a cavity within it. The Gaussian surface lies within the conductor outside the cavity and very close to the cavity's surface.**

The results obtained in this section have many practical applications. We can now answer the question: What should we do when we get caught in a thunderstorm while travelling in a vehicle? From what you have studied in this section, you can answer the question as follows:

We should shut all windows and doors of the vehicle and keep ourselves insulated from all electronic gadgets present in it. If lightning strikes the vehicle, the entire charge will be distributed on its outer metallic surface. Its effects inside of the conductor (vehicle) will be substantially reduced: We will not be struck by lightning if we are sitting in a closed vehicle or any other closed space that is made of conducting material. On the other hand, if we were inside a non-conducting material like a wooden crate, lightning would pass right through it and we would be struck by it. The crate could also catch fire.

The fact that the electric field inside an isolated conductor with a cavity is zero has an interesting application in experimental physics called the **Faraday cage**. It is used in experiments which involve the measurement of very low power electrical signals generated, e.g., in computer chips or in neurons of animals. You can read about it at https://en.wikipedia.org/wiki/Faraday_cage. This is also the reason why your mobile phones, radio receivers, etc. will not work inside metal cages or metallic buildings.

With this we complete the discussion on Gauss's law and its applications. Let us now summarise the contents of this unit.

## 7.5 SUMMARY

| Concept | Description |
|---|---|
| ***Infinite line charge*** | ■ From Gauss's law, the electric field due to conducting and non-conducting **infinite line or wire of charge with uniform line charge density** $\lambda$ is directed perpendicular to the line of charge and its magnitude is given by $$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{at any point } r$$ |
| ***Infinite non-conducting cylindrical charge distribution*** | ■ The electric field due to a **non-conducting infinite solid cylinder of radius** $R$ **with uniform volume charge density** $\rho$ is given by $$\vec{\mathbf{E}} = \frac{\rho R^2}{2\varepsilon_0 r}\hat{\mathbf{r}} \quad \text{for } r \geq R$$ $$\vec{\mathbf{E}} = \frac{\rho r}{2\varepsilon_0}\hat{\mathbf{r}} \quad \text{for } r < R$$ |
| ***Infinite non-conducting sheet of charge*** | ■ The electric field due to a **non-conducting infinite sheet of charge** with **uniform surface charge density** $\sigma$ at any point is given by $$E = \frac{\sigma}{2\varepsilon_0}$$ and points in a direction perpendicular to the sheet. |
| ***Charged isolated conductor without and with a cavity*** | ■ If any excess unbalanced, static charges are placed on an isolated conductor, they must reside on the surface of the conductor. It follows that **if a net charge does reside on an isolated conducting body/object, it can be distributed only over the surface layer of that conductor**. In an isolated conductor having a cavity, all excess charge placed on the conductor will reside only on its outer surface. |
| ***Electric field due to charged isolated conductor at points lying inside and outside the conductor*** | ■ The **electric field at points lying inside** an isolated charged conductor is zero. <br><br> The electric field **at a point lying outside** an isolated charged conductor is the same as that of a non-conductor of the same geometry/symmetry (see Table 7.1). |

***Electric field due to charged isolated conductor at points lying inside and outside the conductor***

- The electric field due to
  - a uniform **conducting spherical charge distribution** (of radius $R$ and carrying charge $Q$) at a point at a distance $r$ from its centre is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad r \geq R$$

  - a thin uniform **conducting spherical shell** (of radius $R$ and carrying charge $Q$) at a point at a distance $r$ from its centre is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad r \geq R$$

  - an **infinite conducting wire** carrying uniform linear charge density $\lambda$ at $r$ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{directed perpendicular to the line of charge}$$

  - an **infinite conducting solid cylinder** having radius $R$ and uniform volume charge density $\rho$ at a point at a distance $r$ is

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \quad \text{in the radial direction for } r \geq R$$

  - an **infinite conducting thin sheet of charge** carrying uniform surface charge density $\sigma$ is

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{directed perpendicular to the sheet.}$$

## 7.6 TERMINAL QUESTIONS

1. What is the magnitude of the electric field at a distance of 2.0 m from an infinitely charged wire given that the linear charge density is $3.6\,\mu\text{C}\,\text{m}^{-1}$?

2. A solid metal wire of length 1000 m and diameter 1.0 cm carries a net charge $q = 5.0\,\mu\text{C}$, which is distributed uniformly in it. Determine the electric field at a distance of a) 5.0 cm and b) 0.50 cm, respectively, from the wire's axis. Assume that the point where the electric field is to be determined is far from the ends of the wire.

3. A thin metal wire of length 30 m and diameter 0.04 cm carries a net charge $6.0\,\mu\text{C}$ distributed uniformly over its surface. Calculate the electric field at the points at the distances of (a) 0.01 cm and (b) 0.09 cm from its axis. Assume that these points lie far away from the ends of the wire.

4. A Gaussian surface of cylindrical shape (of radius 1.0 m and height 20 m) encloses a few positive charges. Assuming that the electric field due to these charges is normal to the Gaussian surface and has magnitude $900\,\text{NC}^{-1}$, calculate the volume charge density of the charge distribution.

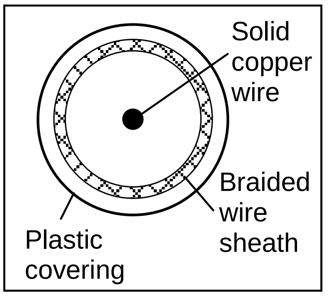



**Fig. 7.14**: **Diagram for TQ 5.**

5. A coaxial cable consists of a thin inner solid copper wire and an outer sheath of braided copper wire (see Fig. 7.14). The linear charge density of the inner wire is $-\lambda$ and that of the outer wire is $\lambda$. Determine the electric

fields at a point (a) in the region inside the inner wire, (b) in the region between the wires and (c) in the region outside the coaxial cable.

6. A flat sheet of charge of surface area $A$ has uniform surface charge density $\sigma$. An electrostatic force of magnitude $3.6\times10^{-12}\,\text{N}$ pointing in a perpendicular direction away from the sheet, is exerted on an electron at a distance of 0.03 m from its centre. Calculate the net charge on the sheet for $A = 2.56\,\text{m}^2$.

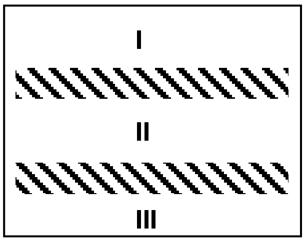



**Fig. 7.15**: **Diagram for TQ 7.**

7. Two identical infinite non-conducting sheets having equal positive surface charge densities $\sigma$ are kept parallel to each other as shown in Fig. 7.15. Determine the electric field at a point in (a) region I above the sheets, (b) region II between the sheets and (c) region III below the sheets.

8. A very long conducting thin solid cylinder of length $L$ carrying a net charge $+q$ is enclosed in a thin conducting cylindrical hollow tube of the same length. The tube carries a net charge $+2q$. Determine the electric fields at

   (a) a point lying outside the conducting tube; and

   (b) a point lying in the region between the solid cylinder and the tube.

   In both cases, the point lies far away from the edges of the conductors.

9. The net charge on an isolated conductor is $q_1 = 15\,\mu\text{C}$. A charge $q_2 = 5.0\,\mu\text{C}$ is later placed inside a cavity in the conductor. Determine the charge on the wall of the cavity. What is the charge on the outer surface of the conductor after $q_2$ is placed inside the cavity?

10. A concentric spherical cavity of radius 3.0 m is created in a conducting sphere of radius 6.0 m. A point charge $Q$ is kept at the centre of the sphere/cavity. The net charge on the conducting sphere is $+9.0\,\text{nC}$. The electric field at a point 2.0 m away from the centre of the sphere is $7.2\,\text{NC}^{-1}$ and points radially inward.

    a) What is the value of the charge $Q$?

    b) What is the charge on the wall of the cavity, i.e., the inner surface of the sphere?

    c) Calculate the value of the charge on the sphere's outer surface.

    d) Determine the electric field at a point 4.0 m away from the centre of the sphere.

## 7.7 SOLUTIONS AND ANSWERS

### Self-Assessment Questions

1. From Eq. (7.3), the linear charge density is $\lambda = 2\pi\varepsilon_0\, r\, E$. Substituting the numerical values of $r$ and $E$ along with the constants, we get

   $$\lambda = 2\pi\times 8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}\times(1.0\,\text{m})\times 9.0\times10^{3}\,\text{NC}^{-1} = 5.0\times10^{-7}\,\text{Cm}^{-1}$$

2. a) The point at a distance of 0.40 m from the cylinder's axis lies inside it. Therefore, we use Eq. (7.7) to calculate the magnitude of the electric field:

$$E = \frac{\rho r}{2\varepsilon_0} = \frac{4.8\,\mu\text{Cm}^{-3} \times 0.40\,\text{m}}{2 \times 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 1.1 \times 10^5\,\text{NC}^{-1}$$

b) The point at a distance of 1.0 m from the cylinder's axis lies outside it. Therefore, we use Eq. (7.6) to calculate the magnitude of the electric field:

$$E = \frac{\rho R^2}{2\varepsilon_0 r} = \frac{4.8\,\mu\text{Cm}^{-3} \times (0.60\,\text{m})^2}{2 \times 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2} \times (1.0\,\text{m})} = 9.8 \times 10^4\,\text{NC}^{-1}$$

3. As explained in Example 7.2, for $\sigma_1 = 9.0 \times 10^{-9}\,\text{Cm}^{-2}$ and $\sigma_2 = 6.0 \times 10^{-9}\,\text{Cm}^{-2}$, the magnitudes and directions of the electric fields in the three regions are given by

Region (1): $$\vec{\mathbf{E}}_1 = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)\,\hat{\mathbf{i}} = \frac{(9.0 - 6.0) \times 10^{-9}\,\text{Cm}^{-2}}{2 \times 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}}\,\hat{\mathbf{i}}$$

$$= 1.7 \times 10^2\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

Region (2): $$\vec{\mathbf{E}}_2 = -\frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)\,\hat{\mathbf{i}} = -\frac{(9.0 + 6.0) \times 10^{-9}\,\text{Cm}^{-2}}{2 \times 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}}\,\hat{\mathbf{i}}$$

$$= -8.5 \times 10^2\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

Region (3): $$\vec{\mathbf{E}}_3 = \frac{1}{2\varepsilon_0}(\sigma_2 - \sigma_1)\,\hat{\mathbf{i}} = \frac{(6.0 - 9.0) \times 10^{-9}\,\text{Cm}^{-2}}{2 \times 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}}\,\hat{\mathbf{i}}$$

$$= -1.7 \times 10^2\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

Refer to Fig. 7.16. If the two sheets are interchanged, then we have negative $\sigma_1 = 9.0 \times 10^{-9}\,\text{Cm}^{-2}$ and positive $\sigma_2 = 6.0 \times 10^{-9}\,\text{Cm}^{-2}$.

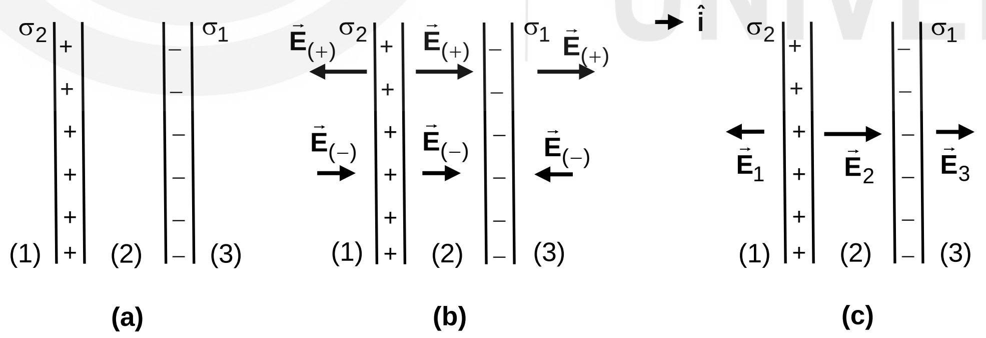



**Fig. 7.16**: **Diagram for answer of SAQ 3. Part (c) is not to scale.**

From Fig. 7.16b, the magnitudes and directions of the electric fields in the three regions are now given by

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = (E_+)(-\hat{\mathbf{i}}) + (E_-)\hat{\mathbf{i}} = \frac{1}{2\varepsilon_0}[\sigma_2(-\hat{\mathbf{i}}) + \sigma_1\hat{\mathbf{i}}] = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)\,\hat{\mathbf{i}}$$

$$= 1.7 \times 10^2\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = E_+\hat{\mathbf{i}} + E_-\hat{\mathbf{i}} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)\,\hat{\mathbf{i}} = 8.5 \times 10^2\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = (E_+)\hat{\mathbf{i}} + (E_-)(-\hat{\mathbf{i}}) = \frac{1}{2\varepsilon_0}(\sigma_2 - \sigma_1)\hat{\mathbf{i}} = -1.7\times10^2\,\text{NC}^{-1}\hat{\mathbf{i}}$$

Of course, when you solve this problem, you have to start from the beginning and follow all steps given in Example 7.2.

4. The net charge on the sphere is $Q = \sigma S$, where $S = 4\pi R^2$ is the area of the surface of the sphere of radius $R$. Therefore,

$$Q = \sigma 4\pi R^2 = 4\pi\times 2.7\,\mu\text{Cm}^{-2}(1.0\,\text{m})^2 = 34\,\mu\text{C}$$

From Gauss's law [Eq. (7.4a)], the net electric flux leaving the surface of the sphere is

$$\Phi_E = \frac{Q}{\varepsilon_0} = \frac{34\,\mu\text{C}}{8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 3.8\times10^6\,\text{Nm}^2\text{C}^{-1}$$

Since the point lies outside the sphere, the electric field due to the conductor at a point 3.0 m from its centre is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = (8.99\times10^9\,\text{C}^{-2}\text{Nm}^2)\times\frac{34\,\mu\text{C}}{(3.0\,\text{m})^2}\hat{\mathbf{r}} = 3.4\times10^4\,\text{NC}^{-1}\hat{\mathbf{r}}$$

5. Substituting $Q_1 = Q$ and $Q_2 = -2Q$ in the results of Example 7.3, we get

a) For $r < r_1$, $\vec{\mathbf{E}} = \vec{\mathbf{0}}$

b) For $r_1 < r < r_2$, $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$, and

c) For $r > r_2$, $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q_1+Q_2)}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q-2Q)}{r^2}\hat{\mathbf{r}} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$

## Terminal Questions

1. From Eq. (7.3), the magnitude of the electric field is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = 2\times(8.99\times10^9\,\text{C}^{-2}\text{Nm}^2)\times\frac{3.6\,\mu\text{Cm}^{-1}}{2.0\,\text{m}} = 3.2\times10^4\,\text{NC}^{-1}$$

2. Although the wire is not infinite, for points close to it and sufficiently far from its ends, we can approximate it as one. This is because at such points we can neglect the contribution of the electric fields due to distant charges.

   a) We use Eq. (7.4b) to calculate the electric field at a point 5.0 cm from the wire's axis, since it lies outside the wire and get

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0 r L} = 2\times(8.99\times10^9\,\text{C}^{-2}\text{Nm}^2)\times\frac{5.0\,\mu\text{C}}{5.0\times10^{-3}\,\text{m}\times1000\,\text{m}}$$

$$= 1.8\times10^4\,\text{NC}^{-1}$$

   b) The metal wire is a conductor. Therefore, the electric field at the point 0.50 cm from the wire's axis is zero, $\vec{\mathbf{E}} = \vec{\mathbf{0}}$ since the point lies inside the conductor.

3. The thin metal wire in the problem cannot strictly be taken as an infinite line charge. But for points close to the wire and sufficiently far from its ends, the contribution of the electric fields from distant charges can be taken to be negligible. Therefore, we can approximate the electric field of the wire to that of an infinite line charge. (a) Since the metal wire is a conductor, the electric field at the point 0.01 cm from the wire's axis, which lies inside it, will be zero: $\vec{\mathbf{E}} = \vec{\mathbf{0}}$. (b) The electric field at the point outside the wire at a distance of 0.09 cm from its axis is given by Eq. (7.2c) and we get

$$E = \frac{Q_{encl}}{2\pi\varepsilon_0 r\,L} = 2\times(8.99\times10^{9}\,\text{C}^{-2}\text{Nm}^2)\times\frac{6.0\,\mu\text{C}}{0.09\times10^{-2}\,\text{m}\times30\,\text{m}} = 4.0\times10^{6}\,\text{NC}^{-1}$$

4. We are given the electric field and the radius and height of the cylindrical Gaussian surface and we have to determine the volume charge density of the charge distribution enclosed by it. Since the surface area of the cylinder is $2\pi r\,h$, the electric flux through the Gaussian surface is

$$\Phi_E = E\,S = E\times(2\pi r\,h) = \frac{Q_{encl}}{\varepsilon_0}$$

or $$Q_{encl} = 2\pi\varepsilon_0 r\,hE$$

The volume charge density $\rho$ of the charge distribution is the net charge enclosed per unit volume.

$$\therefore \rho = \frac{Q_{encl}}{V} = \frac{2\pi\varepsilon_0 r\,hE}{\pi r^2 h} = \frac{2\varepsilon_0 E}{r} = \frac{2\times8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}\times900\,\text{NC}^{-1}}{1.0\,\text{m}}$$

$$= 1.6\times10^{-8}\,\text{Cm}^{-3}$$

5. a) The electric field at a point inside the inner copper wire (region I) is zero since it is a conductor: $\vec{\mathbf{E}} = \vec{\mathbf{0}}$.

b) Refer to Fig. 7.17. We take the Gaussian surface to be a coaxial cylindrical surface of radius $r$ and length $L$ lying in the region II between the wires. Note that the net charge enclosed by the Gaussian surface is $Q_{encl} = -\lambda L$, where $-\lambda$ is the linear charge density of the inner wire. From Gauss's law given by Eq. (7.4a),

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oiint_S E\,dS = E\oiint_S dS = E\,2\pi r\,L = \frac{Q_{encl}}{\varepsilon_0} = -\frac{\lambda L}{\varepsilon_0}$$

So, we have $$\vec{\mathbf{E}} = -\frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector perpendicular to the cylindrical axis pointing away from the axis. So, the electric field in region II is directed radially inward.

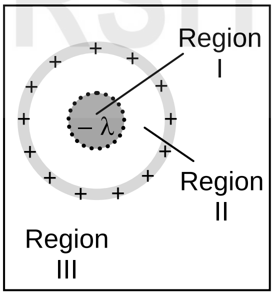



**Fig. 7.17**: **Diagram for the answer of TQ 5.**

c) For the point that lies outside the cable, the electric field is zero. This is because the two wires have equal and opposite linear charge densities and the net charge enclosed by a Gaussian surface outside both wires will be zero: $\vec{\mathbf{E}} = \vec{\mathbf{0}}$.

6. The sheet is effectively infinite for the point at a distance of 0.03 m from its centre assuming that the point lies far from its edges. Let the net charge on the sheet be $q$. The electric field due to the sheet is given by Eq. (7.10):

$$E = \frac{\sigma}{2\varepsilon_0} \text{ directed perpendicular to the sheet, where } \sigma = \frac{q}{A}$$

The magnitude of the electrostatic force on an electron is given by

$$F = -eE \qquad \text{or} \qquad F = -e\frac{\sigma}{2\varepsilon_0} = \frac{-eq}{2\varepsilon_0 A}$$

since the surface charge density is charge per unit area and $A$ is the area of the sheet. Thus,

$$q = \frac{2\varepsilon_0 AF}{-e} = \frac{2\times 8.85\times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}\times 2.56\,\mathrm{m^2}\times 3.6\times 10^{-12}\,\mathrm{N}}{-1.6\times 10^{-19}\,\mathrm{C}} = -1.0\,\mathrm{mC}$$

The negative sign shows that the charge on the sheet is negative. This is expected because the electrostatic force between the sheet and the electron is negative, i.e., the electron is repelled by the sheet.

7. Let $\vec{\mathbf{E}}_1$ be the electric field due to sheet 1 and $\vec{\mathbf{E}}_2$ be the electric field due to sheet 2 at some point in each of the three regions. The magnitudes of the electric fields due to the sheets will be equal since their surface charge densities are equal. Let us denote the magnitudes by $E$. Then from Eq. (7.10),

$$E = \frac{\sigma}{2\varepsilon_0}$$

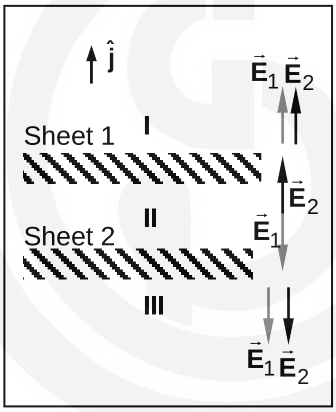



**Fig. 7.18**: **Diagram for answer of TQ 7.**

Since both sheets are charged positively, the electric fields due to them would be directed away from them in each region. The electric fields due to the sheets in the three regions are shown in Fig. 7.18. Now we can determine the net electric field at any given point in each region as follows:

a) Region I above the sheets: The electric fields due to the sheets are in the same direction, say, $\hat{\mathbf{j}}$, as both sheets are positively charged. Therefore, the net electric field at a point in region I is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = 2\times\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{j}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{j}}$$

b) Region II between the sheets: The electric field due to sheet 1 is directed opposite to the electric field due to sheet 2. Therefore, the net electric field at a point in region II is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \frac{\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{j}} = \vec{\mathbf{0}}$$

c) Region III below the sheets: The electric fields are again in the same direction, but opposite to $\hat{\mathbf{j}}$. Therefore, the net electric field at a point in region III is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = 2\times\frac{\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) = -\frac{\sigma}{\varepsilon_0}\hat{\mathbf{j}}$$

8. We use Gauss's law given by Eq. (7.4a) to determine the electric fields in the two regions for conducting cylindrical charge distributions.

a) For a point lying outside the conducting tube, the net charge enclosed by a Gaussian cylindrical surface of radius $r$ and length $L$ passing through the point is the algebraic sum of the total charge on the solid cylinder and the cylindrical tube, i.e., $+3q$. Therefore, from Eq. (7.4a), we get

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oiint_S E\,dS = E\oiint_S dS = E\,2\pi r\,L = \frac{Q_{encl}}{\varepsilon_0} = \frac{+3q}{\varepsilon_0}$$

or $\quad E = \dfrac{3q}{2\pi\varepsilon_0 r\,L}$ directed radially outward

b) For a point lying in the region between the solid cylinder and the tube, the net charge enclosed by a Gaussian cylindrical surface of radius $r$ and length $L$ passing through the point is just the charge on the solid cylinder, i.e., $+q$. Therefore, from Eq. (7.4a), we get

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oiint_S E\,dS = E\oiint_S dS = E\,2\pi r\,L = \frac{Q_{encl}}{\varepsilon_0} = \frac{+q}{\varepsilon_0}$$

or $\quad E = \dfrac{q}{2\pi\varepsilon_0 r\,L}$ directed radially outward

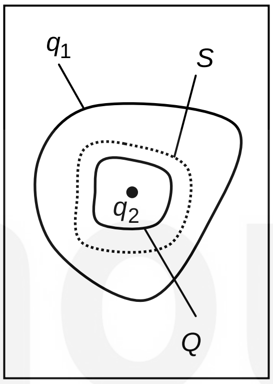



**Fig. 7.19**: **Diagram for the answer of TQ 9.**

9. Refer to Fig. 7.19. The net charge on the conductor is $q_1 = 15\,\mu\text{C}$. Suppose the charge on the wall of the cavity is $Q$. Let $S$ be the Gaussian surface enclosing the cavity. The electric flux $\Phi_S$ through $S$ is zero since the electric field inside the conductor is zero. Since the charge $q_2 = 5.0\,\mu\text{C}$ is placed inside the cavity in the conductor, the net charge enclosed by the Gaussian surface is the algebraic sum of the charge $Q$ on the cavity wall (which is also the inner surface of the conductor) and $q_2$. So, from Gauss's law,

$$\Phi_S = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q + q_2}{\varepsilon_0} = 0$$

$$\Rightarrow \quad Q + q_2 = 0 \Rightarrow Q = -q_2 = -5.0\,\mu\text{C}$$

Let the net charge on the outer surface of the conductor be $q'$ after the charge $q_2 = 5.0\,\mu\text{C}$ is placed inside the cavity. From conservation of charge, the net charge on the conducting sphere is equal to the algebraic sum of the charge on its inner surface and the charge on its outer surface. Therefore, we have

$$q_1 = Q + q'$$

So, the charge on the outer surface of the conductor is

$$q' = 15\,\mu\text{C} - (-5.0\,\mu\text{C}) = 20\,\mu\text{C}$$

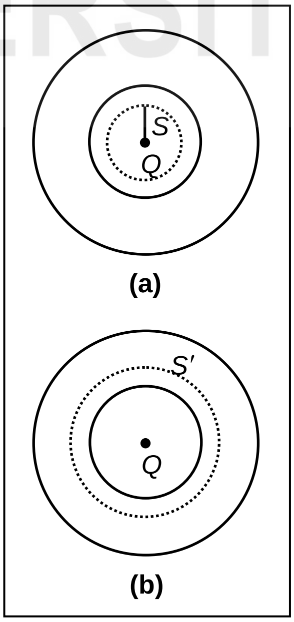



**Fig. 7.20**: **Diagram for the answer of TQ 10.**

10. a) It is given that the electric field at a point 2.0 m away from the centre of the sphere/cavity points inward. Refer to Fig. 7.20a. We draw a spherical Gaussian surface $S$ of radius 2.0 m. So, its surface area is $4\pi(2.0\text{m})^2$ and the net charge enclosed by it is $Q$. Thus, from Gauss's law, we have

$$\Phi_S = -ES = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \Rightarrow Q = -\varepsilon_0 ES = -4\pi\varepsilon_0 (2.0\,m)^2 E$$

or

$$Q = -4\pi(8.85\times10^{-12}\,C^2N^{-1}m^{-2})\times(7.2\,NC^{-1})\,(2.0\,m)^2 = -3.2\,nC$$

b) We follow the same steps as in the solution of TQ 9. The Gaussian surface $S'$ lies inside the conductor and surrounds the cavity as shown in Fig. 7.20b. The net charge enclosed by $S'$ is the algebraic sum of the charge $Q$ and the charge on the wall of the cavity, say $q$. Since the electric field inside the conductor is zero, from Gauss's law, the net charge enclosed by the Gaussian surface is zero. Therefore,

$$Q + q = 0 \Rightarrow q = -Q = 3.2\,nC$$

c) From conservation of charge, the total charge on the conducting sphere is equal to the algebraic sum of the charge $q$ on its inner surface (i.e., the wall of the cavity) and the charge on its outer surface. Therefore, if the charge on the outer surface of the sphere is $q'$, then we have

Net charge on the sphere $= +9.0\,nC = q + q'$

or $q' = +9.0\,nC - 3.2\,nC = 5.8\,nC$

d) Since the point at a distance of 4.0 m from the centre lies inside the conducting sphere, the electric field at that point is zero.