Gauss's law is used to find the electric fields in symmetrical capacitors. The Earth is a huge spherical capacitor that we use all the time. How do we do so? You will learn the answer in this unit!

UNIT 6

GAUSS'S LAW AND APPLICATIONS

Structure

STUDY GUIDE

In Unit 5, you have studied the concepts of charge, electrostatic force, Coulomb's law, electric field and calculated the electrostatic force on charges, electric field of point charges and continuous line charge.

In this unit, you will study Gauss's law that simplifies the calculation of electric fields and electrostatic forces for distributions of discrete point charges and symmetric continuous charge distributions. You will learn how to apply Gauss's law to a point charge and spherically symmetric systems like uniformly charged sphere and spherical shell for which the electric field has spherical symmetry. You have learnt about the divergence theorem in Unit 4, which you will also use in this unit. You should revise Units 1 to 4 of this course as you will be using them all the time to learn the concepts of this unit. Of course, you should also know the concepts of vector algebra thoroughly. We advise you to solve the SAQs and Terminal Questions given in this unit. You should study all sections of this unit thoroughly and make sure you can solve the SAQs and Terminal Questions on your own.

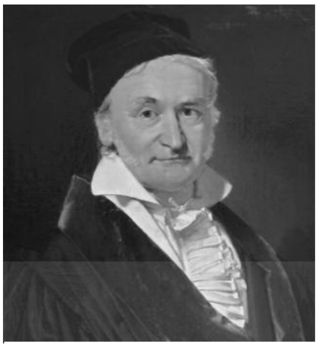
"All the measurements in the world do not balance one theorem by which the science of eternal truths is actually advanced."

Carl F. Gauss

6.1 INTRODUCTION

In Unit 5, you have revised the concept of charge and Coulomb's law. You have learnt the concept of electric field and calculated the electric field due to point charges and continuous line charge. You have also learnt how to calculate the electrostatic force on a charge kept in any given electric field.

This is what electrostatics is about: Calculating electric fields due to charges and electrostatic forces on a charge or distribution of charges placed in an electric field. You also saw how involved the calculation of the electric field of a line charge was. Would you not like to learn simpler methods for doing these calculations? This is what we do in the rest of this block. Most of this block involves **learning the tools** that simplify the calculation of electric fields and electrostatic forces.



Carl Friedrich Gauss (1777 – 1855), a German mathematician and physicist, is referred to as the 'greatest mathematician since antiquity'. He made exceptional contributions in the areas of mathematics such as algebra, number theory, analysis, differential geometry, and physics such as mechanics, electrostatics, magnetic fields, optics, etc. He is known as one of history's most influential mathematicians with equally significant contributions in physics.

In this unit, we describe an alternative to Coulomb's law and the principle of superposition to help us determine electric fields of discrete charges and charge distributions. This is the **Gauss's law** which relates electric charge distributions and electric fields. It gives us a simpler method to determine electric fields associated with symmetric charge distributions. If we know the electric fields in any region, we can also use the law to determine the net charge of the charge distributions that give rise to them.

We begin our study of Gauss's law by defining a new quantity called **electric flux** (Sec. 6.2). We then present the law in Sec. 6.3. You will learn that Gauss's law is particularly useful when applied to systems that possess some symmetry, a concept that you may know but will learn again in this unit. In Sec. 6.4, we apply the law to **spherically symmetric** systems and determine the electric fields due to a point charge, a uniform spherical charge distribution and a uniformly charged spherical shell.

You may ask: Why is it important for you to learn these applications of Gauss's law? One of the most important uses of these applications is in calculating the electric fields in capacitors and consequently their capacitances. You would know from your school physics that capacitors are important devices used to store electric charge and electrical energy. You will learn in detail about them in Unit 11 of Block 3. The Earth is one huge spherical capacitor that we use all the time as you will learn in Sec. 6.5.

In the next unit, we continue the discussion on Gauss's law for systems having cylindrical and planar symmetry such as a uniform line charge, a uniformly charged cylinder and a plane sheet of charge. You will learn some more applications of the law and then you will be able to appreciate the power of this law.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ define electric flux and calculate the electric flux due to an arbitrary distribution of charges;
- ❖ state Gauss's law;

- apply Gauss's law to calculate the electric field due to a point charge;
- apply Gauss's law to calculate the electric field due to a uniformly charged sphere; and
- using Gauss's law, determine the electric field due to a uniformly charged spherical shell.

6.2 ELECTRIC FLUX

You have learnt the concept of flux of a vector field in Sec. 4.2.1 of Unit 4 of this course. Here we briefly explain the concept again so that you can understand the concept of electric flux. You know that flux is defined for any vector field but is most easily pictured for the flow of fluids. So, we begin the discussion with a brief revision of the concept of flux for fluid flow.

The word flux has its origins in the old French word 'flus' and the Latin word 'fluxus' both meaning 'flowing' or 'to flow'. When we say that something is in the state of flux, we mean that it is changing.

Imagine that a stream of water or some fluid is flowing and the velocity of the particles in it is described by the velocity vector field. We now place a very small flat wire loop of area dS in the stream so that it is normal (perpendicular) to the direction of the flow (Fig. 6.1a). We choose this flat element of area to be small enough so that the velocity of all fluid particles flowing through it is constant. The **volume flux** of the fluid through the loop is defined as the **rate of flow** of the fluid through the area (of the loop). Let us determine its value.

Suppose ΔV is the volume of the fluid that passes through the small loop of area dS in time Δt. Since its area is flat and very small, we can take the speed v of the small amount of fluid flowing through it to be constant. So, during the time interval Δt, the fluid moves a length $\Delta x = v\Delta t$. The volume of fluid that flows through the loop during that time interval is then given by

$$\Delta V = dS\Delta x = dS\,v\,\Delta t \qquad (6.1)$$

So, the rate of flow of fluid through the very small area dS is given by

$$\frac{\Delta V}{\Delta t} = v\,dS \qquad (6.2a)$$

This is just the **volume flux** of the fluid when the small area chosen is **normal** (**perpendicular**) to the direction of its flow.

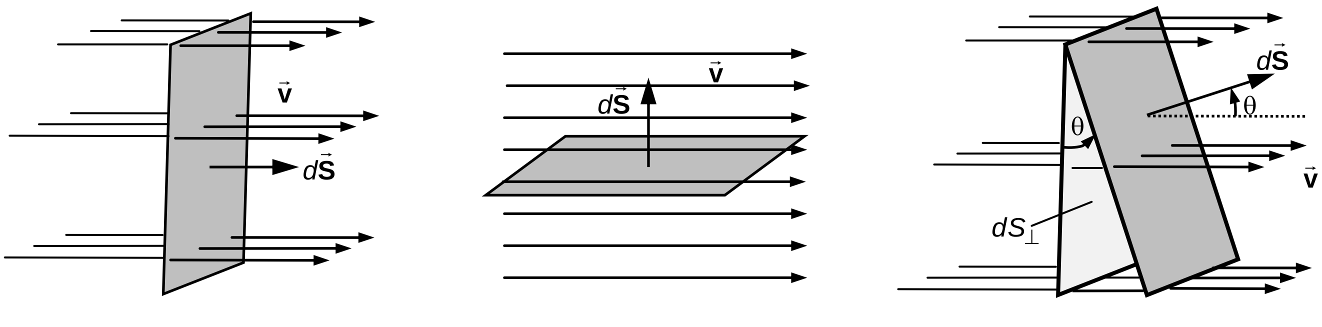


Fig. 6.1: **A wire loop placed in a stream a) normal and b) parallel to the direction of the flow or the velocity field $\vec{v}$; c) the same loop placed at an angle θ to the direction of fluid flow. In parts (a) and (c) of this figure, we have shown only a few lines for the fluid flow but the loop is immersed in the stream.**

What would the flux be if we kept the loop parallel to the direction of fluid flow as shown in Fig. 6.1b? You can see that **no fluid will now flow across the wire loop** or across the area dS. So, the volume flux will be zero in this case.

What would the flux be if we kept the loop at some angle θ to the direction of fluid flow as shown in Fig. 6.1c?

In this case, the fluid will pass through only that component of the area, which is perpendicular to the direction of fluid flow. This is just $dS_{\perp} = dS\cos\theta$.

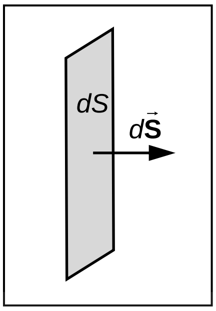


Fig. 6.2: **Area vector** $d\vec{\mathbf{S}}$ **for any surface of area** dS **is directed normal to the surface (refer to Sec. 4.3 of Unit 4 for the sense of the normal vector to the surface).**

Therefore, substituting $dS\cos\theta$ for dS in Eq. (6.2a), the **volume flux** through the loop kept at an angle θ to the direction of the fluid flow will be

$$v\,dS_{\perp} = v\,dS\cos\theta \qquad (6.2b)$$

Now, we use the definition of the scalar product to express the volume flux given by Eqs. (6.2a and b) as

$$\Phi_{dS} = \vec{\mathbf{v}}.d\vec{\mathbf{S}} \qquad (6.3)$$

where $\vec{\mathbf{v}}$ is the velocity field and $d\vec{\mathbf{S}}$, the area vector corresponding to the area dS of the loop (see Fig. 6.2). The area vector gives the magnitude of the area and its direction gives the sense of the flux through the area. In our example (Figs. 6.1a and c), the sense of the flux is from left-hand side of the loop to its right-hand side. If we choose the direction of the area vector to be opposite to this, i.e., from right to left, the sense of the flux would also be from the right-hand side of the loop to its left-hand side. We can choose either direction for the area vector but once chosen, it should remain the same and be specified.

Note that the scalar product of Eq. (6.3) reflects all three situations we have considered: When the loop is normal to the flow, $\theta = 90^\circ$ and Eq. (6.3) gives the volume flux as $v\,dS$, which is just Eq. (6.2a). If the loop is parallel to the flow, $\theta = 0^\circ$ and the flux through the loop is zero. For any other value of θ, Eq. (6.3) gives the volume flux as $v\,dS\cos\theta$, which is just Eq. (6.2b).

The definition of volume flux can be extended to the flux of any vector field including the electric field. **In an electrostatic field, nothing is flowing** but we define the flux of the electric field in analogy to Eq. (6.3).

By definition, the **electric flux** $d\Phi_E$ of an electric field $\vec{\mathbf{E}}$ through a small flat surface of area dS is defined as

$$d\Phi_E = \vec{\mathbf{E}}.\,d\vec{\mathbf{S}} \qquad (6.4)$$

where $d\vec{\mathbf{S}}$ is the area vector of magnitude dS **directed normal to the surface. Its orientation is defined to be outward to the surface**. Note that electric flux is a scalar quantity.

In Eq. (6.4), we have considered a small flat surface of area dS to define electric flux. You may ask: **What is the electric flux through a surface of any arbitrary shape**?

In that case, we divide the surface into a large number (say n) of small flat surfaces represented by area vectors $d\vec{\mathbf{S}}_i$, all pointing outwards from the same side of the surface. Let $\vec{\mathbf{E}}_i$ be the electric field through the element of surface area $d\vec{\mathbf{S}}_i$. Since flux is a scalar quantity, the electric flux through the surface S is just the sum of the electric flux through all such flat surfaces:

$$\Phi_E = \sum_{i=1}^{n} \vec{\mathbf{E}}_i \cdot d\vec{\mathbf{S}}_i \qquad (6.5)$$

We then make the sizes of the flat surfaces smaller and smaller so that $n \to \infty$ and collectively these surface elements approach the surface S. Then as you have learnt in Unit 4, the sum given in Eq. (6.5) approaches a limiting value which is equal to the electric flux through the surface S. In that limit, we can write the sum as a two-dimensional surface integral and the electric flux is given by

$$\Phi_E = \lim_{n \to \infty} \sum_i \vec{\mathbf{E}}_i \cdot d\vec{\mathbf{S}}_i = \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} \qquad (6.6a)$$

As you have learnt in Unit 4, the subscript S under the integral sign tells us that the area of integration is the entire surface S. If the surface is closed, we write the surface integral and Eq. (6.6a) as follows:

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} \qquad (6.6b)$$

In Unit 4, you have learnt how to determine surface integrals for different cases. From Eqs. (6.6a and b), you can see that electric flux is expressed as a surface integral. You may now like to determine the electric flux of an electric field through a surface using Eq. (6.6b). We take up the example of calculating the electric flux of a point charge through a closed surface. In the process, we shall arrive at Gauss's law.

EXAMPLE 6.1 : ELECTRIC FLUX OF A POINT CHARGE

Determine the electric flux for the electric field generated by a point charge q through a closed surface S of a sphere of radius R enclosing the charge such that the charge is placed at the centre of the sphere.

SOLUTION ■ We use Eq. (6.6b) to determine the electric flux through the surface of a sphere (of radius R) enclosing the charge q. From Eq. (6.6b), the electric flux through a closed surface is given by

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

where S is the surface of a sphere of radius R enclosing the charge q, which is kept at its centre. The electric field of the charge q at a point on the surface of the sphere is given from Eq. (5.6a) as

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \hat{\mathbf{r}}$$

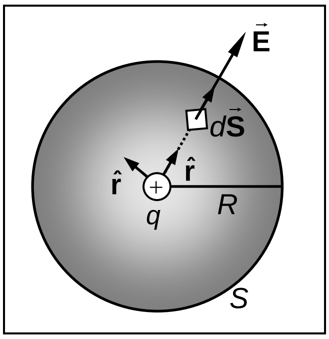


Fig. 6.3: **Calculation of the electric flux through a spherical surface enclosing charge** q**.**

where $\hat{\mathbf{r}}$ is the unit vector along the radial direction. Now, for a sphere, the direction of the area vector $d\vec{\mathbf{S}}$ is along the outward normal to its surface at all points on the surface. From Fig. 6.3 (showing one such point), you can see that it is along the vector $\hat{\mathbf{r}}$. Thus, we have

$$d\vec{\mathbf{S}} = dS\,\hat{\mathbf{r}} \qquad \text{and} \qquad \vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS\,\hat{\mathbf{r}}.\hat{\mathbf{r}} = E\,dS$$

The electric flux of the point charge through the sphere's surface is then

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{4\pi\varepsilon_0 R^2}\iint dS = \left(\frac{q}{4\pi\varepsilon_0 R^2}\right)\times 4\pi R^2 = \frac{q}{\varepsilon_0} \qquad (6.7)$$

Did you note in Example 6.1 that the radius of the sphere cancels out? This is because while the field **decreases** as $\frac{1}{r^2}$, the surface area **increases** as r^2. So, their product is constant. **REMEMBER**: This result arises because of the **inverse square nature of the electrostatic force field and the electric field**.

Also note that we have obtained Eq. (6.7) in Example 6.1 for the electric flux of a point charge across a **spherical surface** enclosing the charge. However, it is true for a surface of any shape enclosing a charge. This is what Gauss's law is about. So you will study it in greater detail in the next section. But before that, you may like to attempt an SAQ to determine electric flux for another simple situation.

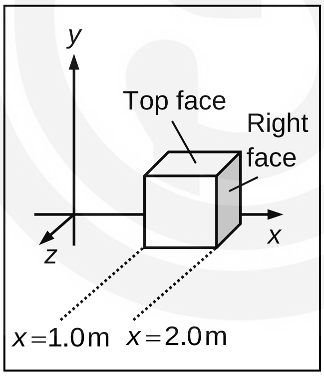
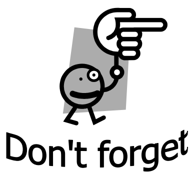


Fig. 6.4: **Diagram for SAQ 1.**

SAQ 1 - Electric flux

A cube of side 1.0 m is kept in an electric field (in units of NC^{-1}) given by $\vec{\mathbf{E}} = 8.0x\,\hat{\mathbf{i}} + 5.0\,\hat{\mathbf{j}}$ as shown in Fig. 6.4. Determine the electric flux through the right and top faces of the cube.

You should always remember the following about electric flux.



- **Electric flux through a surface (of area** S**) represents the summation of electric flux elements** $(\vec{\mathbf{E}}.d\vec{\mathbf{S}})$ **over the entire surface.**
- **Each electric flux element represents the product of a small flat element of area on the surface with the <u>component of the electric field along the normal</u> to that area element.**
- **This product is nothing but the scalar product of the electric field vector and the area element vector.**
- **Electric flux does not represent flow or change the way volume flux does.**

Let us now study Gauss's law.

6.3 GAUSS'S LAW

In Example 6.1, we have enclosed a point charge in a spherical surface and arrived at Eq. (6.7), which relates the electric flux through a spherical surface to the point charge q enclosed by it. This is just Gauss's law for a point charge. However, we have enclosed the point charge in a spherical surface, which is a special case. **Gauss's law applies to any arbitrary surface enclosing a charge or charge distribution**. Any imaginary surface enclosing a charge or a charge distribution is called a **Gaussian surface**. We usually choose the Gaussian surface so that our calculations become easier.

Therefore, in this section, we first generalise Eq. (6.7) for any arbitrary surface enclosing the point charge and arrive at a formal statement of Gauss's law. So, let us find out whether the same equation [Eq. (6.7)] applies to **any arbitrary surface** enclosing a point charge.

Consider the electric field of a positive point charge in free space. Imagine that the charge is enclosed in a **closed Gaussian surface** *S* of an **arbitrary shape** (Fig. 6.5).

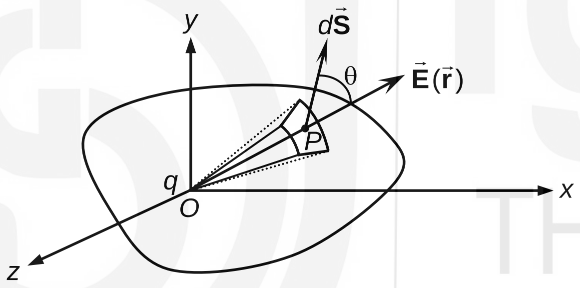


Fig. 6.5: **Gauss's law for a point charge enclosed by an arbitrary surface.**

Note from Fig. 6.5 that we have chosen the origin of the coordinate system to be at the location of the charge. Let *P* be a point on the Gaussian surface, having position vector $\vec{\mathbf{r}} = r\,\hat{\mathbf{r}}$. We choose a small element of area $d\vec{\mathbf{S}}$ centred at the point *P* on the Gaussian surface. As you know from Eq. (5.6a), the electric field at the point *P* is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{\vec{\mathbf{r}}}{r} \qquad (6.8)$$

Then from Eq. (6.4), the element of electric flux passing through $d\vec{\mathbf{S}}$ is given by

$$d\Phi_E = \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{\mathbf{r}}.d\vec{\mathbf{S}} \qquad (6.9)$$

Now, you know that if θ is the angle between $\vec{\mathbf{r}}$ and $d\vec{\mathbf{S}}$, then

$$\vec{\mathbf{r}}\,.\,d\vec{\mathbf{S}} = r\,dS\cos\theta \qquad (6.10a)$$

You also know from vector algebra that $dS\cos\theta$ is the projection of $d\vec{\mathbf{S}}$ along $\vec{\mathbf{r}}$. From Sec. 4.3.5 of Unit 4, you know that the quantity

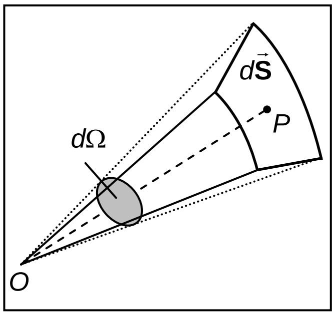


Fig. 6.6: **The solid angle $d\Omega$ subtended by an area element $d\vec{\mathbf{S}}$ at a point O. Recall Sec. 4.3.5 of Unit 4 for the definition of solid angle.**

$$\left(\frac{dS\cos\theta}{r^2}\right) = d\Omega \tag{6.10b}$$

is defined as the solid angle ($d\Omega$) subtended by the area $d\vec{\mathbf{S}}$ at O, the location of the charge (Fig. 6.6). Then using Eq. (6.10b), we can write Eq. (6.9) as

$$d\Phi_E = \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{4\pi\varepsilon_0}\frac{r\,dS\cos\theta}{r^3} = \frac{q}{4\pi\varepsilon_0}\,d\Omega \tag{6.11a}$$

The total electric flux through the surface S is determined by integrating over the entire closed surface as follows:

$$\Phi_E = \oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{4\pi\varepsilon_0}\oiint_S d\Omega \tag{6.11b}$$

Now since the surface S surrounds the point O and the total solid angle around any point is 4π (see Sec. 4.3.5 of Unit 4), we have

$$\oiint_S d\Omega = 4\pi \tag{6.11c}$$

So, we can write Eq. (6.11b) as

$$\Phi_E = \oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{\varepsilon_0} \tag{6.12}$$

Eq. (6.12) is the same as Eq. (6.7) for a spherical surface. Let us see whether we can extend Eq. (6.12) to a distribution of charges. Suppose that instead of a single charge at the centre of a sphere, many charges are situated in some region of space. From the principle of superposition [Eq. (5.11)], you know that the net electric field is the vector sum of all individual electric fields:

$$\vec{\mathbf{E}} = \sum_j \vec{\mathbf{E}}_j \tag{6.13}$$

By definition [Eq. (6.6b)], the electric flux through a closed surface that encloses all these charges is given by

$$\Phi_E = \oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \sum_j\left(\oiint_S \vec{\mathbf{E}}_j.d\vec{\mathbf{S}}\right) = \frac{1}{\varepsilon_0}\sum_j (q_j) \tag{6.14}$$

where we have substituted $\vec{\mathbf{E}}$ from Eq. (6.13) and used Eq. (6.12) for individual charges, i.e., we have written

$$\oiint_S \vec{\mathbf{E}}_j.d\vec{\mathbf{S}} = \frac{q_j}{\varepsilon_0} \tag{6.15a}$$

Let us write the sum of all charges enclosed by the surface as Q_{encl}, i.e., Q_{encl} is the total or net charge enclosed by the surface S:

$$Q_{encl} = \sum_j (q_j) \tag{6.15b}$$

Then, we can write Eq. (6.14) as follows:

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{Q_{encl}}{\varepsilon_0} \qquad (6.16)$$

Eq. (6.16) is the quantitative statement of **Gauss's law**. Let us now give a formal statement of Gauss's law.

GAUSS'S LAW

Gauss's law states that the net electric flux through any imaginary closed surface S (called the **Gaussian surface**) is directly proportional to the net charge (Q_{encl}) enclosed by the surface. In SI units, it is equal to $\frac{Q_{encl}}{\varepsilon_0}$.

The net charge is the algebraic sum (sum with sign of the charge included) *of all charges enclosed within the Gaussian surface*.

Mathematically, we write the law as

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{Q_{encl}}{\varepsilon_0} \qquad (6.16)$$

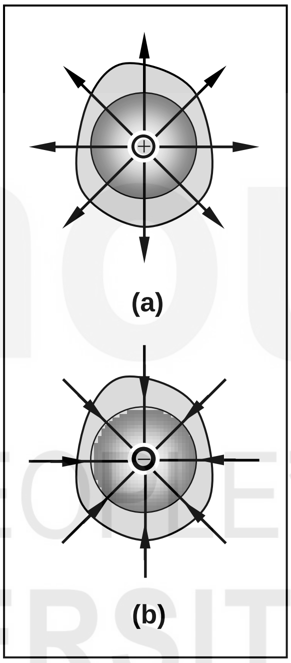


Fig. 6.7: **The same number of electric field lines will pass through surfaces of different shapes. Two Gaussian surfaces, one spherical and the other of arbitrary shape, are shown here for positive and negative charges**.

What Eq. (6.16) tells us is that the flux of the electric field through any surface would be the same regardless of its shape. It is proportional to the charge enclosed by it. This point is easier to visualise for a point charge if you picture its electric field in terms of the field lines passing through a surface. A surface of any shape enclosing the charge would have the same number of field lines passing through as that of the sphere's surface (Fig. 6.7). So the electric flux through any surface enclosing charge q is $\frac{q}{\varepsilon_0}$.

Eq. (6.16) is the integral form of Gauss's law. We can write **Gauss's law in the <u>differential</u> form** using the divergence theorem, which you have studied in Unit 4. For this, we write the charge enclosed by a surface in terms of the volume charge density ρ and substitute it in Eq. (6.16). Then we get

$$Q_{encl} = \iiint_V \rho\, dV \qquad (6.17a)$$

And

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{1}{\varepsilon_0}\iiint_V \rho\, dV \qquad (6.17b)$$

Now you may recall the divergence theorem from Unit 4 given as

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \iiint_V \vec{\nabla}.\vec{\mathbf{E}}\, dV \qquad (6.17c)$$

We substitute the value of $\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}}$ from Eq. (6.17c) in the left hand side of Eq. (6.17b).

Then Eq. (6.17b) becomes

$$\iiint\limits_V \vec{\nabla}.\vec{\mathbf{E}}\, dV = \frac{1}{\varepsilon_0} \iiint\limits_V \rho\, dV \qquad (6.17d)$$

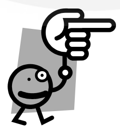
Since Eq. (6.17d) holds for any volume, the integrands must be equal and we have:

$$\vec{\nabla}.\vec{\mathbf{E}} = \frac{1}{\varepsilon_0}\rho \qquad (6.18)$$

Eq. (6.18) gives **Gauss's law in its differential form**.

It is easier to apply Gauss's law in its differential form. **However, note that we have expressed it only for volume charge density. Since the integral form of Gauss's law can be applied to point, line, surface and volume charges, it has wider use.**

In the next section, we consider some applications of Gauss's law to spherically symmetric systems. But before that you may like to remember the following aspects of Gauss's law and then try an SAQ to check your understanding.



Don't forget

- **In Eq. (6.16),** Q_{encl} is the **net charge** enclosed by the surface **taking into account the algebraic sign of the charges** (in case of many charges)**. So, if a surface encloses equal and opposite charges, the net electric flux through it is zero.**
- From the statement of Gauss's law, it is clear that the **charges lying outside the closed surface are not included in** Q_{encl}. **If the closed surface does not enclose any net charge, or if all charges lie outside the closed surface, then the electric flux through the surface is zero**. This implies that the electric field through such a surface is zero.
- We can **calculate the net charge enclosed inside any closed surface** using this law if we know the net electric flux through the surface enclosing the charges.
- **The form and location of the charges inside the closed surface do not matter in the calculations**. **What matters is the total charge enclosed by the closed surface and its sign**. This very fact makes the calculation of electric fields using the Gauss's law far easier in comparison with Coulomb's law.
- Gauss's law essentially follows from Coulomb's law and the principle of superposition. It contains no additional information that was not already present in Coulomb's law. The law follows from the inverse square nature of the electrostatic force. Without that, the cancellation of r^2 would not take place. Then the total flux would also depend on the surface chosen and not only upon the charge enclosed.

SAQ 2 - Gauss's law

a) Can we apply Gauss's law to the surfaces shown in Figs. 6.1a, b and c?

b) A point charge is enclosed by a spherical Gaussian surface. Would the electric flux through the surface change

i) If the Gaussian surface is chosen to be a closed cylinder or a cube?

ii) If the sphere is replaced by a cube that has one-tenth of its volume?

iii) If the charge is located at some other point within the sphere instead of its centre?

iv) If the charge is moved outside the Gaussian surface?

v) If another charge is placed inside the Gaussian surface?

vi) If another charge is placed outside the Gaussian surface?

c) The electric flux through a closed spherical Gaussian surface of radius 0.5 m surrounding a charged particle is equal to $500\,\text{Nm}^2\,\text{C}^{-1}$. Determine the value of the charge on the particle. If the radius of the surface were to be halved, what would the value of the electric flux through it be?

d) Determine the net electric flux through two overlapping closed surfaces S_1 and S_2 shown in Fig. 6.8, given that the values of the charges on the three particles are $q_1 = +3.1\,\text{nC}$, $q_2 = -5.9\,\text{nC}$ and $q_3 = -3.1\,\text{nC}$. The particle P enclosed by the surface S_1 carries no charge.

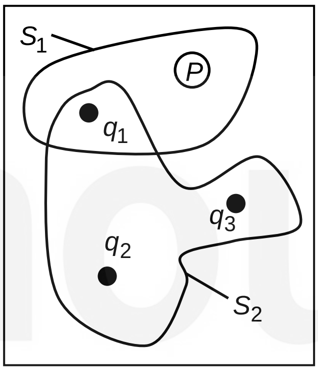


Fig. 6.8: **Diagram for SAQ 2d.**

You may be wondering: **Why do we need another method for calculating electric fields when we already have Coulomb's law**? This is because we can use Gauss's law to calculate the electric fields due to symmetric charge distributions in a much simpler way. You will discover in the next section and the next unit that Gauss's law is a powerful tool for determining electric fields of symmetric continuous charge distributions. Let us explain this point further.

6.3.1 Gauss's Law and Symmetric Charge Distributions

Let us first explain: **What are symmetric charge distributions**?

Symmetric charge distributions are arrangements of charges that **remain unchanged (or invariant) or look the same after a transformation**.

These charge distributions could be translated along some axis, reflected or rotated about some axis and would still appear the same.

Symmetry in physics essentially means that a system or an object remains unchanged (or invariant) under some transformation. You may already know of several examples of symmetric objects, e.g., a straight line, square, plane, sphere, cylinder, etc.

Due to the symmetries of charge distributions, the calculations of electric flux and electric fields due to them become far easier.

We will be dealing with three kinds of symmetry while applying Gauss's law:

1. Spherical symmetry
2. Cylindrical symmetry
3. Planar symmetry

We will talk about each of these symmetries when we apply Gauss's law to symmetric charge distributions in this unit and the next unit.

In the next three sections of this unit, you will learn how to apply Gauss's law. We will determine the electric field due to a point charge. We will also determine the electric fields due to **spherically symmetric** charge distributions such as a uniformly charged sphere and a spherical shell carrying uniform charge using Gauss's law. In the next unit, we will apply Gauss's law to infinitely long line of uniform charge, which has cylindrical symmetry and a plane sheet of charge having planar symmetry. So we will explain both these symmetries in the next unit.

Here we answer the question: **What is a spherically symmetric charge distribution**?

A charge distribution is said to be **spherically symmetric** if it remains **invariant** (**the same**)

- when it is **rotated around any axis passing through its centre**. It is said to possess **rotational symmetry** about that axis.
- when it is **reflected across any plane passing through its centre**. This is the **reflection symmetry**.

For such spherically symmetric charge distributions, we choose a spherical Gaussian surface. For a point charge, the centre of the Gaussian surface lies at the position of the charge. For a spherical charge distribution or a spherical shell, the Gaussian surfaces are concentric with them.

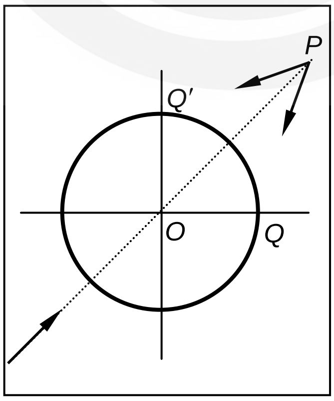


Fig. 6.9: **If the electric field is not radially directed, it will not remain the same under rotation or any other symmetry transformation of the sphere.**

The electric field of a spherically symmetric charge distribution is in the radial direction. It points outward from the centre of the sphere for positive charge and inward for negative charge. The magnitude of the electric field depends only on the distance r from the centre of the sphere. You may ask: Why is it so? Let us answer this question for both the direction and the magnitude of the electric field due to a spherically symmetric charge distribution.

Let us first answer the question: **Why is the electric field due to a spherically symmetric charge distribution directed radially i.e., it either points outward from the centre of the sphere, or inward along the radius of the sphere**?

Suppose the electric field at some point P outside the sphere **is not directed radially**, i.e., along the radius of the sphere. Suppose it points in some other direction, say in the direction of a point Q on the sphere's surface along the line PQ (see Fig. 6.9). Now suppose we rotate the sphere around the sphere's axis that passes through point P by 180°. The point Q shifts to position Q' on the sphere. Note that the sphere remains exactly the same and the point P

would also be in the same place. But the electric field would now point in a different direction – in the direction of Q' along the line PQ'.

This is a contradiction because you know that the electric field **at the same point due to the same charge distribution has to be in the same direction**; it cannot be in two different directions. When will the electric field **at any point** be in the same direction under any symmetry operation performed on the spherical charge distribution? This will happen only **if the electric field is directed along the axis of rotation of the sphere passing through that point**. This means that it must point along the axis of rotation (or the radius) of the sphere, i.e., in the **radial direction**.

Let us now answer the question: **Why does the magnitude of the electric field due to a symmetric charge distribution at any point depend only on its distance *r* from the centre of symmetry**?

Study Fig. 6.10. Suppose we have to determine the electric field at a point *P* at a distance *r* from the sphere. Consider a spherical surface *S* of radius *r* passing through that point, concentric with the spherical charge distribution. Now, consider any two points *P* and *Q* on the surface *S*. Note that these two points have the same radial coordinate but different angular coordinates.

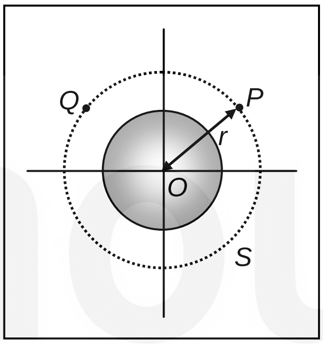


Fig. 6.10: **The magnitude of the electric field at any point *P* on the spherical surface *S* depends only on the radius *r* of the surface, i.e., the radial coordinate of *P*. Due to spherical symmetry, it is independent of the angular coordinates of the point.**

Let us now ask: **What would happen if the magnitude of the electric field depended on the angular coordinates of the points** *P* and *Q*? If this were so, the magnitude of the electric field due to the spherical charge distribution would be different at these two points.

But this is a contradiction **because due to spherical symmetry, the spherical charge distribution looks the same for all points on** *S* and hence for **both these points.** Therefore, for the same charge distribution, **the magnitude of the electric field cannot be different for different points on** *S*. It has to be the same for **all points** on the spherical surface *S*, i.e., all points at the same distance *r* from the centre of the charge distribution.

Hence, the **magnitude of the electric field at any point** on the spherical surface *S* (of a fixed radius) **cannot depend on the angular coordinates of that point**. It will only depend on the radius of the spherical surface, i.e., the radial coordinate of the point, which is just the distance of the point from the centre of the charge distribution. Therefore, we have

$$E(\vec{\mathbf{r}}) = E(r) \text{ for a spherically symmetric charge distribution}$$

So, all points on the spherical surface *S* of radius *r* are equivalent as far as the magnitude of the electric field is concerned. You must always remember the following for any spherically symmetric charge distribution.

- **The electric field due to the spherical charge distribution is directed radially.**
- **The magnitude of the electric field at any point depends only on the distance *r* of the point from the centre of the charge distribution.**

Let us now apply Gauss's law to determine the electric field due to a point charge.

6.4 ELECTRIC FIELD DUE TO A POINT CHARGE

Using Gauss's law, let us determine the electric field due to a positive point charge q at point P situated at a distance r from the charge.

We use Gauss's law given by Eq. (6.16) taking $Q_{encl} = q$.

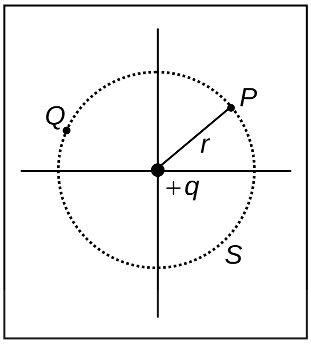


Fig. 6.11: **Spherical Gaussian surface *S* for determining electric field due to a positive point charge.**

We draw a spherical Gaussian surface of radius r passing through the point P with the charge at the centre of the sphere (Fig. 6.11). Now, you have learnt in Sec. 6.3.1 that for spherical symmetry, the electric field points radially outwards for a positive charge, i.e., the direction of the electric field is normal to the sphere's surface. The area vector $d\vec{\mathbf{S}}$ for any surface area element of the sphere is also normal to its surface. So, it is parallel to the electric field $\vec{\mathbf{E}}$ and $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS.$ Then Gauss's law becomes

$$\oint\!\!\oint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \oint\!\!\oint_S E\,dS = \frac{q}{\varepsilon_0}$$

Due to spherical symmetry, the magnitude of the electric field due to the charge would be the same for all points on the spherical surface and we can take it to be constant for S. So, we can take E out of the integral and write

$$\oint\!\!\oint_S E\,dS = E\oint\!\!\oint_S dS = \frac{q}{\varepsilon_0}$$

So, the integral is just the area of the spherical surface, i.e., it is $4\pi r^2$. Thus,

$$E\,4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

and $$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \qquad (6.19)$$

Did you notice that Eq. (6.19) is the same as Eq. (5.6a) of Unit 5 that was obtained from Coulomb's law? This means that Gauss's law and Coulomb's law give us the same result for the electric field due to a point charge. Gauss's law is equally true for a distribution of charges. You have seen it in arriving at Eq. (6.16).

The result for the electric field due to a charge distribution will be the same whether we use Gauss's law or Coulomb's law to calculate it. The only difference between the two laws is this: It is easier to use Coulomb's law for a charge distribution having many discrete point charges. But it is far easier to use Gauss's law if the charge distributions are continuous and symmetric. You have learnt this in this section for a point charge and will learn in the next two sections and next unit for other charge distributions. Otherwise, these two laws are not independent laws but the same law expressed in different ways.

6.5 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERE

Let us now apply Gauss's law to a spherical charge distribution having uniform volume charge density. You can verify that a charged sphere possesses spherical symmetry. It remains **invariant** (**the same**)

- when it is **rotated around any axis passing through its centre**; and
- when it is **reflected across any plane passing through its centre**.

The **volume charge density** (charge per unit volume) of a spherically symmetric charge distribution such as the charged sphere is the same at all points situated at the distance *r* from its centre. At any point, it depends only on the distance of that point from the centre of the sphere and not on the direction. Thus, the volume charge density ρ of a spherically symmetric charge distribution is a function of only *r*.

You have learnt in Sec. 6.3.1 that the **magnitude of the electric field** due to a spherically symmetric charge distribution at any point depends only on *r*. The direction of the electric field is radially outward for positive charge distribution and radially inward for a negative charge distribution. Let us now apply Gauss's law to determine the electric field due to a uniformly charged sphere.

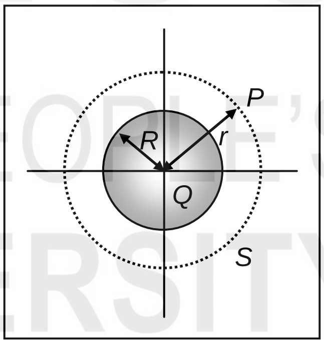


Fig. 6.12: **Determining the electric field due to a uniformly charged sphere of radius *R* carrying net charge *Q* at a point *P* outside the sphere.**

Consider a non-conducting charged sphere of radius *R* carrying total positive charge *Q* (Fig. 6.12). It is uniformly charged, which means that its volume charge density ρ is constant. Let us determine the electric field due to this charge distribution at a point *P* outside it, at a distance *r* from the centre of the sphere.

We draw a spherical Gaussian surface *S* of radius *r* through the point *P*. Since the point *P* lies outside the sphere, $r > R$ and $Q_{encl} = Q.$ From Gauss's law [Eq. (6.16)], we have

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{Q}{\varepsilon_0} \qquad (6.20)$$

Due to spherical symmetry, the magnitude of the electric field is the same on all points on the Gaussian surface. So we can take it to be constant for this Gaussian surface. The direction of the electric field is radially outwards for the positive charge, i.e., in the same direction as $d\vec{\mathbf{S}}$. So, $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are parallel and

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS \qquad (6.21a)$$

Since *E* (the magnitude of the electric field on the Gaussian surface) is constant, we can pull it out of the surface integral.

Therefore, Eq. (6.21a) becomes

$$\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\oiint_S dS = E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \tag{6.21b}$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{for } r \geq R \tag{6.21c}$$

The electric field is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad \text{for } r \geq R \tag{6.22}$$

Notice that we have included the points lying on the surface of the spherical charge distribution in the result because the Gaussian sphere of radius R would enclose the entire charge. Did you also notice that the electric field given by Eq. (6.22) is the same as that due to a point charge [given by Eq. (6.19)]? It is as if the entire charge within the spherical surface is concentrated at the centre of the sphere. **Note that this result is a consequence of spherical symmetry**. So, a uniformly charged sphere would exert the same force on a charge placed anywhere **outside** it as an equivalent single charge would.

The electric field due to a uniformly charged sphere and the electrostatic force exerted by it on a charge situated outside the sphere are the same as the electric field and electrostatic force due to a point charge (equal to the charge of the sphere) situated at its centre.

Let us now determine the electric field at a point **inside** a spherical charge distribution carrying net charge Q , i.e., at points for which $r < R$ (see Fig. 6.13).

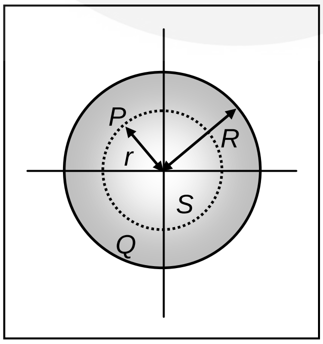


Fig. 6.13: **Determining the electric field of a uniformly charged sphere of radius *R* carrying net charge *Q* at a point *P* inside the sphere.**

For this, we draw a spherical Gaussian surface of radius $r < R$. We apply Eq. (6.20), in which Q has now to be replaced by the charge (q) enclosed by the Gaussian sphere of radius r.

What is the value of the charge enclosed by the Gaussian sphere of radius r?

You know that the volume charge density is uniform for the charged sphere of radius R, i.e., ρ is constant. The volume of the spherical charge distribution is $\frac{4\pi}{3}R^3$. Since the volume charge density ρ (charge per unit volume) is constant, for the sphere of volume $\frac{4\pi}{3}R^3$ carrying charge Q, it is given by

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3} \tag{6.23a}$$

Therefore, the charge enclosed by the Gaussian sphere of volume $\frac{4\pi}{3}r^3$ will be the product of its volume with the volume charge density:

$$q = \rho \frac{4\pi}{3} r^3 = \frac{Q}{\frac{4\pi}{3} R^3} \left[\frac{4\pi}{3} r^3 \right] = Q \frac{r^3}{R^3} \qquad (6.23b)$$

Using Eq. (6.23b) for q and the result $\oint\!\!\oint_S \vec{\mathbf{E}} . d\vec{\mathbf{S}} = E \oint\!\!\oint_S dS = E\,4\pi r^2$ from Eq. (6.21b) in Eq. (6.16), we have

$$E\,4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \frac{r^3}{R^3}$$

or $$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3} \quad \text{for} \quad r < R \qquad (6.24a)$$

The electric field at a point inside the uniformly charged sphere is given by

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3} \hat{\mathbf{r}} \quad \text{for} \quad r < R \qquad (6.24b)$$

Note from Eqs. (6.24a) and (6.22) that the electric field inside the spherical charge distribution increases linearly with distance from its centre ($E \propto r$). However, for points outside the sphere, the electric field falls off as $\frac{1}{r^2}$. We show this behaviour of the electric field in Fig. 6.14.

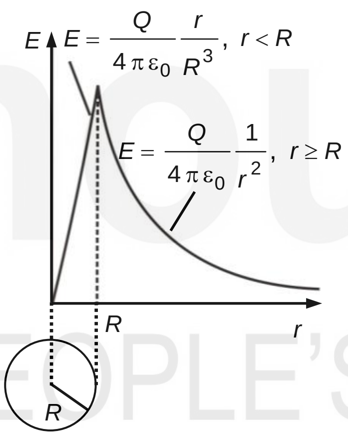


Fig. 6.14: **The behaviour of the electric field due to a uniformly charged sphere of radius *R*.**

We have said in the introduction that these results for the electric field due to a spherical charge distribution will be of use when you determine the capacitance of a spherical capacitor. As we have said on the first page of this unit, the Earth is one huge spherical capacitor that we use all the time. The Earth's capacitance is so large (~ 0.0007 F) that we can dump charge in it or take it out without changing its electric field much. That is why, we 'ground' or 'earth' the electrical circuits in our homes and all electrical appliances and instruments. That is also why we connect the lightning rods in buildings to the Earth so that most excess charge flows into it without hurting people.

Another example of spherical charge distributions is an isolated atom of inert gases. Since the atom is neutral, it carries no net charge and from Gauss's law, the electric field outside it is zero. Even when the atoms of an inert gas are in the neighbourhood of other atoms in it, they depart only slightly from spherical symmetry, and the electric fields near them remain small. So, we can say that the feeble chemical activity of inert gases is related to their spherically symmetric charge distributions. In the next section, you will learn how to apply Gauss's law to determine the electric field due to a spherical shell. Before that, you should solve an SAQ.

SAQ 3 - Applying Gauss's law to charged sphere

The electric field due to a uniformly charged sphere of radius 0.1 m has the magnitude 9.0 NC^{-1} at a distance of 0.3 m from the centre. What is the net charge on the sphere? What is the volume charge density of the charge distribution?

6.6 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL

As a first step, do convince yourself that a thin spherical shell possesses spherical symmetry, i.e., it remains the same under any rotation about its axis and any reflection about a plane passing through its centre and axis of rotation. You can rotate or reflect a hollow sphere with a thin surface (such as a hollow ball) to verify the spherical symmetry of a spherical shell.

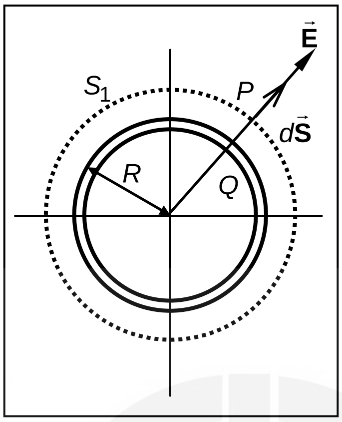


Fig. 6.15: **A thin uniformly charged spherical shell of radius *R* carrying a net charge *Q*. The cross-section of the Gaussian surface S_1 is shown for a point lying outside the shell. It is concentric with the shell.**

Now, consider a non-conducting thin spherical shell of radius R carrying a total positive charge Q that is distributed uniformly over its surface (Fig. 6.15). Let us determine the electric field due to this shell at a point lying outside it.

For a point P lying outside the shell, we draw a spherical Gaussian surface S_1 through the point and concentric with the spherical shell. You can see that the Gaussian surface lies outside the shell. Let us determine the electric field at the point P (see Fig. 6.15).

Due to the spherical symmetry of the charged spherical shell, its electric field has the same magnitude at every point on any spherical Gaussian surface and is directed radially. We apply Gauss's law [Eq. (6.16)] with $Q_{encl} = Q$ to the spherical surface S_1 and note that the electric field $\vec{\mathbf{E}}$ is in the same direction as $d\vec{\mathbf{S}}$ for S_1 so that $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are parallel. Therefore,

$$\vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\,dS \qquad (6.25a)$$

and since E (the magnitude of the electric field on the Gaussian surface) is constant, we can pull it out of the surface integral. Therefore, Eq. (6.16) becomes

$$\oint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = E\oint_S dS = E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad (6.25b)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{for} \quad r \geq R \qquad (6.25c)$$

The electric field at any point lying outside the spherical shell of radius R is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \quad \text{(spherical shell, for } r \geq R) \qquad (6.26)$$

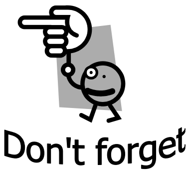
Note that the electric field given by Eq. (6.26) is the same as that due to a point charge [given by Eq. (6.19)].

For the electric field at a point lying outside the spherical shell, it is as if the entire charge *Q* of the spherical shell were replaced by a single equal charge placed at the centre of the shell.

Thus, a uniformly charged spherical shell would exert the same force on a charge placed anywhere **outside** the shell as a single equal charge would.

So always remember,

The electric field due to a spherical shell with a uniform charge distribution and the electrostatic force exerted by it on a charge situated outside the shell are the same as due to a single charge (equal to the charge of the shell) situated at its centre.



What is the electric field at a point **inside** the shell, i.e., at a point lying anywhere in the empty interior part of the shell?

For a point lying inside the shell, we draw a spherical Gaussian surface S_2 concentric with the spherical shell, lying in the empty interior of the shell (see Fig. 6.16). Since this Gaussian surface encloses no net charge, from Gauss's law, the electric field is zero at all points inside the shell:

$$\vec{\mathbf{E}} = \vec{\mathbf{0}} \quad \text{(spherical shell, for } r < R\text{)} \qquad (6.27)$$

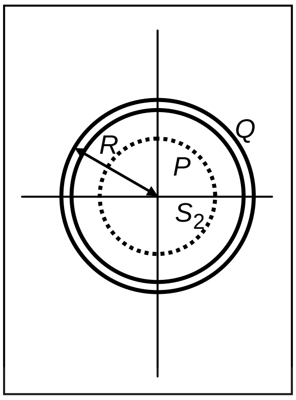


Fig. 6.16: **The cross-section of a Gaussian surface S_2 enclosing the empty interior of the thin uniformly charged spherical shell of radius R carrying a net charge Q.**

So, **always remember, when a charge is enclosed by a uniformly charged spherical shell so that the charge lies inside the shell, no electrostatic force is exerted on the charge by the shell**.

Let us apply what you have learnt in this section to an example of two concentric thin spherical shells.

EXAMPLE 6.2: TWO CONCENTRIC THIN SPHERICAL SHELLS

Two concentric thin spherical shells of radii R_1 and R_2 (with $R_2 > R_1$) carry uniformly distributed charges q_1 and q_2, respectively (Fig. 6.17). Use Gauss's law to determine the electric fields at the points

a) $r < R_1$,

b) $R_2 < r < R_1$ and

c) $r \geq R_2$.

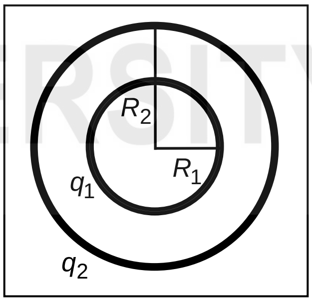


Fig. 6.17: **Diagram for Example 6.2.**

SOLUTION ■ We use Gauss's law along with the results obtained for a thin spherical shell.

a) For the point $r < R_1$, that is, any point A lying inside the inner spherical shell, we can draw the spherical Gaussian surface through it (Fig. 6.18).

You can see that the **charge enclosed by that Gaussian surface is zero**. From Eq. (6.27) obtained using Gauss's law for a point inside the thin spherical shell, we get the result that the electric field for $r < R_1$ is zero:

$$\vec{\mathbf{E}} = \vec{\mathbf{0}} \quad \text{(inside the inner spherical shell, for } r < R_1\text{)}$$

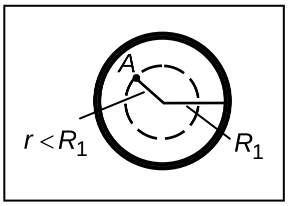


Fig. 6.18: **The electric field at a point inside the inner shell is zero since the charge enclosed by it is zero.**

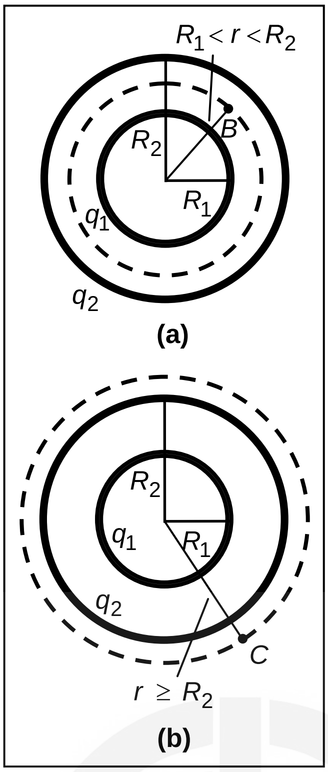


Fig. 6.19: **Diagram for parts (b) and (c) of Example 6.2.**

b) For the point $R_1 < r < R_2$, that is, the point lying between the two concentric shells, the net charge enclosed by the Gaussian surface of radius r is just the charge q_1 on the inner spherical shell (Fig. 6.19a). Therefore, from Eq. (6.26), the electric field at any point between the two thin concentric shells is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{\mathbf{r}} \qquad (\text{for } R_1 < r < R_2)$$

c) For the point $r \geq R_2$, that is, the point lying outside the outer spherical shell (Fig. 6.19b), the net charge enclosed by the Gaussian surface of radius r is the sum of the charges q_1 and q_2. Therefore, from Eq. (6.26), the electric field at any point outside the outer spherical shell is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(q_1 + q_2)}{r^2}\hat{\mathbf{r}} \qquad (\text{for } r \geq R_2)$$

What would your answers be if the charges on the inner shell and outer shell were equal to $+q$? To know this, answer the following SAQ!

SAQ 4 – Uniformly charged thin spherical shell

Each of two concentric thin spherical shells of radii R_1 and R_2 (with $R_2 > R_1$) carries uniformly distributed charge $+q$. Use Gauss's law to determine the electric fields due to the shells at the points a) $r < R_1$, b) $R_2 < r < R_1$ and c) $r \geq R_2$.

With this discussion on the applications of Gauss's law to spherically symmetric charge distributions, we end this unit. In the next unit, we continue our study of the applications of Gauss's law to charge distributions possessing cylindrical and planar symmetry. Let us now summarise what you have learnt in this unit.

6.7 SUMMARY

Concept	Description
Electric flux	■ The **electric flux** through a surface (of area S) represents the sum of electric flux elements $(\vec{\mathbf{E}}.d\vec{\mathbf{S}}_i)$ over the entire surface. Each flux element represents the product of a small flat element of area on the surface and the <u>**component of the electric field along the normal**</u> to that area element. This product is nothing but the scalar product of the electric field vector and the area element vector. Mathematically, electric flux or the flux of an electric field $\vec{\mathbf{E}}$ through a surface of area S is defined as $$\Phi_E = \iint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}}$$

Remember, electric flux does not represent flow or change of any entity.

Gauss's law

- **Gauss's law** states that the **net electric flux** through any imaginary **closed** surface *S* of arbitrary shape (called the Gaussian surface) is directly proportional to the net charge (Q_{encl}) enclosed by the surface. In SI units, it is equal to $\frac{Q_{encl}}{\varepsilon_0}$. *The net charge is the algebraic sum* (sum with sign of the charge included) *of all charges enclosed within the Gaussian surface*.

  Mathematically, the law in its integral form is $\oiint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{Q_{encl}}{\varepsilon_0}$

  The differential form of Gauss's law is $\vec{\nabla}.\vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$

  where ρ is the volume charge density of the charge distribution.

Applications of Gauss's law to spherically symmetric systems

- Using Gauss's law, we can determine the electric field due to a point charge, distribution of discrete charges and continuous charge distributions enclosed by arbitrary surfaces. In this unit, we have considered spherically symmetric charge distributions.

  A charge distribution is said to be **spherically symmetric** if it remains **invariant** (**the same**)

  - when it is **rotated around any axis passing through its centre**. It is said to possess **rotational symmetry** about that axis.
  - when it is **reflected across any plane passing through its centre**. This is the **reflection symmetry**.

  Examples are a point charge, a uniformly charged sphere and a uniformly charged spherical shell.

  The **magnitude** of the electric field of a spherically symmetric charge distribution at any point depends only on *r*, the distance of the point from the centre of symmetry. The **direction** of the electric field is radially outward for positive charge distribution and radially inward for a negative charge distribution.

Point charge

- The **electric field of a point charge** *q* at a distance *r* from it is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

Uniformly charged sphere

- The **electric field due to a uniformly charged sphere of radius** *R* **carrying charge** *Q* at a point located outside the sphere at a distance *r* is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad \text{for } r \geq R$$

For a point inside the sphere, it is given by

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}\hat{\mathbf{r}} \qquad \text{for} \quad r < R$$

Uniformly charged thin spherical shell

- The **electric field due to a uniformly charged thin spherical shell of radius** *R* **carrying charge** *Q* at any point lying outside the shell at a distance *r* from its centre is given by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad \text{(spherical shell, for } r \geq R\text{)}$$

At all points lying anywhere in the empty interior part of the shell, the electric field is zero:

$$\vec{\mathbf{E}} = \vec{\mathbf{0}} \qquad \text{(spherical shell, for } r < R\text{)}$$

6.8 TERMINAL QUESTIONS

1. Calculate the flux of the electric field $\vec{\mathbf{E}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}$ through the surfaces of area $1.0\,\text{m}^2$ situated in the *xy*, *xz* and *yz* planes, respectively.
2. A particle carrying a charge of $2.7\times10^{-9}\,\text{C}$ is enclosed in a cubical Gaussian surface of side 0.5 m. Calculate the electric flux through the surface of the cube and any one of its faces.
3. Consider a system of four charges: 3*q*, *q*, – 3*q* and – *q*. Draw a Gaussian surface enclosing at least two charges of the system so that the net electric flux through it is a) zero, b) $+\left(\frac{4q}{\varepsilon_0}\right)$, c) $+\left(\frac{2q}{\varepsilon_0}\right)$ and d) $-\left(\frac{2q}{\varepsilon_0}\right)$.
4. The electric field in some region of space is given by $\vec{\mathbf{E}} = cr\,\hat{\mathbf{r}}$, where *c* is a constant. Use the differential form of Gauss's law to calculate the volume charge density, which gives rise to this electric field. Obtain the total charge contained in a sphere of radius *R*, centred at the origin in this region of space.
5. Suppose that a Gaussian surface encloses zero net charge. (a) Does Gauss's law require that the electric field be zero for all points on the surface? (b) If the electric field is zero everywhere on the Gaussian surface, does Gauss's law require that the net charge inside the surface be zero?
6. Is Gauss's law useful in calculating the electric field due to three equal charges placed at the corners of an equilateral triangle? Explain.
7. A charge *q* is placed at a corner of a cube as shown in Fig. 6.20. Determine the flux of the electric field of the charge through the right face (*ABCD*) of the cube? (Hint: Solving this problem requires a clever choice of the Gaussian surface.)

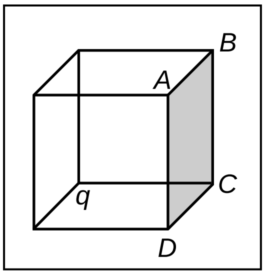


Fig. 6.20: **Diagram for TQ 7.**

8. a) The electric flux due to a point charge passing through a spherical Gaussian surface of radius 0.10 m centred on the charge is

$-900\,\text{Nm}^2\,\text{C}^{-1}$. What is the value of the point charge? What is the electric field due to the point charge at a point on the Gaussian surface? What would the electric flux through the Gaussian surface be if its radius were increased to 0.30 m?

b) The magnitude of the electric field due to a non-conducting charged sphere of radius 0.30 m at a distance of 0.10 m from its centre is $3.0\times10^3\ \text{NC}^{-1}$. What is the net charge on the sphere?

9. A non-conducting sphere of radius *R* carrying net positive charge *Q* is enclosed by a concentric non-conducting thin spherical shell of radius *r* carrying net negative charge *q*. Determine the electric field (a) inside the sphere, (b) between the sphere and the shell, and (c) outside the shell.

10. A charged non-conducting spherical shell having inner radius 3.0 m and outer radius10 m carries a charge of magnitude 9.0 nC distributed uniformly over its volume. Determine the electric field due to it at a distance of 6.0 m from its centre.

6.9 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. We can determine the electric flux through the faces of the cube by using Eq. (6.6a), i.e., by integrating the scalar product $\vec{\mathbf{E}}.d\vec{\mathbf{S}}$ over the right and top faces of the cube. Refer to Fig. 6.21. For the choice of the coordinate axes, the area vector for the right face is $d\vec{\mathbf{S}} = dS\,\hat{\mathbf{i}}$ and for the top face, it is $d\vec{\mathbf{S}} = dS\,\hat{\mathbf{j}}$. So, the electric flux through the right face of the cube is given as:

$$\Phi_E = \iint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \iint_S (8.0\,x\,\hat{\mathbf{i}} + 5.0\,\hat{\mathbf{j}}).dS\,\hat{\mathbf{i}}$$

$$= \iint_S (8.0x)\hat{\mathbf{i}}.\hat{\mathbf{i}}\,dS = (8.0)\iint_S (x)dS \quad (\because\ \hat{\mathbf{i}}.\hat{\mathbf{i}} = 1,\ \hat{\mathbf{i}}.\hat{\mathbf{j}} = 0)$$

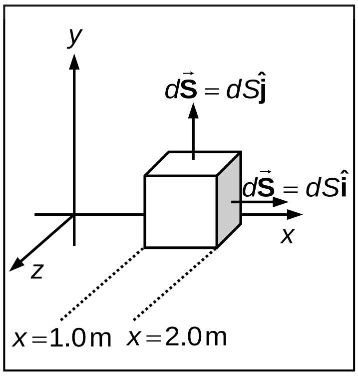


Fig. 6.21: **Diagram for the answer of SAQ 1.**

Now, on the right face of the cube, *x* is constant and has the value *x* = 2.0 m. Therefore, for the right face, we get

$$\Phi_E = (8.0)\iint_S (2.0)dS = (16.0)\iint_S dS$$

Now the integral $\iint_S dS$ is equal to the area of the right face of the cube, which is just $1.0\,\text{m}^2$. Therefore, the electric flux through the right face of the cube is

$$\Phi_E = (16.0)\text{NC}^{-1}\text{m}^2$$

Now, we follow the same steps for the top face of the cube as we followed for the right face of the cube. Since for the top face, $d\vec{\mathbf{S}} = dS\,\hat{\mathbf{j}}$ and $\hat{\mathbf{j}}.\hat{\mathbf{j}} = 1,\ \hat{\mathbf{i}}.\hat{\mathbf{j}} = 0,$ the electric flux through the top face of the cube is given as:

$$\Phi_E = \iint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \iint_S (8.0\,x\,\hat{\mathbf{i}} + 5.0\,\hat{\mathbf{j}}).dS\,\hat{\mathbf{j}} = \iint_S (5.0)dS = (5.0)\iint_S dS$$

Now the integral $\iint_S dS$ is equal to the area of the top face of the cube, which is just $1.0\,\mathrm{m}^2$. Therefore, the electric flux through the top face of the cube is $\Phi_E = (5.0)\mathrm{NC}^{-1}\mathrm{m}^2$.

2. a) We cannot apply Gauss's law to the surfaces shown in Figs. 6.1a, b and c as these are open surfaces (these do not define an enclosed volume) and Gauss's law can be applied to only closed surfaces.

b) i) No, the electric flux through the surface would not change as the Gaussian surface can be of any shape and the electric flux is equal to only the net charge enclosed by it.

ii) No, since the net charge enclosed by the surface is the same.

iii) No, because the location of the charge within the surface does not matter.

iv) Yes, because the net charge enclosed by the surface would change.

v) Yes, because the net charge enclosed by the surface would change.

vi) No, since the net charge enclosed by the surface is the same.

c) From Eq. (6.12), the value of the charge on the particle is given by

$$q_{encl} = \varepsilon_0 \Phi_E = (8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}})\times 500\,\mathrm{Nm^2\,C^{-1}} = 4.42\times10^{-9}\,\mathrm{C}$$

The electric flux through the surface would not change since the net charge enclosed by it remains the same.

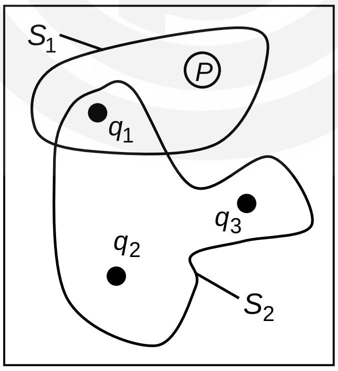


Fig. 6.22: **Diagram for the answer of SAQ 2d.**

d) Refer to Fig. 6.22. The net charge enclosed by the surface S_1 is $q_1 = +3.1\,\mathrm{nC}$. Since the particle P enclosed by the surface S_1 carries no charge, it makes no contribution to the electric flux. The remaining charges are outside the surface. Therefore, from Eq. (6.12),

$$\Phi_E = \frac{q_{encl}}{\varepsilon_0} = \frac{q_1}{\varepsilon_0} = \frac{3.1\times10^{-9}\,\mathrm{C}}{8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}} = 350\,\mathrm{Nm^2\,C^{-1}}$$

The net charge enclosed by the surface S_2 is

$$q_1 + q_2 + q_3 = +3.1\,\mathrm{nC} + (-5.9\,\mathrm{nC}) + (-3.1\mathrm{nC}) = -5.9\,\mathrm{nC}$$

Therefore, from Eq. (6.12),

$$\Phi_E = \frac{q_{encl}}{\varepsilon_0} = \frac{-5.9\times10^{-9}\,\mathrm{C}}{8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}} = -6.7\times10^{2}\,\mathrm{Nm^2\,C^{-1}}$$

3. We use Eq. (6.22) for the electric field of a uniformly charged sphere since the point lies outside the sphere and take the magnitude only. Therefore, the net charge on the sphere is

$$Q = E(4\pi\varepsilon_0 r^2) = \frac{(9.0\,\text{NC}^{-1})\times(0.3\,\text{m})^2}{8.99\times10^9\,\text{C}^{-2}\text{Nm}^2} = 0.09\,\text{nC} = 0.1\,\text{nC}$$

Since the sphere is uniformly charged, its volume charge density is given by Eq. (6.23a):

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3} = \frac{0.09\times10^{-9}\,\text{C}}{\frac{4\pi}{3}(0.1\,\text{m})^3} = 2.1\times10^{-8}\,\text{C m}^{-3}$$

4. Refer to Fig. 6.23. We follow the steps in Example 6.2 with $q_1 = q_2 = +q$.

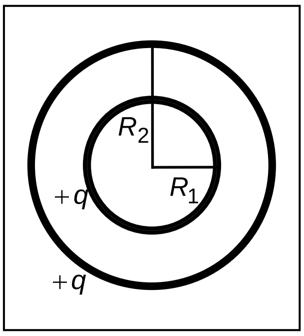


Fig. 6.23: **Diagram for the answer of SAQ 4.**

a) For the point $r < R_1$, the net charge enclosed by a spherical Gaussian surface passing through r, (i.e., a point inside the inner shell) is zero. Hence, for $r < R_1$,

$$\vec{\mathbf{E}} = \vec{\mathbf{0}}$$

b) For $R_2 < r < R_1$, (i.e., a point lying between the two concentric shells), the net charge enclosed by a spherical Gaussian surface passing through r is just $+q$ and hence, for $R_2 < r < R_1$,

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

c) For $r \geq R_2$, (i.e., a point lying outside the outer shell), the net charge enclosed by a spherical Gaussian surface passing through r is $+q+q = +2q$ and hence, for $r \geq R_2$,

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}\hat{\mathbf{r}}$$

Terminal Questions

1. Refer to Fig. 6.24. A surface of area S in the xy plane is represented by the vector $S\hat{\mathbf{k}}$ since $\hat{\mathbf{k}}$ is the unit vector perpendicular to the xy plane. Therefore, the flux of the electric field $\vec{\mathbf{E}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}$ through a surface of area $S = 1.0\,\text{m}^2$ situated in the xy plane is

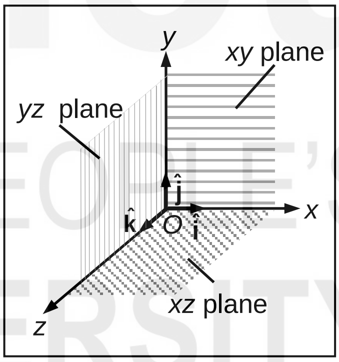


Fig. 6.24: **Area vectors for the answer of TQ 1.**

$$\Phi_E = \vec{\mathbf{E}}.S\hat{\mathbf{k}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}.(1.0\,\text{m}^2)\hat{\mathbf{k}} = 100\,\text{NC}^{-1}\text{m}^2\,(\hat{\mathbf{i}}.\hat{\mathbf{k}}) = 0$$

The area vector in the xz plane is given by $S\,\hat{\mathbf{j}}$ and for $S = 1.0\,\text{m}^2$, the flux of the electric field $\vec{\mathbf{E}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}$ through the xz plane is

$$\Phi_E = \vec{\mathbf{E}}.S\,\hat{\mathbf{j}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}.(1.0\,\text{m}^2)\,\hat{\mathbf{j}} = 100\,\text{NC}^{-1}\text{m}^2\,(\hat{\mathbf{i}}.\hat{\mathbf{j}}) = 0$$

In the yz plane, the area vector is given by $S\,\hat{\mathbf{i}}$ and for $S = 1.0\,\text{m}^2$, the flux of the electric field $\vec{\mathbf{E}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}$ through the yz plane is

$$\Phi_E = \vec{\mathbf{E}}.S\hat{\mathbf{i}} = 100\,\text{NC}^{-1}\,\hat{\mathbf{i}}.(1.0\,\text{m}^2)\,\hat{\mathbf{i}} = 100\,\text{NC}^{-1}\text{m}^2\,(\hat{\mathbf{i}}.\hat{\mathbf{i}}) = 100\,\text{NC}^{-1}\text{m}^2$$

2. Let us choose the surface of the cube as the Gaussian surface. From Gauss's law [Eq. (6.16)], the electric flux through this surface is

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{2.7\times10^{-9}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 3.0\times10^2\,\text{NC}^{-1}\text{m}^2$$

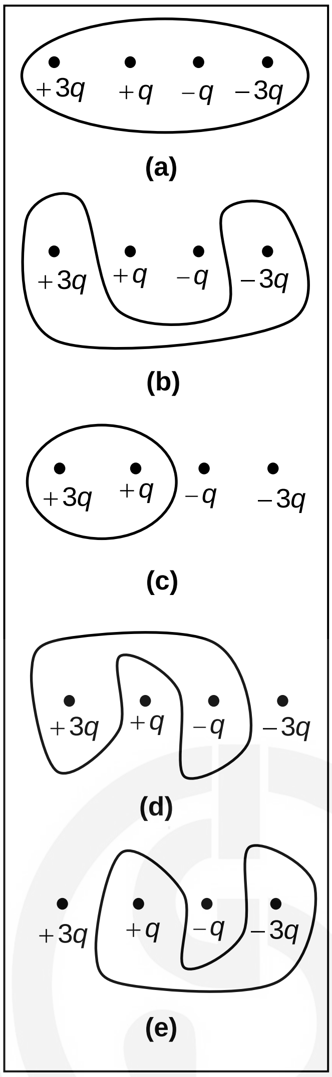


Fig. 6.25: **Diagram for the answer of TQ 3.**

Since the cube has 6 faces, the electric flux through any one of the cube's faces is

$$\Phi_{E'} = \frac{\Phi_E}{6} = \frac{q}{6\varepsilon_0} = \frac{3.0\times10^2\,\mathrm{NC^{-1}m^2}}{6} = 50\,\mathrm{NC^{-1}m^2}$$

3. Refer to Figs. 6.25a to e. We use Eq. (6.16): $\Phi_E = \dfrac{q}{\varepsilon_0}$

a) For the net electric flux through the Gaussian surface to be zero, the net electric charge enclosed by it should be zero. In Figs. 6.24a and b, the two Gaussian surfaces shown enclose the charges so that the net charge within each one of them and hence the electric flux through them is zero. You can draw a third one too enclosing only the charges $+q$ and $-q$.

b) For the net electric flux through the Gaussian surface to be $(+\,4q/\varepsilon_0)$, the net electric charge enclosed by it should be $+4q$. In Fig. 6.24c, the Gaussian surface encloses the charges $+3q$ and $+q$ so that the net charge within it is $+4q$ and the net electric flux through it is $(+\,4q/\varepsilon_0)$.

c) For the net electric flux through the Gaussian surface to be $(+\,2q/\varepsilon_0)$, the net electric charge enclosed by it should be $+2q$. In Fig. 6.24d, the Gaussian surface encloses the charges $+3q$ and $-q$ so that the net charge within it is $+2q$ and the net electric flux through it is $(+\,2q/\varepsilon_0)$.

d) For the net electric flux through the Gaussian surface to be $(-\,2q/\varepsilon_0)$, the net electric charge enclosed by it should be $-2q$. In Fig. 6.24e, the Gaussian surface encloses the charges $-3q$ and $+q$ so that the net charge within it is $-2q$ and the net electric flux through it is $(-\,2q/\varepsilon_0)$.

4. We use Eq. (6.18) to calculate the volume charge density ρ and write

$$\rho = \varepsilon_0\vec{\nabla}.\vec{\mathbf{E}} = \varepsilon_0\vec{\nabla}.(cr\,\hat{\mathbf{r}}) = \varepsilon_0\vec{\nabla}.(c\,\vec{\mathbf{r}}) \qquad \left(\because\ \hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}\right)$$

In Unit 2, you have learnt how to calculate the divergence of a vector field.

$$\therefore\ \rho = \varepsilon_0\vec{\nabla}.(c\,\vec{\mathbf{r}}) = \varepsilon_0 c\,\vec{\nabla}.\vec{\mathbf{r}} = \varepsilon_0 c\,(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}).(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}})$$

$$= \varepsilon_0 c\,(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}) = 3\varepsilon_0 c$$

In this region of space, the total charge contained in a sphere of radius R, centred at the origin is just the volume integral $Q = \iiint_V \rho\, dV$. Since ρ is constant and the volume integral equals the volume of the sphere of radius R, we have

$$Q = 3\varepsilon_0 c\int_V dV = 3\varepsilon_0 c\,\frac{4\pi}{3}R^3 = 4\pi\varepsilon_0 c\,R^3$$

5. a) When the Gaussian surface encloses zero net charge, Gauss's law yields $\vec{\mathbf{E}}.d\vec{\mathbf{S}} = 0$. However, this does not mean that the electric field is zero for all points on the surface. $\vec{\mathbf{E}}.d\vec{\mathbf{S}}$ can be zero even when $\vec{\mathbf{E}}$ and $d\vec{\mathbf{S}}$ are perpendicular to each other.

b) If the electric field is zero everywhere on the Gaussian surface, Gauss's law requires that there should be no net charge inside the surface, i.e., the net charge should be zero.

6. Gauss's law is not useful in calculating the electric field due to three equal charges placed at the corners of an equilateral triangle because it is not possible to find a closed surface of appropriate symmetry over which the electric field can be taken to be constant and its direction can be taken to be either parallel or normal to the surface to evaluate the surface integral.

7. The electric flux through the shaded right face (*ABCD*) of the cube having area, say S', is

$$\Phi_{S'} = \iint_{S'} \vec{\mathbf{E}}.d\vec{\mathbf{S}}$$

To determine $\Phi_{S'}$, the trick is to choose an appropriate Gaussian surface that encloses the charge q. We can put together 8 cubes of the same size as the original cube in the problem to construct the Gaussian surface as shown in Fig. 6.26. It includes the right face *ABCD* of the original cube and encloses the charge q. Note that the area of the Gaussian surface is 24 times the area of the right face *ABCD*. So, now we can apply Gauss's law to this problem.

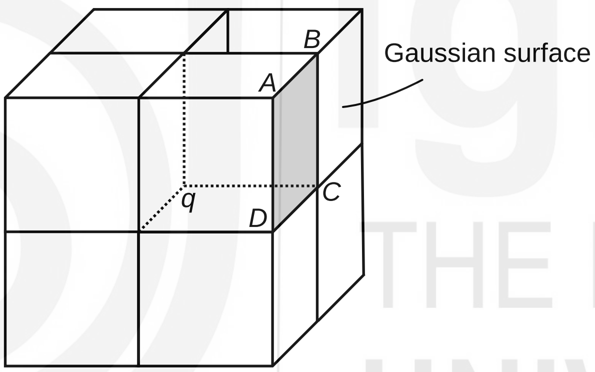


Fig. 6.26: **Diagram for answer to TQ 7.**

From Gauss's law, we have $\oint\!\!\oint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{Q_{encl}}{\varepsilon_0}$ where S is the surface area of the Gaussian surface enclosing the charge. Since the area of the Gaussian surface is 24 times the area S' of *ABCD*, we have

$$\oint\!\!\oint_S \vec{\mathbf{E}}.d\vec{\mathbf{S}} = 24 \times \iint_{S'} \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{\varepsilon_0} \quad \text{or} \quad \iint_{S'} \vec{\mathbf{E}}.d\vec{\mathbf{S}} = \frac{q}{24\,\varepsilon_0}$$

Thus, the electric flux through the right face (*ABCD*) of the cube is

$$\Phi_{S'} = \frac{q}{24\,\varepsilon_0}$$

8. a) We use Eq. (6.12) to determine the value of the point charge:

$$q = \varepsilon_0\,\Phi_E = (8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}})\times(-900\,\mathrm{Nm^2C^{-1}}) = -7.96\,\mathrm{nC}$$

From Eq. (6.19), the electric field of q at a distance of 0.10 m from it is

$$\vec{\mathbf{E}} = (8.99\times10^9\,\mathrm{C^{-2}Nm^2})\times\frac{-7.96\times10^{-9}\,\mathrm{C}}{(0.10\,\mathrm{m})^2}\hat{\mathbf{r}} = (-7.2\times10^3\,\mathrm{NC^{-1}})\hat{\mathbf{r}}$$

The electric flux through the Gaussian sphere of radius 0.30 m would remain the same as the charge enclosed by it is the same. It will be $-900\,\mathrm{Nm^2C^{-1}}$.

b) We use Eq. (6.24b) for the magnitude E to determine the net charge on the sphere since the point at which the electric field is given lies inside the sphere. From Eq. (6.24b) for E, we have $Q = 4\pi\varepsilon_0 \dfrac{R^3}{r} E$.

Upon substituting the numerical values given in the problem, we get

$$Q = \frac{1}{(8.99\times10^9\,\text{C}^{-2}\text{Nm}^2)} \times \frac{(0.30\,\text{m})^3}{(0.10\,\text{m})} \times (3.0\times10^3\,\text{NC}^{-1}) = 90\,\text{nC}$$

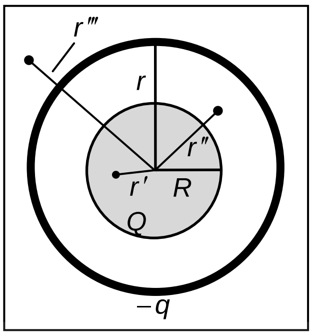


Fig. 6.27: **Diagram for the answer of TQ 9.**

9. a) Since the electric field inside the shell is zero, from Eq. (6.24b), the net electric field at a point r' inside the sphere (see Fig. 6.27) is given as:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0}\frac{r'}{R^3}\hat{\mathbf{r}}$$

b) At a point r'' between the sphere and the shell, the total charge enclosed by a spherical Gaussian surface passing through r'' is the charge Q on the sphere and the electric field is given by Eq. (6.22):

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r''^2}\hat{\mathbf{r}}$$

c) At a point r''' outside the shell, the total charge enclosed by a spherical Gaussian surface passing through the point is the charge Q on the sphere and the charge $-q$ on the spherical shell. The electric field is given by Eq. (6.22) or Eq. (6.26) where the net charge enclosed by the Gaussian surface passing through r''' is $(Q - q)$:

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{(Q-q)}{r'''^2}\hat{\mathbf{r}}$$

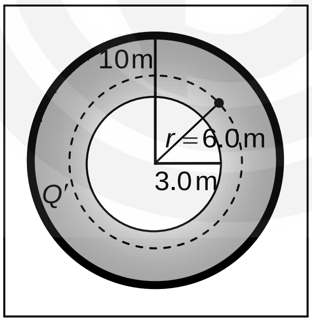


Fig. 6.28: **Diagram for the answer of TQ 10 (not to scale).**

10. We have to first determine the volume charge density of the spherical shell: $\rho = \dfrac{Q}{V}$. For this, we need to calculate the volume of the spherical shell, which is $V = \dfrac{4\pi}{3}\left[(10\,\text{m})^3 - (3.0\,\text{m})^3\right] = 4077\,\text{m}^3$

$$\therefore \quad \rho = \frac{Q}{V} = \frac{9.0\,\text{nC}}{4077\,\text{m}^3} = 2.2\times10^{-12}\,\text{Cm}^{-3}$$

To determine the electric field at the point 6.0 m away from the centre, we draw a spherical Gaussian surface of radius 6.0 m passing through the point (Fig. 6.28). Let us first calculate the total charge Q' enclosed by the Gaussian surface of radius 6.0 m. The volume of the part of the spherical shell that contains the charge Q' is

$$V' = \frac{4\pi}{3}\left[(6.0\,\text{m})^3 - (3.0\,\text{m})^3\right] = 792\,\text{m}^3$$

$$\Rightarrow Q' = \rho V' = 2.2\times10^{-12}\,\text{Cm}^{-3} \times 792\,\text{m}^3 = 1.7\,\text{nC}$$

From Gauss's law, we have

$$\Phi_E = E(4\pi R^2) = \frac{Q'}{\varepsilon_0} \qquad \Rightarrow \qquad E = \frac{Q'}{4\pi\varepsilon_0 R^2} \text{ or}$$

$$\vec{\mathbf{E}} = (8.99\times10^9\,\text{C}^{-2}\text{Nm}^2)\left(\frac{1.7\,\text{nC}}{(6.0\,\text{m})^2}\right)\hat{\mathbf{r}} = 0.42\,\text{NC}^{-1}\hat{\mathbf{r}}$$