**Lightning in clouds is the most powerful display of strong electrostatic forces and electric fields in nature!**

UNIT 5

# ELECTROSTATIC FORCE AND ELECTRIC FIELD

## Structure



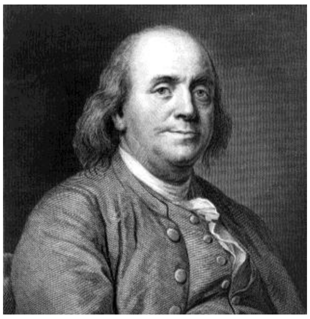
## STUDY GUIDE

We hope that you have studied thoroughly the concepts of vector algebra given in Block 1 of the course BPHCT-131 on Mechanics and the concepts of vector calculus presented in Block 1 of this course. You can revise the basic concepts of vector algebra from the Appendix given in Block 1 of this course. You have to make sure that you know all these concepts very well and only then you should study this block and the remaining blocks of this course. In this unit, you will learn about the basic concept of electrostatic force between charges, its quantitative definition given by Coulomb's law, which you have learnt in school physics. You will also learn the concept of electric field and its relation with the electrostatic force. The presentation of these concepts may be new to you. To help you learn the concepts and their application better, we have given many Examples and SAQs within the unit and Terminal Questions at its end. Most of these should take you at most 5 to 10 minutes to solve. You should study all sections thoroughly and make sure that you can solve the SAQs and Terminal Questions on your own before studying the next unit.

***"Science is beautiful when it makes simple explanations of phenomena or connections between different observations."***

***Stephen Hawking***

## 5.1 INTRODUCTION



Benjamin Franklin (1706- 1790), an American polymath (meaning expert in many subjects), was one of the founding fathers of the United States of America. In physics, he is well known for his pioneering work on electricity. He was also a great inventor. The lightning rod, bifocal glasses and urinary catheter are some of his well known inventions in use today. Franklin coined several terms in electricity which we use today: battery, charge, conductor, plus, minus, positively, negatively, condenser ( ≡ capacitor).

In your school physics, you have studied about electrostatic force between electric charges and Coulomb's law. How are these concepts related to your direct experiences?

During the rainy season, you must have seen flashes of lightning in dark clouds lighting them up. You may have wondered what causes lightning. Do you know that it was Benjamin Franklin who first proved the electric nature of lightning through his experiment with the flying kite? He also gave the idea that clouds possess electric charges, which when discharged in the atmosphere, give rise to a giant spark of lightning.

Actually, human beings have known about the effect of electric charges for thousands of years – the Greeks knew that rubbing amber on a piece of fur made it attract light objects such as feathers. It was later found that many materials such as silk, wax, precious stones, flannel, etc., when rubbed with other materials developed the ability to attract light objects. For example, rubbing glass with silk made it attract pieces of paper. Such materials were called 'electrics'. It was said that the materials became 'electrified' or 'acquired vitreous or resinous electricity'. You may have observed this effect yourself. If you run a comb through your dry hair or rub any dry synthetic fabric, you will notice that small bits of paper or hair cling to them.

The concept of 'positive' and 'negative' charges was developed by Benjamin Franklin and other scientists in the eighteenth century to explain a large number of such observations (as above) made in many experiments. A notable thing about electric charges is that the force between them is extremely large. This force is now known as the **electrostatic force**. As you may recall from Sec. 6.2.5 of Unit 6 of the course Mechanics (BPHCT-131), the electrostatic force is a fundamental force in nature that controls everyday phenomena such as friction, tension, normal force, etc. It helps form electrically neutral stable atoms, molecules, solids and liquids.

'Electrica' is a Latin word coined from elektron, the Greek word for amber. Electrica was translated as electrics in English and later the two words electrical and electricity were coined. All electrical effects due to rubbing together of various materials were ascribed to two forms of electricity – 'vitreous' electricity and 'resinous' electricity. Franklin identified the term 'positive' with vitreous electricity and 'negative' with resinous electricity.

So in Sec. 5.2, we explain the concept of **electrostatic force** between positive and negative charges. To do so, we revise the concept of **electric charge.** Then we give the mathematical expression of the force law known as **Coulomb's law** and use it to calculate the electrostatic force between two charges. We then discuss the concept of **electric field** in Sec. 5.3. You have been introduced to vector fields in the first block of this course. You have learnt that the electric field is a vector field, which is set up due to a charge or distribution of charges in the region surrounding it. You will learn how to calculate the electric field due to different simple charge distributions. In the next unit, you will study the concept of **electric flux**. You will use it to learn the easier and more elegant Gauss's law for determining the electric field due to various charge distributions.

## Expected Learning Outcomes

After studying this unit, you should be able to:

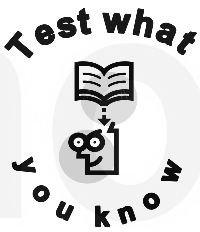
- use Coulomb's law to calculate the electrostatic force between two given charges at rest;

- ❖ apply the principle of superposition of forces to calculate the resultant force due to a system of more than two charges;
- ❖ define electric field due to multiple discrete charges and continuous distribution of charges; and
- ❖ calculate the net electric field due to a distribution of multiple discrete charges and infinite uniform line charge.

## 5.2 ELECTROSTATIC FORCE

Do you recall the concepts of charge and electrostatic forces between like and unlike charges and Coulomb's law from school physics? Do you remember studying that like charges repel each other and unlike charges attract each other? You have studied about positive and negative charges and the forces between them in your school physics. You may like to revise the concepts by solving the problems in the pre-test given below. Otherwise, study this section and then try to solve these problems again.

### PRE-TEST




1. A glass rod rubbed with silk is said to be 'positively' charged and amber or plastic rubbed with fur, 'negatively' charged. Select the correct conclusion for each observation given below:

   Observation 1: An object is repelled by a piece of glass that has been rubbed with silk.

   a) The object is positively charged.

   b) The object is negatively charged.

   Observation 2: Two objects are both attracted to a piece of amber that has been rubbed with fur.

   a) Both objects are positively charged.

   b) Both objects are negatively charged.

2. State whether the following statements are true or false:

   a) The charge on free particles has also been measured to be a fraction of the charge on the electron, $1.6 \times 10^{-19}$ C.

   b) Objects are electrically neutral because they have equal numbers of positive protons and negative electrons.

   c) The total charge in the universe is conserved.

   d) The force between two charges at rest is independent of their magnitude.

   e) The force between two charged particles at rest is proportional to the product of the magnitudes of the charge on them.

   f) The force between two charged particles at rest is an inverse square force.

If you have solved these problems correctly, you know the basic concepts about charges and the force between them. You may like to quickly go through the remaining part of Sec. 5.2 and solve the SAQs given in it. Otherwise, study it thoroughly and try the pre-test and SAQs again.

Actually, electric charge could have been given any other name by scientists. How it came to be used is interesting. In older English language, the word charge was used for a *load* carried by anything, such as a cannon or a horse. Since the property/ substance/'fluid' was 'carried' by matter, it was called 'electric charge'.

## 5.2.1 Electric Charge

In this section, we will quickly revise what you have learnt about electric charges in your school physics, viz., the types of charge, the unit of charge, quantisation of charge and charge conservation.

### Types of Charge and the Unit of Charge

You have learnt in school physics that **charge** is a scalar quantity and **is of two types**: **positive** and **negative**. Electrons and protons are the most familiar examples of negative and positive charges having the same magnitude of charge, i.e., $1.6 \times 10^{-19}$ C. As you can see, the SI unit of charge is **coulomb** (denoted by C) named after the French physicist Charles-Augustin de Coulomb (1736 – 1806).

Coulomb, the unit of charge is defined in terms of magnetic forces and you will learn about them in Block 3.

Atoms and molecules are electrically neutral because they are made up of an equal number of electrons and protons. You may also have read an explanation of how two materials when rubbed together become electrically charged. On rubbing, electrons flow from one material (which becomes positively charged) to another (which is then negatively charged). This way of charge transfer is called **charging by friction** (because you are rubbing one material with another). There are other ways of charging an object about which you have studied in your school physics and we will not go into those details here.

### Quantisation of Charge

In the eighteenth century, scientists (including Benjamin Franklin) thought that electric charge was a continuous invisible fluid present in all matter and could flow in and out of objects to charge them positively or negatively. Later experiments about the nature of matter revealed that it was made up of atoms, and molecules and atoms were made up of electrons, protons and neutrons. Today we know that the **smallest free charge** that is possible to obtain is that of an electron or proton. The magnitude of this charge is denoted by $e$.

Electric charge was first measured in 1909 by an American Nobel Laureate physicist Robert Millikan (1868 – 1953). His famous experiment known as the oil-drop experiment is now performed in school and college laboratories. In this experiment, you can observe the motion of a charged oil drop falling between two electrified metallic plates under the influence of two forces: the force of gravitation and an electric force being exerted on it in a direction opposite to the gravitational force. Millikan made observations on a large number of drops and found that charges on different drops were integral multiples of an elementary charge $1.6 \times 10^{-19}$ C. This is not only true for negative charges but also for positive charges.

Mathematically, any positive or negative charge on a free particle is written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, ..... \qquad (5.1a)$$

where

$$e = 1.6 \times 10^{-19}\ \text{C} \qquad (5.1b)$$

You may know that when a physical quantity can have only discrete values rather than any arbitrary continuous value, we say that it is **quantised**. We do not know why electric charge is quantised. But it is an experimental observation that has had no exception so far. Thus, we say that

**<u>Charge is quantised</u>**; it takes discrete values that are integral multiples of *e*.

For example, we can find a free particle (such as positron, $\alpha$-particle) or charged object (say, a charged sphere or a charged drop) that has a charge equal to an integral multiple of *e*, i.e., $+4e$ or $-4e$, but never a free particle having a charge of, say, $+0.77e$ or $-2.55e$. You may know that protons and neutrons are made up of tightly bound quarks having charges $-e/3$ and $+2e/3$. However, quarks are yet to be detected as free particles. So on the basis of experimental evidence so far, we can say that

> **Charge is quantised, i.e., charges on free particles have always been measured to be integral multiples of $1.6 \times 10^{-19}$ C, never a fraction.**

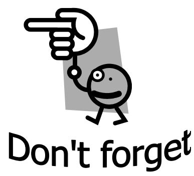

## <u>Conservation of Charge</u>

Experiments on electric charges also show that whenever any two objects are in contact (e.g., due to rubbing, touching, etc.) and there is an excess charge on any one of these two objects after contact, then there is an excess charge on the other object too. These excess charges on the two objects in contact are equal in amount but **opposite** in sign. This means that when electric charge (electrons) is transferred from one object to another, no electrons are destroyed or created. Thus, the amount of charge contained in the two objects is a conserved quantity. This is true for all **isolated** systems in nature.

Actually, based on his experiments Benjamin Franklin was the first scientist to propose the hypothesis of conservation of charge. No violations of this law have been found in countless experiments done on microscopic particles such as elementary particles, nuclei, atoms and molecules as well as large charged objects. So, we can add electric charge to the list of conserved quantities such as linear momentum, energy and angular momentum and state **the law of conservation of charge**. Experimental evidence shows that

> Conservation of total electric charge in the universe also points to the existence of anti-particles.

> **In an isolated system, the <u>total amount of electric charge</u> (that is, the algebraic sum of the positive and negative charge present in the system at any time) never changes. We say that it is <u>conserved</u>. Charge-carrying particles can be transferred from one object to another, but the charge associated with those particles cannot be created or destroyed. It follows that <u>the total electric charge in the universe is conserved</u>.**



You may like to go back to the pre-test and attempt questions 1 and 2a to c before studying further. We now revise Coulomb's law which tells us how much force is exerted by one charged object on another.

Charles-Augustin de Coulomb (1736 – 1806) was a French physicist who is best known for his law describing the electrostatic forces between charged particles. Coulomb's law has been firmly established after countless experiments. It applies to all electrical charges whether free or between the positively charged nucleus and electrons bound within an atom. It accounts for the forces that bind atoms to form molecules, and atoms and molecules to form all types of matter. Thus, it accounts for the stability of matter.

## 5.2.2 Coulomb's Law

The force law for charged particles at rest was arrived at after a series of careful experiments by Coulomb. He discovered that the magnitude of the force (electric, Coulomb or **electrostatic** force as we know it today) between two charged particles $q_1$ and $q_2$ **at rest** is given by

$$F = k\frac{|q_1||q_2|}{r^2} \qquad (5.2)$$

where $r$ is the distance between the charged particles and $k$ is the constant of proportionality. The force is directed along the line joining the two particles. The force on either particle is directed toward the other particle if the two have opposite (unlike) charges and away if the two have similar (like) charges. So we say that like charges repel and unlike charges attract each other. Since force is a vector quantity, let us write down Eq. (5.2) in vector form for both like and unlike charges in one place.

### COULOMB'S LAW

The **electrostatic force on a particle carrying a charge** $q_1$ **by a particle carrying a charge** $q_2$ **situated at a distance** $r$ **from it is given by**

$$\vec{\mathbf{F}}_{21} = k\frac{q_1 q_2}{|\vec{\mathbf{r}}_{21}|^2}\hat{\mathbf{r}}_{21} \qquad (5.3a)$$

where $\hat{\mathbf{r}}_{21}$ is the unit vector along the line joining the particles and directed from $q_2$ to $q_1$ (see Fig. 5.1) and $k$ is called the **Coulomb constant**. Note that $\vec{\mathbf{r}}_{21} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$ and $|\vec{\mathbf{r}}_{21}| = r$. Here $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ are the position vectors of $q_1$ and $q_2$, respectively. Note that the particles are at rest. In SI units, Coulomb's law is written as

$$\vec{\mathbf{F}}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{\mathbf{r}}_{21}|^2}\hat{\mathbf{r}}_{21} \qquad (5.3b)$$

where the units of $q_1$ and $q_2$ are coulomb, those of $\vec{\mathbf{r}}_{21}$ and $\vec{\mathbf{F}}_{21}$ are metre and newton, respectively. Here $\varepsilon_0$ is the permittivity of free space.

**Coulomb constant** $\frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2}$.

Note that Eqs. (5.3a and b) account for the attractive and repulsive nature of the electrostatic force if $q_1$ and $q_2$ include the sign of the charge. So, **if the charges are like**, that is, both charges are either positive or negative, **the force** $\vec{\mathbf{F}}_{21}$ on $q_1$ points away from $q_2$, along $\vec{\mathbf{r}}_{21}$, i.e., it is **repulsive**. **If the charges are unlike**, that is, one of them is positive and the other negative, **the force** $\vec{\mathbf{F}}_{21}$ on $q_1$ is towards $q_2$, in the direction opposite to $\vec{\mathbf{r}}_{21}$, i.e., it is **attractive**.

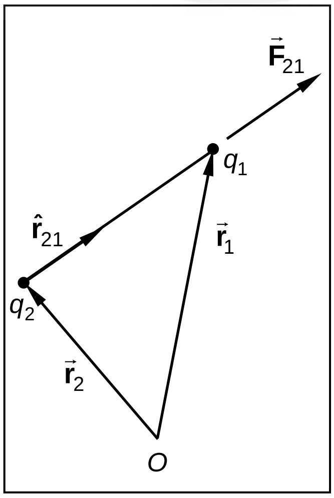
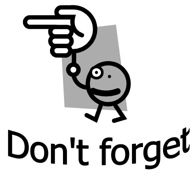



**Fig. 5.1**: **The electrostatic force between two electric charges at rest.**

Did you notice that the expression for the attractive Coulomb force between unlike charges is similar to the expression of the gravitational force you have studied in Unit 7 of Block 2 of the course on Mechanics (BPHCT-131)?

We have used the same sign convention here. The force of repulsion differs only in sign. So, the mathematical expression of Coulomb's law given by Eq. (5.3a or b) sums up four experimental observations:

1. **Unlike charges attract and like charges repel;**
2. **The force between two charged particles is exerted along the line joining them;**
3. **The force between any two charged particles is proportional to the magnitude of charge on each particle; and**
4. **It is an inverse square force, i.e., it is inversely proportional to the square of the distance between the particles.**




Let us now take up an example to show you how to apply Coulomb's law.

## EXAMPLE 5.1 : APPLYING COULOMB'S LAW

Determine the magnitudes and directions of the electrostatic force on the following charged particles at rest and show them on a diagram:

$q_1 = 5.0$ C, $q_2 = -12$ C at a distance of 30 m.

**SOLUTION ■** The electrostatic force on each charge is given by Coulomb's law, i.e., Eq. (5.3b).

We substitute the values of $q_1, q_2$ and $r$ in each case and take

$$\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

The magnitude of the force on each particle is

$$F = (8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times \frac{5.0 \text{ C} \times 12 \text{ C}}{(30\text{ m})^2} = 6.0 \times 10^8 \text{ N}$$

Since the charges on the particles are unlike, they will attract each other. The force on each particle will be directed toward the other particle. Mathematically, we write the forces as:

Force on $q_1$ by $q_2$ is $\vec{\mathbf{F}}_{21} = -6.0 \times 10^8 \text{ N } \hat{\mathbf{r}}_{21}$ and

Force on $q_2$ by $q_1$ is $\vec{\mathbf{F}}_{12} = -6.0 \times 10^8 \text{ N } \hat{\mathbf{r}}_{12} = +6.0 \times 10^8 \text{ N } \hat{\mathbf{r}}_{21}$

since $\hat{\mathbf{r}}_{12} = -\hat{\mathbf{r}}_{21}$. Both forces are shown in Fig. 5.2.

You can see that the force is very large.

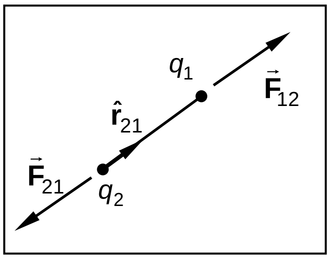



**Fig. 5.2**: **The electrostatic forces for Example 5.1.**

Let us take up another example of applying Coulomb's law and then you can test yourself by solving an SAQ.

## *EXAMPLE 5.2* : APPLYING COULOMB'S LAW

Two point charges $Q_1$ and $Q_2$ are 3.0 m apart and their combined charge is 20 μC. If one charge repels the other with a force of 0.075 N, what are the magnitudes of the two charges?

**SOLUTION ■** Once again we use Coulomb's law given by Eq. (5.3b).

We are given that the charges repel each other. Therefore, they are like charges. Let $Q_1$ and $Q_2$ represent their magnitudes.

Substituting the values of the distance and the force in the scalar form of Eq. (5.3b), we get

$$0.075\text{ N} = (8.99\times10^9\text{ Nm}^2\text{C}^{-2})\times\frac{Q_1Q_2}{(3.0\text{m})^2}$$

or $\quad Q_1Q_2 = 75\times10^{-12}\text{ C}^2 = 75\times(10^{-6}\text{ C})^2 = 75(\mu\text{C})^2 \qquad$ (i)

Also $\quad Q_1 + Q_2 = 20\mu\text{C} \quad \Rightarrow \quad Q_2 = 20\mu\text{C} - Q_1 \qquad$ (ii)

Substituting $Q_2$ from Eq. (ii) in Eq. (i), we get a quadratic equation in $Q_1$:

$$75(\mu\text{C})^2 = Q_1(20\mu\text{C} - Q_1) \quad\Rightarrow\quad Q_1^2 - 20Q_1 + 75 = 0$$

where $Q_1$ is in μC. Solving the equation gives the magnitudes of the charges

$Q_1 = 5.0\mu\text{C}$ and $Q_2 = 15\mu\text{C}$ or $Q_1 = 15\mu\text{C}$ and $Q_2 = 5.0\mu\text{C}$

## *SAQ 1* – Coulomb's law

a) Determine the electrostatic force on $q_1$ due to $q_2$ for :

i) $q_1 = 8.0\ \mu\text{C},\ q_2 = 8.0\ \mu\text{C}$ at a distance of 0.04 m.

ii) $q_1 = 15\text{ mC},\ q_2 = -10\text{ mC}$ at a distance of 3.0 m.

b) The hydrogen atom consists of an electron and a proton separated by an average distance of $5.3\times10^{-11}\text{m}$. Calculate the magnitude of the electrostatic force between the electron and proton taking them to be at rest. Compare it with the magnitude of the gravitational force between them. It is given that the mass of the electron is $9.1\times10^{-31}$ kg, mass of the proton is $1.7\times10^{-27}$ kg and $G = 6.7\times10^{-11}\text{ Nm}^2\text{ kg}^{-2}$.

In Example 5.2, we have used the term point charge. What does it mean? A **point charge** is a hypothetical **charge** located at a single **point** in space. In that sense, it has no size: it is dimensionless. It is a purely abstract mathematical concept used in electrostatics. For many purposes, we consider the electron to be a **point charge**. However, its size can be characterized by a length scale known as the electron radius. We often use the term **point charge in electrostatics** when we do not wish to take the size (dimensions) of the particle into consideration.

So far, you have learnt how to determine the electrostatic force between two static charged particles. How do we calculate the electrostatic force on a charge in a system having more than two charges at rest? We use the principle of superposition. Recall that you have studied this principle for the force of gravitation in Sec. 7.2.1 of Unit 7 of the course BPHCT-131 entitled Mechanics. Let us now explain it for electrostatic forces.

## 5.2.3 The Principle of Superposition

The first thing to understand is that electrostatic forces are two-body forces. This means that the electrostatic force between any pair of charged objects does not change if other charged objects are present in their surroundings. In a system having more than two charged objects, the electrostatic force between each pair of objects is given by Coulomb's law.

To determine the **net electrostatic force** on any given charged particle in a system of charged particles, exerted by the other charged particles in the system, we simply take the vector sum of the forces being exerted on it by the other charged particles in the system.

Suppose there are three charges $q_1$, $q_2$ and $q_3$ at rest in the system. Then the net electrostatic force $\vec{\mathbf{F}}_1$ exerted on $q_1$ by $q_2$ and $q_3$ is the vector sum of the electrostatic force $\vec{\mathbf{F}}_{21}$ exerted on $q_1$ by $q_2$ and the electrostatic force $\vec{\mathbf{F}}_{31}$ exerted on $q_1$ by $q_3$, i.e.,

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} \tag{5.4a}$$

or

$$\vec{\mathbf{F}}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{\mathbf{r}}_{21} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{31}^2}\hat{\mathbf{r}}_{31} \tag{5.4b}$$

Note that while applying Eqs. (5.4b and 5.4c), you have to take into account the sign of the charges as shown in Example 5.3.

In general, the electrostatic force $\vec{\mathbf{F}}_i$ on the *i*th charge $q_i$ due to all other charges $q_1, q_{2,}, \ldots, q_j, \ldots$ in a many-particle system of charged particles is given by

$$\vec{\mathbf{F}}_i = \sum_{j \neq i} \vec{\mathbf{F}}_{ji} = \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \frac{q_i q_j}{r_{ji}^2}\hat{\mathbf{r}}_{ji} \tag{5.4c}$$

Note that the summation in Eq. (5.4c) does not include the *i*th charge. This is indicated by putting $j \neq i$ under the summation signs.

**PRINCIPLE OF SUPERPOSITION**

*Recap*

**According to the principle of superposition, in a many-particle system of charged particles, the resultant electrostatic force on any charged particle is the vector sum of the electrostatic forces exerted by all other charged particles on it** [as given by Eq. (5.4c)].

You may like to work through an example to apply the principle of superposition.

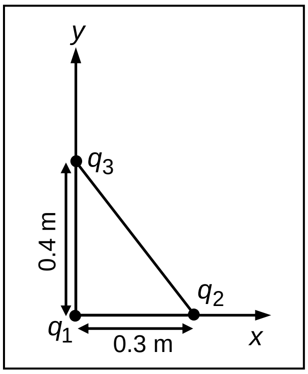



**Fig. 5.3**: **Diagram for Example 5.3.**

### EXAMPLE 5.3 : PRINCIPLE OF SUPERPOSITION

Three charges $q_1 = -2.0\ \mu C$, $q_2 = 9.0\ \mu C$ and $q_3 = 16.0\ \mu C$ are situated at the corners of a right-angled triangle as shown in Fig. 5.3. Calculate the electrostatic force exerted on $q_1$ by $q_2$ and $q_3$.

**SOLUTION ■** We use the principle of superposition given by Eq. (5.4b) for a system of three charges. Since $\hat{\mathbf{r}}_{21} = -\hat{\mathbf{i}}$ and $\hat{\mathbf{r}}_{31} = -\hat{\mathbf{j}}$, we have

$$\vec{\mathbf{F}}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{(r_{21})^2}(-\hat{\mathbf{i}}) + \frac{q_1 q_3}{(r_{31})^2}(-\hat{\mathbf{j}})\right] \qquad \text{(i)}$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along the $x$ and $y$-axes (Fig. 5.3). Substituting all numerical values (with the sign of the charges) in Eq. (5.4b), we get

$$\vec{\mathbf{F}}_1 = (8.99\times10^9\ \text{N m}^2\,\text{C}^{-2})\left[\frac{(-2.0\times10^{-6}\,\text{C})\times(9.0\times10^{-6}\,\text{C})}{(0.3\,\text{m})^2}(-\hat{\mathbf{i}}) + \frac{(-2.0\times10^{-6}\,\text{C})\times(16.0\times10^{-6}\,\text{C})}{(0.4\,\text{m})^2}(-\hat{\mathbf{j}})\right] \qquad \text{(ii)}$$

or $$\vec{\mathbf{F}}_1 = \left(1.8\,\hat{\mathbf{i}} + 1.8\,\hat{\mathbf{j}}\right)\text{N}$$

The magnitude of the force is $\sqrt{(1.8)^2 + (1.8)^2}\ \text{N} = 2.5\ \text{N}$

The direction of the force is given by the angle $\theta$ it makes with the positive $x$-axis: $\theta = \tan^{-1}\left(\frac{1.8}{1.8}\right) = \tan^{-1}(1) = 45^\circ$

So far you have revised the concepts of charge and electrostatic force between charged particles/objects at rest. You have also revised Coulomb's law and the superposition principle, and learnt how to determine the magnitude and direction of electrostatic forces between like and unlike charges. We now discuss the concept of electric field that you have also learnt in school physics.

## 5.3 ELECTRIC FIELD

Although the notion of electric field first figured in the work of British physicist Michael Faraday (1791 – 1867) on electromagnetic induction, he did not develop its concept. This was done by James Clerk Maxwell (1831 – 1879), a Scottish physicist. You will read more about the work of these two physicists in Block 4 of this course. You are familiar with the concept of vector fields from

Block 1. You have studied about the gravitational field in Unit 7 of the course on Mechanics. You know that the concept of electric field is a very powerful concept that gives us a simple tool for determining the electrostatic force on any charge due to another charge.

The advantage of this concept is that to calculate the net electrostatic force on a given charge due to other charges, we need not follow the lengthy process of Coulomb's law (where we need to know the relative positions of these charges) and vector addition. You will appreciate this point better as you study this section further and learn the concept of electric field. You may ask: **How do we define electric field?** We begin with the simplest case of a point charge.

## 5.3.1 Electric Field due to a Point Charge

Let us define the electric field due to a point charge.

**ELECTRIC FIELD**

A point charge $Q$ sets up an electric field in the region surrounding it. If another charge, say $q$, is placed in this region, it experiences the electrostatic force in accordance with Coulomb's law. The **electric field** generated by an electric charge or a group of charges **is a vector field defined as follows**:

Suppose a positive charge $q$ of an infinitesimal (negligibly small) magnitude, called a **test charge**, is placed at a position $\vec{\mathbf{r}}$ relative to a point charge $Q$ (Fig. 5.4). According to Coulomb's law, at that point, the test charge $q$ will experience the electrostatic force

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r^2} \hat{\mathbf{r}} \tag{5.5}$$

where $\hat{\mathbf{r}}$ is the unit vector along $\vec{\mathbf{r}}$. Then the electric field of the point charge $Q$ at a point having position vector $\vec{\mathbf{r}}$ is defined as the electrostatic force on a test charge at that point **divided by** the magnitude of the test charge. It is denoted by $\vec{\mathbf{E}}(\vec{\mathbf{r}})$. Mathematically, it is given by

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{\vec{\mathbf{F}}(\vec{\mathbf{r}})}{q} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}} \tag{5.6a}$$

Its magnitude is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{|Q|}{r^2} \tag{5.6b}$$

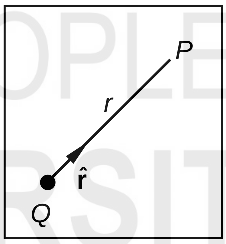
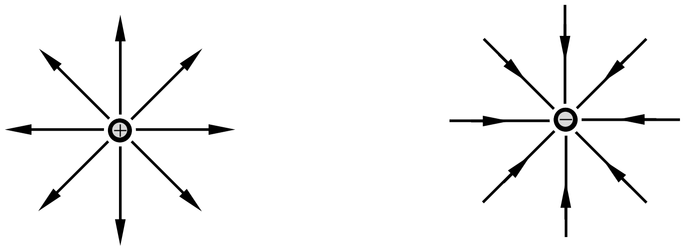



**Fig. 5.4**: **Unit vector for electric field at point** $P$ **due to a point charge** $Q$.

You have learnt how to visualise electric fields due to a point charge $Q$ defined by Eqs. (5.6a and b) in Sec. 2.2.2 of Unit 2. The representations of these electric fields are shown in Figs. 5.5a and b for positive and negative charges.

Note that the magnitude of the electric field is the same for both positive and negative electric charge (+ $Q$ or – $Q$). However, the directions are different as these are given by the direction of the electrostatic force experienced by the respective test charges. The electric field due to a positive point charge is directed away from the charge (Fig. 5.5a). For a negative point charge, it points towards the charge (Fig. 5.5b). The arrows in both Figs. 5.5a and b indicate the direction of the electric field. The continuous lines are called **field lines** (or the lines of force).



**(a)** **(b)**

**Fig. 5.5**: **Electric field lines around a) positive electric charge**; **b) negative electric charge**.

So, to draw electric field lines, you should always remember that


**Electric field lines (or lines of force) begin at positive charges and end at negative charges. Electric field lines may also go to infinity without terminating. These lines do not intersect.**

**These are close together near the point charges where the electric field is strong and far apart at large distances from the charges where the electric field is weak.**

From Eq. (5.6a), you should also note that the electrostatic force on the charge $q$ when it is placed in the electric field of charge $Q$ is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} \tag{5.7}$$

So, if you know the electric field in a region of space (could be due to a charge or system of charges), you can determine the electrostatic force on any charge placed in that electric field using Eq. (5.7).

Before studying further, you may like to calculate the electric field due to a few point charges. Work out SAQ 2.

## *SAQ 2* - Electric field due to point charge

a) Determine the electric field due to the point charges (i) + 5 μC at a point 30 cm from it and (ii) – 10 μC at a point 1 m from it. Show them in properly labelled diagrams.

b) If a point charge + 6μC is placed in the electric fields at the respective points given in part (a), what electrostatic force would be exerted on it in both cases?

You may now like to ask: **How is the electric field due to a group of charges defined**? This is what you will now learn.

## 5.3.2 Electric Field due to Multiple Discrete Charges

Consider a group of point charges $q_j$, having position vectors $\vec{\mathbf{r}}_j$. Let us place a test charge $q_i$ having position vector $\vec{\mathbf{r}}_i$ in the electric field of these charges. From the principle of superposition for electrostatic forces, the net electrostatic force on the test charge $q_i$ due to this group of charges is given by

$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \frac{q_i\, q_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2} \hat{\mathbf{r}}_{ji} \qquad (5.8)$$

The electric field due to the group of charges at the point with position vector $\vec{\mathbf{r}}_i$ is defined as

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_i} = \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \frac{q_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2} \hat{\mathbf{r}}_{ji} \qquad (5.9)$$

**Eq. (5.9) defines the electric field at a point in space due to a group of point charges**. Now, in Eq. (5.9), each charge appears only once. So if only one charge, say $q_j$, were present, we could write the electric field due to it as

$$\vec{\mathbf{E}}_j = \frac{1}{4\pi\varepsilon_0} \frac{q_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2} \hat{\mathbf{r}}_{ji} \qquad (5.10)$$

So, Eq. (5.9) becomes

$$\vec{\mathbf{E}} = \sum_j \vec{\mathbf{E}}_j \qquad (5.11)$$

In other words, **the total electric field due to a group of charges is the vector sum of the individual electric fields of the charges**. This is just the principle of superposition at work. You may like to study Fig. 5.6 to get a sense of the vectors involved in Eq. (5.10) before reading further.

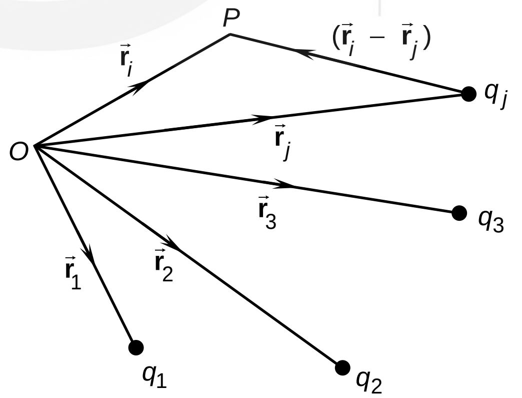



**Fig. 5.6**: **The vectors involved in defining the electric field due to a group of charges. The vector $\vec{\mathbf{r}}_{ji} = (\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j)$ represents the vector joining $q_j$ to the point *P* having position vector $\vec{\mathbf{r}}_i$. The vector $\hat{\mathbf{r}}_{ji}$ is the unit vector along $\vec{\mathbf{r}}_{ji}$.**

Once again, if a charge $q$ is placed in the electric field given by Eq. (5.9), the electrostatic force exerted on it will be given by Eq. (5.7). This makes the calculation of electrostatic force on a charge due to a group of charges much easier than using Coulomb's law. Let us now consider an example to calculate the electric field due to a special arrangement of two charges called the **electric dipole**.

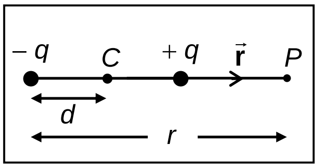



**Fig. 5.7**: **An electric dipole made up of equal and opposite charges, $\pm q$, separated by distance $2d$. The vector $2\vec{d}$ along the axis of the dipole is drawn from the negative to the positive charge. The point $P$ lies on the dipole axis at a distance $r$ from the midpoint $C$.**

## EXAMPLE 5.4 : ELECTRIC FIELD OF AN ELECTRIC DIPOLE

Two point charges $-q$ and $+q$ are separated by distance $2d$ (see Fig. 5.7). Such an arrangement of equal and opposite charges placed at some distance from each other is called an **electric dipole**. Determine the net electric field due to the charges at the point $P$ located on the **dipole axis** (i.e., the line joining the charges) at a distance $r$ from the midpoint $C$ of the dipole axis.

**SOLUTION ■** From Eq. (5.10), we determine the electric field due to each charge at the point $P$ and then use Eq. (5.11).

From Eq. (5.9), the electric fields due to both charges at the point $P$ are, respectively,

$$\vec{\mathbf{E}}_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{(r-d)^2} \quad \text{and} \quad \vec{\mathbf{E}}_{-q} = \frac{(-q)}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{(r+d)^2}$$

Here $\hat{\mathbf{r}}$ is the unit vector pointing from the charge $-q$ to the charge $+q$ along the line joining them and $d$ is the distance of the midpoint from each charge (see Fig. 5.7). From Eq. (5.11), the resultant or net electric field at the point $P$ due to the two charges is:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{+q} + \vec{\mathbf{E}}_{-q} = \frac{q\hat{\mathbf{r}}}{4\pi\varepsilon_0}\left(\frac{4rd}{(r^2-d^2)^2}\right)$$

If we assume that the point $P$ lies far away from the dipole so that $r >> d$, we can neglect the term $d^2$ in comparison to $r^2$ in the denominator of the expression for $\vec{\mathbf{E}}$. Under this assumption, the net electric field at $P$ is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \times \frac{q\hat{\mathbf{r}}(4rd)}{r^4} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{\mathbf{p}}}{r^3} \qquad \text{(i)}$$

where $\vec{\mathbf{p}} = 2qd\hat{\mathbf{r}}\ (= 2q\vec{\mathbf{d}})$ is a vector quantity called **dipole moment.**

You may like to solve an SAQ to determine the electric field of a dipole.

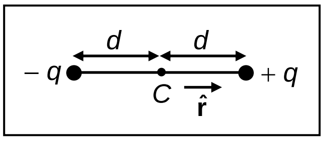



**Fig. 5.8**: **Diagram for SAQ 3.**

## SAQ 3 - Electric field due to an electric dipole

Determine the electric field due to an electric dipole at the midpoint of its axis.

Let us now calculate the electric field of an electric dipole at a point off its axis.

## EXAMPLE 5.5 : ELECTRIC FIELD OF AN ELECTRIC DIPOLE

Determine the net electric field due to the electric dipole of Example 5.4 at a point $P$ situated on the perpendicular bisector at a distance $r$ from the midpoint $C$ of the dipole axis.

**SOLUTION ■** As in Example 5.4, we use Eq. (5.10) to determine the electric field due to each charge at point $P$ and then apply Eq. (5.11).

The distance of the point $P$ from both the charges $+q$ and $-q$ is $\sqrt{(d^2 + r^2}$ and therefore, from Eq. (5.10), the magnitudes of the electric fields at $P$ due to these charges are equal and, respectively, given by:

$$E_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2 + r^2} \quad \text{and} \quad E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2 + r^2}$$

From Fig. 5.9a, you can see that the direction of the field is away from the charge $+q$ and towards the charge $-q$. To obtain the expression for the resultant field at $P$, we take the vector sum of the two electric fields using the **parallelogram law of vector addition**. From Fig. 5.9a, note that the angle between the two electric field vectors is $2\theta$. So we obtain the magnitude and direction of the resultant electric field as follows [Eqs. (A1.3a and b) in the Appendix A1 of Block 1]:

$$E = \sqrt{E_{+q}^2 + E_{-q}^2 + 2E_{+q}E_{-q}\cos 2\theta} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{d^2 + r^2}\cos\theta$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{(d^2 + r^2)^{3/2}} \quad \text{since } \cos\theta = \frac{d}{\sqrt{d^2 + r^2}}$$

The direction of the resultant electric field is given by the angle $\alpha$ it makes with $\vec{\mathbf{E}}_{-q}$ (Fig. 5.9b):

$$\alpha = \tan^{-1}\left[\frac{E_{+q}\sin 2\theta}{E_{-q} + E_{+q}\cos 2\theta}\right] = \tan^{-1}[\tan\theta] = \theta$$

Note that $\vec{\mathbf{E}}$ is anti-parallel to $\vec{\mathbf{p}}$. So, we can express $\vec{\mathbf{E}}$ at point $P$ as

$$\vec{\mathbf{E}} = -\frac{\vec{\mathbf{p}}}{4\pi\varepsilon_0(r^2 + d^2)^{3/2}}$$

If the point $P$ is located far away from the dipole so that $r >> d$, we can express the electric field due to the electric dipole at the point as

$$\vec{\mathbf{E}} = -\frac{\vec{\mathbf{p}}}{4\pi\varepsilon_0 r^3} \qquad \text{(i)}$$

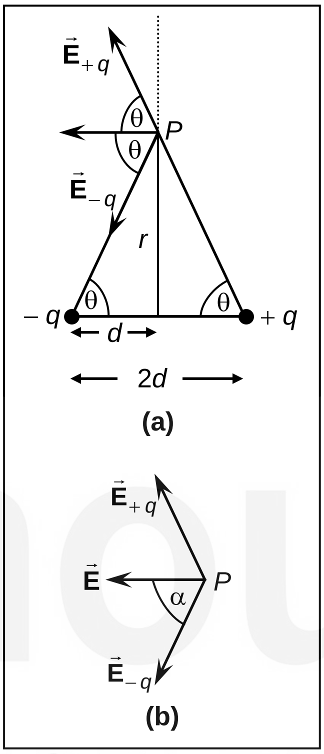



**Fig. 5.9**: **Diagram for Example 5.5.**

You may now like to learn how to determine the electric field due to a system of more than two charges. Consider the following example.

## *EXAMPLE 5.6* : ELECTRIC FIELD OF MANY CHARGES

Three charges $+q$, $+2q$ and $-q$ are kept in the $xy$ plane at three vertices of a square $ABCD$ of side $a$ (as shown in Fig. 5.10). Determine the net electric field due to these charges at the point $B$.

**SOLUTION ■** We use Eq. (5.10) to determine the electric field at point $B$ due to each charge. Then we apply Eq. (5.11) to obtain the net electric field.

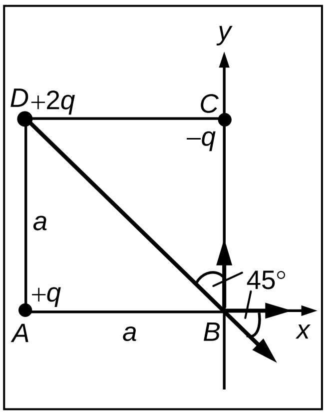



**Fig. 5.10**: **Diagram for Example 5.6.**

The electric field at $B$ due to charge $+q$ is $\vec{\mathbf{E}}_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{a^2}\hat{\mathbf{i}}$ where $\hat{\mathbf{i}}$ is the unit vector along the $x$-axis. To simplify the algebra, we write

$$E_0 = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{a^2} \quad \text{so that} \quad \vec{\mathbf{E}}_{+q} = E_0\,\hat{\mathbf{i}}$$

The electric field at $B$ due to charge $-q$ is $\vec{\mathbf{E}}_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{a^2}(\hat{\mathbf{j}}) = E_0\,\hat{\mathbf{j}}$

Using the geometry of Fig. 5.10, we resolve the electric field at $B$ due to charge $+2q$ along the $x$ and $y$-axes to get

$$\vec{\mathbf{E}}_{+2q} = E_{+2q}\cos 45^\circ\,\hat{\mathbf{i}} - E_{+2q}\sin 45^\circ\,\hat{\mathbf{j}} \quad \text{where} \quad E_{+2q} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(\sqrt{2}a)^2} = E_0$$

The net electric field at $B$ is, therefore,

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{+q} + \vec{\mathbf{E}}_{-q} + \vec{\mathbf{E}}_{+2q} = E_0\,\hat{\mathbf{i}} + E_0\,\hat{\mathbf{j}} + \frac{E_0}{\sqrt{2}}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

or $$\vec{\mathbf{E}} = E_0\left(1 + \frac{1}{\sqrt{2}}\right)\hat{\mathbf{i}} + E_0\left(1 - \frac{1}{\sqrt{2}}\right)\hat{\mathbf{j}}$$

The magnitude of the resultant electric field is [Eq. (A1.3a), Appendix A1 of Block 1]:

$$E = E_0\sqrt{\left[\left(1 + \frac{1}{\sqrt{2}}\right)^2 + \left(1 - \frac{1}{\sqrt{2}}\right)^2\right]} = \sqrt{3}\,E_0$$

The direction of the resultant electric field is given by the angle

$$\theta = \tan^{-1}\left[\frac{\left(1 - \frac{1}{\sqrt{2}}\right)}{\left(1 + \frac{1}{\sqrt{2}}\right)}\right] = \tan^{-1}[0.17] \Rightarrow \theta = 9.6^\circ$$

The resultant electric field has magnitude $\frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{|q|}{a^2}$.

It makes an angle of 9.6° with the $x$-axis.

Before studying further, you may like to practice how to calculate the electric field due to many charges.

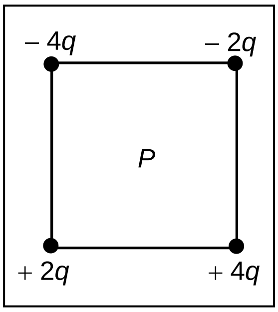



**Fig. 5.11**: **Diagram for SAQ 4.**

## *SAQ 4* – Electric field due to many charges

Four charges $+2q$, $-2q$, $+4q$ and $-4q$ are placed at the vertices of a square of side $a$ (Fig. 5.11). Determine the net electric field due to the charges at the centre $P$ of the square given that $q = 1.0 \times 10^{-9}$ C and $a = 6.0$ cm.

So far, we have defined the electric field and calculated its value for an isolated point charge or an arrangement of two or more point charges. You may like to ask: **What is the electric field of a continuous charge distribution, for example, charge distribution on a wire, lamina or sphere**? Let us find out.

### 5.3.3 Electric Field due to Continuous Charge Distributions

Let us calculate the electric field at point $P$ with position vector $\vec{\mathbf{r}}_i$ due to any continuous distribution of charge (like the one shown in Fig. 5.12). Let us take the continuous charge distribution to be made up of infinitesimal charges $dq_j$. Then from Eq. (5.10), the electric field $d\vec{\mathbf{E}}_j$ due to the infinitesimal charge $dq_j$ (having position vector $\vec{\mathbf{r}}_j$) at the point $P$ is given by

$$d\vec{\mathbf{E}}_j = \frac{1}{4\pi\varepsilon_0}\frac{dq_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2}\hat{\mathbf{r}}_{ji} \qquad (5.12)$$

From the principle of superposition [Eqs. (5.11 and 5.9)], the net electric field $\vec{\mathbf{E}}$ at point $P$ due to the charge distribution will be just the **vector sum of electric fields due to all such infinitesimal charges comprising the distribution**:

$$\vec{\mathbf{E}} = \sum_j d\vec{\mathbf{E}}_j = \frac{1}{4\pi\varepsilon_0}\sum_j \frac{dq_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2}\hat{\mathbf{r}}_{ji} \qquad (5.13)$$

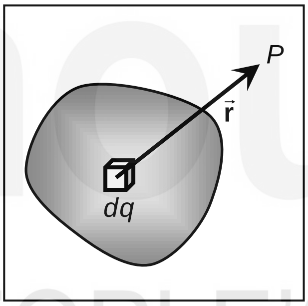



**Fig. 5.12**: **Determining electric field at a point due to a continuous charge distribution.**

But in the limit as the charges are infinitesimally small and tend to zero, the sum in Eq. (5.13) can be written as the following integral:

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{\mathbf{r}} \qquad (5.14)$$

The limits of the integral are defined so that the entire region over which charge is distributed is included. Remember that in Eq. (5.14), $\hat{\mathbf{r}}$ is the unit vector from the charge $dq$ to the point $P$ (having position vector $\vec{\mathbf{r}}$) at which the electric field is being determined (see Fig. 5.12).

Now, the charge may be continuously distributed over a line, a surface or a volume as shown in Figs. 5.13a, b and c. In such distributions, instead of charges, we speak of the density of charges. The charge density (line, surface or volume) will, in general, be a function of the coordinates. However, in this course, we will consider only those charge distributions that have constant charge density.

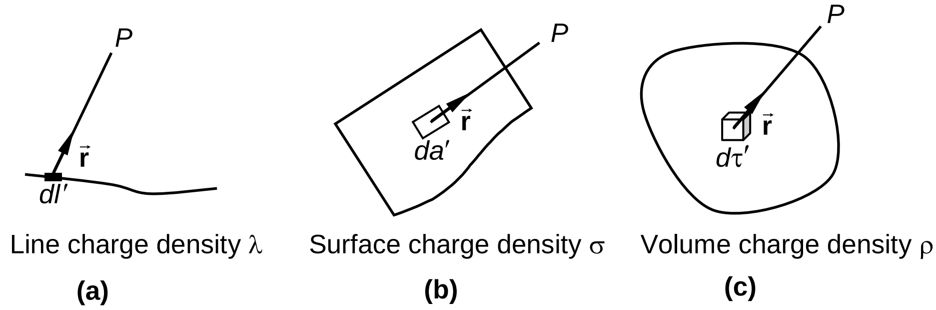



**Fig. 5.13**: **Determining electric field due to a) line charge distribution; b) surface charge distribution; c) volume charge distribution.**

If the charge is distributed over a line, as in a wire, (Fig. 5.13a), then we speak of the **line charge density**, i.e., charge per unit length and usually denote it by $\lambda$. The SI unit of $\lambda$ is $\mathrm{C\,m^{-1}}$.

In general, when the line charge density is not constant, we have

$dq = \lambda(\vec{\mathbf{r}}')dL'$

and $q = \int_C \lambda(\vec{\mathbf{r}}')dL'$ (i)

Suppose we use the Cartesian coordinates to solve these integrals. Then in Eq. (i), we will integrate with respect to only one variable *x*, *y* or *z* depending on whether the line charge is distributed along the *x*, *y* or *z*-axis.

For a non-uniform surface charge distribution, $\sigma$ is not constant, and we have

$dq = \sigma(\vec{\mathbf{r}}')dS'$

$q = \iint_S \sigma(\vec{\mathbf{r}}')dS'$ (ii)

Since an area is defined in two dimensions, we will integrate Eq. (ii) with respect to any two variables *x* and *y*, *y* and *z* or *z* and *x*.

For a non-uniform volume charge distribution, $\rho$ is not constant, and we have

$dq = \rho(\vec{\mathbf{r}}')dV'$

and

$q = \iiint_V \rho(\vec{\mathbf{r}}')dV'$ (iii)

Now we will have to integrate Eq. (iii) with respect to the variables *x*, *y* and *z* since volume is defined in three dimensions. These calculations are beyond the scope of this course.

The line charge is, in general, a function of the position **along** the line. Its expression is given in the margin. If the line charge is distributed uniformly, i.e., the line charge density $\lambda$ is constant, then we have

$$dq = \lambda dL \qquad (5.15a)$$

So, the electric field due to a **uniformly distributed line charge** is defined by

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int_C \frac{\lambda dL}{r^2}\hat{\mathbf{r}} \quad \text{or} \quad \vec{\mathbf{E}} = \frac{\lambda}{4\pi\varepsilon_0}\int_C \frac{dL}{r^2}\hat{\mathbf{r}} \qquad (5.15b)$$

For continuous charge distribution over a surface (Fig. 5.13b), we define the **surface charge density** $\sigma$ as the **charge per unit area**. Its SI unit is $\text{Cm}^{-2}$. It is constant for a uniformly distributed charge on any surface. In this case,

$$dq = \sigma dS \qquad (5.16a)$$

and the electric field due to a **uniformly distributed surface charge** is defined as

$$\vec{\mathbf{E}} = \frac{\sigma}{4\pi\varepsilon_0}\iint_S \frac{dS}{r^2}\hat{\mathbf{r}} \qquad (5.16b)$$

Eq. (5.16b) is a surface integral about which you have studied in Unit 4.

If the continuous charge distribution is spread over a volume (Fig. 5.13c), then we use the **volume charge density** $\rho$, which is the **charge per unit volume**. Its SI unit is $\text{Cm}^{-3}$. For a uniformly distributed charge over any volume, $\rho$ is constant and

$$dq = \rho dV \qquad (5.17a)$$

The electric field due to a **uniformly distributed volume charge** is defined as

$$\vec{\mathbf{E}} = \frac{\rho}{4\pi\varepsilon_0}\iiint_V \frac{dV}{r^2}\hat{\mathbf{r}} \qquad (5.17b)$$

Let us take up an example to apply the simplest of these equations, Eq. (5.15b), to calculate the electric field of a uniform line charge.

## *EXAMPLE 5.7*: ELECTRIC FIELD OF INFINITE LINE CHARGE

A straight line of infinite length carries a uniform charge with line charge density $\lambda$. Determine the electric field at a distance *y* above the midpoint of the line.

**SOLUTION ■** We apply Eq. (5.15b) to determine the electric field due to a uniformly distributed infinite line charge.

Study Fig. 5.14, which shows the charge distribution in the given geometry. Let us choose the *xy* coordinate system to solve this problem with its origin at the midpoint.

Here $dq = \lambda dx$ (i)

By definition, the **magnitude** of the electric field due to $dq$ at the point $P$ directly above the origin is given by

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \quad \text{(ii)}$$

where $r$ is the distance of $dq$ from $P$ and $\hat{\mathbf{r}}$, the unit vector from $dq$ to $P$. Note that the direction of $\hat{\mathbf{r}}$ will be different for different elements of charge. Now to determine the net electric field, we write the electric field $d\vec{\mathbf{E}}$ in terms of its $x$ and $y$-components and then integrate each component over the respective variable $x$ or $y$.

Our choice of the coordinate system simplifies the calculation. Note that for each infinitesimal charge $dq$ placed at the point $+x$ to the right of the origin, we can place a corresponding infinitesimal charge $dq$ at the point $-x$ to the left of the origin. So these form a pair. Now the $x$-components of the electric fields due to this pair cancel out as shown in Fig. 5.14. This will be the case for each pair of points $\pm x$ on the $x$-axis. Therefore, the $x$-component of the electric field $d\vec{\mathbf{E}}$ will be zero. The $y$-component of the electric field due to the element of charge $dq$ is given by

$$dE = dE_y = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\frac{y}{r} \quad \left(\because \cos\theta = \frac{y}{r}\right) \quad \text{(iii)}$$

where $\theta$ is the angle between $\hat{\mathbf{r}}$ and the $y$-axis. We add the $y$-components of the electric fields of the two elements at the points $\pm x$ that will be in the same direction to get the net electric field due to them as,

$$d\vec{\mathbf{E}}_{net} = \frac{1}{4\pi\varepsilon_0}\frac{2dq}{r^2}\frac{y}{r}\hat{\mathbf{j}} = \frac{1}{4\pi\varepsilon_0}\frac{2y\lambda dx}{(x^2+y^2)^{3/2}}\hat{\mathbf{j}} \quad (\because dq = \lambda dx)$$

The net electric field due to the infinite line charge is obtained by integrating $d\vec{\mathbf{E}}_{net}$ with respect to $x$ with the limits from 0 to $\infty$.

Although the line extends from $-\infty$ to $+\infty$, we integrate over only half the line because the expression we are integrating is already the electric field of a charge pair $dq$.

Thus, $$\vec{\mathbf{E}}_{net} = \int d\vec{\mathbf{E}}_{net} = \int_{x=0}^{x=\infty}\frac{1}{4\pi\varepsilon_0}\frac{2y\lambda dx}{(x^2+y^2)^{3/2}}\hat{\mathbf{j}} \quad \text{(iv)}$$

Integrating the right hand side of Eq. (iv) gives (read the margin remark):

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{y}\hat{\mathbf{j}}$$

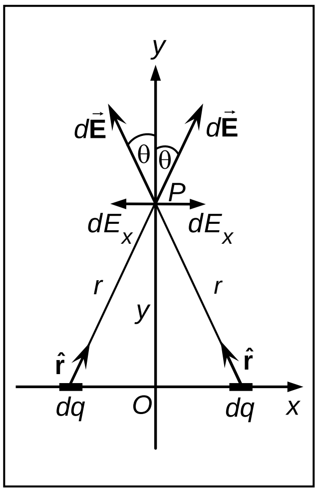



**Fig. 5.14: Electric field of a uniform infinite line charge.**

Let $x = y\tan\theta$. Then $dx = y\,d\theta\sec^2\theta$ with the limits from 0 to $\frac{\pi}{2}$. The electric field is then given by

$$\vec{\mathbf{E}}_{net} = \frac{\lambda y}{2\pi\varepsilon_0}\int_0^{\pi/2}\frac{y\sec^2\theta\,d\theta}{y^3\sec^3\theta}\hat{\mathbf{j}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\int_0^{\pi/2}\cos\theta\,d\theta\,\hat{\mathbf{j}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\sin\theta\Big|_0^{\pi/2}\hat{\mathbf{j}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\hat{\mathbf{j}}$$

You will agree that this way of calculating the electric field is quite lengthy as it involves solving complicated integrals. You will learn a much simpler way of determining the electric field of such continuous charge distributions that have some symmetry of this kind in the next unit.

Let us now stop and review what you have learnt in this section. To sum up, you have learnt the definition of the electric field and calculated it for a point charge, arrangements of discrete point charges and a continuous line charge. But while going through this section, this question may still have puzzled you: **What exactly *is* an electric field**?

You should think of the electric field as a **real** physical entity which exists in the space in the neighbourhood of any charge, groups of charges or continuous charge distributions, which set up the electric field. Any charge kept in the electric field experiences the electrostatic force given by Eq. (5.7). The concept of electric field is abstract and it is difficult to imagine it concretely. But you have learnt how to calculate the electric field and also the electrostatic force experienced by a charge kept in the electric field.

This is actually all that we are supposed to do in electrostatics: Determine the electrostatic forces and electric fields due to a given charge distribution. However, as you may have felt while working through Example 5.7, the integrals involved in calculating electric fields can be quite complicated even for simple charge distributions. So, much of electrostatics is about learning the tools and methods that simplify these calculations so that we have no need to solve such complicated integrals. This is what you will be learning in the remaining units of this block and Units 10 and 11 of the next block.

We now summarise the concepts you have studied in this unit.

## 5.4 SUMMARY

| Concept | Description |
|---|---|
| ***Electric charge*** | ■ From a large number of observations and experiments, it has been deduced that there exist two types of electric charges in nature, which are arbitrarily called positive and negative charges. In SI system, the unit of electric charge is coulomb denoted by C.<br><br>Like charges repel and unlike charges attract.<br><br>In an isolated system, electric charge is always conserved. Thus, the total positive charge is equal to the total negative charge in an isolated system.<br><br>Free electric charge is quantised and can take only discrete values that are integer multiples of the charge on the electron. |
| ***Electrostatic force and Coulomb's law*** | ■ The magnitude of the **electrostatic force** between two charged particles **at rest** is proportional to the product of the magnitudes of charges on them and inversely proportional to the square of the distance between them. The quantitative expression of the electrostatic force between two charges is given by **Coulomb's law**: The **electrostatic force** **on** a particle carrying a charge $q_1$ **by** a particle carrying a charge $q_2$ situated at a distance $r$ from it is given by |

$$\vec{\mathbf{F}}_{21} = k\,\frac{q_1 q_2}{|\vec{\mathbf{r}}_{21}|^2}\,\hat{\mathbf{r}}_{21}$$

where $\hat{\mathbf{r}}_{21}$ is the unit vector along the line joining the particles and is directed from $q_2$ to $q_1$. Note that $|\vec{\mathbf{r}}_{21}| = r$ and $\vec{\mathbf{r}}_{21} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$ where $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ are the position vectors of $q_1$ and $q_2$, respectively. In SI units, Coulomb's law is written as

$$\vec{\mathbf{F}}_{21} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{|\vec{\mathbf{r}}_{21}|^2}\,\hat{\mathbf{r}}_{21}$$

***Principle of superposition***

- According to the **principle of superposition**, in a many-particle system of charged particles at rest, the resultant electrostatic force on any charged particle is the vector sum of the electrostatic forces exerted by all other particles on it. In general, the electrostatic force $\vec{\mathbf{F}}_i$ on the *i*th charged particle due to all other charges $q_1, q_2, ..., q_j, ...$ in a many-particle system of charged particles at rest is given by

$$\vec{\mathbf{F}}_i = \sum_{j \neq i} \vec{\mathbf{F}}_{ji} = \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \frac{q_i q_j}{r_{ji}^2}\,\hat{\mathbf{r}}_{ji}$$

Note that the above summation does not include the *i*th charge and $\hat{\mathbf{r}}_{ji}$ is the unit vector along the line joining the *i*th and *j*th particles and is directed from $q_j$ to $q_i$.

***Electric field due to a point charge***

- The electric field due to a point charge or charge distribution at a point is defined in terms of the electrostatic force experienced by a test charge $q$ placed at that point divided by the magnitude of the test charge:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

The electric field due to a charge $Q$ at a point having position vector $\vec{\mathbf{r}}$ is given by

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{r^2}\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector pointing from the charge to the point at which the electric field is being calculated.

***Electric field due to multiple discrete charges***

- The electric field due to a distribution of charges at the point with position vector $\vec{\mathbf{r}}_i$ is given from the principle of superposition as

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_i} = \frac{1}{4\pi\varepsilon_0} \sum_j \frac{q_j}{|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j|^2}\,\hat{\mathbf{r}}_{ji}$$

where $\hat{\mathbf{r}}_{ji}$ is the unit vector along the line joining the *i*th and *j*th particles and is directed from $q_j$ to $q_i$. We can write this equation as

$$\vec{\mathbf{E}} = \sum_j \vec{\mathbf{E}}_j$$

where $$\vec{\mathbf{E}}_j = \frac{1}{4\pi\varepsilon_0}\frac{q_j}{\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|^2}\hat{\mathbf{r}}_{ji}$$

So, **the total electric field due to a group of charges is the vector sum of the electric fields due to individual charges of the distribution**.

***Electric field due to continuous charge distributions***

- The electric field due to a continuous distribution of charge is given by,

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{\mathbf{r}}$$

The electric field due to a **uniformly distributed line charge** with constant line charge density $\lambda$ is given by

$$\vec{\mathbf{E}} = \frac{\lambda}{4\pi\varepsilon_0}\int_C\frac{dL}{r^2}\hat{\mathbf{r}}$$

The electric field due to a **uniformly distributed surface charge** with constant surface charge density $\sigma$ is given by the surface integral

$$\vec{\mathbf{E}} = \frac{\sigma}{4\pi\varepsilon_0}\iint_S\frac{dS}{r^2}\hat{\mathbf{r}}$$

The electric field due to a **uniformly distributed volume charge** with constant volume charge density $\rho$ is given by the volume integral

$$\vec{\mathbf{E}} = \frac{\rho}{4\pi\varepsilon_0}\iiint_V\frac{dV}{r^2}\hat{\mathbf{r}}$$

## 5.5 TERMINAL QUESTIONS

1. The electrostatic force exerted by two point charges on each other has magnitude 10 N when these are at rest and placed a distance $r$ apart. What would the magnitude of the electrostatic force between them be if the distance between them is a) $4r$, b) $100r$, c) $\frac{r}{4}$ and d) $\frac{r}{100}$?

2. Two identical charged particles are placed at rest at a separation of 1 m. What is the charge on them if the magnitude of the electrostatic force exerted on each particle is 1 N?

3. Three charged particles $A$, $B$ and $C$, each having a charge of 1.0 $\mu$C, are placed at rest on a straight line. The distance between $A$ and $B$ is 0.01 m. What is the net electrostatic force exerted on particle $C$ if it is placed a) at a distance 0.01 m to the right of the particle $B$ along the line $AB$, b) to the left of the particle $B$ along the line $AB$, at the midpoint of $AB$?

4. Two point charges $+4q$ and $+q$ are placed at rest at a distance '$a$' from each other. Determine the position of a charge $+q$ placed on a straight line joining these two charges, if it is in equilibrium.

5. What is the electric field of a particle having charge $-9.0\times10^{-9}$ C at a point 1.0 m away from it? Determine the electrostatic force exerted on a proton placed at that point.

6. The electric field due to a charged particle at a point 0.5 m away from it has magnitude $36\,NC^{-1}$. What is the magnitude of the electric charge on the particle?

7. When a particle having charge $-9\times10^{-9}\,C$ is placed at a certain point in an electric field, the electrostatic force exerted on it is of magnitude $3\times10^{-9}\,N$ and directed along the negative *x*-axis. What is the electric field at this point? What would the magnitude and direction of the electrostatic force acting on an electron placed at this point be?

8. Three particles each having charge $+q$ are placed at the vertices of an equilateral triangle with each side of length *r*. Calculate the magnitude of the net electric field at the midpoint of any side of the triangle.

9. Three particles having charge $+q$, $-q$ and $-2q$ are placed at the same distance *a* from the origin as shown in Fig. 5.15. Calculate the net electric field at the origin.

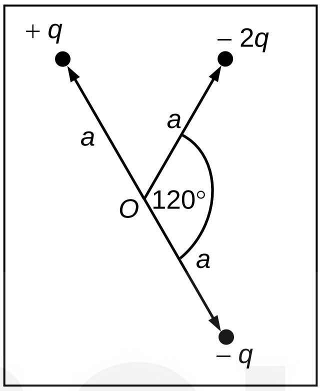



**Fig. 5.15**: **Diagram for TQ 9.**

10. Four charges $+2q$, $+2q$, $-2q$ and $-2q$ are placed at the vertices of a rectangle of sides 3.0 m and 4.0 m. What is the net electric field due to the charges at the point of intersection of the diagonals given that $q = 3.0\times10^{-9}\,C$?

## 5.6 SOLUTIONS AND ANSWERS

### Pre-test

1. **Observation 1**: Correct answer is (a) since the glass rubbed with silk is positively charged. Since the object is repelled by the glass, it must have the same charge as the glass.

   **Observation 2**: Correct answer is (a) since the amber rubbed with fur is negatively charged. Since both objects are attracted to it, therefore, both must have the opposite charge to that of amber.

2. a) False. So far, no such measurements have been made for free particles.

   b) True.

   c) True.

   d) False. It depends on the product of their magnitudes.

   e) True.

   f) True.

### Self-Assessment Questions

1. a) From Eq. (5.3b), the electrostatic force on charge $q_1$ due to charge $q_2$ at rest is given by Coulomb's law as $\vec{\mathbf{F}}_{21} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{|\vec{\mathbf{r}}_{21}|^2}\,\hat{\mathbf{r}}_{21}$ where $\hat{\mathbf{r}}_{21}$ is the unit vector along the line joining the particles and directed from $q_2$ to $q_1$ and $|\vec{\mathbf{r}}_{21}| = r$. Also $\dfrac{1}{4\pi\varepsilon_0} = 8.99\times10^9\,Nm^2\,C^{-2}$.

Substituting the values of $q_1$, $q_2$ and $r$ for both cases, we get

i) $$\vec{\mathbf{F}}_{21} = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2} \times \frac{(8.0\,\mu\text{C})\,(8.0\,\mu\text{C})}{(0.04\,\text{m})^2}\,\hat{\mathbf{r}}_{21} = 360\,\text{N}\,\hat{\mathbf{r}}_{21}$$

ii) $$\vec{\mathbf{F}}_{21} = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2} \times \frac{(15\,\text{mC})\,(-10\,\text{mC})}{(3.0\,\text{m})^2}\,\hat{\mathbf{r}}_{21}$$

$$= -1.5\times10^5\,\text{N}\,\hat{\mathbf{r}}_{21}$$

b) From Eq. (5.3b), the magnitude of the electrostatic force between the electron and the proton is given by

$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{since } |\vec{\mathbf{r}}_{21}| = r.$$

Substituting the values of the magnitudes of the charges of electron and proton, i.e., $|q_1| = |q_2| = 1.6\times10^{-19}\,\text{C}$ and $r = 5.3\times10^{-11}\,\text{m}$, we get

$$F_{elec} = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2} \times \frac{(1.6\times10^{-19}\,\text{C})\times(1.6\times10^{-19}\,\text{C})}{(5.3\times10^{-11}\,\text{m})^2}$$

$$= 8.2\times10^{-8}\,\text{N}$$

The gravitational force between the electron and the proton is given by

$$F_{grav} = G\,\frac{m_1 m_2}{r^2}$$

Substituting the values of the masses of electron and proton, i.e., $m_1 = 9.1\times10^{-31}\,\text{kg}$, $m_2 = 1.7\times10^{-27}\,\text{kg}$, $r = 5.3\times10^{-11}\,\text{m}$ and $G = 6.7\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}$, we get

$$F_{grav} = 6.7\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2} \times \frac{(9.1\times10^{-31}\,\text{kg})\times(1.7\times10^{-27}\,\text{kg})}{(5.3\times10^{-11}\,\text{m})^2}$$

$$= 3.7\times10^{-47}\,\text{N}$$

Hence, $$\frac{F_{elec}}{F_{grav}} = \frac{8.2\times10^{-8}}{3.7\times10^{-47}} = 2.2\times10^{39}$$

Thus, the electrostatic force is much stronger (~ $10^{39}$ times stronger) than the gravitational force.

2. a) Substituting for $Q$ (with its sign) and $r$ in Eq. (5.6a), we get

(i) For $Q = +5\,\mu\text{C}$ and $r = 0.30\,\text{m}$,

$$\vec{\mathbf{E}}\,(\vec{\mathbf{r}}) = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2} \times \frac{(+5\times10^{-6}\,\text{C})}{(0.30\,\text{m})^2}\,\hat{\mathbf{r}} = 5\times10^5\,\text{NC}^{-1}\,\hat{\mathbf{r}}$$

(ii) For $Q = -10\,\mu\text{C}$ and $r = 1\text{m}$,

$$\vec{\mathbf{E}}\,(\vec{\mathbf{r}}) = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2} \times \frac{(-10\times10^{-6}\,\text{C})}{(1\text{m})^2}\,\hat{\mathbf{r}} = -9\times10^4\,\text{NC}^{-1}\,\hat{\mathbf{r}}$$

Note that the electric field due to the negative charge is directed towards it. The electric fields at the points *P* and *R* are shown in Figs. 5.16a and b. Note that the tails of the electric fields are placed at *P* and *R*, respectively.

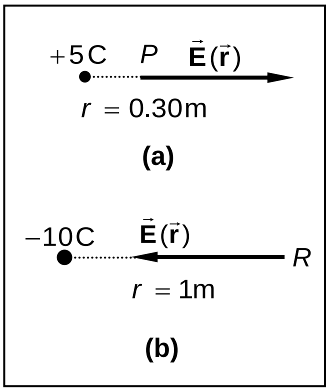



**Fig. 5.16**: **Diagrams for the answers of SAQ 2a (i) and (ii). The diagrams are not to scale.**

b) From Eq. (5.7), the electrostatic force is given by $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = q\vec{\mathbf{E}}(\vec{\mathbf{r}})$. For $q = +6\,\mu C$, it is given as follows:

(i) $5 \times 10^5 \times 6 \times 10^{-6}\,N\,\hat{\mathbf{r}} = 3\,N\,\hat{\mathbf{r}}$

(ii) $-9\times10^4 \times 6\times10^{-6}\,N\,\hat{\mathbf{r}} = -0.5\,N\,\hat{\mathbf{r}}$ up to one significant digit.

3. See Fig. 5.17. The midpoint *C* of the dipole axis is at equal distance *d* from each charge. From Eq. (5.6b), the magnitudes of the electric fields of both charges at the midpoint are, respectively,

$$\left|\vec{\mathbf{E}}_{-q}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|(-q)|}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2} \quad \text{and} \quad \left|\vec{\mathbf{E}}_{+q}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}$$

The directions of the electric fields at the point *C* due to both charges are opposite to $\hat{\mathbf{r}}$, the unit vector along the line joining the two charges as shown in Fig. 5.17. From Eq. (5.11), the resultant or net electric field at the midpoint *C* due to the two charges is:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{+q} + \vec{\mathbf{E}}_{-q} = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{d^2}\,\hat{\mathbf{r}}$$

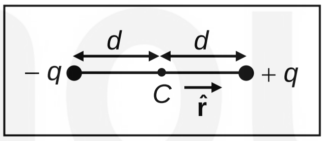



**Fig. 5.17**: **Diagram for the answer of SAQ 3.**

4. Let us choose the *x* and *y*-axes as shown in Fig. 5.18 by the dashed arrows.

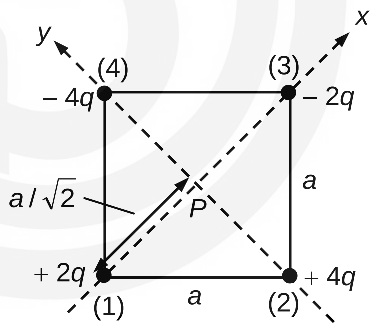



**Fig. 5.18**: **Diagram for answer to SAQ 4.**

Note from Fig. 5.18 that the distance of the point *P* from any of the four charges is $\frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$, where *a* is the side of the square. Now the net electric field at the point *P* is the vector sum of the electric fields due to all charges at that point:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \vec{\mathbf{E}}_4 \qquad \text{(i)}$$

where $\vec{\mathbf{E}}_1$ is the electric field due to the charge $+2q$, $\vec{\mathbf{E}}_2$, the electric field due to the charge $+4q$, $\vec{\mathbf{E}}_3$, the electric field due to the charge $-2q$ and $\vec{\mathbf{E}}_4$, the electric field due to the charge $-4q$. We use Eq. (5.6a) to determine each one of these electric fields and then take their vector sum. Note that with the choice of axes in Fig. 5.18, the direction of the position vector $\hat{\mathbf{r}}$ of the point *P* with respect to each charge can be expressed in terms of $\hat{\mathbf{i}}$ and

$\hat{\mathbf{j}}$ or their combinations. Also $r = \frac{a}{\sqrt{2}}$. So, from Fig. 5.18, we can write the electric field at $P$ due to the charge 1 as

$$\vec{\mathbf{E}}_1 = \frac{1}{4\pi\varepsilon_0}\frac{(+2q)}{(a/\sqrt{2})^2}\hat{\mathbf{i}} = \frac{1}{4\pi\varepsilon_0}\frac{2\times 2q}{a^2}\hat{\mathbf{i}} = \frac{1}{4\pi\varepsilon_0}\frac{4q}{a^2}\hat{\mathbf{i}} \;(\because \hat{\mathbf{r}} = \hat{\mathbf{i}}) \qquad \text{(ii)}$$

We write $E_0 = \frac{1}{4\pi\varepsilon_0}\frac{4q}{a^2}$ so that the expressions become simpler to write. The electric fields at $P$ due to the charges 2, 3, 4 are:

$$\vec{\mathbf{E}}_2 = \frac{1}{4\pi\varepsilon_0}\frac{2\times 4q}{a^2}\hat{\mathbf{j}} = 2E_0\,\hat{\mathbf{j}}, \qquad (\because \hat{\mathbf{r}} = \hat{\mathbf{j}}) \qquad \text{(iii)}$$

$$\vec{\mathbf{E}}_3 = \frac{1}{4\pi\varepsilon_0}\frac{2\times(-2q)}{a^2}(-\hat{\mathbf{i}}) = E_0\,\hat{\mathbf{i}} \qquad (\because \hat{\mathbf{r}} = -\hat{\mathbf{i}}) \qquad \text{(iv)}$$

and
$$\vec{\mathbf{E}}_4 = \frac{1}{4\pi\varepsilon_0}\frac{2\times(-4q)}{a^2}(-\hat{\mathbf{j}}) = 2E_0\,\hat{\mathbf{j}} \qquad (\because \hat{\mathbf{r}} = -\hat{\mathbf{j}}) \qquad \text{(v)}$$

Substituting Eqs. (ii) to (v) in Eq. (i), we can write

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \vec{\mathbf{E}}_4 = 2E_0\,\hat{\mathbf{i}} + 4E_0\,\hat{\mathbf{j}} = 2E_0(\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \qquad \text{(vi)}$$

Now for $q = 1.0\times 10^{-9}\,\text{C}$ and $a = 0.06\,\text{m}$,

$$E_0 = \frac{1}{4\pi\varepsilon_0}\frac{4q}{a^2} = 8.99\times 10^9\,\text{Nm}^2\,\text{C}^{-2}\times\frac{4\times 1.0\times 10^{-9}\,\text{C}}{(0.06\,\text{m})^2}$$

$$\therefore \quad E_0 = 1.0\times 10^4\,\text{NC}^{-1} \quad \text{and} \quad \vec{\mathbf{E}} = 2.0\times 10^4\,\text{NC}^{-1}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$

## Terminal Questions

1. We use Eq. (5.2) for the magnitude of the electrostatic force with $k = \frac{1}{4\pi\varepsilon_0}$. It is given that $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} = 10\,\text{N}$ is the magnitude of the electrostatic force exerted by two point charges on each other when these are placed a distance $r$ apart. The magnitudes of the electrostatic force between them for various distances will be, respectively,

   a) $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{(4r)^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{16(r)^2} = \frac{F_1}{16} = \frac{5}{8}\text{N}$ since

   $$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} = 10\,\text{N}$$

   b) $F_3 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{(100r)^2} = \frac{F_1}{10000} = \frac{10}{10000}\text{N} = 10^{-3}\,\text{N}$

   c) $F_4 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{(r/4)^2} = 16F_1 = 160\,\text{N}$ and

d) $$F_5 = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{(r/100)^2} = 10000 F_1 = 10^5 \text{ N}$$

2. Let the charge on the identical particles be $q$. We use Eq. (5.2) for the magnitude of the electrostatic force with $k = \frac{1}{4\pi\varepsilon_0}$ . It is given that the charges are identical and the distance between them is 1m. Substituting these values in Eq. (5.2), we have

$$1\text{N} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{(1\text{m})^2}$$

or $$q = \sqrt{\frac{1\text{N} \times (1\text{m})^2}{8.99 \times 10^9 \text{ Nm}^2 \text{C}^{-2}}} = 0.33\,\text{mC}$$

3. a) Refer to Fig. 5.19a. The net electrostatic force exerted on particle $C$ is the vector sum of the electrostatic forces exerted on it by the particles $A$ and $B$ as given by Eq. (5.4b). In terms of the unit vector $\hat{\mathbf{i}}$ along the $x$-axis, it is given by

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{AC} + \vec{\mathbf{F}}_{BC} = \frac{1}{4\pi\varepsilon_0}\left(\frac{(1.0\,\mu\text{C})^2}{(0.02\,\text{m})^2} + \frac{(1.0\,\mu\text{C})^2}{(0.01\text{m})^2}\right)\hat{\mathbf{i}}$$

$$= (8.99 \times 10^9 \text{ Nm}^2 \text{C}^{-2} \times 10^{-12} \text{ C}^2 \times \frac{5}{4} \times 10^4 \text{ m}^{-2})\,\hat{\mathbf{i}} = 1.1 \times 10^2 \text{ N}\,\hat{\mathbf{i}}$$

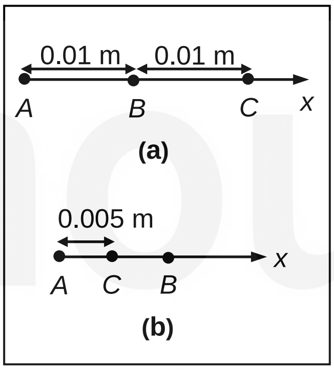



**Fig. 5.19**: **Diagram for the answer of TQ 3.**

b) Refer to Fig. 5.19b. The net electrostatic force exerted on particle $C$ is the vector sum of the electrostatic forces exerted on it by the particles $A$ and $B$. In this case, the electric field due to $B$ will be in the opposite direction to that of $A$ since it points away from the positive charge. In terms of the unit vector $\hat{\mathbf{i}}$ along the $x$-axis, it is given by Eq. (5.4b):

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{AC} + \vec{\mathbf{F}}_{BC} = \frac{1}{4\pi\varepsilon_0}\left(\frac{(1\,\mu\text{C})^2}{(0.005\,\text{m})^2}\,\hat{\mathbf{i}} - \frac{(1\,\mu\text{C})^2}{(0.005\,\text{m})^2}\,\hat{\mathbf{i}}\right) = \vec{\mathbf{0}}$$

4. Refer to Fig. 5.20. Let the charges lie along the $x$-axis. Let the position of the charge 3 ($= +q$) be at a distance $x$ from the charge 1 ($= +4q$) such that $x < a$. At this point the charge 2 ($= +q$) is at a distance $(a - x)$ from the charge 3. Therefore, the net electrostatic force exerted on the charge 3 due to the charges 1 and 2 is given by Eq. (5.4b) as

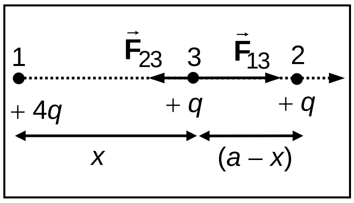



**Fig. 5.20**: **Diagram for the answer of TQ 4.**

$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q \times q}{x^2}\,\hat{\mathbf{i}} + \frac{q \times q}{(a-x)^2}(-\hat{\mathbf{i}})\right)$$

When the charge 3 is in equilibrium, the net force on it is zero. Thus,

$$\vec{\mathbf{F}}_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q \times q}{x^2}\,\hat{\mathbf{i}} + \frac{q \times q}{(a-x)^2}(-\hat{\mathbf{i}})\right) = \vec{\mathbf{0}}$$

or $$\frac{4}{x^2} = \frac{1}{(a-x)^2} \Rightarrow 4(a-x)^2 = x^2 \Rightarrow 2(a-x) = \pm x$$

For the positive sign of $x$, $x = \dfrac{2a}{3}$ and for the negative sign of $x$, $x = 2a$.

Since $x < a$, $x = \dfrac{2a}{3}$ is the only possible value of $x$. Therefore, for the charge 3 $(+q)$ to be in equilibrium, it should be placed at a distance $\dfrac{2a}{3}$ from the charge $+4q$.

5. From Eq. (5.6a), the electric field of a particle having charge $Q = -9\times10^{-9}\,\text{C}$ at a point 1 m away from it is given by

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = 8.99\times10^9\,\text{N m}^2\,\text{C}^{-2}\,\frac{(-9\times10^{-9}\,\text{C})}{(1\text{m})^2}\,\hat{\mathbf{r}} = -81\,\text{N C}^{-1}\hat{\mathbf{r}}$$

It is directed towards the negatively charged particle. The electrostatic force experienced by a proton placed at that point is an attractive force directed towards the charge $Q$ and is given by

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = e\vec{\mathbf{E}}(\vec{\mathbf{r}}) = (1.6\times10^{-19})\,\text{C}\times(-81\,\text{NC}^{-1})\hat{\mathbf{r}} = -1.3\times10^{-17}\,\text{N}\,\hat{\mathbf{r}}$$

up to 2 significant digits.

6. Substituting $E = 36\,\text{NC}^{-1}$ and $r = 0.5\,\text{m}$ in Eq. (5.6a), we have

$$E(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = 8.99\times10^9\,\text{N m}^2\,\text{C}^{-2}\,\frac{|Q|}{(0.5\,\text{m})^2} = 36\,\text{N C}^{-1}$$

$$\therefore \quad |Q| = 1\times10^{-9}\,\text{C} = 1\text{nC}$$

7. From Eq. (5.7), we have

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = Q\vec{\mathbf{E}}(\vec{\mathbf{r}}) \qquad \text{(i)}$$

where $Q = -9\times10^{-9}\,\text{C}$ and $\vec{\mathbf{F}} = -3\times10^{-9}\,\text{N}\,\hat{\mathbf{i}}$. Substituting these values in Eq. (i), we get

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{\vec{\mathbf{F}}(\vec{\mathbf{r}})}{Q} = \frac{-3\times10^{-9}\,\text{N}}{-9\times10^{-9}\,\text{C}}\,\hat{\mathbf{i}} = 0.3\,\text{NC}^{-1}\,\hat{\mathbf{i}}$$

It is directed along the positive $x$-axis. The electrostatic force exerted on an electron placed at this point is given by

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = e\vec{\mathbf{E}}(\vec{\mathbf{r}}) = (-1.6\times10^{-19}\,\text{C})\times(0.3\,\text{NC}^{-1})\,\hat{\mathbf{i}}$$

$$= -0.48\times10^{-19}\,\hat{\mathbf{i}}\,\text{N} = -5\times10^{-20}\,\hat{\mathbf{i}}\,\text{N}$$

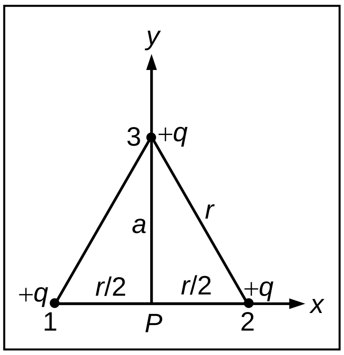



**Fig. 5.21**: **Diagram for the answer to TQ 8.**

8. Refer to Fig. 5.21. Let us take the $x$ and $y$-axes as shown in the figure. Then the electric fields at the midpoint $P$ due to two charges 1 and 2 of

magnitude $+q$ along the $x$-axis will be equal in magnitude and opposite in direction to each other:

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r/2)^2}\hat{\mathbf{i}} \quad \text{and} \quad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r/2)^2}\hat{\mathbf{i}}$$

The magnitude of the net electric field will be just the magnitude of the electric field due to charge 3 on the $y$-axis. The distance of the charge 3 from the midpoint of the side of the triangle along the $x$-axis is given by

$$a = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}}{2}r$$

Therefore, the magnitude of the net electric field at the midpoint of the base of this equilateral triangle is given by

$$E_3(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{4q}{3r^2} = \frac{q}{3\pi\varepsilon_0 r^2}$$

This result holds for any side of the equilateral triangle.

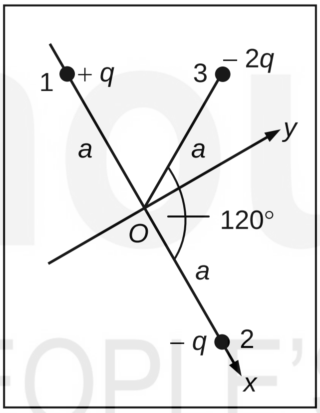



**Fig. 5.22**: **Diagram for the answer of TQ 9.**

9. Refer to Fig. 5.22. Let us choose the coordinate axes so that the problem becomes simplified. We choose the $x$-axis to be along the line joining the charges 1 and 2 as shown in the figure. The net electric field at the origin is the vector sum of the electric fields due to the charges 1, 2 and 3:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) + \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) + \vec{\mathbf{E}}_3(\vec{\mathbf{r}}) \qquad \text{(i)}$$

Let us determine the electric fields due to the three charges at the origin. You can see that the charges $+q$ and $-q$ are at the same distance ($a$) from the origin. So, the origin is at the midpoint of the line joining them. Therefore, for our choice of the $x$-axis, we get

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) + \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{\mathbf{i}} \qquad \text{(ii)}$$

The magnitude of the electric field due to the third charge $-2q$ is

$$E_3(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}$$

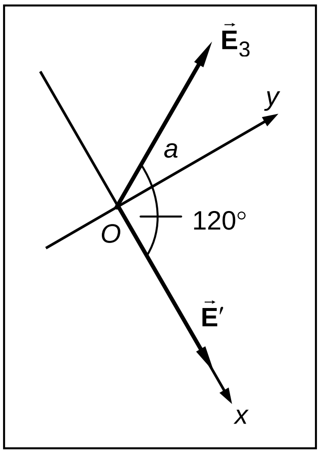



**Fig. 5.23**: **Electric fields for the answer of TQ 9.**

Since the charge 3 is a negative charge, the direction of the electric field due to it is towards the charge. The net electric field $\vec{\mathbf{E}}'$ due to the charges 1 and 2 and the electric field $\vec{\mathbf{E}}_3$ due to charge 3 are shown in Fig. 5.23. Note that the tails of the vectors are placed at the point where the net electric field is to be determined. To determine the net electric field at the origin, we resolve the electric field $\vec{\mathbf{E}}_3(\vec{\mathbf{r}})$ along the $x$ and $y$-axes:

$$E_{3x} = E_3\cos 120^\circ = -\frac{E_3}{2} \quad \text{and} \quad E_{3y} = E_3\sin 120^\circ = \frac{\sqrt{3}}{2}E_3 \qquad \text{(iii)}$$

Therefore, combining the results of Eqs. (ii) and (iii) with Eq. (i), the net electric field at $O$ is given as:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{\mathbf{i}} - \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{\mathbf{j}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}}\right)$$

The magnitude of the net electric field is given by

$$E(\vec{\mathbf{r}}) = \sqrt{E_x^2 + E_y^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\sqrt{1+3} = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}$$

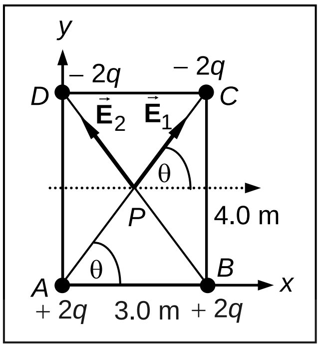



**Fig. 5.24**: **Diagram for the answer of TQ 10.**

10. Refer to Fig. 5.24 showing the four charges $A$, $B$, $C$, $D$, viz. $+2q$, $+2q$, $-2q$ and $-2q$ placed at the vertices of a rectangle of sides $AB = 3.0\,\text{m}$ and $BC = 4.0\,\text{m}$. The net electric field due to the charges at the point of intersection of the diagonals is the vector sum of the electric fields of the respective charges at that point. Let us choose the $x$ and $y$-axes as shown in the figure. The length of the diagonal of the rectangle is $\sqrt{(3.0)^2 + (4.0)^2}\,\text{m} = 5.0\,\text{m}$. Note from Fig. 5.24 that the electric fields due to the charges placed at the vertices $A$ and $C$ point in the same direction since the charges are unlike. So is the case for the charges placed at the vertices $B$ and $D$. The magnitudes of the electric fields due to all four charges are the same since the magnitudes of the charges are equal and their distances from the point $P$ are equal. Thus, the magnitude of the electric field due to each charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|2q|}{r^2} = 8.99\times10^9\,\text{Nm}^2\,\text{C}^{-2}\,\frac{6.0\times10^{-9}\,\text{C}}{(2.5\,\text{m})^2} = 8.6\,\text{NC}^{-1}$$

The net electric field is the resultant of the electric fields $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ shown in Fig. 5.24 with their tails at the point $P$. Note that their magnitudes are:

$$E_1 = E_2 = 2E$$

Note also from Fig. 5.24 that the $x$-components of these electric fields are equal and opposite so they cancel out. Their $y$-components are equal in magnitude and in the same direction and are given by:

$$E_{1y} = E_{2y} = E_1 \sin\theta = 2E\frac{BC}{AC} = 2\times8.6\,\text{NC}^{-1}\frac{4}{5} = 13.8\,\text{NC}^{-1}$$

So, the magnitude of the net electric field is

$$E = E_{1y} + E_{2y} = 13.8\,\text{NC}^{-1} + 13.8\,\text{NC}^{-1} = 28\,\text{NC}^{-1}$$

up to 2 significant digits. It is directed along the $y$-axis.