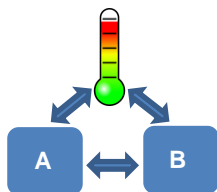


UNIT 6



The Zeroth law of thermodynamics introduces us to the concepts of thermal equilibrium and temperature about which you will learn in this unit.

THE ZEROTH LAW

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STUDY GUIDE

In the last unit, you have learnt the basic terminology used to describe a thermodynamic system. In this unit, you will learn about the Zeroth law of thermodynamics and how it helped to introduce the concept of temperature. You will also learn how to obtain parametric equation of state of a system. Parametric equation can be used to relate quantities such as the coefficient of volume expansion, compressibility and elasticity to thermodynamic variables. This would involve use of elementary calculus, including partial differentiation. You may have learnt these in your school mathematics classes. It will be useful if you refresh your knowledge about the differential equations before starting to study this unit.

We reiterate that you should answer SAQs and TQs yourself in order to check your understanding and enjoy the subject.

Heat energy of uniform temperature [is] the ultimate fate of all energy. The power of sunlight and coal, electric power, water power, winds and tides do the work of the world, and in the end, all unite to hasten the merry molecular dance.

***Frederick
Soddy***

6.1 INTRODUCTION

The study of thermodynamics is based on four empirical laws, which are derived from experience and need no proof. That is why thermodynamics is a phenomenological science. Its laws find wide applications in the design of combustion engines, refrigeration and air conditioning systems, power generation and the like. With the help of these laws, we can determine the efficiency of all types of thermo-mechanical devices such as steam engines, thermal power plants and automobiles. These laws can also be applied to analyse energy transformations in chemical and geological systems.

This nomenclature is due to RH Fowler.

In Block 1, we have used the term ‘temperature’ quite freely, since we are quite familiar with it. But have you ever thought as to how the concept of temperature was first introduced? Its basis lies in the *Zeroth* law of thermodynamics. We have discussed it in Sec. 6.2. The genesis of nomenclature of this law was in its formulation after the first and the second laws of thermodynamics had been established. And the concept of temperature was considered more basic than the internal energy and entropy. This law leads us to the equation of state. This equation can be represented in parametric form as discussed in Sec. 6.3. In Sec. 6.4, you will learn how to use the parametric equation of state to obtain mathematical relations between physical quantities such as coefficient of volume expansion, isothermal and adiabatic compressibilities/elasticities, etc.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ state the Zeroth law of thermodynamics;
- ❖ explain how the Zeroth law introduces the concept of temperature;
- ❖ apply the parametric equation to describe different thermodynamic processes; and
- ❖ use the equation of state to solve problems in thermodynamics.

6.2 THE ZEROth LAW OF THERMODYNAMICS

Refer to Fig. 6.1. Here, the adiabatic walls are denoted by solid lines and the diathermal walls are indicated by shaded lines.

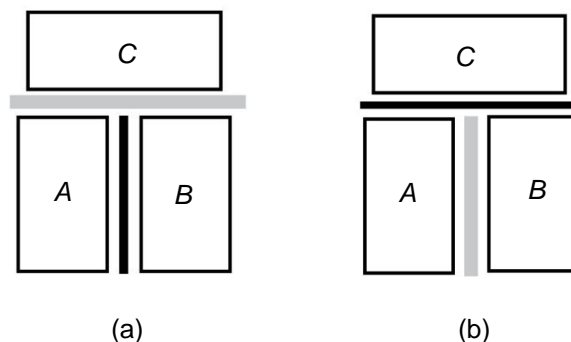


Fig. 6.1: The zeroth law of thermodynamics: a) A and B are in thermal contact with C *separately*; b) A and B are in thermal contact with one another.

In Fig. 6.1a, *A* and *B* are separated by an adiabatic wall but are individually in thermal equilibrium with *C*. When these systems are insulated from *C* but put in thermal contact with one another as shown in Fig. 6.1b, they are seen to remain in thermal equilibrium.

We can summarise this observation as follows:

If two systems A and B are in thermal equilibrium independently with a third system C, then A and B will also be in thermal equilibrium with one another.

This is known as the **Zeroth law of thermodynamics**.

You may now like to know: What determines whether a number of bodies will be in thermal equilibrium or not when they are put in thermal contact? They must have a common property which has the same value for all systems. This property is called '**temperature**'. Thus, the temperature of a body is that property which determines whether or not it will be in thermal equilibrium with other bodies.

The phenomenon that two bodies in thermal contact tend towards a common temperature is so common that its importance was initially overlooked. When physicists finally appreciated its significance and fundamental nature, it was elevated to the status of a 'Law of Thermodynamics.' By that time the first and second laws of thermodynamics had already been enunciated. So, in order to place it ahead of the first and second laws, it was named the 'zeroth law.' The significance of this law lies in the fact that **it introduces the concept of temperature**.

You are familiar with the equation of state. The relation $pV = nRT$ (Eq.(5.1)) is one such equation of state for n moles of an ideal gas. Can such relations exist for other thermodynamic systems as well? From the zeroth law it can be established mathematically that relations do exist between the temperature and the other thermodynamic variables associated with a system. All such relations are the **equations of state**. Let us now study these equations in detail.

6.3 THE EQUATION OF STATE

You know that temperature of a system can be expressed in terms of two other thermodynamic variables. If x and y (pressure p , volume V , say) are the two parameters, then mathematically we can write

$$T = f_1(x, y) \quad (6.1a)$$

Eq. (6.1a) can be solved for x in terms of T and y or for y in terms of T and x . Then we get the relations of the form

$$x = f_2(T, y) \quad (6.1b)$$

$$\text{and } y = f_3(T, x) \quad (6.1c)$$

Eqs. (6.1 a, b and c) can be expressed as a composite relation as

$$f(x, y, T) = 0 \quad (6.2)$$

Eq. (6.2) is parametric as the form of the function f is not given. But the equation of state for an ideal gas (Eq. 5.1) is exact. Likewise, we have

equations of state of other systems like a real gas, a stretched wire, etc. In Table 6.1, we quote equations of state for a few typical physical systems.

Table 6.1: Equations of state for various systems

System (variables)	Equation of State	
	Parametric	Exact
Ideal gas (p, V, T)	$f(p, V, T) = 0$	$pV = RT$, where R is the universal gas constant
Real gas (p, V, T)	$f(p, V, T) = 0$	$(p + \frac{a}{V^2})(V - b) = RT$, where a and b are constants
Paramagnetic solid (M, B, T)	$f(M, B, T) = 0$	$M = CB / T$, where M is the intensity of magnetisation and B is the flux density of the magnetic field in which the solid is placed. C is a constant.
Stretched wire (L, F, T)	$f(L, F, T) = 0$	$L = L_0 [1 + q_1 F + q_2 (T - T_0)]$ where L and L_0 are the lengths of the wire at temperatures T and T_0 , respectively. F is the tension in the wire, q_1 and q_2 are constants.

The parametric forms of the equations of state may be used to study some typical characteristics like coefficient of thermal expansion, elasticity, compressibility and so on of any substance. We shall do so now. This exercise will enable you to handle thermodynamic relations involving partial derivatives. You will encounter several such relations, particularly in Block 3 of this course.

6.4 DEDUCTIONS FROM THE EQUATION OF STATE

Before we proceed to study the characteristics using the equation of state, we need to learn about partial differentiation, which is an extension of the idea of ordinary differentiation. You have learnt about it in the second semester course entitled Electricity and Magnetism (BPHCT 133). We come across such differentiation when a quantity is 'a function of more than one variable and we need to know the change in the quantity when any one of the variables changes by a small amount'. For example, the temperature of a gaseous system is a function of pressure and volume. Now we may like to obtain the rate of change of temperature with respect to pressure for an isochoric (volume = constant) process. In this case, we are seeking the partial derivative of T with respect to p at constant V , denoted by $(\partial T / \partial p)_V$. On the other hand, if we seek the rate of variation of T with respect to V for an isobaric (pressure = constant) process, the appropriate partial derivative will be $(\partial T / \partial V)_p$.

Note that instead of using the symbol ' d ', as in case of ordinary differentials, we use ' ∂ ' (pronounced as 'del') in case of partial differentials.

We shall now work out the mathematical relationship between partial derivatives involving any three variables. These will be very useful to you in doing the relevant deductions involving any three variables.

Let us assume that a gas is defined by pressure p , volume V and temperature T . So, V may be taken as a function of p and T , i.e., we can write $V = V(p, T)$. The change in volume can be expressed as

$$dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT \quad (6.3a)$$

where the first term indicates the change of volume due to change of pressure alone and the second term indicates the change of volume due to change of temperature alone. Similarly, we can write $p = p(V, T)$ and the total differential dp for change of pressure due to changes in volume and temperature as

$$dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT \quad (6.3b)$$

On substituting the expression for dp from Eq. (6.3b) in Eq. (6.3a), we get

$$dV = \left(\frac{\partial V}{\partial p}\right)_T \left[\left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT \right] + \left(\frac{\partial V}{\partial T}\right)_p dT$$

On collecting the coefficients of dV and dT , we can write

$$\left[1 - \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T \right] dV - \left[\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_p \right] dT = 0 \quad (6.4)$$

Note that of the three variables p , V and T , only two are independent. Let us choose V and T as the independent variables. Then, Eq. (6.4) must be true for all sets of values of dV and dT . So, for any two states which are at the same temperature ($dT = 0$), but have different volumes ($dV \neq 0$), Eq. (6.4) implies that

$$1 - \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T = 0$$

$$\text{or} \quad \left(\frac{\partial V}{\partial p}\right)_T = \frac{1}{\left(\frac{\partial p}{\partial V}\right)_T} \quad (6.5)$$

This means that the partial derivative of any variable can be replaced by the reciprocal of the inverted partial derivative with the same variable (here T) held constant. There is another important relation which you will derive in the following SAQ.

SAQ 1 – Equation of state

Imposing the conditions $dV = 0$ and $dT \neq 0$ in Eq. (6.4), prove that thermodynamic variables are connected through the relation

$$\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = -1 \quad (6.6)$$

We shall now discuss how to use Eqs. (6.5) and (6.6) to obtain some results of physical interest. To do so, we first define thermal expansivity or the isobaric coefficient of volume expansion, α , the bulk modulus or isothermal elasticity, E_T and the isothermal compressibility ($\beta_T = 1/E_T$):

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p \quad (6.7)$$

and
$$E_T = \frac{1}{\beta_T} = -V \left(\frac{\partial p}{\partial V} \right)_T \quad (6.8)$$

Note the negative sign in Eq. (6.8). It has been put to give E_T [or (β_T)] a positive value since pressure decreases as volume increases, i.e., $(\partial p / \partial V)_T$ is negative for all systems.

You may like to attempt an SAQ to find out the values of β_T and α for an ideal gas.

SAQ 2 – Isothermal compressibility and volume expansion coefficient

Show that for an ideal gas

$$\beta_T = \frac{1}{p} \quad \text{and} \quad \alpha = \frac{1}{T}$$

On combining Eqs. (6.7) and (6.8), we can write

$$\alpha E_T = - \left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial p}{\partial V} \right)_T$$

or
$$\left(\frac{\partial T}{\partial V} \right)_p \left(\frac{\partial V}{\partial p} \right)_T = - \frac{1}{\alpha E_T}$$

Using this result in Eq. (6.6), we get

$$\left(\frac{\partial p}{\partial T} \right)_V = \alpha E_T \quad (6.9)$$

Similarly, an infinitesimal change in pressure may be expressed in terms of changes in temperature and volume as

$$dp = \left(\frac{\partial p}{\partial T} \right)_V dT + \left(\frac{\partial p}{\partial V} \right)_T dV$$

Using Eqs. (6.8) and (6.9), we can write

$$dp = \alpha E_T dT - E_T \frac{dV}{V}$$

At constant volume, if change in temperature is small but finite, β and E_T may be assumed to be constant. Hence, the change in pressure Δp , corresponding to a finite change in temperature, ΔT , at constant volume is given by

$$\Delta p = \alpha E_T \Delta T \quad (6.10)$$

We now take up a numerical example to illustrate Eq. (6.10).

EXAMPLE 6.1: ISOCHORIC PROCESS

The temperature of a block of copper is increased from 400K to 410K. What change in pressure is necessary to keep the volume constant? Given, for copper, $\alpha = 5 \times 10^{-5} \text{K}^{-1}$ and $E_T = 1.3 \times 10^{11} \text{Nm}^{-2}$.

SOLUTION ■ In Eq. (6.10), we have $\Delta T = 10\text{K}$.

$$\begin{aligned} \text{So } \Delta p &= (5 \times 10^{-5} \text{K}^{-1})(1.3 \times 10^{11} \text{Nm}^{-2}) \times 10\text{K} \\ &= 6.5 \times 10^7 \text{Nm}^{-2} \end{aligned}$$

Let us now sum up what you have learnt in this unit.

6.5 SUMMARY

Concept

Description

Zeroth law of thermodynamics and temperature

- The **zeroth law of thermodynamics** states that *if systems A and B are separately in thermal equilibrium with a system C, then A and B will be in thermal equilibrium with one another*. This nomenclature has genesis in the fact that it introduces the concept of **temperature**, which has fundamental significance.

Thermal equilibrium

- All systems in **thermal equilibrium** are characterized by the same temperature.

The general equation of a gaseous system

- The general equation of any gaseous system is given by

$$f(p, V, T) = 0,$$

where f is a single valued function of pressure p , volume V and absolute temperature T .

Coefficients of isobaric volume expansion, isothermal elasticity and isothermal compressibility

- In terms of partial derivatives, the coefficients of isobaric volume expansion (α), isothermal elasticity (E_T) and isothermal compressibility (β_T) are defined as

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p, \quad E_T = -V \left(\frac{\partial p}{\partial V} \right)_T, \quad \beta_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$$

These are related to pressure through the relations

$$\Delta p = \alpha E_T dT - E_T \frac{dV}{V}, \quad dp = \alpha E_T dT - \frac{1}{\beta_T} \frac{dV}{V}$$

6.6 TERMINAL QUESTIONS

- Write down the relation between thermodynamic variables expressed by Eq. (6.6) for (i) a paramagnetic solid and (ii) a stretched wire.
- A stretch of railway track is laid without expansion joints in Thar desert where day and night temperatures differ by 25K. The cross-sectional area of the rails is $3.6 \times 10^{-3} \text{m}^2$. The Young's Modulus, Y , of its material is $2 \times 10^{11} \text{Nm}^{-2}$ and the coefficient of linear expansion, α , is $8 \times 10^{-6} \text{K}^{-1}$.
 - If the length of the track is kept constant, what is the difference in the tension in the rails between day and night?
 - If the track is 15 km long and is free to expand, calculate the change in its length between day and night.

(Hint: Use $Y = \frac{L}{A} \left(\frac{\partial F}{\partial L} \right)_T$ and $\alpha = -\frac{1}{L} \left(\frac{\partial L}{\partial T} \right)_F$ where L , F and T denote length, tension and temperature, respectively.)

- The density of a substance is ρ . Show that

$$(i) \quad \beta_T = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_T \quad \text{and} \quad (ii) \quad \alpha = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

- Prove that for a pVT -system

$$\frac{dV}{V} = \alpha dT - \beta_T dp$$

- Calculate the coefficient of volume expansion and isothermal compressibility for a van der Waals' gas.
- The isothermal compressibility and expansivity of a substance are, respectively, given by

$$\beta_T = \frac{aT^3}{p^2} \quad \text{and} \quad \alpha = \frac{bT^2}{p}$$

Obtain the equation of state of the system under consideration.

6.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

- Imposing the condition $dV = 0$, $dT \neq 0$ in Eq. (6.4), we get

$$\left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial p}{\partial T} \right)_V + \left(\frac{\partial V}{\partial T} \right)_p = 0 \quad \text{or} \quad \left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial p}{\partial T} \right)_V = - \left(\frac{\partial V}{\partial T} \right)_p$$

Since $\left(\frac{\partial V}{\partial T} \right)_p = + \frac{1}{\left(\frac{\partial T}{\partial V} \right)_p}$, we can write

$$\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = -1$$

2. We know that isothermal compressibility is defined as

$$\beta_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T \quad (i)$$

For a perfect gas, $V = \frac{RT}{p}$

so that $\left(\frac{\partial V}{\partial p}\right)_T = -\frac{RT}{p^2}$

On using this result in Eq. (i), we get

$$\beta_T = \frac{RT}{p^2 V} = \frac{1}{p} \cdot \frac{RT}{pV} = \frac{1}{p}$$

Similarly, volume expansion coefficient is defined as

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p \quad (ii)$$

For one mole of a perfect gas, $pV = RT$

so that

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p}$$

$$\therefore \alpha = \frac{R}{pV} = \frac{1}{T}$$

Terminal Questions

1. (i) Paramagnetic solid: $\left(\frac{\partial M}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_M \left(\frac{\partial T}{\partial M}\right)_B = -1$

(ii) Stretched wire: $\left(\frac{\partial L}{\partial F}\right)_T \left(\frac{\partial F}{\partial T}\right)_L \left(\frac{\partial T}{\partial L}\right)_F = -1$

2. a) Let L = length, F = tension and T = temperature of the track.
Since $f(L, F, T) = 0$, following the relation for stretched wire obtained in TQ-1 (ii), we can write

$$\left(\frac{\partial F}{\partial L}\right)_T \left(\frac{\partial T}{\partial F}\right)_L \left(\frac{\partial L}{\partial T}\right)_F = -1$$

Using the relations for Y and α given in the question, we can write,

$$\left(\frac{YA}{L}\right) \left(\frac{\partial T}{\partial F}\right)_L (\alpha L) = -1$$

$$\therefore \left(\frac{\partial F}{\partial T}\right)_L = -YA\alpha$$

So, if the length of the track is kept constant, difference in tension corresponding to a difference of temperature ΔT is given by

$$\Delta F = -YA\alpha\Delta T$$

When we consider change from day to night, $\Delta T = -25\text{K}$.

$$\begin{aligned}\therefore \Delta F &= -YA\alpha\Delta T \\ &= (2 \times 10^{11} \text{ Nm}^{-2}) \times (3.6 \times 10^{-3} \text{ m}^2) \times (8 \times 10^{-6} \text{ K}) \times 25\text{K} \\ &= 1.44 \times 10^5 \text{ N}\end{aligned}$$

As the temperature falls during night, the track tries to contract. However, since it is not permitted to contract, the tension increases to keep it stretched to its original length. Therefore, ΔF is positive.

$$\text{b) Since } L = L(F, T), \quad dL = \left(\frac{\partial L}{\partial F}\right)_T dF + \left(\frac{\partial L}{\partial T}\right)_F dT$$

If the track is free to expand, $dF = 0$. $\Delta T = -25\text{K}$.

The change in length in terms of the coefficient of linear expansion is defined as

$$\Delta L = \alpha L \Delta T = (8 \times 10^{-6} \text{ K}^{-1}) \times (15 \times 10^3 \text{ m}) \times (-25\text{K}) = -3 \text{ m}$$

Thus the track will contract by 3m.

3. (i) We know that isothermal compressibility is defined as

$$\beta_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial p}\right)_T \quad (\text{i})$$

Since $V = \frac{m}{\rho}$, we can write

$$\left(\frac{\partial V}{\partial p}\right)_T = -\frac{m}{\rho^2} \quad (\text{ii})$$

Using this result in (i), we get

$$\beta_T = -\frac{1}{V} \left(-\frac{m}{\rho^2}\right) \left(\frac{\partial p}{\partial p}\right)_T = \frac{1}{\rho} \left(\frac{\partial p}{\partial p}\right)_T \quad (\text{iii})$$

(ii) We know that volume expansivity at constant pressure is given by

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_p \left(\frac{\partial p}{\partial T}\right)_p \quad (\text{iv})$$

On substituting for $(\partial V/\partial p)_p$ from (ii), we get

$$\alpha = -\frac{m}{V\rho^2} \left(\frac{\partial p}{\partial T}\right)_p = -\frac{1}{\rho} \left(\frac{\partial p}{\partial T}\right)_p \quad (\text{v})$$

4. For a pVT -system, we choose volume as independent variable and write

$$V = V(T, p)$$

$$\begin{aligned}\therefore dV &= \left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_T dp \\ &= V\alpha dT - V\beta_T dp\end{aligned}$$

$$\text{or } \frac{dV}{V} = \alpha dT - \beta_T dp$$

5. We know that for one mole of a gas, van der Waals equation of state is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT \quad (\text{i})$$

Further the coefficient of volume expansion and isothermal compressibility are defined as

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p \quad (\text{ii})$$

$$\text{and } \beta = \frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T \quad (\text{iii})$$

Hence, from Eq. (i) we can write

$$\begin{aligned}\left[\left(p + \frac{a}{V^2}\right) - \frac{2a}{V^3}(V - b)\right] \left(\frac{\partial V}{\partial T}\right)_p &= R \\ \text{or } \left(\frac{\partial V}{\partial T}\right)_p &= \frac{R}{\left(p + \frac{a}{V^2}\right) - \frac{2a}{V^3}(V - b)}\end{aligned}$$

To simplify, we multiply the numerator as well as the denominator on the right-hand side by $(V - b)$ and use Eq. (i), this gives

$$\begin{aligned}\left(\frac{\partial V}{\partial T}\right)_p &= \frac{R(V - b)}{RT - \frac{2a}{V^3}(V - b)^2} = \frac{V - b}{T \left(1 - \frac{2a(V - b)^2}{RTV^3}\right)} \\ &= \frac{RV^3(V - b)}{RTV^3 - 2a(V - b)^2} \\ \therefore \alpha &= \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = \frac{RV^2(V - b)}{RTV^3 - 2a(V - b)^2} \quad (\text{iv})\end{aligned}$$

To obtain the expression for β_T , we rewrite Eq. (i) as

$$\begin{aligned}p &= \frac{RT}{V - b} - \frac{a}{V^2} \\ \therefore \left(\frac{\partial p}{\partial V}\right)_T &= -\frac{RT}{(V - b)^2} + \frac{2a}{V^3} \\ \text{and } -V \left(\frac{\partial p}{\partial V}\right)_T &= E_T = +\frac{RTV}{(V - b)^2} - \frac{2a}{V^2} = \frac{RTV^3 - 2a(V - b)^2}{(V - b)^2 V^2}\end{aligned}$$

$$\text{Hence } \beta_T = \frac{1}{E_T} = \frac{V^2 (V - b)^2}{RTV^3 - 2a(V - b)^2} \quad (\text{v})$$

6. The equation of state is a relation between any two variables out of p , V and T . Therefore, we first establish the relation between p and V . To do so, we write

$$V = V(p, T) \quad (\text{i})$$

An infinitesimal change in V corresponding to infinitesimal change in p and T can be expressed as

$$dV = \left(\frac{\partial V}{\partial p} \right)_T dp + \left(\frac{\partial V}{\partial T} \right)_p dT \quad (\text{ii})$$

In terms of α and β_T , we can rewrite this equation as

$$dV = -V\beta_T dp + V\alpha dT$$

$$\text{or } \frac{dV}{V} = -\beta_T dp + \alpha dT \quad (\text{iii})$$

For an isothermal process, $dT = 0$ and Eq. (iii) reduces to

$$\frac{dV}{V} = -\beta_T dp$$

On substituting the given value of β_T , we get

$$\frac{dV}{V} = -\frac{aT^3}{p^2} dp$$

On integration, we get the following expression:

$$\ln V = \frac{aT^3}{p} + \ln V_0$$

On taking antilogarithm, we get the equation of state for an isothermal process as

$$V = V_0 \exp\left(\frac{aT^3}{p}\right) \quad (\text{iv})$$

For an isobaric process, $dp = 0$ and Eq. (iii) reduces to

$$\frac{dV}{V} = \alpha dT = \frac{b}{p} T^2 dT$$

On integration we get

$$\ln V = \frac{b}{3p} T^3 + \ln V_0$$

On taking antilogarithm, we get the equation of state for an isobaric process as

$$\therefore V = V_0 \exp\left(\frac{b}{3p} T^3\right). \quad (\text{v})$$