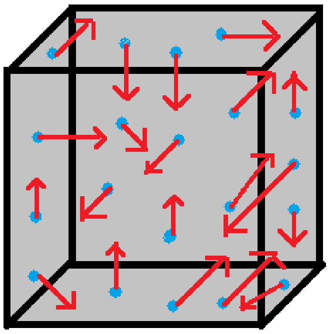


Brownian motion is continuous and random motion of molecules. You will learn about its importance in kinetic theory in this unit.

UNIT 4

BROWNIAN MOTION

Structure

STUDY GUIDE

In the previous unit, you have learnt about mean free path, coefficient of viscosity, thermal conductivity and diffusion coefficient. These physical properties helped us to estimate the size of gas molecules in terms of directly measurable quantities. The conformity of theoretical and experimental results provided indirect evidence in favour of kinetic theory. In this unit, you will learn about Brownian motion, which provided the first direct evidence in favour of kinetic theory. For this reason, this concept is extremely important and we have discussed the physics of this phenomenon. However, you should answer the SAQs and TQs, which will help you develop conceptual clarity.

"We cannot solve our problems with the same thinking we used when we created them."

Albert Einstein

4.1 INTRODUCTION

Robert Brown (1773 – 1858) was a Scottish botanist. He reported the initial observation of continuous random motion of pollen particles in a viscous liquid observed under a microscope. He is also credited with coining the word 'nucleus' in reference to living cells.

You now know that elementary kinetic theory successfully explains many observed properties of gases and has great aesthetic appeal; well defined laws are used to describe chaotic motion. Moreover, it helps us to estimate the size of gas molecules in terms of directly measurable quantities fairly well. The agreement between theoretically predicted behaviour with observed results constituted indirect evidence in favour of kinetic theory. However, direct evidence for the *existence of molecules and their motion was lacking* and prominent scientists were reluctant to accept the realities of atoms and molecules.

The first experimental evidence for the existence of molecules and their continuous chaotic motion was provided by Robert Brown while he was observing the motion of particles suspended in a fluid. These suspended particles were seen to move completely haphazardly. This irregular motion was termed **Brownian motion**. The nature of Brownian movements (motion) of suspended particles was seen to depend on the properties of the fluid in which the particles were suspended. In view of its importance, in Sec. 4.2, we review the developments that led to the discovery of Brownian motion.

In the study of Brownian motion, scientists from three diverse disciplines were involved: Brown, a botanist, observed the phenomena; Einstein, a physicist, provided theoretical explanation and Perrin, a chemist, provided experimental evidence through determination of Avogadro's number. This is an excellent example of unified nature of knowledge.

Albert Einstein explained the phenomenon of Brownian motion theoretically in 1905 in terms of the effects of collisions between fluid molecules and the suspended particles. He argued that although each impact is very small, the net result of a large number of random collisions gives rise to haphazard motion. Einstein quantified this problem by relating the diffusion of particles to the properties of the molecules responsible for collisions. In this way, he related the molecular theory of gases to the observed motion of particles. His predictions were verified by Perrin in 1908. This work convinced everyone about the reality of molecular/atomic nature of matter. Several efforts have been made to generalise Einstein's theory of Brownian motion. In 1908, Langevin re-derived Einstein's formula for mean square displacement by considering the equation of motion of suspended particles. In this unit, we shall not go into mathematical aspects of these effects. In Sec. 4.4, we have discussed examples of Brownian motion and Sec. 4.5 is devoted to the discussion of Perrin's experiments, which provided a convenient method to determine Avogadro number.

Expected Learning Outcomes

After studying this unit, you should be able to:

- explain the significance of Brownian motion;
- write expression for the mean square displacement;
- discuss sedimentation as an example of Brownian motion; and
- explain how Perrin determined Avogadro's number and discuss its significance for kinetic theory of gases.

4.2 BROWNIAN MOTION

In 1827, Scottish botanist Robert Brown observed the motion of pollen grains suspended in an aqueous solution through a high-power microscope. These pollen grains were seen to exhibit a continuous and completely erratic movement. This irregular motion is termed as **Brownian motion** (Fig. 4.1).

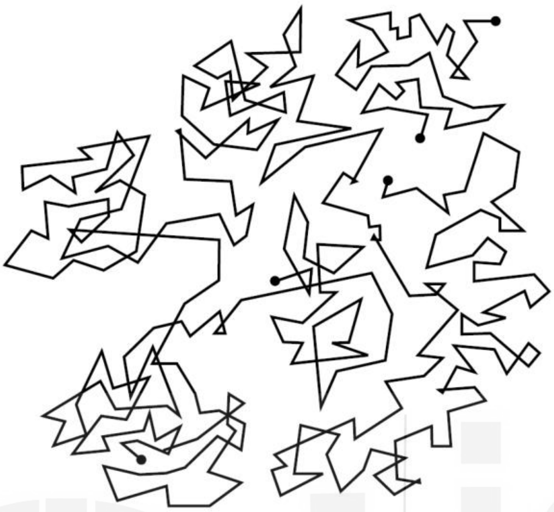


Fig. 4.1: **Brownian motion of suspended particles in an aqueous solution.**

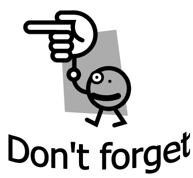
The observed characteristics of Brownian motion are:

1. The motion is continuous, completely random and irregular.
2. No two particles execute the same motion.
3. Smaller particles execute faster and hence more noticeable motion.
4. The movement is about the same in all directions.
5. The motion is independent of external influences.
6. Smaller the viscosity, faster is the motion.
7. Motion is more rigorous at higher temperatures.

A proper explanation of this phenomenon eluded scientists for a long time. Studies of Guy Williams and other physicists led to the view that Brownian motion arises due to collisions of suspended particles with the molecules of the surrounding fluid. This phenomenon provided a very elegant picture of the gaseous state wherein gas molecules were in random motion and frequently collided against each other. In a way, Brownian motion provided us a mechanism for visualising the behaviour of matter at microscopic scale. Only for this reason, it has been of such great interest to physicists. But it required the genius of Einstein to work out a detailed theory of Brownian motion. Einstein's predictions were found to be precisely correct by the beautiful experiments of Perrin. This also paved the way for accurate determination of molecular masses and convinced everyone of the reality of the molecular nature of matter.

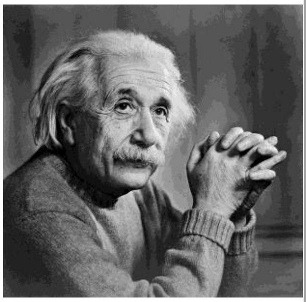
BROWNIAN MOTION

A perpetual, irregular and completely random motion of particles suspended in a viscous solution is known as **Brownian motion**. This phenomenon provides an elegant picture of gaseous state.



SAQ 1 – Brownian motion

State the importance of Brownian motion.



Albert Einstein (1879 – 1955) is regarded as the greatest physicist of 20^{th} century. 1905 was a wonder year for Albert Einstein. He published four very important papers in this year. Each of these papers significantly contributed to enhance our knowledge at that point of time. The diverse topics covered in these papers were photoelectric effect, Brownian motion, special theory of relativity and matter energy equivalence ($E = mc^2$) relation. In 1921, Nobel prize was conferred on him for his work on photoelectric effect. In the paper on Brownian motion, Einstein gave an exact description of the effects of random collisions with the molecules of the liquid on the motion of the suspended particles.

4.3 THEORETICAL ANALYSIS

Einstein's Theory

Einstein gave an exact description of Brownian motion by relating Brownian motion to physical processes. In particular, he considered the effects of random collisions between molecules of the liquid and suspended particles. To quantify this problem, he obtained expressions for diffusion coefficient, D, from the random motion of suspended particles as well as from the osmotic pressure difference in different parts of the solution caused by difference in concentrations of suspended particles (see box below). He then equated these expressions to calculate mean squared displacement of a Brownian particle. We just quote the result without going into mathematical details.

Osmotic Pressure

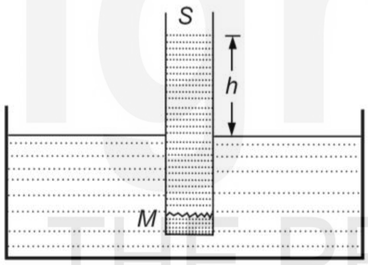


Fig. 4.2: **Glass tube with semi-permeable membrane *M* at its end filled with sugar solution dipped in water.**

Osmotic pressure of a solution is the pressure required to prevent osmosis when the solution is separated from pure solvent by a semi-permeable membrane. van't Hoff's law of osmotic pressure states that the osmotic pressure of a dissolved substance in a solution is numerically equal to the pressure which it would exert if it were assumed to behave like a gas having same volume as occupied by the given solution at the same temperature. Suppose we take an open glass tube and cover its one end with a 'semi-permeable membrane' that is permeable to water but not to sugar in solution. Let us fill this tube with a dilute sugar solution and dip it into a beaker of water, as shown in Fig. 4.2 above. We observe that the solution rises to a height above the level of the water.

This means that the solution has a pressure $\rho\, g\, h$ higher than that of pure water at the same temperature. This pressure, exerted by the sugar dissolved in solution, is called **osmotic pressure**. For dilute solutions, i.e., in which the number of solute molecules is very small compared to the solvent molecules, van't Hoff established that

$$p_{osmotic} = nk_B T$$

where n is concentration of the solution. That is, for dilute solutions, the osmotic pressure is equal to the pressure which the solute would exert if it were assumed to behave like an ideal gas having the volume and temperature as that of the solution.

$$\ell^2 = \frac{k_B T}{3\pi\eta r_0}\tau = \frac{RT}{N_A}\frac{1}{3\pi\eta r_0}\tau \quad (4.1)$$

This is the famous **Einstein's formula** for mean square displacement of a Brownian particle.

It is instructive to note that Brownian mean square displacement is independent of the mass of particles. Through a brilliant series of experiments, Perrin confirmed this prediction by varying mass through a factor of 15,000. We further note that diffusion of particles is related to molecular motion. Moreover, since ℓ^2, τ, η and r_0 are measureable quantities, Eq. (4.1) proved a ready tool for determination of Avogadro number. In fact, this equation was verified by Perrin, and was found to be precisely correct. His experiments, therefore, established the existence of molecules beyond any doubt and provided general acceptability to kinetic theory of gases.

You must have realised that Einstein laid greater emphasis on relating Brownian motion to physical processes.

Langevin Theory

A somewhat more elegant explanation was given by Langevin. He assumed that the average force acting on a suspended particle due to molecular bombardment is made up of a frictional and a fluctuating component.

Langevin argued that a suspended particle undergoes, on an average, one collision in about 10^{-21}s with the molecules of the liquid. So the mean free path of the molecules is small compared with the size of the suspended particles. It means that the surrounding medium can be considered continuous. Langevin also assumed that all suspended particles are spherical in shape. Then using Stoke's law and the fact that the direction of motion of each suspended particle changes after each collision, he arrived at Einstein's expression for mean square displacement of Brownian particles.

$$\Delta(\overline{x^2}) = \frac{k_B T}{3\pi\eta r_0}\tau = \frac{RT}{N_A}\frac{1}{3\pi\eta r_0}\tau \quad (4.2)$$

Note that $\Delta(\overline{x^2})$ is not the actual displacement of Brownian particles. We have to take a snapshot of the suspension at time $t = 0$ and again at time $t = \tau$. Then we measure the component of displacement along any arbitrarily chosen direction, say *x*-axis, and determine $\Delta(x^2)$ for each particle. A sum over all the $\Delta(x^2)$ and division by the number of particles gives $\Delta(\overline{x^2})$. In his experiments, Perrin worked with 100 different particles of known size. If we closely re-examine Fig. 4.1 we find that the motion is so complex that an experimentalist may not find it convenient to work with such a large number. However, we can make use of the fact that if one particle is followed for *N* succesive intervals of time (when *N* is a large number), the motion is almost equivalent to the motion of *N* particles during a single time interval.

SAQ 2 – **Size of a particle**

In an experiment with colloidal particles suspended in water at temperature 32°C, the mean square displacement in unit time was found to be $1.8 \times 10^{-6}\,\text{cm}^2$. Calculate the value of the radius of a suspended particle. Take $R = 8.31 \times 10^7$ erg mol $^{-1}$ K^{-1} $N_A = 6.023 \times 10^{23}$ mol $^{-1}$ and $\eta = 0.01$ poise.

4.4 EXAMPLES OF BROWNIAN MOTION

We have just now seen that colloidal suspensions in a fluid exhibit Brownian motion. We come across many other interesting examples of Brownian motion. These include sedimentation, diffusion of pollutants or smoke particles in air and Johnson noise in electrical devices. We will discuss sedimentation now.

4.4.1 Sedimentation

From common experience we know that if we take sandy water in a beaker, the sand settles down at the bottom. This natural process is known as sedimentation. It is responsible for automatic cleaning of rainwater stored in ponds and lakes. In sedimentation, the distribution of particles is determined by the influence of gravity and diffusion. Whereas gravity tends to settle them, diffusion brings about homogenisation. (The same is true of pollutants in our atmosphere, which give rise to acid rain, greenhouse effct and climate change.)

To calculate the number of particles at a given height, we consider a shallow box of depth Δz enclosing layers of particles bound at heights z and $z + \Delta z$. Let the pressure on the lower and upper faces be p and $p + \Delta p$ respectively as shown in Fig. 4.3.

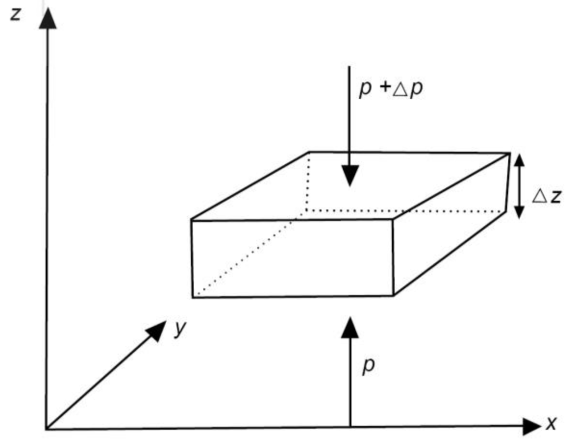


Fig. 4.3: **A shallow box of depth Δz and cross-sectional area A. The pressures on the lower and upper faces are assumed to be p and $p + \Delta p$, respectively.**

If ρ and g, respectively, denote the density of particles and acceleration due to gravity, the pressure difference between the two surfaces in equilibrium is given by

$$\Delta p = -g\,\rho\,\Delta z \tag{4.3}$$

The negative sign signifies that pressure decreases as height increases. If the mass of a single particle is m and number density is n, we can write $\rho = mn$. Then we can rewrite the expression for pressure difference as

$$\Delta p = -\,g\,m\,n\,\Delta z \;= -\frac{mgN}{V}\Delta z$$

If we assume that Brownian particles obey gas laws, we can replace V by $\mu RT/p$. This gives

$$\frac{\Delta p}{p} = -\frac{mgN}{\mu RT}\Delta z$$

where μ denotes the number of moles.

This expression can be readily integrated to obtain

$$\ln p = -\frac{mgN}{\mu RT}z + \ln p_0$$

where $\ln p_0$ is constant of integration.

On taking antilog, we get

$$p = p_0 \exp\left(-\frac{mgN}{\mu RT}z\right) \tag{4.4}$$

where $p = p_0$ at $z = 0$. From Unit 1, you may recall that the pressure exerted by the molecules of a gas on the walls of the container is proportional to number density ($p = \frac{1}{3}mn\overline{v^2}$). Using this result, we can rewrite Eq. (4.4) as

$$n = n_0 \exp\left(-\frac{mgN}{\mu RT}z\right) = n_0\,\exp\left(-\frac{mgN_A}{RT}\,z\right) \tag{4.5}$$

where $N_A = N/\mu$ is Avogadro number.

This result shows that during sedimentation, particle concentration decreases exponentially as height increases. In practice, the suspended particles experience upward bouyant force due to difference in the densities of the solute ρ and the solvent ρ'. As a result, the effective mass of the suspended particles is reduced to

$$m_{eff} = \frac{4\pi}{3}(\rho - \rho')r_0^3 \tag{4.6}$$

where r_0 is the radius of a particle. If $\rho \approx \rho'$, m_{eff} would be substantially small. We now take natural log of both sides of Eq. (4.5) and use the value of m_{eff} given in Eq. (4.6). Then we can write the expression for Avogadro's number in terms of the microscopic properties of the suspended paricles:

$$N_A = \frac{3RT}{4\pi r_0^3(\rho - \rho')gz}\ln\left(\frac{n_0}{n}\right) \tag{4.7}$$

Eq. (4.7) suggests that if we study the variation of n with height for fine suspensions, we can conveniently determine Avogadro number. Perrin

A. Avogadro, an Italian scientist, had proposed in 1811 that equal volume of all gases at a given pressure and temperature contains the same number of molecules. The number of molecules in one mole of a gas is known as Avogadro number. We denote it by N_A. Jean Perrin proposed this nomenclature in 1905.

worked with emulsions of gamboge and mastic (resin pigments obtained from trees) and obtained a value very close to the presently accepted value. His results lent support to molecular theory of gases.

In the next section, we will discuss Perrin's work. However, for now, go through the following example.

EXAMPLE 4.1: AVOGADRO'S NUMBER

In an experiment, motion of 49 particles per cm^2 is observed in a layer of gamboge suspended in water at one level and 14 particles per cm^2 in a layer 60 microns higher $(1\,\text{micron} = 10^{-6}\,\text{m})$. If the density of gamboge is 1.194 g cm^{-3} and radius of each particle is 0.212 micron, calculate Avogadro number. Take the temperature of the solution as 20°C, $R = 8.31\,\text{J mol}^{-1}\text{K}^{-1}$, and $g = 9.8\,\text{ms}^{-2}$.

SOLUTION ■ From Eq. (4.7), we have

$$N_A = \frac{3RT}{4\pi r_0^3(\rho - \rho')gz}\ln\left(\frac{n_0}{n}\right)$$

On substituting the values of various quantities, we get

$$N_A = \frac{3 \times (8.31\,\text{J mol}^{-1}\text{K}^{-1})}{4 \times 3.14 \times (0.212 \times 10^{-6}\,\text{m})^3 \times (1.194 - 1.0) \times 10^3\,\text{kgm}^{-3}} \times \frac{(293\,\text{K})}{(9.8\,\text{ms}^{-2}) \times (60 \times 10^{-6}\,\text{m})}\ln\left(\frac{49}{14}\right)$$

$$= \frac{3 \times 8.31 \times 293 \times 1.25}{4 \times 3.14 \times (0.212)^3 \times 0.194 \times 9.8 \times 60} \times 10^{21}\,\text{mol}^{-1}$$

$$= 6.7 \times 10^{23}\,\text{mol}^{-1}.$$

Let us now recapitulate what we have discussed in this sub-section.

Recap

EXAMPLES OF BROWNIAN MOTION

- Sedimentation and Johnson Noise are two very familiar examples of Brownian motion.
- During sedimentation, particle density decreases exponentially as height increases:

$$n = n_0 \exp\left(-\frac{mgN_A}{RT}z\right)$$

Perrin carried out a series of beautiful experiments on colloidal suspensions to obtain the value of Avogadro number. This work signified a great triumph of molecular theory. For these investigations Perrin was awarded Nobel Prize for Physics in 1926. We now discuss Perrin's experiments.

4.5 DETERMINATION OF AVOGADRO NUMBER

To determine Avogadro number, we have to measure $\overline{x^2}$, the mean square displacement of a Brownian particle. Perrin observed the motion of a single gamboge grain suspended in water at intervals of thirty seconds with the help of a microscope and a camera. To locate the particle, the microscope had in its field of view a series of mutually perpendicular lines, as shown on a graph paper in Fig. 4.4. The projections of successive displacements along the x-axis give a set of values of x from which $\overline{x^2}$ can be calculated.

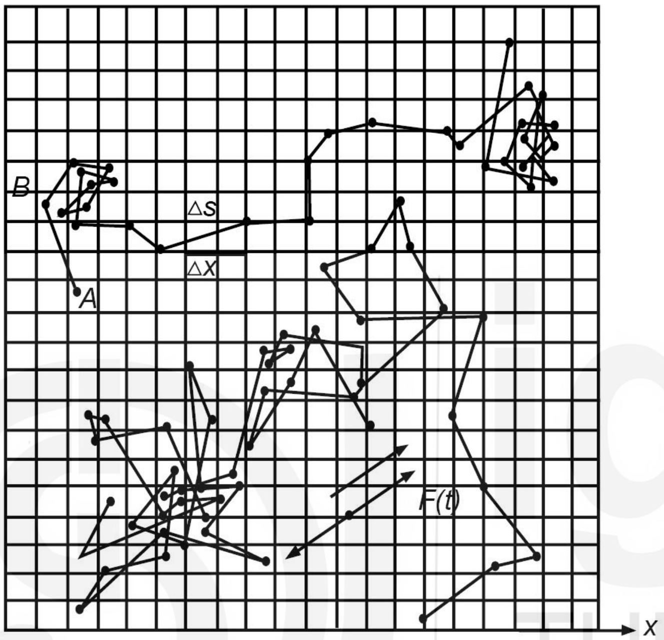


Fig. 4.4: **Calculation of $\overline{x^2}$ for a Brownian particle.**

It should be realised that the straight-line segments in Fig. 4.4 are in no way a representation of the actual path of the particle. The particle is hit millions of times in a second, and hence, its trajectory has a jagged and irregular structure. For example, if we magnify the part *AB* of the trajectory, it will appear as shown in Fig. 4.5.

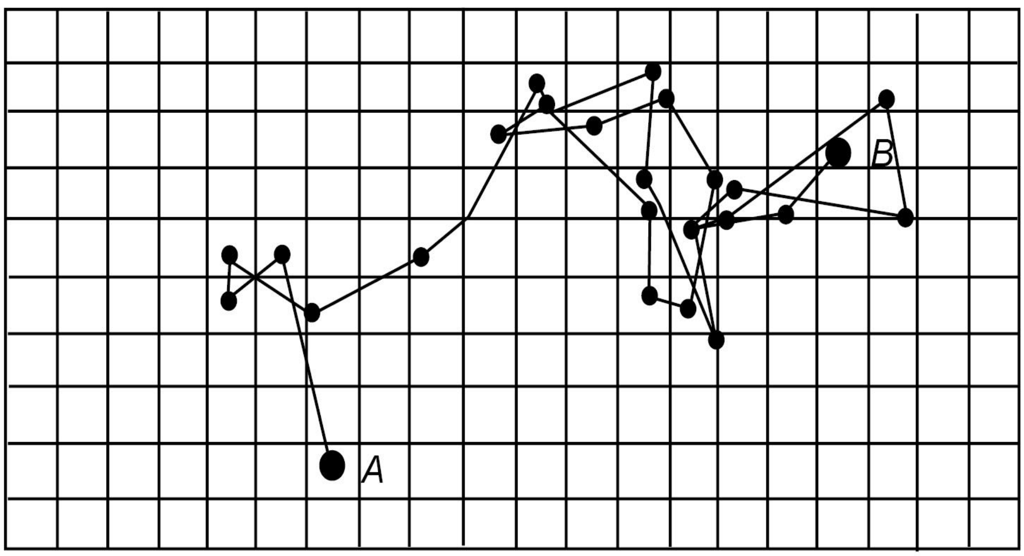


Fig. 4.5: **The path *AB* after magnification.**

You may now ask: How could Perrin make such wonderful observations with a simple arrangement? The physical basis of his work was very sound. Perrin derived his argument from the fact that at 300 K, $v_{rms} = 2 \times 10^{-2}\,\text{ms}^{-1}$ for a 2×10^{-7} m radius grain of gamboge having a mass of about 3×10^{-17} kg (which is 10^9 times the mass of H_2O molecule). This combination of slow speeds and large size was used by Perrin to observe the motion of suspended particles. It justifies the popular belief that Nature likes simplicity: most natural laws have been unfolded using very simple arguments. (Sir C. V. Raman explained the blue colour of Deep Ocean in terms of scattering of light by H_2O molecules and used a very modest apparatus, to vividly demonstrate it.)

From his measurements, Perrin obtained a value of 6.85×10^{26} molecules kmol^{-1} for Avogadro number, which is fairly close to the currently accepted value of $6.022 \times 10^{26}\,\text{kmol}^{-1}$.

From this value of Avogadro number, we can estimate the mass of a molecule. For example, one kilo-mole of nitrogen gas has a mass of 14 kg. Hence, mass of a nitrogen molecule

$$m_{N_2} = \frac{14\,\text{kg}}{6.02 \times 10^{26}} = 2.32 \times 10^{-26}\,\text{kg}$$

Perrin is, therefore, said to be the first person to have weighed the atom with kinetic theory as the tool!

4.6 SUMMARY

Concept	Description
Brownian motion	■ **Brownian motion** is perpetual and irregular motion of the particles immersed in fluid. It is caused by their continuous bombardment by the surrounding molecules of much smaller size. Sedimentation and Johnson noise are familiar examples of Brownian motion. ■ Brownian motion provided direct experimental evidence in support of kinetic theory of gases.
Einstein's relation	■ **Einstein's relation** for mean square displacement of a Brownian particle is $$\ell^2 = \frac{RT}{N_A}\frac{1}{3\pi\eta r_0}\tau$$
Sedimentation	■ The variation of particle concentration with height, z, during sedimentation is given by $$n = n_0 \exp\left(-\frac{mgN_A}{RT}z\right)$$

Avogadro number

- Perrin determined the value of Avogadro number and used it to determine the mass of nitrogen atom. That is, Perrin weighed the atom with kinetic theory as tool.

4.7 TERMINAL QUESTIONS

1. W. Pospisil observed the motion of soot particles of radius 0.4×10^{-4} cm in a water-glycerine solution with $\eta = 0.0278$ poise at $T = 292$ K. The observed value of $\overline{\Delta x^2}$ was $3.3 \times 10^{-8}\,\text{cm}^2$ for $t = 10$ s. Use this information to calculate Boltzmann constant and hence N_A.
2. The mean kinetic energy of molecules of hydrogen at 0°C is 5.60×10^{-21} J and molar gas constant is $8.31\,\text{J mol}^{-1}\text{K}^{-1}$. Calculate Avogadro's number.
3. A Brownian particle of radius 2.10×10^{-7} m moves in a liquid at 20°C. If the value of RMS displacement in 32s is 6.5×10^{-6} m, calculate the value of Boltzmann constant. The coefficient of viscosity of the liquid is $1.2 \times 10^{-3}\,\text{N s m}^{-2}$.

4.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. The Brownian motion is the experimental evidence in support of the basic assumption of kinetic theory of a gas that it consists of molecules moving at random.
2. Given $N_A = 6.023 \times 10^{23}\,\text{mol}^{-1}$, $\eta = 10^{-2}$ poise,

$T = 32°\text{C} = 32 + 273 = 305\,\text{K}$, $\tau = 1\text{s}$

$\Delta(\overline{x^2}) = 1.8 \times 10^{-6}\,\text{cm}^2$, $R = 8.31 \times 10^7\,\text{erg mol}^{-1}\text{K}^{-1}$.

From Eq. (4.2), we recall that

$$\Delta(\overline{x^2}) = \frac{RT}{N_A}\frac{1}{3\pi\eta r_0}\tau$$

On rearranging the terms, we can express r_0 in terms of the given physical parameters:

$$r_0 = \frac{RT}{\Delta(\overline{x^2})N_A \times 3\pi\eta}\tau$$

On substituting the given values, we get

$$r_0 = \frac{(8.31 \times 10^7\,\text{erg mol}^{-1}\text{K}^{-1}) \times (305\,\text{K}) \times 1\text{s}}{(6.023 \times 10^{23}\,\text{mol}^{-1}) \times (1.8 \times 10^{-6}\,\text{cm}^2) \times (3 \times 3.14 \times 0.01\text{dyne cm}^{-2}\text{s})}$$

$$= 2479.53 \times 10^{-10}\,\text{cm} = 2.48 \times 10^{-7}\,\text{cm}$$

Terminal Questions

1. We have $\Delta\langle x^2\rangle = \dfrac{k_B T}{3\pi\eta r_0}\tau$

   On rearrangement, we can write

$$k_B = \frac{3\pi\eta r_0}{T\tau}\Delta\langle x^2\rangle = \frac{3\pi \times (0.0278\,\text{poise}) \times (0.4\times 10^{-4}\,\text{cm}) \times 3.3 \times 10^{-8}}{(292\,\text{K}) \times (10\,\text{s})}$$

$$= 1.18 \times 10^{-16}\ \text{erg K}^{-1}$$

   Using this result, we can write

$$N_A = \frac{R}{k_B} = \frac{8.31 \times 10^7\,\text{erg mol}^{-1}\text{K}^{-1}}{1.18 \times 10^{-16}\,\text{erg K}^{-1}}$$

$$= 7.04 \times 10^{23}\,\text{mol}^{-1}$$

2. Given: $E = 5.60 \times 10^{-21}\,\text{J}$, $R = 8.31\,\text{J mol}^{-1}\text{K}^{-1}$ and $T = 273\,\text{K}$.

   We have

$$E = \frac{3}{2}k_B T \qquad \text{(i)}$$

   But $k_B = \dfrac{R}{N_A}$ (ii)

   On combining Eqs. (i) and (ii), we get the following expression for energy:

$$E = \frac{3}{2}\left(\frac{R}{N_A}\right).T$$

$$\Rightarrow \quad N_A = \frac{3RT}{2E} \qquad \text{(iii)}$$

   On substituting the values in Eq. (iii), we get

$$N_A = \frac{3 \times 8.31\,\text{J mol}^{-1}\text{K}^{-1} \times 273\,\text{K}}{2 \times 5.60 \times 10^{-21}\,\text{J}} = 6.08 \times 10^{23}\,\text{mol}^{-1}$$

3. From Eq. (4.2), we can write

$$\Delta(\overline{x^2}) = \frac{k_B T}{3\pi\eta r_0}\tau$$

   We are given that RMS displacement is $6.5 \times 10^{-6}\,\text{m}$, $\eta = 1.2 \times 10^{-3}\,\text{Ns m}^{-2}$, and $r_0 = 2.10 \times 10^{-7}\,\text{m}$. To use the value of RMS displacement in the above formula for $\Delta(\overline{x^2})$, we have to calculate its square. Hence, we can write

$$(6.5 \times 10^{-6}\,\text{m})^2 = \frac{k_B \times (293\,\text{K}) \times 32\text{s} \times 7}{3 \times 22 \times (1.2 \times 10^{-3}\,\text{Ns m}^{-2}) \times (2.10 \times 10^{-7}\,\text{m})}$$

   or $$k_B = \frac{42.25 \times 10^{-12} \times 166.32 \times 10^{-10}\,\text{Nm}}{65632\,\text{K}}$$

$$= \frac{7027.02 \times 10^{-22}}{65632}\frac{\text{J}}{\text{K}} = 1.07 \times 10^{-23}\,\text{JK}^{-1}$$