UNIT 12 PLANNING AND CONTROL OF PROJECTS

Objectives

- After completion of this unit you should be able to :
- describe a project in terms of its activities
- represent the inter-relationships among the activities as network
- differentiate between CPM and PERT
- compute activity times, critical paths and slacks
- use the above information for the time management of the project
- use PERT when activity times are probabilistic
- specify project cost curve and crashing of ,the activities for reduction in the time duration of the project
- schedule resources

Structure

12.1 INTRODUCTION

We are sure that, both in your professional career and in your personal life, you have handled projects. In this unit we will deal with efficient management of projects. We will describe a project as consisting of inter-related activities. Networks will be used as visualisation of these inter-relationships. A successful implementation of the project will involve planning, coordination and control of the activities constituting the project. We will discuss the time-management of project using Critical Path Method and, subsequently, in situations where activity times are probabilistic, Programme Evaluation and Review Technique (PERT). We shall also describe the relationship between cost and time for implementation of the projects. Each project involves consumption of certain raw materials and use of certain resources. We shall briefly look into the resource allocation problems.

While this unit will outline methods and techniques which are useful in Project Management, a successful implementation of a project will depend on the skill and efficiency of the manager in using these techniques.

12.2 PROJECTS

Before we formally define projects, it will be a good idea if you describe, in your own words, your impression of the word project.

Activity A

In the space provided (or on separate sheet) write down your impressions of the term project. Also prepare a list of a few projects, which you have handled.

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Now consider the following two situations:

1 Preparing tea
2 Arranging for a tea party

If we look at the above two situations, then arranging for a tea party will require preparing tea as one of the tasks. In fact, arranging for a tea party will require organisation of other tasks, such as issuing invitations, procurement of snacks, procurement of cutlery, table arrangement, serving tea, etc. You will agree that arranging for a tea party is a much more complex task consisting of a large number of simpler tasks or activities such as preparing tea. We will call arranging for a tea party as a project while we will refer to individual tasks such as preparing tea as activities. A list of the activities for this project is as follows:

Example 1

List of activities for the project **Arranging a tea party**:

A Procure material
B Prepare snacks
C Arrange for the cutlery
D Prepare tea
E Set table with snacks and cutlery
F Serve tea
G Clean cutlery

We shall also make following observations:

a) Each activity needs physical time for its completion.
b) Certain activities are inter-related in the sense that tea cannot be actually served until its preparation is completed.
c) Each activity will require one or more of the resources such as manpower, equipment, raw material, etc.
d) A project will be completed only if all of its activities are completed.

We will describe formally an **activity** as a physical independent action which requires time for its completion and will consume one or more of the resources, and a project as a set of inter-related activities that are organised for a common goal or objective.

Activity B

With the above definition in mind, identify at least one project with which you have been associated and list the activities of that project.

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Some of the examples of projects are: erection of a manufacturing plant, preventive maintenance of a chemical plant, launching of a space vehicle, construction of a building, etc. These projects have a very large number of activities and can be successfully completed only if the various activities are properly planned, time schedules are prepared, resources allocated and a proper control is exercised during the implementation. Project management essentially deals with these aspects of the project.

12.3 NETWORK REPRESENTATION OF PROJECTS

First step in the management of the projects is to understand the inter-relationship between the various activities which constitute the project. The activities may be inter-linked with each other in various ways. For example, the activities preparation of tea and preparation of snacks may be inter-linked with each other by the type' of resource required (in this case a gas stove). On the other hand, the activity serving tea cannot be started unless and until the activity preparation of tea is completed. Unlike the other example this dependence cannot be freed by providing additional resources. Thus in any feasible time-schedule, activity serving tea has to be scheduled only after the completion of the activity preparation of tea. This kind of interdependence will be referred to as precedence requirement. An activity B is said to succeed activity A if, in any feasible time-schedule, B has to be scheduled only after completion of A. Then, A is said to be the predecessor of B.

Project Networks are used to visually depict through arrow diagrams these precedence requirements.

Drawing Project Network

The input required to draw project network is the list of the activities and their precedence requirements. When considering the precedence requirements, only immediately preceding activities will be listed. These are also called immediate predecessors. Consider the precedence requirements of the Example 1.

To start the activity prepare snacks, material has to be procured and hence A will precede B. Similarly A will also precede C. Activity E can start only when A, B, D are completed. However, as A precedes B, it is not necessary to list A as the preceding activity to E. Thus, B, D as a set of preceding activities to E, is sufficient to enforce all the precedence requirements. For the above example, you may check that the list of precedence requirements is as follows.

Activity C

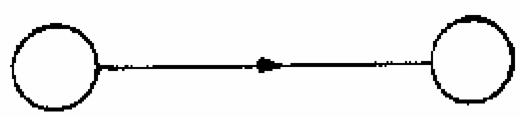
For the example 1, verify the following list of predecessors.

Table 1
List of predecessors for example 1

	All preceding **Activities**	Immediate Predecessors
A	-	
B	A	A
C	A	A
D	-	
E	A.B.D	B.D
F	A.B.C.D.E	C.E
G	A,B,C,D,E,F	F

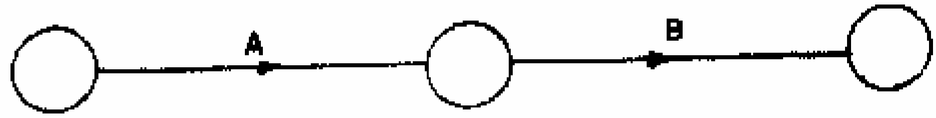
From the list of precedence activities, project network is drawn using arrows, Each activity is represented as an arrow, which is a line with a small circle at each end called node. Figure 1 represents one such activity.

Figure I: Activity in a Network



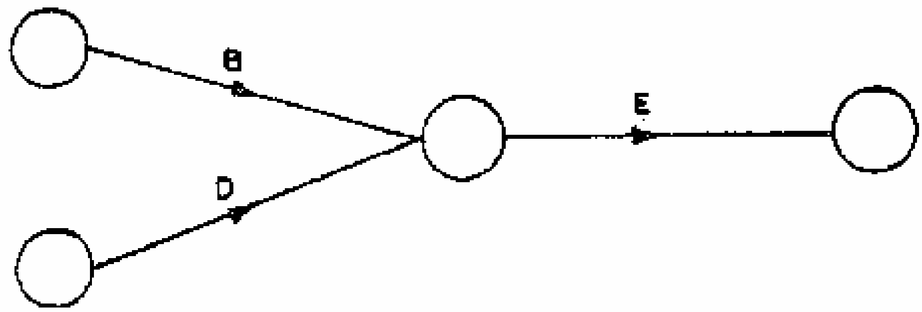
If an activity A precedes another activity B then the end node (circle) of activity A is merged with the start node (circle) of activity B. Thus the following diagram, will represent the precedence requirement A precedes B.

Figure II: Precedence Representation



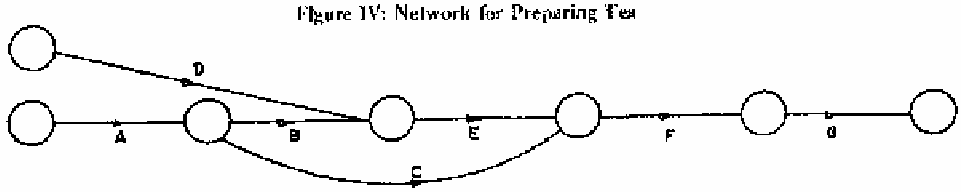
If an activity such as activity E in the project arranging a tea party has more than one predecessor (B, D in this case) the end points of all the predecessors are merged with the start node of this activity. Thus Figure III represents B and D precede E.

Figure III: Two Activities Preceding One Activity



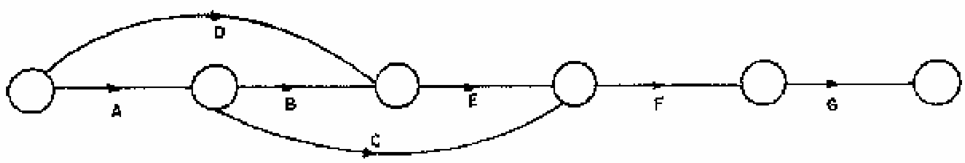
Using the above method the project network for the project arranging a tea party is as in Figure IV.

Figure IV: Network for Preparing Tea



For further convenience, the starting node of all the activities with no predecessors are joined together, as they can be started at the same time. With this modification the network will look like as follows:

Figure V: Correct Network for Preparing Tea



The node (or circles) in the arrow diagram are referred to as event. Events are well defined points in time, at which an activity can be started.

As another example consider a project with following precedence requirement:

Example 2

Activity	Immediate predecessors
A	-
B	-
C	-
D	C
E	A,B
F	E,D
G	D
H	F,G

Activity D

Try to draw a network for the example 2.

-While you will be able to draw the diagram up■u the activity F without any problem (Figure VI), you will find difficulty in correct .bowing G, which has only D as its predecessor. Representation in Figure VII is not correct, as in it both D and E are required to precede G -while it can start as soon as D is completed.

Figure VI: Partial Network for Example 2

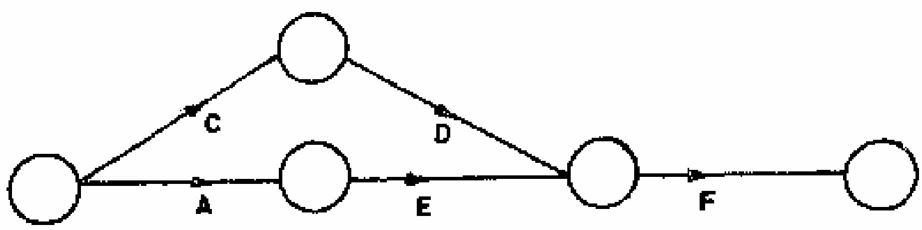
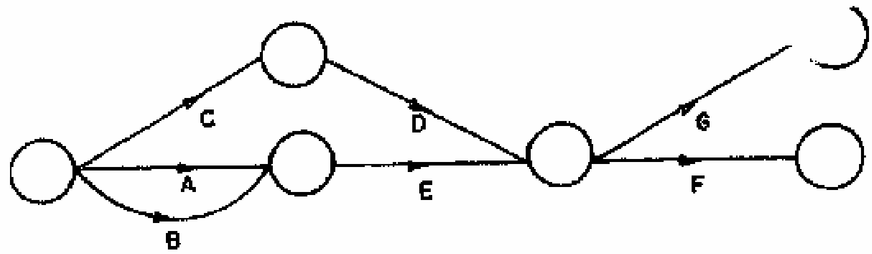
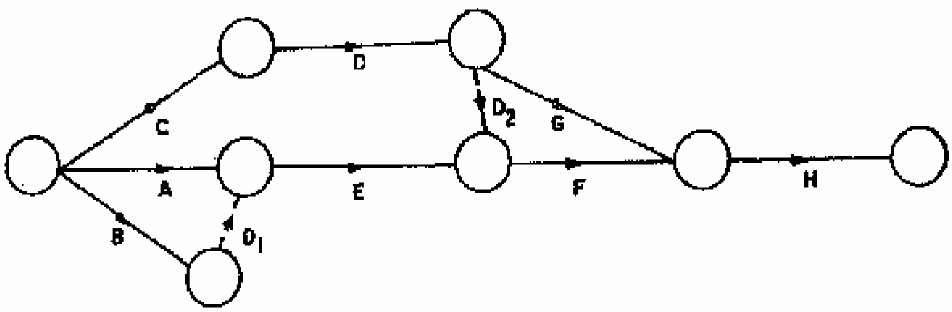


Figure VII: Incorrect Precedence Representation for Example 2



The correct representation is shown in Figure VIII.

Figure VIII: Correct Network for Example 2



The dotted arrows are called dummy activities and are activities which take zero time. Thus the activity F can be started only when both D and 7. are completed but G can be started as soon as D is completed. In addition another dummy is added after activity B. The purpose here is to make sure that one pair of events connects only one activity.

Thus in project network dummy activities are. added to:

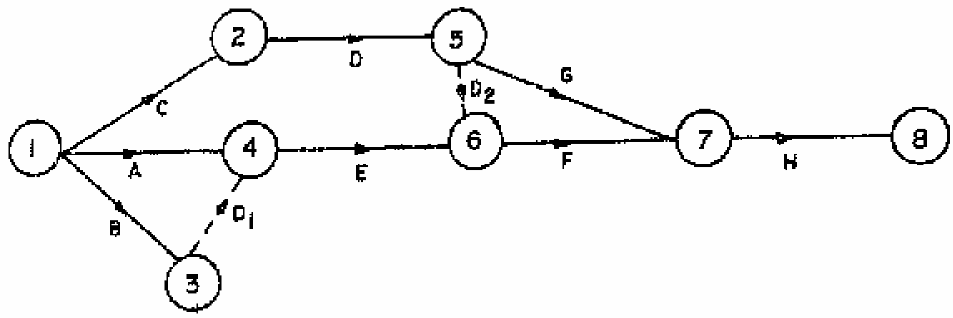
1 Ensure correct precedence relationship.
2 Make sure that two events are joined at most by one arrow.

Node Labelling

Once the network is drawn, it is a good practice to label the events systematically. A standard procedure is as follows:

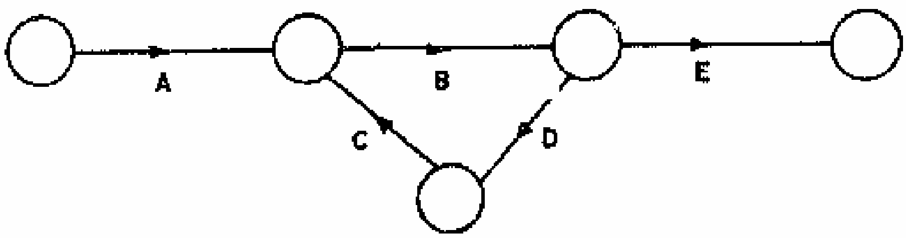
1 A start event is the one which has arrows emerging from it but none entering into it. Find the start event and number it 1.
2 Delete all the arrows emerging from all numbered events. This will create at least one new start event out of the remaining events.
3 Number the new start events as 2, 3 and so on.
4 Repeat steps 2 and 3 until end event is reached. Consider the diagram in Figure VIII. The above procedure will lead to the following node numbering.

Figure IX: Node Numbers for the Network for Example 2



Activity E
Number the nodes of the Figure V.
Activity F
Consider the network shown in Figure XC Try to number the. node as per procedure. You will find that step 2 cannot be carried out. This implies that the precedence relationship is not correct. Look at the precedence requirement of activity B, C, D. Do you see any inconsistency?

Figure X: Network for Activity 2



12.4 TIME MANAGEMENT OF THE PROJECT

One of the major requirements of the project management is a feasible time-scheduling of the various activities with the objective that project is completed at the earliest possible time. Feasibility of time-schedule will require that, an activity he scheduled only when all its predecessors are scheduled. For preparation of such a schedule, information will be needed about the duration in which the various activities can be completed. The duration of each activity is referred to as activity time. Depending on the nature of the activity times, the projects will get classified in two categories:

1 Projects in which activity times can be estimated with sufficient certainty, for example-building a house, or erecting a plant.

2 Project in which there is high degree of uncertainty about the activity times. For example-launching a space vehicle, developing a new product, etc.

Time Management of the project is the first case is usually done by using Critical Path Method (CPM) and in the second case using Programme Evaluation and Review Techniques (PERT).

Activity G
Consider a simple activity like preparing tea. Estimate activity time for this activity; clearly write down the method which you have used for arriving at this estimate. Repeat this for the following activities:

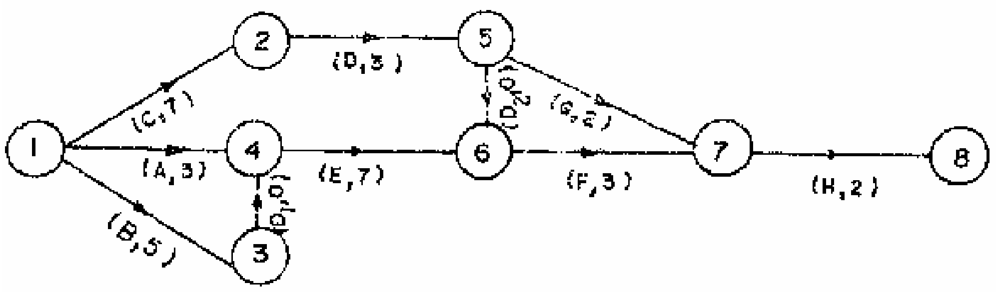
a) Changing punctured wheel of a scooter.
b) Getting a cheque encashed from the Bank.
c) Completing this exercise (before completing it).
d) Travelling from your home to office.
e) Preparing a report for your boss.
f) Travelling from your home to a new place (which you have not visited before)

Summarise various methods which can be used to arrive at the activity times.

12.5 CHITICAL PATH METHOD (CPM)

In this method it is assumed that activity durations are deterministic or are known with certainty. The method will compute the earliest possible start time and latest possible start time for each activity. In addition it will identify the critical activities. These activities are critical in the sense that if completion of any of these activities is delayed even by a short period of time, the whole project will be delayed. For other activities, slacks (floats) will be computed which will give some idea of the relative importance of these activities in terms of their time-management. To illustrate the computations at various times we will discuss the example 2 whose network representation with node number is shown in Figure XI.

Figure XI: Network with Activity Times for Example 2



Let the activity times of activities A, B, C, D, E, F, G, H be 3, 5, 7, 3, 7, 3, 2, 2 weeks respectively.

Figure XI shows the project network with activity times. In this Figure (C,7) on the 'arrow 1,2. indicates the activity name C and its duration as .7 weeks.

Computations of Early Start and Early Finish Times for the Activities

Once the network is developed and the activity times are compiled, we wish to compute the earliest time by which an activity can be started. For any activity (j) we shall refer this time as ES(j). Let the activity time for the activity j be denoted by T (j). In example 2 activity A, B, C can begin at time zero. Hence we can assign ES (A)=0, ES(B)=0, ES(C)=0. You may note that all the three activities can start at the same time. Earliest time by which an activity (j) can be completed will be denoted by Earliest Finish Time, EF(j). As the activity time for activity (j) is '1(j) then, its Earliest Finish Time EF(j) is,

Thus

EF (^{j})	=	ES(^{j})		T(^{j})
EF (A)	=	0	+	=3
EF (B)	=	0	+	= 5
EF (C)	=	0	+	=7

Activity D can be started only when activity C is finished, while activity E can be started only i-f both the activities A and D_I are finished. We can compute the times associated with these activities as follows:

ES (D) = EF (C) = 7 EF (D) = ES (D) + T (D) = 7+3 = 10
ES (D_1)= EF (B) = 5 EF (D_1) = ES (D_1)+T (D_1) = 5+0 = 5
ES (E) = maximum (EF (A), EF (D_1)) = maximum (3,5) = 5
EF (E) = ES (E) + T (E) = 5+7 = 12.

Similarly,

ES (G) = EF (D) = 10 EF (G) = 10+2 = 12
ES (D_2)= EF (D) = 10 EF (D_2) = 10+0 = 10
ES (F) = max (EF(D_2), EF (E)) = max (12,10) = 12,
EF (F) = 12+3 = 15
ES (H) = max (EF (G), EF (F)) = max (15,12) = 15, EF (H)=15+2=17

ES (H) = max EF (G), EF (F) = max (15, 12) = 15, EF (H) = 15 + 2 = 17.

Thus the activity H will be completed on the 17th week and hence the project can be completed by the 17th week from its start.

You may note that while all the activities together require 32 weeks but they can be completed in 17 weeks as some of them can be completed simultaneously. However, as the precedence relationship require certain activities to run sequentially, the project cannot be completed before 17th week.

We can summarise the rules for computations of Early Start Time and Early Finish Time as follows:

a) All the activities which do not have any preceding activity can be started at time zero.

b) Earliest Start Time for an activity can be computed only if the Earliest Finish Times (and hence Earliest Start Times) for all its preceding activities are computed.

c) Earliest Start Time for an activity (j) is the earliest time by which all its preceding activities are completed,

ES (j) = largest finish time among all its preceding activities
=maximum {EF (i)} over all preceding activities i

Critical Path

A path in a network is a sequence of activities which have to be completed sequentially due to precedence requirements. Thus C, D, F, H is a path and so are (A, E, F, H), (B, D, E, F, H), (C, D, D_2, F, H). Time duration of the path is the sum of the activity times of the activities forming the path. The path or paths with the largest time duration in the network is (are) called critical path(s). All the activities on the critical path(s) are called critical activities. The earliest time by which a project can be completed is the time duration of the critical path(s). If an activity on the critical path is delayed the whole project will get delayed. Thus for timely completion of the project it is essential that all critical activities be started at the earliest start time and be completed in the planned duration.

Activity H

For each of the paths mentioned above, compute its length. Which are the critical paths?

Activity I

Consider the example 2 again with the difference that the activity time of activity G is changed to 3 weeks and of D to 5 weeks. Recompute the length of each 'path. How many paths are critical?

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A method of computing critical path(s) can be to enumerate all the paths in a network and to compute the time duration of eacn of these paths and then to select the path(s) with largest time duration. However, in a large network such enumeration will take large amounts of computation time even on very fast computers. Another mechanism will be to identify those activities which are critical. This can be done by finding the Latest Start Time for each activity. This is the time by which if the activity is started, the project will be completed in time. For critical activities Latest Start Time and Earliest Start Time will be identical.

Computations of Latest Start Time and Latest Finish Time of Activities

Latest Start Time of an activity j, LS(j) is the time by which, if it is started the project will get completed in time. Any delay beyond this time in the starting of the activity or in its execution will delay the completion of the project. In our example, if the project has to be completed by 17th week, the activity H has to be finished by 17th week. Hence if the latest finish time of the activity H, LF(H), is 17 weeks,

$$LF(H) = 17$$

and $LS(H) = 17 - T(H) = 17 - 2 = 15$

It may be noted that for activity H :

$$ES(H) = LS(H) = 15$$

However, activity G can be completed by 15th week, without delaying the project as in that case H can still be started on the 15th week and completed by 17th week.

Hence, $LF(G) = LS(H) = 15 \quad LS(G) = 15 - 2 = 13$

While for the activity G ES(G) is 10, LS(G) is 13 weeks. Thus it is possible to delay the start of G upto 13th week without delaying the project completion.

Similarly.

$$LF(F) = 15 \quad LS(F) = 15 - 3 = 12 \quad LF(D_2) = LS(F) = 12$$
$$LS(D_2) = 12 - 0 = 12$$
$$LF(E) = LS(F) = 12 \quad LS(E) = 12 - 7 = 5$$

However, latest point in time by which the activity D should be completed is the point in time such that both D_2 (hence F) and G can be started by their Latest Start Time.

Thus $LF(D) = \text{minimum } \{LS(D_2), LS(G)\} = \{13, 12\} = 12.$

$$LS(D) = 12 - 3 = 9$$

If D is not completed by 12th week, F has to be started later than its Latest Start Time resulting in the project delay.

Similarly, $LF(C) = LS(D) = 9 \quad LS(C) = 9 - 7 = 2$

$$LF(A) = LS(E) = 5 \quad LS(A) = 5 - 3 = 2$$
$$LF(D_1) = LS(E) = 5 \quad LS(D_1) = 5 - 0 = 5$$

The table gives the ES, LS, EF, LF for all the activities. In the fifth column, the difference between LS and ES is listed as slack.

Table 2
Computations of Activity Times

Activity	Activity Duration	ES	EF	LS	LF	S
A	3	0	3	2	5	2
B	5	0	5	0	5	0
C	7	0	7	2	9	2
D	3	7	10	9	12	2
E	7	5	12	5	12	0
F	3	12	15	12	15	0
G	2	10	12	13	15	3
H	2	15	17	15	17	0
D_1	0	5	5	5	5	0
D,	0	10	10	12	12	2

The activities B,G,E,F,H are critical and will form the critical path. We can summarise the procedure for the computation of latest start and latest finish time for the activities as follows:

a) All the activities without any successor are required to be completed by the time project is completed and hence for all these activities

$$LF(j) = \text{Project completion time}$$

$$LF(i) - T(j)$$

b) The latest an activity can be completed without delaying the 'project is the time such that all its succeeding activities can be started latest by their latest start time LF (j)=maximum {LS (i)}
overall successor i

and LS (j) =LF (j) - T (j)

c) Latest Start Time for an activity can he computed only if the Latest Start Times of all its successor have been computed.

In all we have observed that all activities, for which Early Start Time and Latest Start Time are same or which have zero slack, are critical activities and the path(s) formed by these activities will be the critical path(s).

Slack (Float)

In the computation of the activity times, we observed that slack or total float in an activity is the difference between its Latest Start Time and its Earliest Start Time. It signifies the delay which is permitted in the completion time of that activity without affecting the project completion. The delay can he either due to delay in completion of its preceding activities or in the execution of this activity itself. You may note that while both the activities C and D have slack of 2 days each, both of them cannot be delayed simultaneously. If C is completed 1 week late i.e. on 8th week then slack available on D will be only one week. In some cases it may be useful to compute free slack (float) which is the slack available in an activity such that early start of the successors are not affected. In other words, free slack is the delay which, if occurs, will not effect the early start of the successors. By this definition, the free slacks for the various activities for the example 2 are as follows:

$$\text{Free slack (C)} = ES(D) - EF(C) = 7 - 7 = 0$$

$$\text{Free slack (D)} = \begin{bmatrix} ES(D_2) - EF(D) \\ ES(G) - EF(D) \end{bmatrix} = \begin{bmatrix} 10 - 10 \\ 10 - 10 \end{bmatrix} = 0$$

$$\text{Free slack (G)} = ES(H) - EF(D) = 15 - 12 = 3$$

$$\text{Free slack (A)} = ES(E) - EF(A) = 5 - 3 = 2$$

On other activities free slack will be zero. Why ?

The total slack and free slack can be effectively used for better management of projects. For example, if there is a resource constraint then it will be better to allocate resources first to activities with zero or small slacks.

12.6 PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

Critical Path Method (CPM) is an effective tool for project planning and control when activity times are known with certainty. However, in certain projects like Research and Development projects, it seems unrealistic to assume that we can know with certainty the time durations in which the activities can be completed. In such cases PERT can be used.

Time Estimates in PERT

In PERT for each activity, three time estimates are made. These estimates are:

a) Most Likely Time (TM): The time which is taken most frequently by the activity.

b) Optimistic Time (TO): The time by which activity can be completed, if everything went well.

c) Pessimistic Time (TP): The time by which the activity will get completed even under adverse conditions.

Activity 3

For the activity, `Prepare a report for the boss', estimate the three times. The TO will be the time under most favourable conditions, i.e. when all data is available, report is short' etc., while TP will; be the time under most adverse Conditions, i.e. when data has to be compiled and report is a long report 'M will be the time which is taken by most of the reports.

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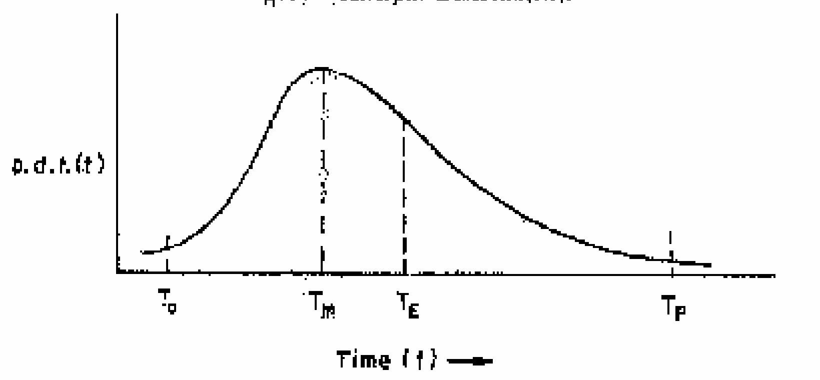
From the above time estimates expected time of the activity (j), TEO), is computed using following relationship:

$$TE(j) = \frac{TO(j) + 4TM(j) + TP(j)}{6}$$

The expected time for an activity represents the average time it would take if the activity is performed over and over again. It is different from most likely time. The above relationship is based on the fact that the probability distribution function of the activity time can be approximated by' a Beta distribution which is of the shape shown in Figure XII

Figure XII: Beta Distribution



Further, consider two distributions, with the following activity times, Activity A

	Activity B	Activity B
TO	3	2
TM	5	4
TP	7	12

Both the activities have same expected time of'5 weeks but there is a higher degree of uncertainty associated with Activity B. This variability can be measured by computing the standard deviation of the activity, SD(i), as

$$SD(i) = \frac{TP(i) - TO(i)}{6}, \text{thus}, SD(A) = \frac{7-3}{6} = 4/6$$

$$SD(B) = \frac{12-2}{6} = 10/6$$

Variance is defined as the square of the standard deviation and hence

$$V(i) = [SD(i)]^2 = \left[\frac{TP(i) - TO(i)}{6}\right]^2$$

For the example 2 the three times estimates, expected time and the variance of the activity are given in the Table 3

Table3
Estimates of the Activity Times

Activity	TO	TM	TP	TE	SD	V
A	I	2	9	3	4/3	16/9
B	2	4	12	5	5/3	25/9
C	4	6	14	7	5/3	25/9
D	1	3	5	3	2/3	4/9
E		8	9	7	8/3	64/9
F	2	3	4	3	2/3	4/9
G	1	2	3	2	1/3	1/9
H	1	2	3	2	1/3	1/9

Time of the Project Completion

Unlike CPM, in the case when there is uncertainty in activity times, it is not possible to compute the time of completion of the project with certainty. Instead, we shall estimate the expected time and variance of the project completion time. The expected project completion time E(T), can be computed as the length of critical path in the CPM, when activity times are replaced by the expected times (TE) of the activities. As in the case of example 2, TE are same as the activity time in Figure XII, the expected time of the completion of the project is 17 weeks. Further a crude estimate of the variance of the completion time can be obtained by adding the variances of the activities on the critical path.

Thus V (completion time) = Sum of variances of the activities on the critical path

$= V(B)+V(E)+V(F)+V(H)$

$$=\frac{25}{9}+\frac{64}{9}+\frac{4}{9}+\frac{1}{9}=\frac{94}{9}=10.44$$

$$\text{SD(Completuiontime)}=\sqrt{V}=\sqrt{10.44}=3.23$$

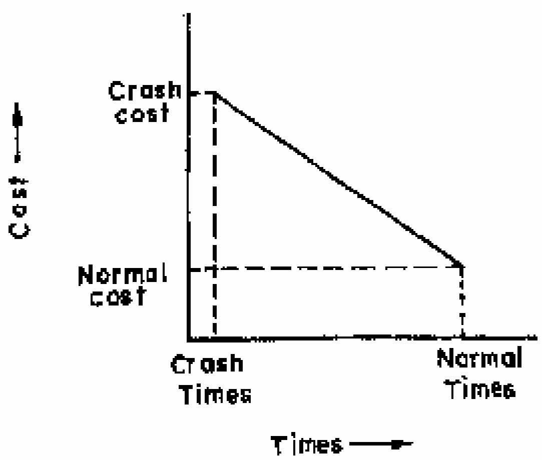
Probability estimates of the completion time of the project can be made using the fact that the project completion time has normal probability distribution with mean as expected completion time and Variance as variance of the Critical-Path.

Further analysis of the network can be done using Simulation from which an estimate of the probability that an activity will become critical can be obtained. This is called critical index of the activity.

12.7 TIME COST RELATIONSHIP AND PROJECT CRASHING

In addition to time-management, cost plays an important role in any project. The project costs can be classified in two groups: direct activity costs and indirect project costs. Direct activity costs are those components of the cost which can be directly linked with the activity. Thus direct labour, material consumed, rental charges for the equipment etc. will form part of these costs. Usually, the activity durations can be reduced by increasing the direct activity cost. For example, by asking the labour to work with OT or with efficiency bonus for faster work etc.

Figure XIII: Times Cost Relationship for an Activity



Crash time is the minimum possible time in which the activity can be completed and the cost associated with this time is the crash cost. Normal cost is the cost incurred when activity is completed in its normal time (or the time used as activity time in the critical path computations).

On the other hand, indirect costs are the overheads associated with the entire project including loss of revenue/benefits due to late completion of the project. This cost will decrease directly with the decrease in the project completion time. An optimal project completion time will be the time for which sum of these two costs is minimum. To obtain the optimal time, we have to compute direct activities cost when project duration is reduced by one unit time each time. This is called project crasing. We shall explain the. project crashing by the following simple example.

Example 3

Let the information regarding project be summarised in Table 4.

Table 4
Data for Example 3

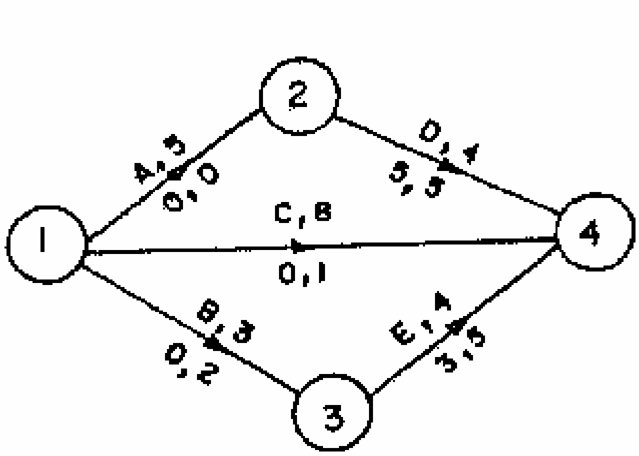
Activity	Preceding activities	Normal time (week)	Crash time (week)	Normal Cost (Rs.)	Crash Cost Rs.
A	-	5	4	600	800
B	-	3	1	400	600
C	-	8	5	900	1200
D	A	4	2	600	1200
E	B	4	3	500	700

We shall assume that cost of crashing an activity by 1 week

$$= \frac{\text{Crash cost - Normal cost}}{\text{Normal time - Crash time}}$$

The project network is shown in Figure XIV.

Figure XIV: Network for Example 3



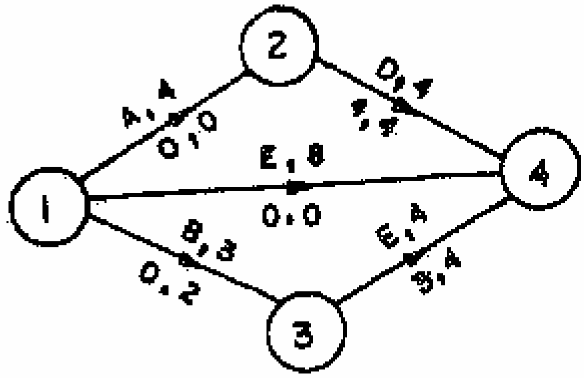
In Figure XIV, E,4/3,5 denote Activity E, T(E) = 4

ES (E) = 3 LS (E) = 5, For this example project completion time T is 9 weeks and the direct activity cost is Rs. 3000 (600+400+900+600+500).

If the project completion time is to be reduced by 1 week, it can be done only by reducing the activity duration of one of the activities on the critical path. If there are more than one critical parts then at least one activity on each path has to be crashed. You may note that reducing activity time of any activity other than critical activities will not reduce the length of critical path and hence the project duration cannot be reduced.

In our example critical path consists of activities A and D. If the duration of either A or D is reduced by 1 week the length of this path will be reduced to 8 weeks and hence project duration will be reduced to 8 weeks. The cost of reducing the activity A by 1 week is Rs. 200 and of D Rs. 300 and hence it will be beneficial to crash activity A by 1 week.. So for T=8, direct activity cost is Rs, 3200 (3000+200). Further the new project network is shown in Figure XV.

Figure XV: Network for Example 3 with Crashing



Both the paths A-D and C are critical in this network. To reduce the project duration further, at least one activity on each of the critical paths has to be reduced. It implies that activity C with either activity A or activity D has to be crashed. However, it is not possible to crash activity A as already it is being done in its crash time and no further reduction is possible. Hence, we can reduce the project duration by 1 week by crashing C and D, which will cost Rs. 400. We can repeat this, process for further reduction in the project duration. We can summarise the method for reducing the project completion time as follows:

Step 1: Compute for each activity
Cost of reducing activity time by one unit time

$$= \frac{\text{Crash cost - Normal cost}}{\text{Normal time - Crash time}}$$

Step 2: Identify all the critical paths. Select one activity on each critical path (same activity can be on more than one critical paths) such that the total cost of crashing all these activities by one unit time is minimum among al such combinations of the activities. Further the activities selected should be such that their current activity time is higher than the crash time, i.e. no activity reduction in activity duration can take place beyond its crash time. If there is at least one critical path, on which none of the activities can be crashed, then no further reduction in the project completion time can take place. In that case stop.

Step 3: Reduce the activity time for the activities selected for crashing by unit time period and recompute the Early Start Time, Late Start Time and critical paths for the network.

Step 4: Repeat Step I to Step 3.

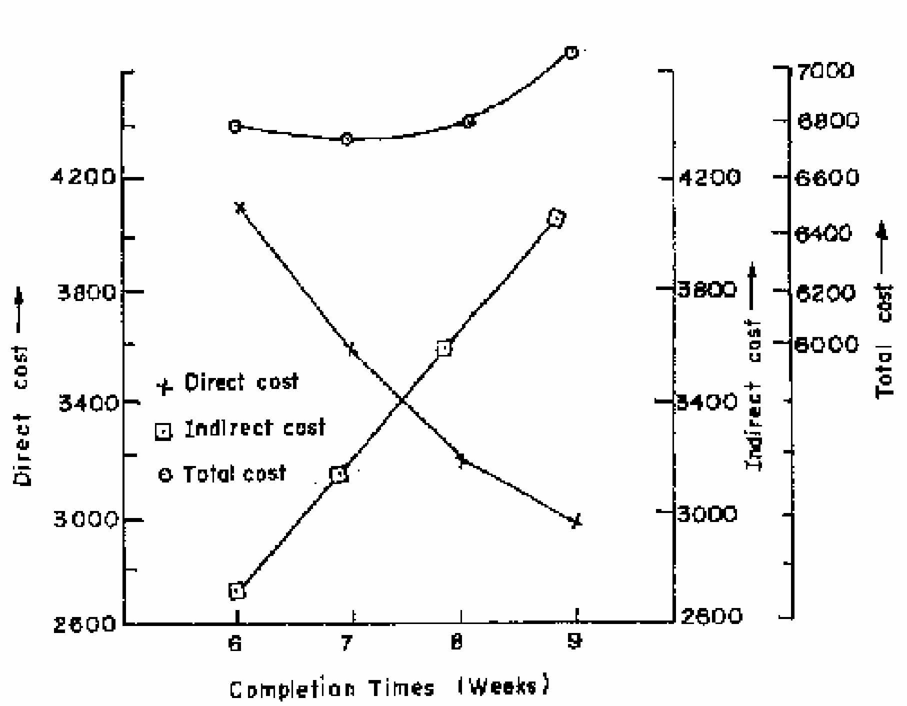
Using the above method the computation for the example 3 are as follows:

Table 5
Project Crashing

Current Project Completion time	Critical Path(s)	Activities which can be crashed	Cost of crashing	Activities selected	Cost	Total Cost
9	A-D	A,D	200,300	A	200	3200
g	A-D	D	300	D	300	
	C	C	100	C	100	3600
7	A-D	D	300	D	300	
	C	C	100	C	100	4100
	B-E	B.E	100.200	B	100	
6	A-D	-	--			
	C	C	100		cannot be	
	B-E	B,E	100,200		crashed	

The direct project cost curve is given in Figure XVI. If the indirect project costs are. Rs. 450 per week, then the optimal project duration will be 7 weeks.

Figure XVI: Project Time-cost Relationship



12.8 RESOURCE ALLOCATION

The analysis up to this point has been based on the assumption that if its predecessors were completed, an activity could begin. Underlying this assumption is the fact that sufficient resources will be available to start activities simultaneously. However, in actual practice some of the resources may be available in limited quantities. Examples of such resources are expensive equipment, skilled manpower, finances etc. For example, let each of the activities A, B, C, D, E in the example 3 require 2 carpenters and let there be only 4 carpenters available. It is obvious that activities A, B, C cannot be started at 0 time period as, they will require 6 carpenters. In fact, only 2 of these activities can be selected. Third activity can be started only when one of these two will get completed. Depending on which two activities are selected, project completion time will change. An efficient manager will like to apportion resources in such a way that increase in project duration is as low as possible. Many heuristic methods (simple rules) have been developed for such resource allocation problems. While such method cannot guarantee that the solution obtained is best possible, they usually provide good solutions. A simple heuristic rule will be as follows:

Step 1: Allocate resources serially in time. That is, start on the first time period and schedule all activities possible with the resources available, then do the same for the second time period and so on.

Step 2: When several activities compete for the same resources, give preference to the activities with the least slack. Recompute the slacks in all the activities.

Step 3: Reschedule non-critical activities, if possible, to free resources for scheduling critical or non-slack activities.

The computation for Example 3 are shown in Table 6. This project will get completed in 12 weeks instead of 9 weeks as scheduled earlier.

Table 6
Resource Allocation

Week	Activities which can be scheduled	Activity time slack		Carpenter needed	Carpen allocat	
	A	5	0	2	2	
0	C	8	1	2	2	
	B	3	2	2	-	
	A	4	0	2	2	
1	C	7	1	2	2	
	B	3	1	2	-	
	A	3	0	2	2	
2	C	6	1	2	-	(reschedule)
	B	3	0	2	2	
	A	2	0	2	2	
3	C	6	0	2	2	
	B	2	0	2	2	
	A	I	1	2	2	T= 10
4	C	6	0	2	2	
	B	I	1	2	-	(reschedule)
	D	4	1	2	-	
5	C	5	0	2	2	
	B	I	0	2	2	
	D	4	0	2	2	
6	C	4	0	2	2	
	E	4	0	2	-	
	D	3	1	2	2	(T = it
7	C	3	1	2	-	(reschedule)
	E	4	0	2	2	
	D	2	I	2	-	(reschedule)
8	C	3	0	2	2	
	E	3	0	2	2	
	D	2	0	2	2	
9	C	2	0	2	2	
	E	2	0	2	-	(reschedule)
	D	1	I	2	2	
10	C	1	1	2	-	(reschedule)
	E	2	0	2	2T= 12	
	C	1	0	2	2	
11	E	I	0	2	2	

12.9 PROJECT UPDATING AND MONITORING

For effective use of the above techniques it is essential that project progress should be continuously monitored. As and when there is a change in time schedule of any activity, the project network should be updated and new time schedule finalised.

Now a days with most of the computers, including PCs, well written CPM1 PERT packages including project crashing and resource allocation are available. These packages will provide all the necessary information needed for efficient project management.

12.10 SUMMARY

Projects are non-repetitive large tasks. A systematic way of describing a project is through identification of activities and their inter-relationships. These inter-relationships can be visualised using arrow diagrams or networks. Project management deals with time-scheduling and resource allocation for these activities. If the activity times are deterministic then Critical Path Method can be used as a systematic method to compute early start and late start times of activities and to, identify critical activities. The slacks associated with each activity can be used for better control of the project. When activity times are probabilistic, PERT is an effective tool. In this method three time estimates are made for each activity and these are used to compute expected project completion time and its variance. Project duration can be reduced by incurring additional cost in executing activities. A project cost-time relationship can be obtained using Project crashing. This can also be used to obtain the optimal project duration. Resource allocation is one of the major problems in project management. A simple heuristic is suggested to handle this problem.

12.11 KEY WORDS

Projects: Set of activities which are inter-related with each other and are to be organised for a common goal or objective.

Activity: Physical independent action which requires time for its completion and will consume one or more of the resources.

Event: Point in time schedule at which an activity can be started (or is finished).
Activity Time: Physical time required to complete an activity.

Immediate Predecessor: An activity which should immediately precede the given activity in any feasible time schedule.

Project Network: A visual representation of the interdependence between different activities of a project.

Earliest Start Time: The earliest time at which an activity can begin. All the activities preceding the given activity should be completed by this time.

Latest Start Time: The latest time by which the activity can be started without delaying the project completion. time.

Earliest Finish Time: The earliest time by which an activity can be finished. **Latest Finish Time:** The latest time by which an activity can be completed.

Total Slack (float): The length of time up to which an activity can be delayed without affecting the start of the succeeding activities.

Free Slack (float): The length of time up to which an activity can be delayed without affecting the starts of the succeeding activities.

Critical Path: The longest sequence of activities or path in the project network. The time it takes to traverse this path is the estimated project completion time.

Critical Activities: Activities on critical path. Any delay in execution or start of these activities will delay the whole project.

CPM: Project management technique used when activity times are deterministic.
PERT: Project management technique used when activity times are probabilistic.

Optimistic Time: Time by which an activity can be completed, if everything goes well with it.

Pessimistic Time: Time by which an activity will get completed under adverse conditions.

Most Likely Time: Most frequently occurring time for an activity.

Expected Activity Time: The average activity time.

Variance: Measure of the deviation of the time distribution for an activity.

Crashing: The process of reducing an activity time by adding resources and hence usually increasing cost.

Normal Cost: Cost associated with an activity when it is completed in normal time.

Crash Cost: Cost associated with an activity when it is completed in the minimum possible time (crash time).

Resource Allocation Methods: Allocation of resources to the activities such that project completion time is as small as possible.

Updating: Revision of the project schedule after partial completion with revised information.

12.12 SELF-ASSESSMENT EXERCISES

1 Under what circumstances would you use PERT as opposed to CPM in project management? Give some example of projects where each would be more applicable than the other.

2 What do you understand by slack? Construct an example and show how you can use the knowledge of slacks for better project management.

3 Identify the information needed for the project crashing. For a project with which you are familiar, try to identify the various items of information. Criticise the various assumptions made in the crashing method.

4 Draw the complete CPM network according to the following table:

Activity	Starts at Event	End at Event
1-2	1	2
1-3	1	3
1-4	1	4
2-3	2	3
2-4	2	4
3-4	3	4

5 Draw the following logic network:
Activities C and D both follow A
Activity E follows C Activity F follows D E and F precedes B.

6 In putting a job together to run at a data-processing centre, certain steps need to be taken. These jobs can be described as follows:

Job	Time (minutes)	Immediate predecessors	Description
A	180	-	Design flow chart and FORTRAN statements
B	30	A	Punch control cards
C	20	A	Punch comment cards
D	60	A	Punch programme cards
E	10	B,C,D	obtain brown folder
F	20	B,C,D	Put deck together
G	10	E,F	Submit deck

a) Draw a critical-path scheduling diagram and indicate the critical path. What is the minimum time to completion?

b) What is the free slack of job C?

c) Assuming the table accurately represents the jobs to be done and their times, if you were performing this project, would the minimum time to , completion obtain above be the minimum time for you to complete the project. If yes, what conditions would change your answer? If no, why not, and what would the correct time be?

7 The following table lists a set of nine activities together with their sequence requirements, estimated activity times, and the daily number of men required for each activity. These nine activities make up a complete project.

Activity Code	Code of Immediate Predecessor	Time Required (days	Men Required per day
A	-	10	3
B	-	8	4
C	-	5	7
D	A	6	5
E	B	4	2
F	C	10	4
G	F	4	3
H	F	8	3
I	D,E,G	7	3

a) Use this information to develop a network diagram.

b) Determine the earliest start (ES), earliest finish (EF), latest start (LS), and latest finish (LF) times for each activity: '

c) List the activities on the critical path.

d) What is the earliest finish time of the project without resource constraints (assuming that we have an unlimited number of men)?

e) Assume that we have only 11 men available but that each man is completely interchangeable. That is, any man can do any task. Also assume that the activities much use exactly the number of men and days specified. For example, you cannot use twice the manpower and complete the activity in one-half the time. Now determine (if possible) a schedule of activity starting times that will allow these 11 men to complete the project by the earliest finish date. If this is not possible, show a schedule that will finish the project as soon as possible with 11 men. (Start with day 1, not day 0).

12.13 FURTHER READINGS

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