
UNIT 8 CAPITAL BUDGETING DECISIONS AND THE CAPITAL ASSET PRICING MODEL

Objectives

Objectives of the Units are :

- to examine the relevance of risk in capital budgeting decisions,
- to understand the application and usefulness of capital asset pricing model in capital budgeting decisions,
- to study the certainty equivalent and risk adjusted discounting rate approaches.

Structure

- 8.1 Introduction
- 8.2 Capital Asset Pricing Model
- 8.3 Measuring Betas and Capital Asset
- 8.4 Stability of Betas over Time
- 8.5 Business and Financial Risk
- 8.6 What determines Asset Betas
- 8.7 Discounted Cash Flow Approach
- 8.8 Summary
- 8.9 Self Assessment Questions
- 8.10 Further Readings

8.1 INTRODUCTION

Long before the development of capital asset pricing theory¹, which says that the equilibrium rates of return on all risky assets are a function of their covariance with the market portfolio; smart financial managers adjusted for risk in capital budgeting. They realized intuitively that, if other things being equal, risky projects are less desirable than safe ones. Therefore, they demanded a higher rate of return from risky projects or they based their decisions on conservative estimates of the cash flows.

Various rule of thumb are often used to make these risk adjustments. For example, many companies estimate the rate of return required by investors in its securities and use this required rate of return to discount the cash flows on all new projects. Since investors require a higher rate of return from a very risk company, such a firm will have a higher company cost of capital and will set a higher discount rate for its new investment opportunities.

You can use the capital asset pricing model as a rule of thumb for estimating the company's cost of capital. For instance ABC Ltd. has a beta of 1.38, the risk free rate is 7.8 per cent and expected market risk premium 8.3 per cent then, the corresponding expected rate of return would be 20.3 or about 20 per cent. Therefore, according to the company cost of capital rule, ABC should have been using a 20 per cent discount rate to compute project net present values².

1 CAPM was first developed by William Sharpe in 1963, 64, and later on developed by J.Mossin, 1963, J. Lintner, 1965, F. Black, 1972.

2 ABC did not use any significant amount of debt financing. Thus its cost of capital is the rate of return investors expect on its common stock.

This is a step in the right direction. Even though we can't measure betas or the market risk premium with absolute precision, it is still reasonable to assert that ABC faced more risk than the average firm and, therefore, should have demanded a higher rate of return from its capital investments.

But the company cost of capital rule can also get a firm into trouble if the new projects are more or less risky than its existing business. Each project should be evaluated at its own opportunity cost of capital. This is a clear implication of the value-additivity principle. For a firm composed of assets A and B, firm value will be:

Firm's value = $PV(AB) = PV(A) + PV(B)$ = sum of separate asset values.

Here $PV(A)$ and $PV(B)$ are valued just as if they were mini-firms in which stock-holders could invest directly. Note: Investors would value A by discounting its forecasted cash flows at a rate reflecting the risk of A. They would value B by discounting at a rate reflecting the risk of B. The two discount rates will, in general, be different.

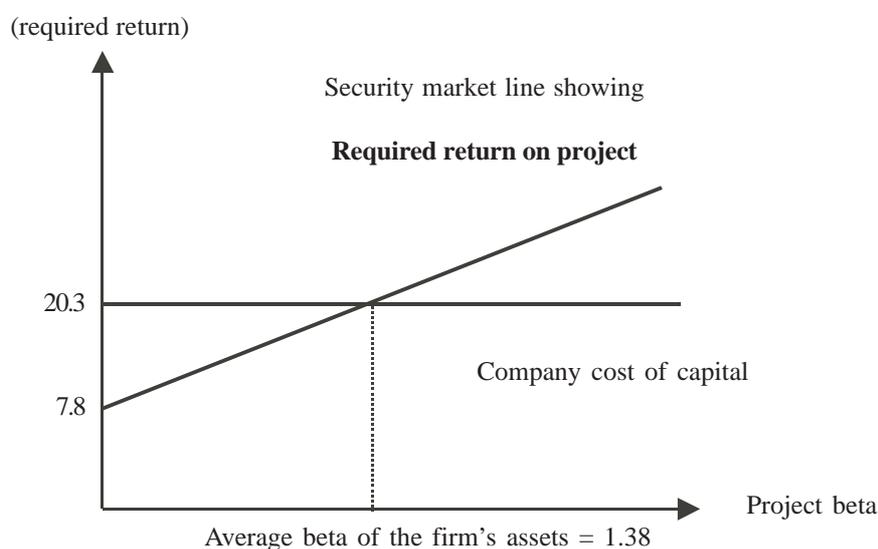


Figure 8.1: Relationship between required return and Project Beta

Figure 8.1 exhibits a comparison between the company cost of capital rule and the required return under the capital asset pricing model. ABC's company cost of capital is about 20 per cent. This is the correct discount rate only if the project beta is 1.38. In general, the correct discount rate increases as project beta increases. ABC should accept projects with rates of return above the security market line relating required return to beta.

If the firm considers investing in a third project C, it should also value C as if it were a mini-firm. That is, it should discount the cash flows of C at the expected rate of return investors would demand to make a separate investment in C. *The true cost of capital depends on the use to which the capital is put.*

8.2 CAPITAL ASSET PRICING MODEL (CAPM)

CAPM approach is used to estimate risk adjusted discount rate for making investment decisions. It is a theory about how the prices of risky financial assets (securities) are determined in the capital market.

Capital asset pricing theory tells us to invest in any project offering a return that more than compensates for the project's beta. This means that ABC

should have accepted any project above the upward-sloping market line in Figure 8.1. If the project had a high beta ABC needed a higher prospective return than if the project had a low beta. Now contrast this with the company cost of capital rule, which is to accept any project regardless of its beta as long as it offers a higher return than the company's cost of capital. In terms of Figure 8.1, it tells ABC to accept any project above the horizontal cost-of-capital line-that is, any project offering a return of more than 20 per cent.

It would be silly to suggest that ABC should demand the same rate of return from a very safe project as from a very risky one. If ABC used the company cost of capital rule, it would reject many good low-risk projects and accept many poor high-risk projects. It is also fully to suggest that, just because XYZ Ltd. has a low company cost of capital, it is justified in accepting projects that ABC would reject. If you followed such a rule to its seemingly logical conclusion, you would think it possible to enlarge the company's opportunities by investing a large sum in risk free securities. That would make the common stock safe and create a low company cost of capital.

The notion that each company has some individual discount rate or cost of capital is widespread, but far from universal. Many firms require different returns from different categories of investment. Discount rates might be set for different investment purposes as given below :

Category	Discount Rate Percent
Speculative ventures	30
New product	20
Expansion of existing business	15 (Company cost of capital)
Cost improvement, known technology.	10

In brief, the main insights of CAPM are :

- Investors need be rewarded for systematic risk only because insystematic risk can be reduced to zero through diversification of investment profolio.
- A security's systematic risk is measured by beta value.
- The required rate of return on a security depends on riskless rate of interest the market risk premium and the security's beta value.

8.3 MEASURING BETAS AND CAPITAL ASSET

Now the question arises how to use beta pricing model to help cope with risk in capital budgeting situation the main problem in how to estimate the discount rate.

$$r = r_f + \beta_p (r_m - r_f)$$

r = Discounting rate

r_f = Risk free rate

r_m = Market rate of return

β_p = Project beta

And in order to do that you have to figure out the project beta. It is a difficult problem so much so that many people hope it will go away if they ignore it. They may go away but unfortunately the problem won't - any investment decision that is made contains an implicit assumption about project risk.

We will start by reconsidering the problems you would encounter in using beta to estimate a company's cost of capital. It turns out that beta is difficult to measure accurately for an individual firm, much greater accuracy can be achieved by looking at an average of similar companies. But then we have to define the 'similar.' Among other things we will find that a firm's borrowing policy affects its stock's beta. It would be incorrect, for example, to average the beta of a company which has borrowed heavily and the one which has not, although, they may have similarity otherwise.

The company's cost of capital is the correct discount rate for projects that have the same risk as the company's existing business but not for those that are safer or riskier than the company's average. The problem is to judge the relative risk of the projects available to the firm. In order to handle that problem, we will need to dig a little deeper and look at what features make some investments riskier than others.

There is still another complication: project betas can shift over time. Some projects are safer in youth than in old age, others are riskier. In this case, what do we mean by the project beta? There may be a separate beta for each year of the project's life. To put it another way, can we jump from the capital asset pricing model, which looks out one period into the future, to the discounted-cash-flow formula for valuing long-lived assets? Most of the time it is safe to do so, but you should be able to recognize and deal with exceptions.

Capital asset pricing theory supplies no mechanical formula for measuring and adjusting for risk in capital budgeting. These tasks of financial management will be among the last to be automated. The best a financial manager can do is to combine an understanding of the theory with good judgement and a good nose for hidden clues.

Suppose that you were considering an across-the-board expansion by your firm. Such an investment would have about the same degree of risk as the existing business. Therefore, you should discount the projected flows at the company cost of capital. To estimate that, you could begin by estimating the beta of the company's stock.

Table 8.1

Sharpe and Cooper divided stocks into risk classes according to their betas in one 5-year period (class 10 contains high betas, class 1 contains low betas). They then looked at how many of these stocks were in the same risk class 5 years later.

Risk	Percent in Same Risk Class 5 Years Later	Percent Within Class 5 Years Later
10	35	69
9	18	54
8	16	45
7	13	41
6	14	39
5	14	42
4	13	40
3	16	45
2	21	61
1	40	62

3 W.F. Sharpe and G.M. Cooper, "Risk-Return Classes of New York Exchange Common Stocks, 1931-1967", *Financial Analysis Journal*, 28:46-54, 81 (March-April 1972)

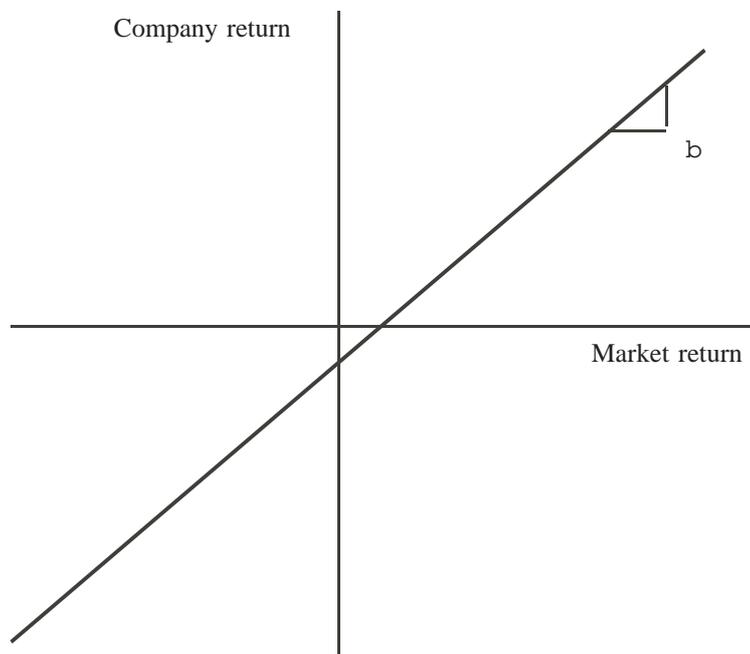


Figure 8.2: Estimating Beta

An obvious way to measure the beta of a stock is to look at how its price has responded in the past to market movements. We may plot monthly rates of return of Company against market returns for the same months. We may fit a line through the points. Beta is the slope of the line (See Figure 8.2). It could vary from one period to the other. If we use the past beta of a stock to predict its future beta, we would, in most cases, not have been too far off.

8.4 STABILITY OF BETAS OVER TIME

Of course, evidence from two (carefully selected) stocks is not worth much, but betas appear to be reasonably stable. An extensive study of stability was provided by Sharpe and Cooper³. They divided stocks into 10 classes according to the estimated beta in that period. Each class contained one-tenth of the stocks in the sample. The stocks with the lowest betas went into class 1. Class 2 contained stocks with slightly higher betas, and so on. They then looked at the frequency with which stocks jumped from one class to another. The more jumps, the less stability. You can see from Table - 8.1 that there is a marked tendency for stocks with very high or very low betas to stay that way. If you are willing to stretch the definition of stable to include a jump to an adjacent risk class, then from 40 to 70 percent of the betas were stable over the subsequent 5 years.

One reason that these estimates of beta are only imperfect guides to the future is that the stocks may genuinely change their market risk. However, a more important reason is that the betas in any one period are just estimates based on a limited number of observations. If good company news coincides by chance with high market returns, the stock's beta will appear higher than if the news coincides with low market returns. We can twist this the other way around. If a stock appears to have a high beta, it may be because it genuinely does have a high beta, or it may be because we have over estimated it.

This explains some of the fluctuation in betas observed by Sharpe and Cooper. Suppose a company's true beta really is stable. Its apparent (estimated) beta will fluctuate from period to period due to random measurement errors. So the stability of true betas is probably better than Sharpe and Cooper's results seem to imply.

Asset Betas and Equity Betas

Think again of what the company cost of capital is and what it is used for. We define it as the opportunity cost of capital for the firm's existing assets; we use it to value new assets which have the same risk as the old ones. The right beta for calculating the company cost of capital is the beta of the firm's existing assets, its asset beta.

Let us take a simple balance sheet with assets on the left and debt and equity on the right.

Asset value	Debt Value (D) Equity value (E)
Asset value	Firm value (V)

Note that the values of debt and equity add up to firm value ($D + E = V$), and that firm value equals asset value.

Stockholders own the firm's equity but they can't claim all of the asset value; they have to share it with debt holders. The debt holders receive part of the cash flows generated by the firm's assets, and they may bear part of the asset's risks. (For example, if the assets turn out to be worthless, there will be no cash to pay stockholders or debt holders.) But debt holders of big firms such as TISCO bear much less risk than stockholders. Debt betas are typically close to zero—close enough that for large blue-chip companies many financial analyst just assume $\beta_{debt} = 0$. But we want the asset beta, β_{asset} . How do we get it?

Suppose you buy all the firm's securities - 100 per cent of the debt and 100 percent of the equity. You would own the assets lock, stock, and barrel. You wouldn't have to share the firm's asset value with anyone; every rupee of cash the firm pays out would be paid out to you. You wouldn't share the risks with anyone else, either; you bear them all. Thus the beta of your debt plus equity portfolio would equal the firm's asset beta.

The beta of this hypothetical portfolio is just a weighted average of the debt and equity betas⁴.

$$\beta_{asset} = \beta_{portfolio} = \beta_{debt} \frac{debt}{debt + equity} + \beta_{equity} \frac{equity}{debt + equity}$$

Calculating LMN Ltd. Asset Beta and Company's Cost of Capital

Now that we know how to derive the beta of a firm's assets from the beta of its stock, we can return to the problem of figuring out company's cost of capital. Say LMN Ltd. common stock β is 0.36 and its common stock accounted for 35 percent of its market value. The remaining 65 percent consisted of debt and preferred stock. To keep matters simple we will just

⁴ Here we ignore certain tax complications. If debt interest generates valuable tax savings, then the formula for β_{asset} changes somewhat.

lump the preferred stock in with the debt and assume both are risk-free. This gives us the following estimates for the beta of LMN's assets:

$$\begin{aligned}\beta_{\text{asset}} &= \beta_{\text{debt}} = \beta_{\text{debt}} \frac{\text{debt}}{\text{debt} + \text{equity}} + \beta_{\text{equity}} \frac{\text{equity}}{\text{debt} + \text{equity}} \\ &= 0(.65) + .36(.35) = .13\end{aligned}$$

Of course, this is a very low number. The reason for this is that LMM's stock beta is low (only 0.36) despite its heavy use of debt and correspondingly high financial risk. When the financial risk is removed, we find the remaining business risk to be small.

With a risk-free rate of 7.8 per cent and an expected market risk premium of 8.3 percent, LMN's cost of capital is

$$\begin{aligned}r &= r_f + \beta_{\text{asset}} (r_m - r_f) \\ &= .078 + .13(.083) = .089, \text{ or } 8.9 \text{ per cent.}\end{aligned}$$

This estimate is probably low, because we arbitrarily assumed LMN's debt was totally risk-free. If we had used $\beta_{\text{debt}} = 0.15$,

$$\begin{aligned}\beta_{\text{asset}} &= \beta_{\text{debt}} + \beta_{\text{equity}} \\ &= .15(.65) + .36(.35) \\ &= .22\end{aligned}$$

$$\begin{aligned}r &= r_f + \beta_{\text{asset}} (r_m - r_f) \\ &= .078 + .22(.083) = .097, \text{ or } 9.7\%\end{aligned}$$

8.5 BUSINESS AND FINANCIAL RISK

A firm's asset beta reflects its business risk. The difference between its equity and asset beta reflects financial risk. More debt means more financial risk.

What would happen if LMN decided to use more debt and correspondingly less equity? It would not affect the firm's business risk. There would be no change in the firm's asset beta, and no change in the beta of a portfolio of all the firm's debt and equity security. The equity beta would change, however.

Let's go back to the formula for β_{asset} ,

$$\beta_{\text{asset}} = \beta_{\text{debt}} \frac{\text{debt}}{\text{debt} + \text{equity}} + \beta_{\text{equity}} \frac{\text{equity}}{\text{debt} + \text{equity}}$$

and solve the formula for β_{equity}

$$\beta_{\text{equity}} = \beta_{\text{asset}} + (\beta_{\text{asset}} - \beta_{\text{debt}}) \frac{\text{debt}}{\text{equity}}$$

If we assume LMN's debt is risk-free, we have

$$\beta_{\text{equity}} = .13 + (.13 - 0) = .36$$

But if the company switched to 75 percent debt, β_{equity} would go up to .52. On the other hand, if LMN paid off all its debt, we would expect to find:

$$\begin{aligned}\beta_{\text{equity}} &= \beta_{\text{asset}} (\beta_{\text{asset}} - \beta_{\text{debt}}) \frac{\text{debt}}{\text{equity}} \\ &= .13 + (.13 - 0) \\ &= .13\end{aligned}$$

With no debt, the firm's asset and equity betas would be exactly the same.

In general, the observed equity beta depends on the firm's asset beta, β_{asset} , the spread between the asset and debt betas, $\beta_{\text{asset}} - \beta_{\text{debt}}$, and the ratio of debt to equity. Figure 8.3 plots the relationship assuming risk free debt ($\beta_{\text{debt}} = 0$).

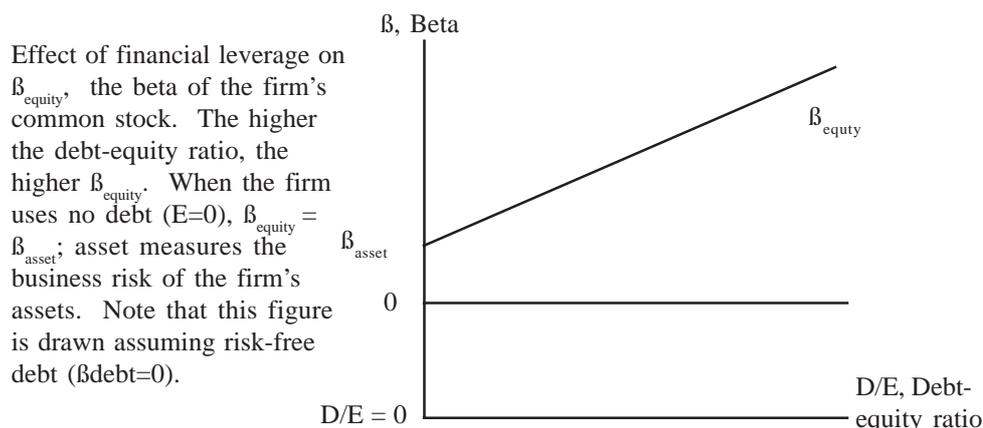


Figure 8.3: Effect of Financial Leverage on β of equity

In many ways we have given an oversimplified version of how financial leverage affects equity risks and returns. We have said nothing about taxes, for example. The finer points can wait. For now, there are really just two points to remember. First, financial leverage creates financial risk. The beta of the firm's stock increases in proportion to the amount borrowed. Second, asset betas can always be calculated as a weighted average of the betas of the various debt and equity securities issued by the firm. Of course, it is the asset beta that is relevant in capital-budgeting decisions, not the beta of the firm's stock.

8.6 WHAT DETERMINES ASSET BETAS?

Stock or industry betas provide a rough guide to the risk typically encountered in various lines of business. But an asset beta for, say, the steel industry can take us only so far. Not all investments made in the steel industry are "typical." What other kinds of evidence about business risk might a financial manager examine?

In some cases the asset is publicly traded. If so, we can simply estimate its beta from past price data. For example, suppose a firm wants to analyze the risks of holding a large inventory of copper. Because copper is a standardized, widely traded commodity, it is possible to calculate rates of return from holding copper and to calculate a copper beta.

What should we do if our asset has no such convenient price record? The advice is to search for characteristics of the asset that are associated with high or low betas. We wish we had a more fundamental scientific understanding of what these characteristics are. We see business risks surfacing in capital markets, but as yet there is no completely satisfactory theory describing how those risks are generated. Nevertheless, some things are known.

Cyclicality

Many people intuitively associate risk with the variability of book or accounting earnings. But much of this variability reflects diversifiable or unique risk. Lone prospectors in search of gold look forward to extremely uncertain future earnings, but whether or not they strike it rich is unlikely to depend on the performance of the market portfolio. Even if they do find gold, they do not bear much market risk. Therefore an investment in gold has a high standard deviation but a relatively low beta.

What really counts is the strength of the relationship between the firm's earnings and the aggregate earnings on all real assets. We can measure this either by the accounting beta or by the cash-flow beta. These are just like a real beta except that changes in book earnings or cash flow are used in place of rates of return on securities. We would predict that firms with high accounting or cash-flow betas should also have high stock betas - and the prediction is correct.

This means that cyclical firms - firms whose revenues and earnings are strongly dependent on the state of business cycle - tend to be high-beta firms. Thus you should demand a higher rate of return from investments whose performance is strongly tied to the performance of the economy.

Operating Leverage

We have already seen that financial leverage - in other words, the commitment to fixed debt charges - increases the beta of an investor's portfolio. In just the same way, operating leverage - in other words, the commitment to fixed production charges - must add to the beta of a capital project. Let's see how this works.

The cash flows generated by any productive asset can be broken down into revenue, fixed costs, and variable cost.

Cash flow = revenue - fixed cost - variable cost.

Costs are variable if they depend on the rate of output. Examples are raw materials, sales commission, and some labor and maintenance costs. Fixed costs are cash outflows that occur regardless of whether the asset is active or idle - property taxes, for example, or the wages of workers under contract.

We can break down the asset's present value in the same way:

$PV(\text{asset}) = PV(\text{revenue}) - PV(\text{fixed cost}) - PV(\text{variable cost})$

Or equivalently:

$PV(\text{revenue}) = PV(\text{Fixed Cost}) + PV(\text{variable cost}) + PV(\text{asset})$

Those who receive the fixed costs are like debtholders in the project. Those who receive the net cash flows from the asset are like holders of levered equity in $PV(\text{revenue})$.

We can now figure out how the asset's beta is related to the betas of the values of revenue and costs. We just use our previous formula with the betas relabeled:

$$\beta_{\text{revenue}} = \beta_{\text{fixed cost}} \frac{PV(\text{fixed cost})}{PV(\text{revenue})} + \beta_{\text{variable cost}} \frac{PV(\text{variable cost})}{PV(\text{revenue})} + \beta_{\text{asset}} \frac{PV(\text{asset})}{PV(\text{revenue})}$$

In other words, the beta of the value of the revenues is simply a weighted average of the beta of its component parts. Now the fixed-cost beta is zero by definition: whoever receives the fixed costs holds a safe asset. The betas of the revenues and variable costs should be approximately the same, because they respond to the same underlying variable, the rate of output. Therefore, we can substitute β_{revenue} for $\beta_{\text{variable cost}}$ and solve for the asset beta. Remember that $\beta_{\text{fixed cost}} = 0$.

$$\begin{aligned}\beta_{\text{asset}} &= \beta_{\text{revenue}} \frac{PV(\text{revenue}) - PV(\text{variable cost})}{PV(\text{asset})} \\ &= \beta_{\text{revenue}} \left(1 + \frac{PV(\text{fixed cost})}{PV(\text{asset})} \right)\end{aligned}$$

Thus, given the cyclical nature of revenue (reflected in β_{revenue}), asset beta is proportional to the ratio of the present value of fixed costs to the present value of the project.

Now we have a rule of thumb for judging the relative risks of alternative designs or technologies for producing the same product. Other things being equal, the alternative with the higher ratio of fixed costs to project value will have the higher project beta.

Firms or assets whose costs are mostly fixed are said to have high operating leverage. As we have seen, the analogy between financial and operating leverage is almost exact. The beta of the stock increases in proportion to the ratio of debt to equity, and the beta of the asset increases in proportion to the ratio of the value of the fixed costs to the value of the asset. Empirical tests confirm that companies with high operating leverage actually do have high betas.

Searching for Clues

Recent research suggests a variety of other factors that affect an asset's beta. But going through a long list of these possible determinants would take us too far afield.

You cannot expect to estimate the relative risk of assets with any precision, but good managers examine any project from a variety of angles and look for clues as to its riskiness. They know that high market risk is a characteristic of cyclical ventures and of projects with high fixed costs. They think about the major uncertainties affecting the economy and consider how projects are affected by these uncertainties.

8.7 DISCOUNTED CASH FLOW APPROACH

We have spent the bulk of this unit discussing how you might estimate the risk and required return on a project. We now have to worry a little about what happens as risk changes over the life of a project.

We have implied that an expected rate of return calculated from the capital asset pricing model

$$r = r_f + \beta(r_m - r_f)$$

This could be plugged into the standard discounted cash flow formula as

$$PV = \sum_{t=1}^T \frac{C_t}{(1+r)^t} = \sum_{t=1}^T \frac{C_t}{[1+r_f + \beta(r_m - r_f)]^t}$$

You should not take step without thinking about it first. In capital budgeting we must usually value cash flows extending over several future periods. The discounted cash flow formula does this in one step, but the capital asset pricing model looks at rates of return and prices over one period at a time.

One-period projects pose no problems.

$$PV = \frac{C_1}{1+r} = \frac{C_1}{1+r_f + \beta(r_m - r_f)}$$

Longer-lived assets likewise pose no problem if you have an estimate of PV₁, future value 1 period hence

$$PV = \frac{C_1 + PV_1}{1+r} = \frac{C_1 + PV_1}{1+r_f + \beta(r_m - r_f)}$$

But suppose that your company is evaluating the construction of a nuclear power station and asks your advice on how to calculate its present value. Would you tell it not to bother about anything other than the cash flow in the first period and the end-of-period value? Of course not. The end-of-period value depends on the cash flow in later periods. You would want a formula that explicitly took this into account. This is formally correct only if we know that the discount rates that will prevail in future periods will be the same as this year's. Among other things, this requires the asset's beta to be constant over the asset's entire future life. Only under that crucial assumption it is strictly proper to write down the discounted cash flow formula with a single discount rate for all future cash flows.

What does that assumption mean in practical terms? In order to answer that question we must develop alternative formulae for calculating present value when beta and r do vary.

Certainty Equivalent

Let us start with a single future cash flow C₁. If C₁ is certain, its present value is found by discounting at the risk-free rate r_f

$$PV = \frac{C_1}{1+r_f}$$

If the cash flow is risky, the normal procedure is to discount its forecasted (expected) value at a risk-adjusted discount rate r which is greater than r_f.⁵

Another approach is to ask, "What is the smallest certain return for which I would exchange the risky cash flow C₁? This is called the certainty equivalent of C₁, denoted by CEQ₁

Suppose that the forecasted value of the risky cash flow is Rs.1000, but that you would be willing to trade it for a safe cash flow of as little as Rs.800. Then Rs.800 is the certainty equivalent of the risky cash flow. You are indifferent between an Rs.800 safe return and an expected but risky cash flow of Rs.1000.

⁵ The quantity r can be less than r_f for assets with negative betas. But the betas of the assets which corporations hold are almost always positive.

What is the present value of Rs.1000 forecasted cash flow? It must be the same as the present value of a certain Rs.800, because by definition you are indifferent between the two flows. Suppose the risk-free rate of interest is $r_f = 0.08$. Then

$$\text{PV of forecasted Rs.1000 cash flow} = \frac{\text{CEQ}_1}{1+r_f} = \frac{800}{1.08} = \text{Rs. } 740.74$$

We could have gotten the same answer by discounting Rs.1000 at a risk-adjusted rate. We can figure out what the proper discount rate is. If

$$\text{PV} = \frac{1000}{1+r} = \text{Rs.}740.74$$

then $r = 0.35$, or 35 per cent

Now we have two equivalent expressions for PV.

$$\text{PV} =$$

As long as you look only one period into the future, the two formulas are exactly the same. But there are important differences when the concept of certainty equivalents is applied to cash flows generated by long-lived assets.

Relationship of Certainty Equivalent and Risk-Adjusted Discount Rate Formulas for Long-Lived Assets

We can easily extend the concept of certainty equivalents to long-lived assets:

$$\text{PV} = \sum_{t=1}^T \frac{\text{CEQ}_t}{(1+r_f)^t} = \sum_{t=1}^T \frac{a_t C_t}{(1+r)^t}$$

where a_t is the ratio of the certainty equivalent of a cash flow to its expected value ($a_t = \text{CEQ}_t/C_t$). Normally a_t will be positive, but less than 1.0⁶.

Table 8.4
Example showing certainty equivalents implied by use of constant risk-adjusted discount rate

Period	Expected Cash Flow = C_t	Present Value Using 10% Risk-Discount Rate, $\text{PV} = C_t/(1.10)^t$	Certainty Equivalent CEQ_t Implied By use of 10% Discount $\text{CEQ}_t = C_t \frac{1+r_f}{1+r}$	a_t Ratio of CEQ_t TO C_t	Present Value of CEQS at 4% Risk-Free Rate
0	-350	-350	-350	1.00	-350
1	100	91	95	0.945	91
2	100	83	89	0.894	83
3	100	75	85	0.845	75
4	100	68	80	0.799	68
5	100	62	76	0.755	62
Net present value =					29

Note: By using a constant risk-adjusted discount rate of 10 per cent the finance manager is implicitly making larger deductions for risk from the later cash flows. Notice that discounting the cash flows at 10 per cent or the certainty equivalents at 4 per cent would give $\text{NPV} = 29$.

⁶ The quantity a_t would be greater than 1.0 for negative-beta assets.

When we discount at a constant risk-adjusted rate r_t we are implicitly making a special assumption about the coefficients a_t . Consider an asset offering cash flows in 2 periods. If the certainty equivalent and risk-adjusted discount rate formulas are really equivalent, they should give the same present value for each cash flow:

$$\frac{C_1}{1+r} = \frac{a_1 C_1}{1+r_f} \quad \text{and} \quad \frac{C_2}{(1+r)^2} = \frac{a_2 C_2}{(1+r_f)^2}$$

But this implies that

$$a_1 \frac{1+r_f}{1+r} = \text{and } a_2 \frac{1+r_f^2}{1+r^2} = (a_1)^2$$

In general, you are justified in using a constant risk-adjusted discount rate r to value the cash flow for each period only if the value of a_t decreases over time at a constant rate. The formula is

$$a_t = \frac{1+r_f^t}{1+r} = (a_1)^t$$

Using Risk-Adjusted Discount Rates — An Example

Consider a project requiring Rs.350 today ($t = 0$) and offering expected cash flows of Rs.100 per year for 5 years. The risk-free rate is 4 percent, the market risk premium is 9 percent, and the estimated beta is 0.67; therefore, the financial manager settles on a discount rate of

$$\begin{aligned} r &= r_t + \beta(r_m - r_f) \\ &= 0.04 + .67(0.09) = 0.10, \text{ or } 10\% \end{aligned}$$

The project's net present value is calculated as

$$\text{NPV} = \text{PV} - 350 = \sum_{t=1}^5 \frac{100}{(1.10)^t} - 350 = \text{Rs.29}$$

What is the finance manager implicitly assuming about the values of a_t ? The answer is given by Table 8.4. By using a constant discount rate the finance manager is effectively making a much larger deduction for risk from the later cash flows. The larger deduction is reflected in lower values for a_t . Notice also that a_t decreases at a constant compound rate of about 5.5 percent per year.

It is usually reasonable to assume that risk increases at a constant rate. For example, if you are willing to assume that beta is constant in each future period, then the risk borne per period will be constant but cumulative risk will grow steadily as you look further into the future.

When You Cannot Use a Single Risk-Adjusted Discount Rate for Long-Lived Assets

The scientists at XYZ Ltd. have come up with an electric mop, and the firm is ready to go ahead with pilot production and test marketing. The preliminary phase will take a year and cost Rs.125,000. Management feels that there is only a 50 percent chance that pilot production and market tests will be successful. If they are, then XYZ will build a Rs.1 million plant which would generate an expected annual cash flow in perpetuity of Rs.250,000 a year after taxes. If they are not successful, the project will have to be dropped.

The expected cash flows (in thousands of Rupees) are

$$C_0 = -125$$

$$C_1 = 50\% \text{ chance of } -1000 \text{ and } 50\% \text{ chance of } 0 \\ = .5(-1000) + .5(0) = -500$$

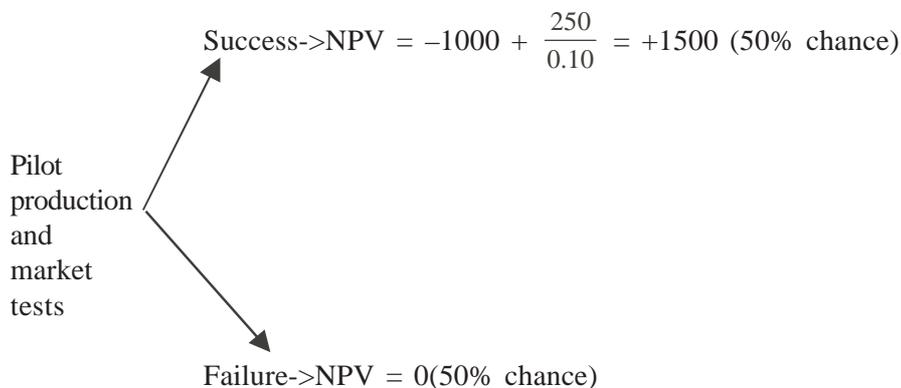
$$C_t, \text{ for } t = 2, 3, \dots \\ = 50\% \text{ chance of } 250 \text{ and } 50\% \text{ chance of } 0. \\ = +.5(250) + .5(0) = 125$$

Management has little experience with consumer products and considers this a project of extremely high risk. Therefore, they discount the cash flows at 25 percent, rather than XYZ's normal 10 percent standard:

$$NPV = -125 - \frac{500}{1.25} + \sum_{t=2}^{\infty} \frac{125}{(1.25)^t} - 125 \text{ or } -\text{Rs.}125,00$$

This seems to show that the project is not worthwhile.

Management's analysis is open to criticism if the first year's experiment resolves a high proportion of the risk. If the test phase is a failure, then there's no risk at all - the project is certainly worthless. If it is a success, there could well be only normal risk from there on. That means there is a 50 percent chance that in 1 year XYZ will have the opportunity to invest in a project of normal risk, for which the normal discount rate of 10 per cent would be appropriate. As such, they have a 50 percent chance to invest Rs.1 million in a project with a net present value of Rs.1.5 million:



Thus, we could view the project as offering an expected payoff of $.5(1500) + .5(0) = 750$ or Rs.750,000 at $t = 1$ on a Rs.125,000 investment at $t = 0$. Of course, the certainty equivalent of the payoff is less than Rs.750,000 but a_t would have to be very small to justify rejecting the project. For example, if the CEQ is half the expected value ($a_t = 0.5$), and the risk-free rate is 7 per cent, the project is worth Rs.225,500;

$$NPV = C_0 + \frac{a_1 C_1}{1 + r_f} \\ = -125 + \frac{0.5(750)}{1.07} = 225.5, \text{ or } \text{Rs. } 225,500$$

Not bad for a Rs.125,000 investment - and quite a change from the negative NPV that management got by discounting all future cash flows at 25 percent.

A Word of Caution

We sometimes hear people say that because distant cash flows are “riskier”, they should be discounted at a higher rate than earlier cash flows. That’s quite wrong: any risk-adjusted discount rate automatically recognizes the fact that more distant cash flows have more risk. The reason is that the discount rate compensates for the risk borne per period. The more distant the cash flows, the greater the number of periods and the larger the total risk adjustment.

Activity I

- a) Mention the most fundamental difference between Certainty Equivalent and the Risk Adjusted Discount Rate approaches of incorporating risk in Project evaluation.

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- b) Given a choice between Certainty Equivalent and Probability Distribution approach, to analyze risk, which one would you prefer? Why?

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- c) Identify two critical factors that affect asset betas.

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- d) Given a choice between Certainty Equivalent and Probability Distribution approach, to analyze risk, which one would you prefer? Why?

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8.8 SUMMARY

Capital Asset Pricing Model is used to estimate risk adjusted discount rate for making investment decisions. In order to find an appropriate discount rate it is necessary to find project beta which may be different from the firm’s beta executing that project. The beta of the assets is weighted average of debt and equity betas. A firm’s asset beta reflects its business risk, whereas the difference between its equity and asset beta reflects financial risk.

Certainly equivalent is the smallest certain return which one would exchange for a risky cash flow. Empirically it has been found that the betas are stable over time.

8.9 SELF ASSESSMENT QUESTIONS

1. “For a high beta Project, you should use a high discount rate to value positive cash flows and a low discount rate to value negative cash flows.” Is this statement correct? Should the sign of the cash flow affect the appropriate discount rate?
2. “The errors in estimating beta are so great that you might just as well assume that all betas are one.” Do you agree?
3. A Project has a forecasted cash flow of Rs.100,000 in year 1 and Rs.120,000 in year 2. The risk free rate is 8 percent, the estimated risk premium on the market is 10 per cent, and the project has a beta of 0.5 If you use a constant risk adjusted discount rate, what would be:
 - (a) The present value of the project?
 - (b) The certainty-equivalent cash flows in year 1 and 2?
 - (c) The ratio (a₁) of the certainty - equivalent cash flows to the expected cash flows in years 1 and 2?
4. (a) PQR Ltd has the following capital structure:

Security	Beta value	Total market value
Debt	0	100,000
Preferred stock	0.20	40,000
Common stock	1.20	200,000

What is the firm’s asset beta (that is, beta of its stock if it were all equity financed)?

- (b) Assume the Capital Asset Pricing Model is correct. What discount rate should PQR Ltd. set for investments that expand the scale of its operations without changing its asset beta? Assume any new investment is all-equity financed. Plug in numbers that are reasonable today. Specify two discount rates, one real and one nominal.

8.10 FURTHER READINGS

Brigham E.F., “*Financial Management Theory and Practice*”, Dryden Press, New York.

Levy H., Sarnat M., “*Financial Decision Making under Uncertainty*”. Academic Press, New York.

Sharpe W.F., Alexander G.J., Bailey J.V., “*Investments*,” Prentice Hall, New Delhi.