UNIT 9 PROJECT EVALUATION UNDER RISK AND UNCERTAINTY

Objectives

The objectives of this unit are:

- to provide conceptual understanding of state of certainty, risk and uncertainty,
- to explain various methods of measuring project risk.

Structure

9.1 Concept of Certainty, Risk and Uncertainty
9.2 Measurement of Project Risk :
    9.2.1 Probability Distribution
    9.2.2 Sensitivity Analysis
    9.2.3 Scenario Analysis
    9.2.4 Monte Carlo Simulation
    9.2.5 Decision Tree Analysis
9.3 Summary
9.4 Self Assessment Questions
9.5 Further Readings

9.1 CONCEPT OF CERTAINTY, RISK AND UNCERTAINTY

In the preceding chapters we considered investment decision without referring to the risk elements in projects. Rule for acceptance or rejection of projects was that any project with positive net present value will be accepted and the one with the negative net present value would be rejected. However, the mere fact that the finance manager does not know before he makes the investment as to what would be the gains from the project indicates that uncertainty is attached to every investment project and different projects have varying degrees of risk. Since the valuation of the firm is very likely to be affected by the amount of risks assumed by it in accepting a project, a finance manager must take cognizance of risk factor while taking investment decision and additional adjustments must be made to cover risks.

Certainty is a state of nature which arises when outcomes in terms of cash flows are known and determinate. For example, if one invests Rs. 20,000 in five-yearly government bonds which is expected to yield 7% tax free returns, then the return on the investment @ 7% can be estimated quite precisely. Thus, the outcome is known to have probability of 1.

Risk involves situations in which the probabilities of cash flows occurring are known and these probabilities are objectively or subjectively determinable. The main attribute of risk situation is that the event is repetitive in nature and possesses a frequency distribution. It is the inability to predict with perfect knowledge the course of future events that introduces risk.

In contrast, when an event is not repetitive and is unique in character and the finance manager is not sure about probabilities of cash flows themselves, uncertainty is said to prevail. Uncertainty is subjective phenomenon. In such a situation no observation can be drawn from frequency distributions. Practically no generally accepted methods could so far be evolved to deal with situation of uncertainty while there are a number of techniques to deal with risk. As such, the term risk and uncertainty will be used interchangeably in the following paragraphs.

Measuring the Risk of a Project

Since prime objective of a finance manager is to maximize value of the firm and the degree of risk in a project affects the value, he must measure degree of risk in different projects under consideration. The following paragraphs throw light on this aspect.

9.2 MEASUREMENT OF PROJECT RISK

9.2.1 Probability Distribution

The Problem of Project Risk

As noted above, "riskiness" of an investment project is the variability of its cash flows from those that are expected. The greater the variability, the riskier project is said to be. For each project under consideration, we can make estimates of the future cash flows. Rather than estimate only the most likely cash-flow outcome for each year in the future, we estimate a number of possible outcomes. In this way we are able to consider the range of possible cash flows for a particular future period rather than just the most likely cash flow.

An Illustration

To illustrate the formunation of multiple cash-flow forecasts for a future period, suppose that we had two investment proposals under consideration. Suppose further that we were interested in making forecasts for the following alternative states of the economy : deep recession, mild recession, normal, minor boom, and major boom. After assessing the future under each of these possible states, we estimate the followin net cash flows for the next year :

State of the Economy	Annual Cash Flows Year1	
	Proposal A (Rs.)	Proposal B (Rs.)
Deep recession	3,000	2,000
Mild recession	3,500	3,000
Normal	4,000	4,000
Minor boom	4,500	5,000
Major boom	5,000	6,000

We see that the dispersion of possible cash flows for proposal B is greater than that for proposal A. Therefore, we might say that it is relatively riskier. To quantify our analysis of risk, however, we need additional information. More specifically, we need to know the likelihood of the occurrence of various states of the economy. Assume that our estimate of the probability of a deep recession occurring next year is 10 per cent, of a mild recession 20 per cent, of a normal economy 40 per cent, of a minor boom 20 per cent, and of a major economic boom 10 per cent. Given this information, we are now able to formulate a probability distribution of possible cash flows for proposals A and B, as follows :

We can graphically depict the probability distributions, as shown in Figure 9.1. As we see, the dispersion of cash flows is greater for proposal B than it is for proposal A, despite the fact that the most likely outcome is the same for both investment proposals - namely Rs. 4,000. The critical question is whether dispersion of cash flows should be considered. If risk is associated with the probability distribution of possible cash flows, such that the greater the dispersion, the greater the risk, proposal B would be the riskier investment. If management, stockholders, and creditors are averse to risk, proposal A would then be preferred to proposal B.

State of the Economy	Proposal A		Proposal B	
	Probability	Cash Flow (Rs.)	Probability	Cash Flow (Rs.)
Deep recession	.10	3,000	.10	2,000
Mild recession	.20	3,500	.10	3,000
Normal	.40	4,000	.40	4,000
Minor boom	.20	4,500	.20	5,000
Major boom	.10	5,000	.10	6,000

Expectation and measurement of Dispersion: A Cash-Flow Example

The probability distributions shown in Figure 9.1 can be summarized in terms of two parameters of the distribution: (1) the expected value and (2) the standard deviation.

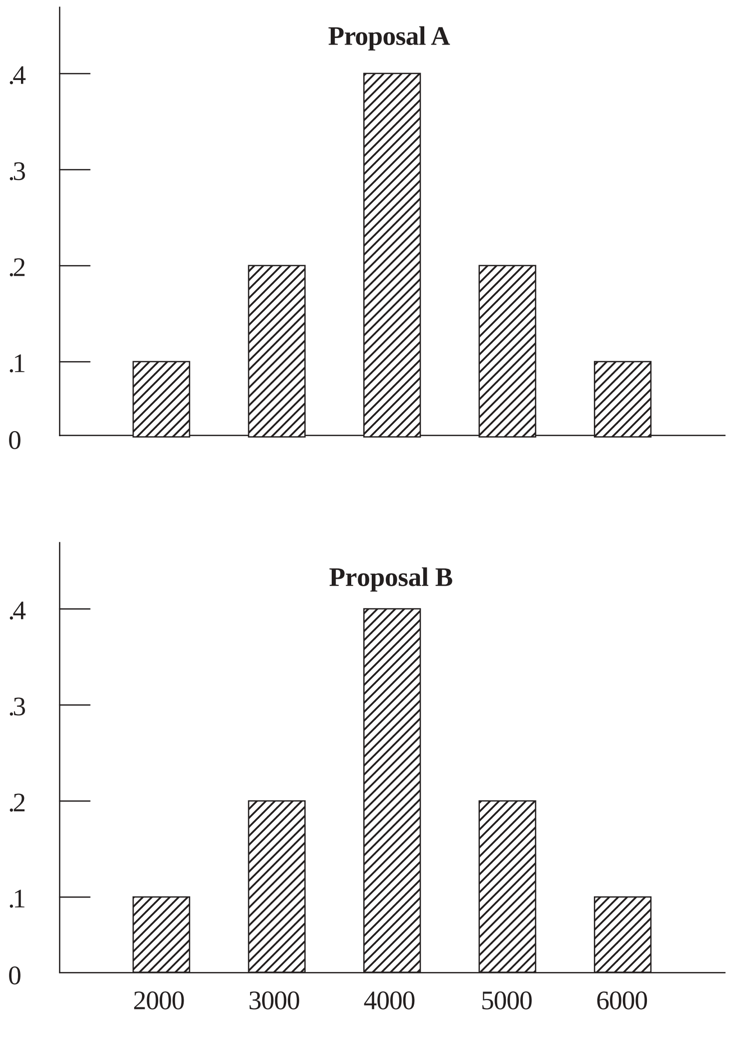

Figure 9.1: Comparison of two proposals using probability distributions of possible cash flows.

The expected value of a cash-flow probability distribution for time period t, CF_t, is defined as

$$\overline{CF}_t = \sum_{x=1}^{n} = (CF_{xt})\,(P_{xt})$$

where CF_{xt} is the cash flow for the xth possibility at time period t, P_{xt} is the probability of that cash flow occurring, and n is the total number of cash-flow possibilities occurring at time period t. Thus, the expected value of cash flow is simply a weighted average of the possible cash flows, with the weights being the probabilities of occurrence.

The conventional measure of dispersion is the standard deviation which completes our two-parameter description of a cash-flow distribution. The tighter the distribution, the lower this measure will be; the wider the distribution, the greater it will be. The cash-flow standard deviation at time period t, σ_t, can be expressed mathematically as where $\sqrt{\ }$ represents the square-root sign. The square of the standard deviation, , is known as the variance of the distribution.

$$\sigma_t = \sqrt{\sum_{x=1}^{n} (CF_{xt} - \overline{CF}_t)^2\,(P_{xt})}$$

The standard deviation is simply a measure of the tightness of a probability distribution. It is a statistical measure of the variability of a distribution around its mean. It is the square root of the variance. For a normal, bell-shaped distribution, approximately 68 per cent of the total area of the distribution falls within one standard deviation on either side of the expected value. This means that there is only a 32 per cent chance that the actual outcome will be more than one standard deviation from the mean. The probability that the actual outcome will fall within two standard deviations of the expected value of the distribution is approximately 95 per cent, and the probability that it will fall within three standard deviations is over 99 per cent.

An Illustration. To illustrate the derivation of the expected value and standard deviation of a probability distribution of possible cash flows, consider again our previous two-proposal example.

Proposal A

Possible Cash Flow, CF_{x1} (Rs)	**Probability of Occurrence, P_{x1}**	**$(CFx1)\,(P_{xt})$ (Rs.)**	**$(CF_{x1}) - \overline{CF}_1)^2\,(P_x)$ (Rs)**
3,000	.10	300	$(3{,}000 - 4{,}000)^2$ (.10)
3,500	.20	700	$(3{,}500 - 4{,}000)^2$ (.20)
4,000	.40	1600	$(4{,}000 - 4{,}000)^2$ (.40)
4,500	.20	900	$(4{,}500 - 4{,}000)^2$ (.20)
5,000	.10	500	$(5{,}000 - 4{,}000)^2$ (.10)
	a = 1.00	a = 4,000 = $\overline{CF}_1$	a = 300,000 = s^2_1 = $(300{,}000)^{.5}$ = 548 = s_1

Proposal B

Possible Cash Flow, CF_{x1} (Rs)	Probability of Occurrence, P_{x1}	$(CFx1)\ (P_{xt})$ (Rs.)	$(CF_{x1}) - CF_1)^2\ (P_x)$ (Rs)
2,000	.10	200	$(2,000 - 4,000)^2\ (.10)$
3,000	.20	600	$(3,500 - 4,000)^2\ (.20)$
4,000	.40	1600	$(4,000 - 4,000)^2\ (.40)$
5,000	.20	1000	$(5,500 - 4,000)^2\ (.20)$
6,000	.10	600	$(6,000 - 4,000)^2\ (.10)$
	a = 1.00	a = 4,000 = CF_1	a = 1,200,000 = s^2_1
			= $(1,200,000)^{.5}$ = 1,095 = s_1

The expected value of the cash-flow distribution for proposal A is Rs. 4000, the same as for proposal B. However, the standard deviation for proposal A is Rs. 548, Thus, proposal B has a higher standard deviation, indicating a greater dispersion of possible outcomes – so we would say that it has greater risk.

Coefficient of Variation : A measure of the relative dispersion of a distribution is the coefficient of variation (CV). Mathematically, it is defined as the ratio of the standard deviation of a distribution to the expected value of the distribution. Thus, it is simply a measure of risk per unit of expected value. For proposal, A, the coefficient of variation is

$$CV_A = \text{Rs. } 548/\text{Rs. } 4,000 = .14$$

While that for proposal B is

$$CV_B = \text{Rs. } 1,095/\text{Rs. } 4,000 = .27$$

Because the coefficient of variation for proposal B exceeds that for proposal A, it has a greater degree of relative risk.

Total Project Risk

If investors and creditors are risk averse – and all available evidence suggests that they are - it is necessary for management to incorporate the risk of an investment proposal into its analysis of the proposal's worth. Otherwise, capital budgeting decisions are unlikely to be in accord with the objective of maximizing share price. Having established the need for taking risk into account, we proceed to measure it for individual investment proposal. But remember that the riskiness of a stream of cash flows for a project can, and often does, change with the length of time in the future that the flows occur. In other words, the probability distributions are not necessarily the same from one period to the next.

Activity 1

a) Define the following.

- Certain Projects
- Risky Projects
- Uncertain Projects

....

....

....

....

....

....
....
....
....
....
....
....
....
....

b) Try to know from some finance managers the methods that they have used to incorporate risk while evaluating investment Projects.

....
....
....
....
....
....
....
....
....
....
....
....

9.2.2 Sensitivity Analysis

Intuitively, we know that most of the variables which determine a project's cash flows are based on some type of probability distribution. We also know that if a key input variable, such as units sold, changes, so will the project's NPV. Sensitivity analysis indicates exactly how much NPV will change in response to a given change in an input variable, other things remaining constant. Sometimes called "what if" analysis, sensitivity analysis answers questions such as these: "What if sales are only 20,000 units rather than 25,000? What then will happen to NPV?"

Sensitivity analysis begins with a base case situation based on expected input values. To illustrate the procedure, let us consider the data given in Table 9.1, where projected income statements for ABC Ltd. project were shown. The values for unit sales, sales price, fixed costs, and variable costs are the expected, or base case, values, and the resulting Rs. 11,465,923 NPV shown in Table-9.2 is called the base case NPV. Now we ask a series of "what if" questions: "What if sales quantity is 20 per cent below the expected level?" "What if sales prices fall?" "What if variable costs are 70 per cent of total sales rather than the expected 65 percent?" Sensitivity analysis is designed to provide the decision maker with answers to questions such as these.

In a sensitivity analysis, we change each variable by specific percentages above and below the base case value, calculate new NPVs, holding other things constant, and then plot the derived NPVs against the variable in question. Figure 9.2 shows the ABC's project sensitivity graphs for three of the key input variables. The Table 9.3 gives the NPVs that were used to construct the graphs. The slopes of the lines in the graphs show how sensitive NPV is to changes in each of the inputs' the steeper the slope, the more sensitive the NPV is to the change in the variable. Here we see that the project's NPV is very sensitive to changes in variable costs, fairly sensitive to changes in sales volume, and relatively insensitive to changes in the cost of capital.

If we were comparing two projects, then, other things, held constant, the one with the steeper sensitivity lines would be regarded as riskier-a relatively small error in estimating variables such as the variable cost per unit or demand for the product would produce a large error in the project's projected NPV. Thus, sensitivity analysis provides useful insights into the relative riskiness of different projects.

Table 9.1: ABC : Operating and Networking Capital Cash Flows, 1988-1993

	1988	1989	1990	1991	1992	1993
Unit Sales	25000	25000	25000	25000	25000	25000
Sales Price[a]	2000	2332	2472	2620	2777	2944
Net Sales[a]	55000000	58300000	61800000	65500000	69425000	73600000
Variable Costs[b]	35750000	37895000	40170000	42575000	45126250	47840000
Fixed Cost (Overhead)[a]	8000000	8480000	8988800	9528128	10099816	10705805
R&D expenses[c]	12500000	1250000	1250000	1250000	1250000	1250000
Depreciation (building)[d]	400000	800000	720000	640000	560000	560000
Depreciation (equipments)[d]	1425000	2090000	1995000	1995000	1995000	0
Earnings before taxes	8175000	7785000	8676200	9511872	10393934	13244195
Taxes (46%)	3760500	3581100	3991052	4375461	4781210	6092330
Project net income	4414500	4203900	46855148	5136411	5612724	7151865
Noncash expenses[e]	3075000	4140000	3965000	3885000	3805000	1810000
Cash flow from operations[f]	7489500	8343900	8650148	9021411	9417724	8961865
Addition to NWC[g]	(396000)	(420000)	(444000)	(471000)	(501000)	(8832000)
NWC cash flows	7093500	7923900	8206148	8550411	8916724	17793865

a 1988 estimate increased by an assumed 6 per cent inflation rate.

b 65 per cent of net sales.

c If the project is accepted, ABC will amortize the Rs. 7.5 million of capitalized R&D cost over 6 years, so it will have a noncash, deductible expenses for Rs. 7,500,000/6 = Rs. 1,25,000 per year.

d ACRS depreciation rates are follows

Year	1	2	3	4	5	6
Building	5%	10%	9%	8%	7%	7%
Equipment	15%	22%	21%	21%	21%	-

e Sum of R&D expenses and depreciation on building and equipment.

f Net income plus noncash expenses.

g 12 per cent of next year's increase in sales. For example, 1989 sales are estimated at Rs. 3.3 million over 1988 sales, so the addition to NWC in 1988 to prepare for the 1989 sales increase in 0.12 (3,300,000) = 396,000. The cumulative working capital investment will be recovered in 1993.

Table 9.2: ABC : Time Line of Consolidated Cash Flows
End-of-year net Cash Flows

1985	1986	1988	1987	1989	1990	1991	1992	1993
4650000	4000000	19600000	7093500	7923900	8206148	8550411	8916724	22961065

Payback period : 5.6 years from first outflow.
IRR : 20.4% versus a 10% cost of capital.
NPV : 11,465,923
PI : 1.5

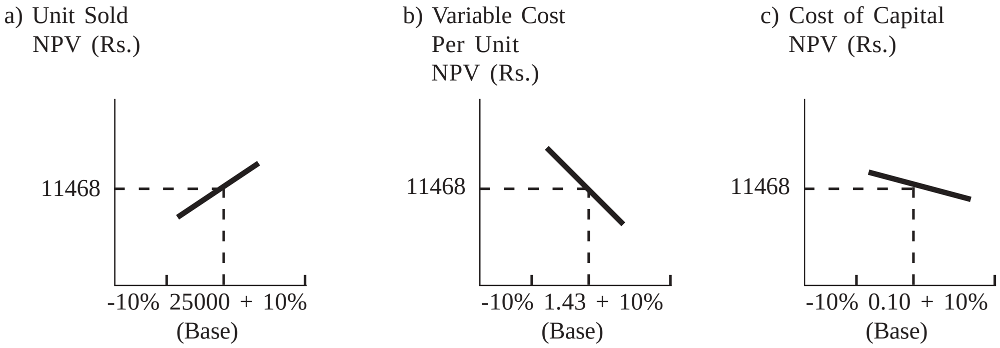

Figure 9.2: Sensitivity Analysis for ABC (Thousands of Rupees)

Table 9.3

	Net Present Value		
Change from Base Value	**Units Sold (Rs.)**	**Variable Cost per Unit (Rs.)**	**Cost of Capital (Rs.)**
-10%	7549	19416	13109
-5	9463	15442	12273
0	11468	11468	11468
+5	13472	7493	10692
+10	15476	3519	9946

9.2.3 Scenario Analysis

Although sensitivity analysis is widely used in industry, it does have limitations. Consider, for example, a proposed coal mine whose NPV is highy sensitive to changes in both output and sales prices. However, if a utility company has contracted to buy most of the mine's output at a fixed price per ton, plus inflation adjustments, then the mining venture may not be very risky in spite of the steep sensitivity lines. In general, a project's risk depends on both (1) its sensitivity to changes in key variables and (2) the range of likely values of these variables as reflected in their probability distributions. Because sensitivity analysis considers only the first factor, it is incomplete.

A risk analysis technique which considers both the sensitivity of NPV to changes in key variables and also the range of likely variable values is scenario analysis. Here, the operating executives pick a "bad" set of circumstances (low unit sales, low sales price, high variable cost per unit, high construction cost, and so on) and a "good" set. The NPV under the "bad" and "good" conditions would be calculated and compared to the exepcted, or base case, NPV.

Table 9.4: Scenario Analysis Results Summary

Scenario	Sales Volume (Units)	Sales Price (Rs.)	NPV (Thousands) (Rs.)
Worst case	5,000	1,700	22,421
Base case	25,000	2,200	11,468
Best case	40,000	2,700	50,093

As an example, let us return to the ABC project. Assume that ABC executives are fairly confident in their estimates of all the project's cash flow variables except price and unit sales. Further, suppose they regard a drop in unit sales below 5,000, or a rise above 40,000 units, as being extremely unlikely. Similarly, they expect the sales price as set in the marketplace to fall within the range of Rs. 1,700 to Rs. 2,700. Thus, 5,000 units at a price of Rs. 1,700 defines the lower bound or the worst case scenario, while 40,000 units at a price of Rs. 2,700 defines the upper bound or the best case scenario. Remember that the expected, or base case, values are 25,000 units at a price of Rs. 2,200. Also, note that the indicated sales prices are for 1988, with future years' prices expected to rise because of inflation.

To carry out the scenario analysis, we use the worst case variable values to obtain the worst case NPV and the best case variable values to obtain the best case NPV. Table 9.4 summarizes the results of the analysis. We see that the base case forecasts a positive NPV; the worst case, a negative NPV; and the best case, a very large positivie NPV. However, it is not easy to interpret this scenario analysis, or to make a decision based on it. In our example, we can say that there is a chance of losing on the project, but we cannot easily attach a specific probability to this loss. Clearly, what we need is some idea about the probability of occurrence of the worst case, the best case, the most likely case, and all the other cases that might arise. This leads us directly to Monte Carlo simulatioin, which is described in the next section.

9.2.4 Monte Carlo Simulation

Monte Carlo simulation, so named because this type of analysis grew out of work on the mathematics of casino gambling, ties together sensitivities and input variable probability distributions. However, simulation requires a relatively sophisticated computer, coupled with an efficient financial planning software package, while scenario analysis can be done using a hand-held calculator.

Table 9.5: Probability Distribution for ABC's Project Sales Price

Sales Price (1)	Probability (2)	Associated Random Numbers (3)
Rs. 1,700	0.05	00-04
2,000	0.20	05-24
2,200	0.50	25-74
2,400	0.20	75-94
2,700	0.05	95-99

The first step in a computer simulation is to specify a probability distribution for each of the key variables in the analysis. To illustrate, suppose we have estimated the probability distribution of the ABC's Project sales price as represented by Columns 1 and 2 of Table 9.5. The expected sales price is Rs. 2,200, but the price can range from Rs. 1,700 to Rs. 2,700. The third column gives a set of random numbers associated with each price estimate. Notice that in column 2, there is a 5 percent probability that sales price will be as low as Rs. 1,700; therefore, 5 digits (0,1,2,3, and 4) are assigned to this price. Twenty digits are assigned to a price of Rs. 2,000 and so on for the other possible prices. Once the distributions and associated random numbers have been specified for all the key variables - in other words, once a table such as 9.5 has been set up for sales quantity, unit variable costs, construction costs, and so on- the computer simulation can begin. These are the steps involved:

1. Computers have stored in them, or they can generate random numbers. First, on Trial Run 1, the computer will select a different random number for each uncertain variable. For example, it might select 44 for units sold, 17 for the sales price, and 16 for labour costs.
2. Depending on the random number selected, a value is determined for each variable. The 17 associated with the sales price indicates in Table 9.5 that the appropriate sales price for use in the firs turn Rs. 2,000. Values for all the other variables are set in like manner.
3. Once a value has been established for each of the variables, the computer generates a set of income statements and cash flows. These cash flows are then discounted at the cost of capital (which may also be treated as a random variable), and the result is the net present value of the project on the computer's first run.
4. The NPV generated on Run 1 is stored in memory, and the computer then goes on to Run 2. Here a different set of random numbers, and hence cash flows, is used. The NPV generated in Run 2 is again stored, and the model proceeds on for perhaps 500 runs. Modern computers can complete this operation almost instantaneously for a vary low cost.
5. The stored NPVs (all 500 of them) are then printed out in the form of a frequency distribution, together with the expected NPV and the standard deviation of this NPV.

Using this procedure, we can perform a simulation analysis on ABC's project. As in our scenario analysis, we have simplified the illustration by specifying the distributions for only two key variables, sales quantity and sales price. For all the other variables, we merely specify their expected values.

In our simulation analysis, we assume that sales prices can be represented by a continuous normal distribution. Further, suppose that the expected value is Rs. 2,200 and that the actual sales price is very unlikely to vary by more than Rs. 500 from the expected value, i.e., fall below Rs. 1,700 or rise above Rs. 2,700. We know that in a normal distribution, the expected value plus or minus three standard deviations will encompass virtually the entire distribution. Thus, for sales price, three standard deviations should equal Rs. 500, so as a reasonable approximation, we assume that $s_{\text{sales price}}$ = Rs. 500/3 = Rs. 166.67 = Rs. 167. Therefore, we tell the computer that the sales price distribution is a normal distribution with an expected value of Rs. 2,200 and a standard deviation of Rs. 167.

Next, we assume that the estimated distribution of unit sales is also symmetric. Further, the expected value is 25,000 units, but sales could be as high as 40,000 units, given our production capacity, but if public acceptance is poor, sales could be as low as 10,000 units. We could have again specified a normal distribution, but in this case, management feels that a triangular distribution, with an expected value of 25,000 a lower limit of 10,000, and an upper limit of 40,000 is most appropriate.

Table 9.6: Summary of Simulation Result

Probability of NPV or IRR being Greater than the Indicated Value (Thousands of Rupees)									
	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
NPV	-1860	1825	5144	8397	11393	13683	16516	19639	25286
IRR	0.080	0.119	0.151	0.179	0.204	0.221	0.242	0.263	0.299
NPV Sample Statistics									
Mean			Standard Deviation			Skewness			
11228			10124			0.1			

We used the Interactive Financial Planning System (IFPS) to conduct a simulation analysis of the project. Then the key results of our simulation are presented in Table 9.6. The top line of the table shows the cumulative probability distribution. Suppose someone asks, "What is the probability that the project will have an NPV greater than Rs. 5,000,000?" The answer is, "About 70 per cent," because NPV = 5,000,000 lies between 70 and 80 percent but much closer to 70 per cent.

The primary advantage of simulation is that it shows us the range of possible outcomes, with attached probabilities, not just a point estimate of the NPV. The expected NPV can be used as a measure of the projects profitability, while the variability of this NPV as measured by s_{NPV} can be used to measure risk. To illustrate, the ABC's project's expected NPV is Rs. 11,288,000 and the standard deviation of this NPV, as calculated by the computer in the simulation, is s_{NPV} = Rs. 10,124,000. If we assume that ABC's average project has an expected NPV of Rs. 975,000 and sNPV = Rs. 370,000, then we can calculate the coefficient of variation (CV) for this project and compare it with the CV of the average project as follows:

$$\text{Coefficient of variation} = \text{CV}\ \frac{\text{Standard deviation}}{\text{Expected value}} = \frac{\sigma_{NPV}}{\text{Expected NPV}}$$

$$CV_{Project} = \frac{\text{Rs.}10{,}124{,}000}{\text{Rs.}11{,}228{,}000} = 0.90$$

$$CV_{Average\,Project} = \frac{\text{Rs.}370{,}000}{\text{Rs.}975{,}000} = 0.38$$

Since the coefficient of variation is a standardized risk measure, it can be used to compare the relative riskiness of projects which differ in size. We see that the CV of this project is much larger than the CV of ABC average project. To account for risk, ABC adds two percentage points to the cost of capital of such high-risk projects as this particular project. Our analysis thus far was based on ABC average cost of capital as 10 per cent. Therefore, we must now reevaluate the project with a project cost of capital of 12 per cent. When evaluated at a cost of capital of 12 per cent, the NPV of the project, using expected values of all variables, is Rs. 8,533,722. Thus, even at the high-risk cost of capital, the office robot project has a large positive NPV. Thus, it appears to be acceptable.

Activity 2

A) Write two points of difference between Sensitivity Analysis and Scenario Analysis of determination degree of risk in investment projects.

....

....

....

....

....

....

....

....

B) List out the five steps involved in Monte Carlo Simulation of Analyzing risk in investment projects.

....

....

....

....

....

....

....

....

....

....

9.2.5 Decision Tree Analysis

Many capital budgeting decisions are not made at a single point in time. Rather, they consist of two or more sequential decisions, which are made as the project progresses through stages. For example, suppose ABC is considering the production of a product X for some other company. The capital budgeting decision for this project will be broken down into three stages, as set forth in Figure 9.3.

Stage 1 : At t = 0, conduct a Rs. 500,000 study of the market potential for this product X.

Stage 2 : If it appears that a sizable market for this product X does exist, then at t = 1, spend Rs. 1,000,000 to design and fabricate several prototype of Product X.

Stage 3 : If reaction to the Prototype Product X is good, then at t = 2, build a production plant with a net cost of Rs. 10,000,000. If this stage were reached, the net payoff is expected to be either Rs. 13,000,000 or Rs. 16,000,000 depending on the state of the economy, competition, and so forth.

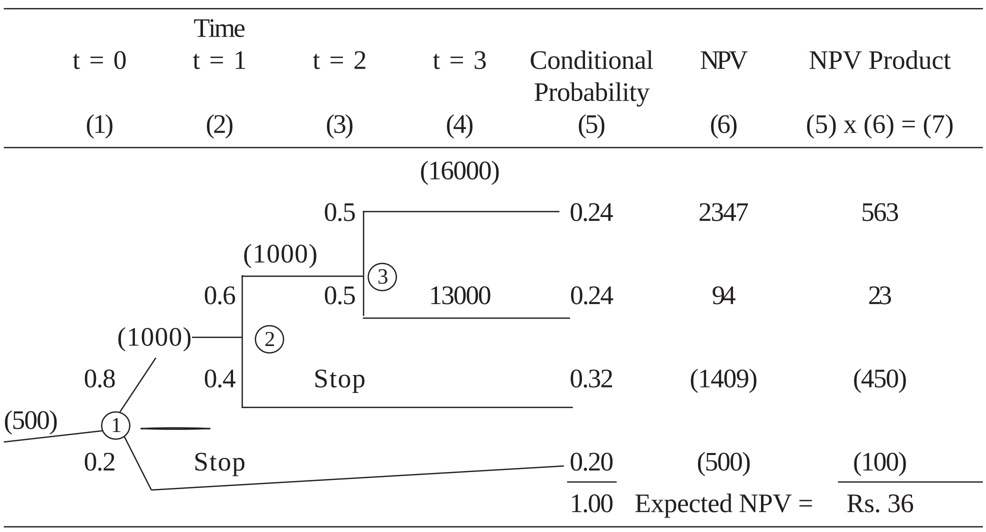

Figure 9.3: ABC Decision Tree (Thousands of Rs.)

The diagram in Figure 9.3 is called a decision tree, a procedure often used to analyze multi-stage, or sequential, decisions. A decision tree lays out the analysis like the branches of a tree. In Figure 9.3, we assume that as on year goes by between decisions and that a single net cash inflow from the project would occur one year after the final decision to go into production. Each circle represents a decision points, or stage. The rupee value to the left of each decision point represents the investment required if the decision is "go" at that point. Each diagonal line represents a branch of the decision tree, and each branch has an estimated probability. For example, if ABC decides to "go" with the project at Decision Point 1, it will have to spend Rs. 500,000 on a marketing study. Management estimates that there is a probability of 0.8 that the study will produce favourable results, leading to the decision to move on to Stage 2, and a 0.2 probability that the marketing study will produce negative results, indicating that the project should be cancelled after State 1. If the project is stopped here, the cost of ABC will be Rs. 500,000 for the initial market study.

If the marketing study is undertaken, and if it does yield positive results, then ABC will go on to Decision Point 2 and spend Rs. 1,000,000 on the product. Managemetn estimates (before even making the initial Rs. 500,000 investment) that there is a 60 per cent probability that product will be useful and a 40 per cent probability that it will not be useful. If the management accept the product, then company would spend the final Rs. 10,000,000 while if the management do not like it, the project would be dropped. Finally, if ABC does go into production, the payoff is assmed to be either Rs. 16,000,000 or Rs. 13,000,000, with each outcome having a 50 per cent probability. (Although we used only two production outcomes for simplicity, we could have used any number of outcomes or even a continuous distribution of outcomes.)

Column 5 of Figure 9.3 gives the conditional probability of occurrence of each final outcome. Each conditional probability is obtained by multiplying together all probabilities on a particular branch. For example, the probability the ABC will, if Stage 1 is undertaken, move through Stage 2 and 3, and that a strong economy will produce a Rs. 16,000,000 net cash inflow, is (0.8) (0.6) (0.5) = 0.24

Column 6 of Figure-9.3 gives the NPV of each final outcome. ABC has a cost of capital of 10 per cent, and management assumes initially that all projects have average risk. The NPV of the top (most favourable) outcome is about Rs. 2,347,000:

$$\text{NPV} = \frac{\text{Rs.}16{,}000{,}000}{(1.10)^3} - \frac{\text{Rs.}10{,}000{,}000}{(1.10)^2} - \frac{\text{Rs.}1{,}000{,}000}{(1.10)^1} = \text{Rs.}\,500{,}00$$

Other NPVs were calculated similarly

Column 7 of Figure 9.3 gives the product of the NPVs in Column 6 times the probabilities in Column 5. The sum of the NPV products is the expected NPV of the project. Based on the expectations set forth in Figure 9.3, and a cost of capital off 10 per cent, the expected NPV is approx. Rs. 36,000.

Since the expected NPV is positive, should ABC initiate Stage-1? Not necessarily, since management only assumed that the project is of average risk, and hence used the unadjusted cost of capital to evaluate it. However, ABC must now reconsider and decide whether this project is more, less, or as risky as an average project. The decision tree provided a distribution of NPVs, similar to the one we obtained using simulation analysis for the single-stage project. With this project, there is a probability of 0.52 of losing money and a probability of 0.32 of losing almost Rs. 1.5 million. With those high loss probabilities, and the small NPV relative to the amount required to undertake the project, chances are good that it would be rejected.

9.3 SUMMARY

Risk involves situations in which the probabilities of cash flow occuring are known and are objectively or subjectively determinable. The event is repetitive in nature and the inability to accurately predict the future course of events introduces risk. In contrast when an event is not repetitive in nature and the probability of occurance is not determinable uncertainity is said to prevail. There are sevral techniques to measure project risk. The most basic and widely used technique is Standard Deviation. Another technique is Sensitivity Analysis which measures the effect of change of variables on NPV of the project.

A risk analysis technique which considers both the sensitivity of NPV to changes in key variables and also the range of likely variables value is Scenario Analysis. Monte Carlo Simulation ties together sensitivities and input variable probability distribution. Another technique is that of Decision Tree Analysis which is used when Capital Budgeting Decisions are dependent on the outcome of a precedent event.

9.4 SELF ASSESSMENT QUESTIONS

1. Why should we be concerned with risk in capital budgeting? Is the standard deviation an adequate measure of risk? Can you think of a better measure?
2. If project A has an expected value of net present value of Rs. 20,000 and a standard deviation of Rs. 4000, is it more risky than project B, whose expected value is Rs. 14000 and standard deviation is Rs. 3000? Explain.
3. a) In a probability tree approach to project risk analysis, what are initial, conditional, and joint probabilities?

   b) What are the benefits of using simulation to evaluate capital investment projects?
4. Naughty Pine Lumber Company is evaluating a new saw with a life of two years. The saw costs Rs. 3000, and future after-tax cash flows depend on demand for the company's products. The tabular illustration of a probability tree of possible future cash flows associated with the new saw is as follows:

Year 1		**Year 2**		
Initial Probability P (1)	**Net Cash Flow (Rs.)**	**Conditional Probability P (2/1)**	**Net Cash Flow (Rs.)**	**Branch**
		.30	1000	1
.40	1500	.40	1500	2
		.30	2000	3
		1.00		
		.40	2000	4
.60	2500	.40	2500	5
		.20	3000	6
1.00		**1.00**		

a) What are the joint probabilities of occurrence of the various branches?

b) If the risk-free rate is 10 per cent, what is (i) the net present value of each of the six complete branches and (ii) the expected value and standard deviation of the probability distribution of possible net present values?

c) Assuming a normal distribution, what is the probability that the actual net persent value will be less than zero? What is the significance of this probability?

5. XYZ Inc., can invest in one of two mutually exclusive, one-year projects requiring equal initial outlays. The two proposals have the following discrete probability distributions of net cash inflows for the first year :

Project A		**Project B**	
Probability	Cash Flow	Probability	Cash Flow
.20	2000	.10	2000
.30	4000	.40	4000
.30	6000	.40	6000
.20	8000	.10	8000
1.00		**1.00**	

a) Without calculating a mean and a coefficient of variation, can you select the better proposal, assuming a risk-average management?

b) Verify your intuitive determination.

9.5 FURTHER READINGS

Brealy R., Myers, *"Principles of Corporate Finance"*, McGraw Hill Company.

VanHorne J.C., J.M. Wachowicz, *"Fundamentals of Financial Management"*, Prentice - Hall.

Stern J.M., D.H. Chew (Jr.), *"The Revolution in Corporate Finance"* Basil Blackwell.

Srivastava, R.M., *Financial Management and Policy,* Himalaya Publishing House, 2003, Chapter 12.