UNIT 10 RISK ANALYSIS IN INVESTMENT DECISIONS

Objectives

The major objective of this unit is to discuss and show the application of some advanced techniques of risk analysis in investment decisions.

Structure

10.1 INTRODUCTION

Most of the literature on capital budgeting decisions has been woven around the assumption of certainty and single goal. In unit two of this block, we have discussed various techniques of risk analysis presuming that true certainty in expected cash flow and the required rate of return do not exist in the real world. However, we have continued to assume that a company has a single objective, i.e., higher rate of return. As we know, in real world not only we are faced with lot of uncertainty but companies tend to have multiple goals like rate of return, sales, employment, etc. The multi-objective criteria and the problem of risk and uncertainty could be taken care of with the help of a stochastic goal programming model by incorporating priority coefficients for different objectives.

10.2 STOCHASTIC GOAL PROGRAMMING MODEL

Assumptions

1. Random variables *ui* are normally distributed with mean 0 and variance-covariance matrix S.
2. 'a' is a matrix of estimates of some unknown parameters, and it has rank >*q*.
3. The non-singular variance-covariance matrix S is known since only non-singular matrices have ordinary inverses.

Given the goals $(x_1, x_2, \ldots, x_n)$ we analyse the goals such that

¦$(x_1, x_2, \ldots, x_n) = bi$ (1)

where b represents given goals.

In order to convert into a real linear functional, this may be written as:

¦$(x_1, x_2, \ldots, x_n) = a_1x_1 + a_2x_2 + \ldots + a_nx_n = bi$ (2)

where $a_1, a_2, \ldots, a_n$ are any real numbers such that ¦ is a real linear functional.

In order to simplify the notation, we use the ideas of matrix algebra. Let x be a column vector with components $x_1, x_2, ..., x_n$ and let 'a' be a row vector with components $a_1, a_2, ..., a_n$ then the (2) may be represented equivalently by

Axi = bi (3)

If a matrix A has an ordinary inverse A^{-1} then we have a unique solution x to Axi = bi which is given by xi = a^{-1}bi. Geometrically, the transformation defined by *A* transforms *x*i into *b*i, whereas the transformation defined by A^{-1} transforms *b*i precisely back into *x*i. Therefore, if the matrix in our problem of goal analysis has its ordinary inverse, then the solution sub-goal which attains the given goal can be obtained by using the ordinary inverse of the matrix. When a matrix has the ordinary inverse, (i.e. when a matrix is non-singular), it is identical with generalized inverse of matrix.

By the generalized inverse matrix the possible solution to axi=bi is given by

Bi = axi (4)

In the presence of linear constraints on the variables, i.e.

Bxi £ h, (4.1)

xi ³ 0. (4.2)

The problem may be formulated in a linear programming format as follows:

Minimise

$$z = (Pid^{+}_{t} + Pid^{-}_{i}) \quad (5)$$

$$axi + d^{-}_{t} - d^{+}_{i} = bi \quad (5.1)$$

$$xi\ ,\ d^{-}_{2} - d^{+}_{i}\ ³\ 0 \quad (5.2)$$

when the assumption of certainty is dropped, we formulate the problem into a generalized inverse matrix form as follows:

*b*i = axi + ui (6)

in the presence of linear constraints on the variables, i.e.

Bxi < h (6.1)

xi > 0 (6.2)

where *b*i refers to a given set of goals and xi are linearly related to bi variables and ui are random variables.

The stochastic equation bi = axi + ui has the form of a linear model in reduced form if we consider the goals bi as endogenous and jointly determined and variable ai as exogenous variables.

Therefore, it follows from the assumptions that bi is normally distributed with means (ax) and variance-covariance matrix S, i.e.

$$¦(b_1, b_2, ... b_q) = 2II)^{q-/2} \mid S \mid^{-½} e^{-q/2} \quad (7)$$

where the quadratic form Q is defined as

$$Q = (b-ax)' S^{-1} (b-ax) \quad (8)$$

A simple criterion function analogous to that used in the goal programming model may be interpreted by analysing the statement b^{*}_{t} = axi as follows:

Let B^* be an appropriately defined region which covers the point *b*. Such that b e *B**. Then one chooses the goals xi for which the probability that the random vector bi = axi + ui will lie inside the region *B** is maximized.

Assumption I suggests that B^* should be taken to be falling in defined region and centred at b^* of the form :

$$Q^* £ c^2 \quad (9)$$

Where Q^* is the quadratic form

$$Q^* = (b - b^*)' S^{-1} (b-b^*) \quad (10)$$

We know that if y is normally distributed and S is non-singular then quadratic form (10) has the chi-square distribution with q degrees of freedom. Thus B^* can be interpreted as confidence region for b at level a because if we choose a set of goals x° such that E(b) = ax° = b* then the probability of b falling in the region B^* is given by a. Region B^* may be restricted to be of the form $Q^* = c^2$.

Therefore, the stochastic goal programming model may be formulated as follows:

Minimize

$$Z=(K+xAx' + 2p'x) \quad (11)$$

subject to

$$Bxi < h \quad (11.1)$$
$$Xi > 0 \quad (11.2)$$

where $K = b^{*'} S^{-1} b^*$, $A = a' S^{-1} a$, $p = a' S^{-1} b^*$ and A is (mxn) positive definite if q=n, and semi positive definite if $q < n$.

The above stochastic goal programming models equivalent to a quadratic programming problem in standard form since the constraints (11.1) and (11.2) are linear, and the problem can be solved by any of the existing algorithms.

The above stochastic goal programming (11) may be formulated into a linear programming format as follows:

Minimize

$$z =(Pidi^{+}_{t} +- Pid^{-}_{i}) \quad (12)$$

subject to

$$axi + uixi + d^{-}_{i} - d^{+}_{t} = bi \quad (12.1)$$
$$xi , d^{-}_{i,} d^{+}_{i} > 0 \quad (12.2)$$

where Pi refer to priority coefficients and $d^{\pm}_{i}$ refer to positive and negative devotional variables.

Using (12), Lorie and Savage modified problem may be formulated under the conditions of uncertainty as follows:[1]

Minimize

$$z = p_1d^-_1 + p_2d^-_2 + p_3d^-_3 + p_2d^-_4 +4 p_2d^-_5 +p_4d^-_6$$

$$p_4d^-_7 + p_5d^+_4 + p_6d^+_6 + p_6d^+_7 \quad (13)$$

Subject to

(*A*) *Present value of investment goal*

$$14x_1 + 17x_2 + 17x_3 + 15x_4 + 40x_5 + 12x_6 + 12x_7 + 10x_8 + 12x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_1 = 32.4 \quad (13.1)$$

1 See Appendix for detail of Lorie and savage problem.

(B) *Budget ceiling goals*

$12x_1 + 54x_2 + 6x_3 + 6x_4 + 30x_5 + 6x_6 + 48x_7 + 36x_8 + 18x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_2 = 50.0$ (13.2)

$3x_1 + 7x_2 + 6x_3 + 12x_4 + 35x_5 + 6x_6 + 4x_7 + 3x_8 + 3x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_3 = 20.0$ (13.3)

(*C*) *Sales goals*

$14x_1 + 30x_2 + 13x_3 + 11x_4 + 53x_5 + 10x_6 + 32x_7 + 21x_8 + 12x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_4 - d^+_4 = 70.0$ (13.4)

$15x_1 + 42x_2 + 16x_3 + 12x_4 + 52x_5 + 14x_6 + 34x_7 + 28x_8 + 21x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_5 = 84.0$ (13.5)

(*D*) *Employment goals*

$10x_1 + 16x_2 + 13x_3 + 9x_4 + 19x_5 + 14x_6 + 7x_7 + 15x_8 + 8x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_6 - d^+_6 = 40.0$ (13.6)

$12x_1 + 16x_2 + 13x_3 + 13x_4 + 16x_5 + 14x_6 + 9x_7 + 22x_8 + 13x_9 + u_1x_1 + u_2x_2 + u_3x_3 + u_4x_4 + u_5x_5 + u_6x_6 + u_7x_7 + u_8x_8 + u_9x_9 + d^-_7 - d^+_7 = 40.0$ (13.7)

$x_1, x_2, ..., x_9, d_1, d_2, ..., d_n$ ³ 0. (13.8)

When ui = 0), (where i = 1,2,..., N=9)

Computational Procedure

The Wolfe's algorithm of a simplex type of a quadratic problem which apply to the stochastic goal programming problem formulated for capital budgeting decision under uncertainty is of the following form:

Let the variables of the problem constitute the n vector x = $(x_1, ... x_n)'$ where x is to be taken to be a column vector, with n x 1 matrix. If A be an m x n matrix and *b* an m x 1 the linear constraints of the problem for l ³ 0.

Minimize f (l,x) = l_{px} 1/2x' cx (14)

May be specified as x ³), Ax = b

Where

Xi ³ 0(*j* = 1, ..., n)

$\sum_{j=1}^{n}$ aij xj = bi(i = 1, ..., m).

The conditions that the n vector x solve the problem for l > 0 may be written as:

Ax=b

Cx – *V* + *A*'u+ *p*'u + *p*'l = 0 (15)

x ³ 0, *v* ³ 0

or in the detached coefficient form as

x ³ 0	v ³ 0	l	l	(16)
A	0	0	0	= b
C	$-I$	A'	p'	= o

Constituting m x n equations in 2 n non-negative variables and m unrestricted variables and where l is not considered as a variable.

Let z^1,and z^2 and w be respectively n-, x- and m component vectors.

An initial basis for this system can be formed from the coefficients z^1,z^2 and w since b ³ 0 by using the simplex method to minimize

$$\sum_i wt = 0.$$

Keeping v and u zero, we would discard w and unused components z^1, z^2; let the remaining n components be denoted by z and their coefficients by E. The solution of the system would be:

$$Ax = b$$

$$Cx - v + A'u + Ez = -p' \quad (17)$$

$$X, v, z ³ 0.$$

Given a basis and basic solution satisfying (17), v'x = 0 and $\sum_{K=1}^{n} Z_k > 0$, make one change of basis in the simplex procedure minimizing the linear form

$$\sum_{K=1}^{n} X_k \quad (18)$$

under the side condition for k=1, ..., n, if xk is in the basis, we do not admit vk; and if vk is in the basis, we do not admit xk.

If $\sum_{K=1}^{n} Z_k > 0$, then we repeat the step (18) and the form will

vanish in subsequent iterations yielding z=0. The x part of the terminal basic solution in a solution of the quadratic programming for l.

The computational procedure, when the number of constraints is larger may be presented in the following manner alongwith line discussed above:

Let the constraints be

$$\begin{aligned} A_{11}x_1 + A_{12}x_2 &= b_1 \\ A_{21}x_1 + A_{22}x_2 + y_2 &= b2 \\ A_{31}x_1 + A_{32}x_2 + y_3 &= b3 \\ X_1, y_2, y_3 &> 0 \end{aligned} \quad (20)$$

the new system of linear constraints (corresponding to 16) will be:

x_1 ³0	x_2	$y_2$³0	$y_3$³0	$v_1$³0	u_1	$u_2$³0	$u_3$³0	l
A_{11}	A_{12}							$=b_1$
A_{21}	A_{22}	I						$=b_2$
A_{31}	A_{32}		$-I$					$=b_3$
C				$-I$	A'_{11}	A'_{21}	A'_{31}	$p'_1=0$
					A'_{12}	A'_{22}	A'_{32}	$p'_2=0$

The algorithm would proceed the same way as in (18) above. If $(x_1)\,k$ is in the basis, we do not admit $(v\text{-}_1)k$, and vice versa ; if $(y\text{-}_2)k$ is in the basis we do not admit $(u_3)k$, and vice versa.

However, if number of variables involved or the number of iterations required is large, computer package programme may be used to arrive at the solution. Our emphasis here has primarily been to suggest a model of stochastic goal programming for capital budgeting decision problem under risk and uncertainty and the computational procedure for its solution.

10.3 GAME THEORY

Since game theory can be used as a technique for dealing with cases of complete ignorance of the initial probabilities of possible outcomes, the reader might be inclined to pitch his hopes for useful guidance a little higher. Again, he is likely to be disappointed. But before pronouncing judgement we shall illustrate with one or two simple examples the relevant techniques known as the 'two-person zero-sum game', so called for the rather obvious reason (1) that the game is played between two persons or groups, one of which may be 'nature', and (2) that there are no mutual gains to be made; the gains to one party being exactly equal to the losses suffered by the other party.

Consider first a reservoir which is full at the beginning of the season and can be used both for irrigation and for flood-control. Without any prior knowledge of whether or not a flood will occur, a decision is required on the amount of water to be released. If a little water is released now it will be good for the harvest, but it will be ineffectual as a contribution to preventing future flood damage. If, instead, a lot of water is released now it will make flood damage virtually impossible, but it will damage the harvest to some extent.

Now the amount of water that can be released from the reservoir can range, in general, from nothing at all to the whole lot. As for the flood, if it occurs it can be either negligible or highly destructive. In order to illustrate the principle, however, we can restrict ourselves to two possible outcomes, (b_1) full flood and (b_2) no flood. The options open to the decision-maker are also to be restricted for simplicity of exposition : they will be (a_1)release one-third of the water in the reservoir, a_2 release two-thirds of the water in the reservoir, and (a_3) release all the water in the reservoir. In addition to the possible states of nature, (b_1) and (b_2), and the options open to the decision maker, (a_1), (a_2), and (a_3), we are also assumed to have a clear idea of the quantitative result corresponding to the particular outcome and the option adopted. If, for example, a decision is taken to release two-thirds of the reservoir, which is option (a_2), and a full flood, (b_1), happens to occur, the net benefit – that is the value of the harvest less the value of the damage done by the flood — is assumed to be known. In this example we shall assume it is equal to Rs.140,000. Again, if instead we choose the (a_3) option, that of releasing all the water, the net benefit that arises if the full flood (b_1) occurs is assumed equal to Rs.80,000. Since there are three options, or strategies, and two possible occurrences, or states of nature, there will be altogether six possible outcomes each identified by a net benefit figure. The scheme is depicted in Table 10.1 below.

Table 10.1: Net benefit figures for various options and various outcome

	***b*1** Flood (Rs.)	***b*2** No Flood (Rs.)
a_1	130,000	400,000
a_2	140,000	260,000
a_3	80,000	90,000

A glance at the Table will convince the reader that, provided the six figures above are all accepted as correct estimates of net benefits, option a_3 – requiring the release of all the water in the reservoir – will never be adopted. Whether b_1 or b_2 occurs the net benefits of adopting the a_3 option will be lower than those of either a_1 or a_2. In the Jargon, option a_3 is dominated by the other options, a fact that is revealed by the figures in the a_1 row (Rs.130,000 and Rs.400,000) and those in the a_2 row (Rs.140,000 and Rs.260,000), both sets of figures being larger than the a_3 row figures (Rs.80,000 and Rs.90,000). We could then save some unnecessary calculation by eliminating the dominated option a_3, since there are no circumstances in which it would pay to adopt it. Nevertheless we shall retain it in this simplified example, as the additional exercise will be useful while the additional calculation will be slight complex.

Given no information other than in Table 10.1 we could employ either of two standard methods to produce a decision: a *maximin* procedure and a *minimax* procedure. We shall only illustrate the former for understanding the application of the game theory.

The maximum procedure : If he looks along the first row of Table 10.1 showing the net revenues, Rs.130,000 and Rs.400,000, corresponding to each of the two possible alternative states of nature, b_1 and b_2, when the decision-maker chooses option a_1, it will be realized that the worst that can happen is the occurrence of b_1, yielding a revenue of only Rs.130,000. Assuming that the decision-maker is a conservative person, he will want to compare this worst result, or minimal net revenue, that he can obtain from choosing a_1 with those minimal he might obtain if instead he adopts the a_2 or a_3 option. Now the choice of a_2 or b_2 occurs respectively. He can then be sure of at least Rs.140,000. Similarly if he chooses option a_3 he can be sure of obtaining at least Rs.80,000. These three row minima, Rs.130,000 for a_1, Rs.140,000 for a_2, and Rs.80,000 for a_3 are all shown in the third column of Table 10.2 which is the same as Table 10.1 except for the addition of two columns.

Table 10.2: The Maximum Procedure

	b_1	b_2	Row minima	Maximin (maximum of row minima)
	Rs.	Rs.	Rs.	Rs.
a_1	130,000	400,000	130,000	
a_2	140,000	260,000	140,000	140,000
a_3	80,000	90,000	80,000	

Down this third column he reads off the worst possible outcome corresponding to each option. If he chooses a_1, he can be sure of not getting less than

Rs.130,000. If he chooses a_2, he can be sure of not getting less than Rs.140,000. If he chooses a_3, he can be sure of not getting less than Rs.80,000. It will then occur to him that if he chooses any option *other than* a_2 he might get less than Rs.140,000; for example, if having chosen a_1, b_1 occurs, he will receive only Rs.130,000, whereas if he chooses a_3 he will receive only Rs.80,000 or Rs.90,000 according as event b_1 or event b_2 occurs. The largest net revenue he can be *sure* of obtaining is, then Rs.140,000. The maximin principle, therefore, requires that he chooses option a_2 (releasing two-thirds of the water in the reservoir), and assure himself of no less than Rs.140,000.

The guiding idea has been to pick out the maximum figure from column three, which column contains the minimum possible net revenues corresponding to each option. Hence the figure chosen Rs.140,000 in column four of Table 10.2 is spoken of as the Maximin.

One feature of the above example is that capital costs are taken to be constant for each of the alternative options. This enables us to compare directly the net revenues—annual revenues *less* annual loss in each of the first two columns. If we assume instead that revenues are fixed and that costs alone vary according to the decision made and the event which takes place, we can go through the same sort of exercise.

An example would be the installation of a boiler in a works. Again we can suppose three options: a_1, installing a coal-fired boiler, a_2, installing an oil-fired boiler, or a_3, installing a dual boiler, one that could be switched from using coal to using oil, and vice versa, at negligible cost. Three possible occurrences are to be considered: b_1, coal prices rise relative to oil prices over the next twenty years by an average of 25 per cent; b_2, the reverse of this; and b_3, the relative prices of the two fuels remain on the average unchanged.

The outcomes of the relevant calculations are summarized in Table 10.3, the figures being the present discounted values (in thousands of rupees) of the streams of future costs associated with each option for each of the three possible outcomes.

Table 10.3: Net benefit figures for various options and outcomes; the maximum procedure

	b_1	b_2	b_3	Row Maximum	Maximin
a_1	-13.0	-12.0	-12.0	-13.0	
a_2	-11.3	-12.5	-11.3	-12.5	-12.5
a_3	-12.8	-12.8	-12.8	-12.8	

By convention costs are to be regarded as *negative* revenues, so the figures in Table 10.3 are all negative. Looking along the a_1 row the worst outcome is -13.0. If a_1 is chosen and b_1 should occur, the cost would be 13. (13 is the highest absolute figure in the row but, seen as a negative revenue and considered algebraically, -13 is less than -12. Thus -13 is the lowest figure in the row.) The largest costs, or the smallest gains, corresponding to options a_2 and a_3 are, respectively, -12.5 and -12.8-, which figures are entered in the fourth column. Of these row minima, the maximum (or least cost) is -12.5 corresponding to option a_2 which, on the maximum principle, would be the one to be chosen. Having chosen a_2, we can be sure that the cost to which the firm can be subjected cannot exceed 12.5, this cost would be incurred if event b_2 took place. If, however, event b_1 or b_3 occurred the cost would be only 11.3.

10.4 THE EXPECTED UTILITY APPROACH

Here we will develop a model for consumption investment decisions by individual under conditions of uncertainty. The general problem can be described as follows:

Consider an individual who must make a consumption - investment decision at each of r discrete points in his lifetime. At the first decision point he has a quantity of wealth W_1 that represents the maximum possible level of consumption during time period 1. At the beginning of period 1, W1 must be split between current consumption C_1 and investment h1=W_1-C_1.

At the beginning of period 2, the individual's wealth level is

$$\bar{W}_2 = h_1 (1 + \bar{R}_2) = (W_1 - C_1)(1+R_2)$$

R_2 is the % expected rate of return in period 2. R_2 is a random variable, therefore, W_2 is also a random variable.

At the beginning of period 2, W_2 must in turn be allocated to consumption and investment, and the consumption - investment decision problem is faced at the beginning of each subsequent period until period r, the last period of the individual's life at which time the entire available wealth is consumed.

The individual is assumed to derive satisfaction only from consumption and his problems is to map out a consumption – investment strategy that maximizes the level of satisfaction provided by anticipated consumption over his lifetime.

Under uncertainty the decision problem is of course complicated by the fact that the actual lifetime consumption sequence is to some extent unpredictable, because as indicated above the wealth levels produced through time by any given investment strategy are usually random variables. Thus in order to solve the individual's sequential consumption - investment problem, we need a theory of choice under uncertainty that defines the criteria that the individual uses in choosing among different probability distributions of lifetime consumptions.

10.5 THE EXPECTED UTILITY MODEL

The theory of choice under uncertainty that we apply to the consumption investment problem is the 'Expected Utility hypothesis'.

In general terms, the expected utility hypothesis states that when faced with a set of mutually exclusive actions, each involving its own probability distributions of 'outcomes' the individual behaves as if he attaches numbers called, purely for convenience, utilities to each outcome and then chooses that action whose associated probability distribution of outcomes provides maximum expected utility.

In the r period consumption - investment problem, an outcome is a complete sequence of lifetime consumption Cr = (C1,C2,...Cr), and an action is a r period consumption - investment strategy that produces a probability distribution for different possible lifetime consumption sequences. But because the consumption investment problem is just one possible application of the expected utility model, first see the model in its most general form.

Since we are confronted here with how the individual ranks outcomes and probability distribution of outcomes, just as behaviour in conformity with the ordinary utility model. So we, have five axioms that can help produce a better understanding of the model.

The Axiom System

The set of axioms we use is as follows:

Axiom 1 : (Comparability) The individual can define a complete preference ordering over the set of prospects in S; that is, for only two prospects X and Y in S, he can say that X>Y and Y>X or X ~ Y.

Axiom 2 : (Transitivity) The ordering of prospective assumed in Axiom 1 is also completely transitive. For example X>Y and Y>Z imply X>Z or X~Y and Y~Z and so on.

Axiom 3 : (Strong Independence) The rankings of two prospects are not changed when each is combined in the same way into a a gamble or probability distribution involving a common third prospect.

If X~Y, then for any third prospect Z in S,G(X,Z:μ) ~ G(Y,Z:μ). Here G(X,Z:μ) represents a gamble, that is, a random prospects, in which the individual gets either X, with probability X, or Z, with probability 1-μ, and G(Y, Z:μ) likewise represents a gamble that produces either Y or Z, with probabilities μ and 1-μ. We also assume that if X>Y, then G(X,Z: μ) > G(Y,Z:μ); or if X ³ Y then G(X,Z:μ) ³ G(Y,Z:μ).

Axiom 4 : If the prospects X, Y and Z are such that either X>Y ³ Z or X ³ Y>Z, there is a unique μ such that Y~G(X,Z:μ).

Axiom 5 : If X ³ Y ³ Z and X ³ U ³ Z, and Y~G(X,Z:μ_1) and U~G(X,Z:μ_2) then $\mu_1 > \mu_2$ implies Y>U and $\mu_1 = \mu_2$ implies Y~U.

Intuitively it is clear that ranking random prospects which are just probability distributions of elementary prospectus, according to expected utility requires a utility function in which the differences between the utility levels assigned to different elementary prospects have some meaning; that is, if the utility of a random prospect is just the expected or average value of the separate utilities of each of its component then elementary prospects differences in utility levels must have meaning.

10.6 SUMMARY

As the number of variables and number of years in planning horizons increases it becomes very tedious and cumbersome to evaluate risk in investment decisions. To cope with a problem of this kind it is helpful to resort to mathematical programming models which aids in determining the optimal solution without explicitly evaluating each feasible combination. Stochastic Goal Programming is one on them. Game theory is a technique for dealing with cases of complete ignorance of the initial probabilities of possible outcomes. The expected utility approach deals with consumption investment decisions under conditions of uncertainty.

10.7 SELF ASSESSMENT QUESTIONS

1. Discuss the capital budgeting techniques without probabilities.
2. Describe the general formulation of a goal programming model.
3. What is meant by utility? Do you feel financial managers should be risk averse? Why or why not?

10.8 FURTHER READINGS

Agarwal J.D., *"Reading in Financial Management"*, IIF, Delhi Publication, 1994.

Hiller F.S., *"The Evaluation of Risk Interrelated Investments"*, North-Holland Pub. Co., London.

Knight F.H., *"Risk, Uncertainty and Profit"*, University of Chicago Press, Chicago.

APPENDIX

A Goal Programing Model for Capital Budgeting

Weingartner (1963) suggested a linear programming formulation taking up Lorie and Savage (1955) problem and suggested an optimal solution. Lorie and Savage problem specifies for consideration nine mutually exclusive projects with given present values of outlays for period I and II and given prsent values of investments. The budget constraints for two periods were Rs.50 and Rs.20 respectively. The problem may be specified as follows:

Exhibit - I
Lorie-Savage Problem of Investment Proposals

Investment Project	*PV of outlays* Period I (Rs.)	Period II (Rs.)	*PV of Investment* (Rs.)
1	12	3	14
2	54	7	17
3	6	6	17
4	6	2	15
5	30	35	40
6	6	6	12
7	48	4	14
8	36	3	10
9	18	3	12

In order to examine the effect of priority coefficients on the CBD, the GP model (12) for the above problem is reformulated here below changing the priority coefficients for the different objectives of the same objective set. The different priority coefficients assumed for the following reformulated GP model (13) are as follows:

Goals	Priority Coefficient
Net present goal (over achievement)	1
Budget constraint goal I	7
Budget constraint goal II (under achievement not desired)	8
Sales goal I (under achievement not desired)	2
Sales goal II (under achievement not desired)	3
Employment goal I (under achievement not desired)	4
Employment goal II (under achievement not desired)	4
Sales goal I (over achievement undesirable)	5
Employment goal I (over achievement not desired)	6
Employment goal II (over achievement not desired)	6

The GP model for capital budgeting decisions with modified priority coefficients may be specified as follows:

Minimize

$$z = p_1 d^-_1 + p_7 d^-_2 + p_8 d^-_3 + p_2 d^-_4 + 4p_3 d^-_5 + p_4 d^-_6 + p_4 d^-_7 + p_5 d^+_4 \,\_\, p_6 d^+_6 + p_6 d^+_7, \qquad (10)$$

Subject to

(*A*) *Present value of investment goal*

$$14x_1 + 17x_2 + 17x_3 + 15x_4 + 40x_5 + 12x_6 + 14x_7 + 10x_8 + 12x_9 + d^-_1 = 32.4 \qquad (10.1)$$

(B) *Budget ceiling goals*

$$12x_1 + 54x_2 + 6x_3 + 6x_4 + 30x_5 + 6x_6 + 48x_7 + 36x_8 + 18x_9 + d^-_2 = 50.0 \quad (10.2)$$

$$3x_1 + 7x_2 + 6x_3 + 2x_4 + 35x_5 + 6x_6 + 4x_7 + 3x_8 + 3x_9 + d^-_3 = 20.0 \quad (10.3)$$

(*C*) *Sales goals*

$$14x_1 + 30x_2 + 13x_3 + 11x_4 + 53x_5 + 10x_6 + 32x_7 + 21x_8 + 12x_9 + d^-_4 - d^+_4 = 70.0 \quad (10.4)$$

$$15x_1 + 42x_2 + 16x_3 + 12x_4 + 52x_5 + 14x_6 + 34x_7 + 28x_8 + 21x_9 + d^-_5 = 84.0 \quad (10.5)$$

(*D*) *Employment goals*

$$10x_1 + 16x_2 + 13x_3 + 9x_4 + 19x_5 + 14x_6 + 7x_7 + 15x_8 + 8x_9 + d^-_6 - d^+_6 = 40.0 \quad (10.6)$$

$$12x_1 + 16x_2 + 13x_3 + 13x_4 + 16x_5 + 14x_6 + 9x_7 + 20x_8 + 13x_9 + d^-_7 - d^+_7 = 40.0 \quad (10.7)$$

$$x_1, x_2, ..., x_9, d^-_1, ... d_7, d^+_4, d^+_6, d^+_7 \geq 0.$$

The optimal solution of (13) is presented in exhibit II.

Exhibit II reveals that to attain the optimal solutions for each of the seven objectives four projects, i.e., x_3, x_4, x_7 and x_9 should be selected. The respective units of these projects to be chosen are $x_3 = 2.00933$, $x_4 = 2.65466$, $x_7 = 0.53334$. On comparison of the solutions presented in exhibit I and exhibit II it may be observed that the basic difference which the modified priority coefficients made is the choice of the projects and their respective units. The optimal solutions under the two situations remained to be almost the same because the overall formulation with regard to different objectives and their over achievement was the same. However, if the formulation with regard to the under achievement and/or over achievement is also simultaneously changed the optimal solution would also be different.

Thus, it may be concluded that a goal programing solution is certainly better than the linear programming solution since a GP model allows (i) a simultaneous solution of a system of complementary and conflicting objectives rather than a single objective only, (ii) that more than any one period can be included in the final programme of choosing projects.

Exhibit II
Goal Programming Solution with Modified Priority Coefficients

Projects	X_1=0.0000	X_2=0.0000	X_3=2.00933	X_4=2.65466	X_5=0.0000
X_6=0.0000	X_7=0.25867	X_8=0.0000	X_9=0.53334		

		Goal Constraints	Total	Goal Programming Solution			
				X_2	X_4	X_7	X_9
Net Present Value Goal	³	32.4	83.99997	34.15861	39.81990	3.62138	6.40008
Budget Constraint Goal I	£	50.0	50.00022	12.05598	15.92796	12.41616	9.60012
Budget Constraint Goal II	£	20.0	20.00000	12.05598	5.30932	1.0469	1.60002
Sales Goal I	=	70.0	70.00007	26.12129	29.20126	8.27744	6.40008
Sales Goal II	=	84.0	84.00012	32.14928	31.85592	8.79478	11.200014
Employment Goal I	=	40.0	56.09064	26.12129	23.89194	1.81069	4.26672
Employment Goal II	=	40.0	69.89332	26.12129	34.51058	2.32803	6.93342

The GP Solution is computed on IBM 360 and involved 24 iterations.