UNIT 12 SETS, RELATIONS, FUNCTIONS AND GRAPHS

Structure

12.1 INTRODUCTION

Mathematics has been defined as the study of sets with structures. A set provides a language which can be used in the rest of mathematics. This language is sometimes more expressive than the ordinary language.

Recently, the vocabulary of sets has been given importance because it facilitates integration of ideas in algebra, geometry etc. Set language also illustrates the use of mathematical symbolism in formulating proofs and illustrating the structures in mathematical logic, probability, Boolean algebra, Switching circuits etc., which will be learnt in higher classes/college level.

The idea of sets was developed towards the end of the 19th century. George Boole (1815-1864) and George Cantor (1845-1918) are the two mathematicians credited with the development of the idea of Sets. Cantor is considered to be founder of the set theory.

The concept of relation and function are fundamental in Mathematics, Science and Social Sciences. In Social Sciences, the set consists of human beings and the structure is provided by the relationships existing between members of the set of human beings such as being father or mother or son or daughter etc. A function essentially shows how one quantity varies with variation of one or more of other quantities. For example, it may show how the area of a rectangle varies with its length and breadth, or how the volume of a sphere varies with its radius or how pressure of a gas varies with the volume and temperature and so on.

The graph of a function gives us a visual picture of the behaviour of the function. It can help us in understanding the properties of function.

It has been felt by many teachers that the ideas being new, it is difficult to inculcate their understanding in the mind of the student. This unit illustrates how simple methods can be used to make learning interesting and useful.

12.2 OBJECTIVES

At the end of this unit, the learner would be able to:

- explain the meaning of basic terms used in set theory;
- analyse the content in terms of concept, subconcept and relation between them;
- determine one or more key concepts from which other concepts follow;
- illustrate concepts with the help of class room situation and daily life problems;
- compare the laws of algebra of sets with laws of algebra of real numbers and thereby identify points of similarity and dissimilarity in the two algebras;
- encourage logical way of thinking to generalize important properties;
- apply the concepts for solving daily life problems; and
- construct objective based test items for diagostic purposes and for planning remedial teaching.

12.3 SETS

12.3.1 Introduction to Sets

Main Teaching Point

Basic definitions related to sets.

Teaching Learning Process

It is well known that, a set is a well defined collection of objects. Here it should be made clear that the term 'collection' and 'object' will not have any formal definition. They are synonyms for the word 'set' and 'elements' respectively. The teacher will do well to explain these undefined concepts through illustrations such as 'a set of books', 'a collection of stamps', 'a chain of' 'stores', 'a set of exercises' a bunch of keys etc. The objects of a set may have some common properties. The set of books may all be encyclopedias, the stamps may be issued by the same country, the stores in chain may be owned by the same company the exercises may be covering the same kind of materials. However, the mere fact of being put together is enough to qualilfy them as elements of a set. A set has elements which are distinct and distinguishable and a set is deemed to have been constituted if we can decisively say whether any particular object belongs to the set or does not belong to the set.

The expression 'well defined' should be explained. For example, 'The collection of all tall boys in a class' is not a set because we cannot decide whether particular boy is an element of the set or not. The idea of tallness is vague and may differ from person to person. But if we define the collection as 'A collection of all boys in a class whose height is above 160cm' is a set because it is now well defined.

The unit should be introduced by reviewing the fundamental notions on sets which the students already know. The procedure outlined in the unit is logical in nature. Sample activities for a few selected concepts are given below:

Activities

1. **Ask :** What are the odd numbers from 1 to 9.

   Write answer 1, 3, 5, 7, 9

   Ask : Write it in set-theoretic form $\{1,3,5,7,9\}$, and also as $\{x : x \text{ is an odd number and } x \leq 9\}$

Explain: The notation { } is used to collect the members of a set. If a set is described by listing all the members of the set within the notation { }, such as {1,3,5,7,9}, it is called **Tabular form** or **Roster form**. However if a set is described by using some rule such as {x : x is an odd number and $x \leq 9$ it is called **set builder form** or **rule form** of describing the set.

2. **Ask :** Repeat the process of change from roster to rule form or vice versa for some other collection, such as Roster form {a,e,i,o,u};

Rule {x : x is vowel in the English alphabet}

Roster form {January, June, July};

Rule {x : x is a month beginning with letter J}

Explain : Mathematicians like to translate statements like these using symbols which we call language of sets. Give more exercises on sets.

3. **Ask** : Consider (i) collection of good students (ii) collection of delicious dishes

(iii) collection of a boy, a girl, a pen, a pencil, a chair and a table.

Which of the above is a set?

Write answer : (iii)

Explain : The collections given in (i) and (ii) do not form a set because they are not well defined.

4. **Ask** : Can you write the set A = {x : x is a prime number} in the roster form.

Write answer : No

Explain : To write it in roster form is not possible because the number of prime numbers is infinite. Thus writing A in set builder form is necessary. We can not write this set in the form {2, 3, 5, 7....} because it is not possible to write all the prime numbers. Thus every set written in set builder form cannot be written in Roster form.

5. **Ask** : Think of some pairs of sets with three elements arranged differently. Write them on the board. Explain that these pairs of sets of three elements are called **equal sets.**

Explain : Any two sets with the same elements are equal (irrespective of the order in which elements are arranged).

Ask : Are the sets {a, b, c} and {p, q, r} equal ?

Write answer : No

Explain : They are different because they have different members. But sets {a, b, c} and {b, c, a} are equal, because they have the same members.

6. **Ask** : Can you match the set {a, b, c} with the set {e, f, g}

Write answer

$$\begin{array}{ccc} a, & b, & c \\ \updownarrow & \updownarrow & \updownarrow \\ e, & f, & g \end{array}$$

Explain ; This matching or pairing of elements is called a one-to-one correspondence.

Whenever one set has a one-to-one correspondence to another set, the sets are called equivalent.

Ask : Are the sets {a,e,i,o,u} and {1,2,3,4,5} equal?

Write answer : No

Explain : They are not equal because elements are different. But they are equivalent. Remember that equivalent and equal have different meanings. **All equal sets are equivalent but equivalent sets need not be equal.**

7. **Ask** : How many elements are there in the following sets?

i) Set of natural numbers less than 10

ii) Set of natural numbers

iii) Set of natural numbers between 5 and 6

Write the answer in i) 9 elements

ii) Countless numbers

iii) No elements

Explain : In case (ii) we can not count the number of elements so it is called an **infinite** set. When a set is not **infinite**, we can count the number of elements in the set and we call it a **finite set.** In contrast to infinite sets, there is also a set with no elements in it. (iii) This set is called the null set, empty set or void set. Explain symbol f is used for the null set. Give more examples for null set like the set of living persons more than 210 years old, 'the set of even prime numbers greater than 2' etc.

Methodology used : Discussion method is used along with a number of illustrations.

Check Your Progress

Notes: a) Write your answer in the space given below.

b) Compare your answer with the one given at the end of the unit

1. Which of the following collections are sets? If a collection is a set, is it finite or not?

i) The collection of all intelligent students in a class.

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ii) The collection of students of class VII who have got above 80% marks in a school.

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12.3.2 Subsets and Universal Set

Main Teaching Points

(a) Relation between subsets and universal set.

(b) Power set.

Teaching Learning Process

8. **Ask** : Write a set "A" of primary, trained graduate and post-graduate teachers in a senior secondary school. Think of the teachers of different levels separately.

Explain : The collection "B" of all primary teachers, "C" of trained graduate teachers and "D" of post graduate teachers are **Subsets** of A.

A is the universal set for the subsets B, C and D.

Explain : If every element of a set B is also an element of a Set A, then B is called a **Subset** of A. Explain that many subsets can be formed from a given set. The symbol 'C' is used for the relation 'is a subset of '.

Explain that the set of all subsets from a set is called power set or original set.

9. **Ask** : What are the subsets of the set {Ram, Sita, Imran}

Write the answer
{Ram, Sita, Imran}
{Ram, Sita}
{Ram, Imran}
{Sita, Imran}
{Ram}
{Sita}
{Imran}, null set

Explain :
i) The null set is considered as a subset of every set.
ii) Every set is a subset of itself.
iii) For all these subsets the set [Ram, Sita, Imran] is the universal set.

Ask : Write down all the subsets of a given set and find the number of subsets formed. (Taking sets of 1,2, 3 and 4 objects.) Let the students discover a pattern. Explain if a set consists of 3 elements, then there will be 2^3 subsets'. More generally **A set having n elements will have 2^n subsets.**

10. **Ask** : Which of the following sets is the largest :
i) The students of the school
ii) The students of Class IX of the school
iii) The students of Class X of the school

Write answer (i).

Explain : The students of Class IX and X are subsets of the students of the school which is the largest set. The subsets have been made out of this largest set. This largest set is called universal set. Explain the set of rectangles is a subset of the universal set of all geometric figures.

Explain that set R of real numbers is a universal set for the set of all positive, negative whole numbers, rationals and irrationals.

Explain that universal set is not unique where as the null set is unique.

Methodology used: Discussion method is mainly used. Inductive method is used to show that the total number of subsets in a set of n elements is 2^n.

Check Your Progess

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit

2. Write all the subsets of the set of natural numbers less than 4. Which is the universal set?

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12.3.3 Operations on Sets

Main Teaching Points

(a) Union and Intersection.

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Teaching Learning Process

11. **Ask** : What is $5 + 6$ and 15×2 ?

Write answer 11 and 30

Explain : In arithmetic we have operations of addition and multiplication. In a similar way we have operations with sets. These are called union and intersection. The symbol '$\cup$' for union and '$\cap$' for intersection is used.

12. **Ask** : What is $A \cup B$ if $A = \{1, 2, 3\}$

and $B = \{3, 4, 5\}$

Write answer $A \cup B = \{1, 2, 3, 4, 5\}$

Explain : In the union of two sets, we write all the elements of sets A and B. Common elements are written only once.

13. **Ask** : Repeat the above process for some other sets such as

A = {chair, table}

B = {pen, pencil} $\quad$ A ∪ B = {chair, table pen, pencil}

14. **Ask** : Does $A \cup \phi = A$?, $A \cup U = U$?

Write answer Yes.

Explain that the union of any set with the null set is always the set itself and union of any set with the universal set is always universal set.

Give more exercises on union.

15. Ask : What is $A \cup B$, if $A = \{4, 5, 6\}$

$B = \{6, 7, 8\}$?

What about $E \cup F$ if

E = {Δ, ▭, ◇, ⌂}, F = {Δ, O, ▭}?

Bring out that the union of two sets will always be a set.

Write number of elements of each union set in the question.

Explain that the symbol n(A) denotes the number of elements in any set A.

16. **Ask** : What are the common elements in the sets A and B?

$A = \{3, 4, 5\}$, $B = \{5, 6, 7\}$?

Write answer = 5

Explain : **The intersection of two sets includes only the common elements of both the sets. It is denoted by $\cap$.**

Ask : What is $A \cap B$ if A = {Suresh, Geeta, Rahman}

B = {Rahman, Dinesh}

Bring out $A \cap B$ = {Rehman}

17. **Ask** : Does $A \cap \phi = \phi$, $A \cap U = A$?

Write answer: Yes

Explain : The intersection of any set with the null set is always the null set and intersection of any set with the universal set is the set itself.

Give more exercises on intersection.

Ask : What is $A \cap B$ if

$A = \{\Delta, \square, \circ, ⌂\}$, $B = \{\Delta, \square\}$? etc.

Write on the black board $A \cap B = \{\Delta; \square\}$

Bring out that the intersection of two sets is always a set.

Write the number of elements of intersection set in the question.

Explain union and inte.,ection using Venn diagram.

18. **Ask** : Can we think of some formula for the number of elements of the set A B ?

Have students give their own arguments. If no good argument is produced. Write the formula $\mathbf{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$.

Explain if A and B are disjointed sets the formula will take the form $n(A \cup B) = n(A) + n(B)$.

Ask : If in a High School

A = the set of boys in the ninth grade Mathematics class, B = the set of boys on the football team such that $n(A) = 15$, $n(B) = 10$, $n(A \cap B) = 3$.

What is $n(A \cup B)$?

Write answer 22.

Methodology used: Intuitive logic with discussion is used to get the relation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

12.3.4 Applications of Union and Intersection

Main Teaching Point

Word problems based on $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Teaching Learning Process

Word problems are useful for practising problem solving. These are used as practice material upon which a student can apply already learnt formulae.

Tell :, The students to read the problem every carefully. Analyse the given data and understand what is to be obtained. Encourage students to translate daily life problems into mathematical sentence and to discover the method to solve them.

Example : In a class of 35 students, 25 like to play football and 20 like to play cricket. If each student likes to play at least one of the two games, how many students like to play both cricket and football?

Given : If A denotes the set of students who like to play football, then $n(A) = 25$. Similarly if B is the set of students who like to play cricket, then $n(B) = 20$.

$A \cup B$ is the set of students who like to play at least one game i. e. $n(A \cup B) = 35$.

Analyse : A B is the set of students who like to play both games. Using formula, the required number is $= 25 + 20 - 35 = 10$.

Example: In a committee, 30 people speak Hindi, 35 speak English and 15 speak both Hindi and English. How many speak at least one of the two languages ?

Given : The set A of people speaking Hindi has 30 elements i. e. n (A) = 30.The set B of people speaking English has 35 elements i. e., n (B) = 35.

$A \cap B$ is the set of people who speak both Hindi and English i. e., $n(A \cap B) = 15$.

Analyse : A B is the set of people who speak at least one of the two language.

The required no. of people is 30 + 35 − 15 = 50.

Complement of a set : If U is the universal set & A is any subset of U, then complement of setA is the set of elements of U which are not in A. $\sim A = \{x \in U : x \notin A\}$.

Methodology used: Heuristic approach is the best to solve word problems.

12.3.5 Comparison of Laws of Algebra of Sets and Algebra of Real Numbers

Main Teaching Point: To compare real numbers with sets.

Teaching Learning Process: Give a comparison of laws of algebra of sets and algebra of real numbers. These create interest in learning.

	Sets	Real Numbers	Laws
1.	$A \cup B = B \cup A$,	$a + b = b + a$	Commutative law
2.	$A \cap B = B \cap A$,	$a.b = b.a$	Commutative law
3.	$(A \cup B) \cup C = A \cup (B \cup C)$,	$(a + b) + c = a + (b + c)$	Associative law
4.	$(A \cap B) \cap C = A \cap (B \cap C)$,	$(a.b).c = a.(b + c)$	Associative law
5.	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,	No Corresponding	Distributive law
6.	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,	$a.(b + c) = a.b + a.c$	Distributive law
7.	$A \cup \phi = A$	$a + 0 = a$	
8.	$A \cap \phi = \phi$	$a.0 = 0$	
9.	$A \cup U = U$	No corresponding result	
10.	$A \cap U = A$	$a.1 = a$	
11.	$A \subset B, B \subset C \Rightarrow A \subset C$	$a < b, b < c \Rightarrow a < c$	
12.	$A \subset B \Rightarrow A \cup C \subset B \cup C$	$a < b \Rightarrow a + c < b + c$	
13.	$A \subset B \Rightarrow A \cap C \subset B \cup C$	$a < b \Rightarrow a.c < b.c$, (if $c > 0$)	
14.	$\phi \subset A$	$0 < a$, if a is positive.	

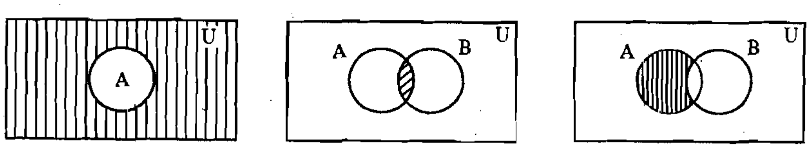
Methodology used: Lecture method combined with deductive logic is used.

Check Your Progress

Notes: a) Write your answer in the space given below.

b) Compare your answer with the one given at the end of the unit.

3. What do the shaded portions in the following Venn diagrams represent?



(i) (ii) (iii)

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12.4 RELATIONS

Main Teaching Points

(a) Cartesian Product of sets.

(b) What is a relation?

Teaching Learning Process

19. **Ask:** Write pair of objects x and y. Think of arranging them in two distinct ways.

Explain that these are (x, y) and (y, x) if $x \neq y$

Explain that in the first case we want x to occur as the first element and y as the second element, while in the second case, we want y to occur as the first element and x as the second element.

Explain that in (x, y) and (y, x) order of x and y is not the same. We treat them as two different pairs; and call them ordered pairs.

Explain : $(x, y) = (y, x)$ if $x = y$

20. **Ask:** How many ordered pairs will be formed by taking first element from set A and the second element from the set B if $A = \{2, 3\}$, $B = \{4, 5\}$?

Write answer (2,4), (2,5), (3,4), (3,5)

Explain: The collection of all the ordered pairs from set A to set B is called cartesian product of two sets A and B and is written as

$A \times B \{(a, b) : a \in A, b \in B\}$

Explain $A \times B \neq B \times A$.

Explain if either A or B equal to f then $A \times B = \phi$.

Ask: What is $R \times R$ and $R \times R \times R$?

Write answer R^2 and R^3.

Explain R^2 is defined as Euclidean plane of two dimension. R^3 is defined as Euclidean space of three dimensions. Explain that the word 'Euclidean' indicates that the idea was conceived by Euclid a Greek mathematician.

21. **Ask:** Write $A = \{3,4,5\}$

$B = \{2,8\}$.

Think of ordered pairs in which first element is less than the second element. Write these ordered pairs (3,8), (4,8), (5, 8) on board. Bring out that this collection of ordered pairs is a subset of $A \times B$ and is called a relation from the set A to set B.

Explain: A relation between two sets X and Y is a set R of ordered pairs (x,y) with $x \in X$ and $y \in y$ such that x R y (i.e. x is related to y by relation R.)

Explain $R \subset X \times Y$ and $(x, y) \in R$

22. **Ask:** What are the sets of first and second elements in the relation R = {(3,8), (4,8), (5,8)}

Write answer {3, 4, 5} and {8}

Explain: The set {3, 4, 5} is domain of R and {8} is range of R. Explain the set of first elements of all the ordered pairs of the relation is called the **domain** of R (dom R) while the set of second elements **of all the ordered pairs** of the relation is called the **range** of R (ran R).

Explain dom $R \subset X$ and ran $R \subset Y$

Ask : Can you tell what is dom R and ran R if $R = X \times Y$

Write on board dom R = X and ran R = Y

Explain if R is an empty relation,

dom $R = \phi$ and ran $R = \phi$

Give more exercises on relation.

23. **Ask :** If A = {2,3,4,5} and B = {3,6,7,10} and R is a relation from A to B defined by x R y if " x divides y ".

What is R ?

Have students give their own reply. If reply is not correct, write the answer R = { (2,6), (2,10), (3,3), (3,6), (5,10) }

Explain: The number of ordered pairs in $A \times B$ is 16. Choose those ordered pairs in which the first element divides the second element.

Methodology used: Discussion is used to arrive at the mathematical meaning of 'Relation'.

12.5 FUNCTIONS

Main Teaching Points

(a) What is a function?

(b) Domain and range of a function.

Teaching Learning Process

24. **Ask :** Write X = {x, y, z} and Y = {1, 2, 3}

Think of some relations from X to Y. Write these relations as

f_1 = {(x, 1), (y, 2), (z, 3) },

$f_2 = \{(x, 1), (x, 2), (z, 1)\}$,

$f_3 = \{(x, 3), (y, 1), (z, 3)\}$.

Explain : In f_1 and f_3 every element of X is associated with one and only one element of Y. but in f_2 it is not so. Explain f_1 and f_3 are examples of functions.

Explain a relation f from set X to Y is called a function if

i) dom f = X and

ii) to each $x \in X$ there exists a unique element $y \in Y$ i.e. if (x, y_1) and $(x, y_2) \in f$ then $y_1 = y_2$.

Alternatively, a relation is called a function if no two ordered pairs have the same first element. Explain if f is a function from the set X to Y, then X is called the domain of f (dom f) and Y is called codomain of f.

The image of x under f is denoted by y = f (x) and the set of all images viz $\bigcup_{x \in X} \{f(x)\}$ is called the range of f.

Explain every function is a relation but the converse is not true.

25. **Ask :** Which functions are real functions ?

Explain if the domain and co-domain of a function are the sets (or subsets) of real numbers, the function is called a real function.

Explain: If f is a real function of x such that y = f (x), where x can take any real value in the domain of f, then **x is called independent variable and y is called dependent variable.**

26. **Ask:** Why $y = \pm\sqrt{x}$ is not a function for a given positive value of x ?

Bring out that two values of y corresponding to a given value of x are obtained which contradicts the definition of function.

Explain that definition of function includes only single valued function of x.

Explain: The concept of a function of x is needed for many branches of mathematics such as calculus, functional analysis and in higher mathematics such as topology.

27. **Ask :** Can you find the range of a function $f : A \to R$ defined by $f(x) = x^2 + 1$ where $A = \{-2, -1, 0, 1, 2\}$?

Compute the image of each element of A :

$f(-2) = (-2)^2 + 1 = 5$

$f(-1) = (-1)^2 + 1 = 2$

$f(0) = (0)^2 + 1 = 1$

$f(1) = (1)^2 + 1 = 2$

$f(2) = (2)^2 + 1 = 5$

Write the range of f is the set of image points

$\{5, 2, 1, 2, 5\}$, i.e. the set $\{5, 2, 1\}$.

Methodology used: Discussion is used to illustrate the meaning of function. Mainly the lecture method is used to define domain and range.

12.6 GRAPHS

Main Teaching Point

Geometrical representation of a function.

Teaching Learning Process

28. **Ask:** What is graph of U × U if U = {1,2,3} is the set of three numbers.

Bring out the graph of U × U on board:

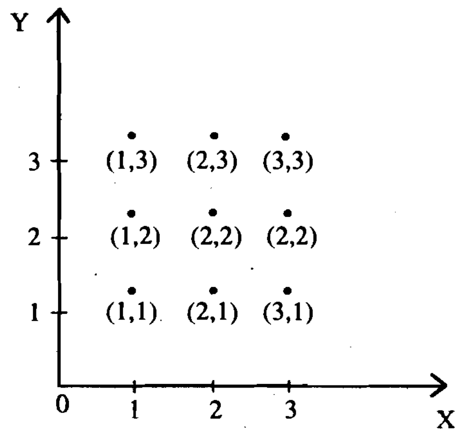


Fig. 12.1

Explain: The set of all possible ordered pairs is

{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) }.

This set is denoted by U × U.

Explain that the point indicated by the circle represents the number x = 1, y = 3, and is said to have the coordinates (1,3).

Explain that there is a one-to-one relationship between every indicated point in the graph and the set U × U. Explain it was Rene' Descartes, a French mathematician, who showed the relationship of algebra and geometry by associating number pairs with points in plane. The ordered pair associated with a point on the graph is also called cartesian coordinates after the name of the mathematician.

Explain, if U denotes the set of all real numbers, then the graph of U × U is an infinite geometric plane called Euclidean plane.

29. **Ask:** What is graph of a function and its importance ?

Explain: Tell the students that the graph of a function represents a visual picture of the behaviour of the function. A function is usually defined in terms of algebraic symbols. The graph of a function may consist of an infinity of points, out of which we draw/plot only a small number of points. By joining these points, an approximate idea of the curve represented by the function is obtained.

Explain: For every value of x, there is a corresponding unique value of y i.e. y = f (x). The set of ordered pairs is

$$f = \{(x_1, f(x_1)), (x_2, f(x_2)), \ldots (x_n, f(x_n)) \ldots\}$$

Explain each ordered pair can be represented by a point in the Euclidean plane. The points (x_1, f (x_1) (i = 1, 2...., n,....) are called points on the graph of the function y = f (x).

Explain, formally the graph of a function

f : X → Y is the set of points represented by { (x, f (x)): x ∈ X, f (x) ∈ Y } in the Euclidean plane.

$$\{(x, f(x) : x \in X, f(x) \in Y)\}$$

30. **Ask:** Can you correlate the concept of a function with a daily life situation?

Explain: Bring out that the temperature conversion graph used to convert Celsius to Farenheit or vice versa are examples of continuous graphs i.e. without breaks. Rain fall graphs and graphs correlating weight and height, where weight approximating to the nearest Kilograms, increases with height, can be given as examples of discontinuous function.

Methodology used: Discussion method is used.

Check Your Process

Notes: a) Write your answer in the space given below.

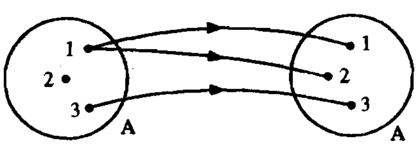
b) Compare your answer with the one given at the end of the unit.

4. Which of the following relations is a function from {1, 2, 3} to itself?

i) { (1, 2), (2, 3), (3, 1)}

ii) {(1, 1), (2, 1), (3, 1) }

iii) {(1, 1), (1, 2), (1, 3)}

iv) 

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12.6.1 Graphs of Linear Functions

Main Teaching Point

To draw graph of ax + by + c = 0.

Teaching Learning Process

31. **Ask :** What is graph of function 2y = 3x + 4 ?

Bring out the table

x	0	2	4
y	2	5	8

The graph of functions is:

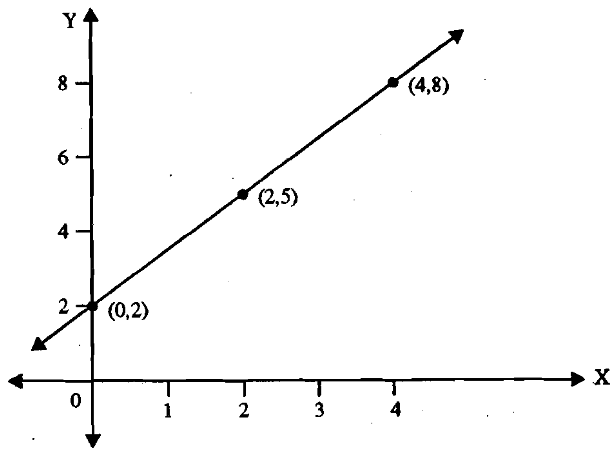


Fig. 12.2

Explain the graph of the function 2y = 3x +4 is the straight line PQ.

32. **Ask :** What is graph of function y = |x| ?

$$|X| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0? \end{cases}$$

Bring out the table

x	–4	–3	–2	–1	0	1	2	3	4
y	4	3	2	1	0	1	2	3	4

Plot the graph as

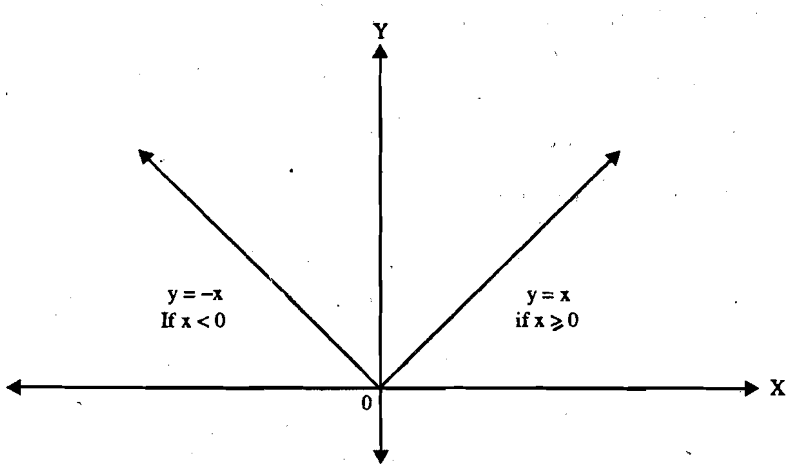


Fig. 12.3

Explain the graph is a ray lying in the first quardrant when x ≥ 0 and a ray in second quadrant when x < 0.

33. **Ask :** What is graph of the function y = [x], where [x] is the greatest integer less than or equal to x ?

Show/develop/write out the table

x	−2.5	−2	−1.5	−1	−.5	0	.5	1	1.5	2	2 5
y	−3	−2	−2	−1	−1	0	1	1	1	2	?

Plot the graph

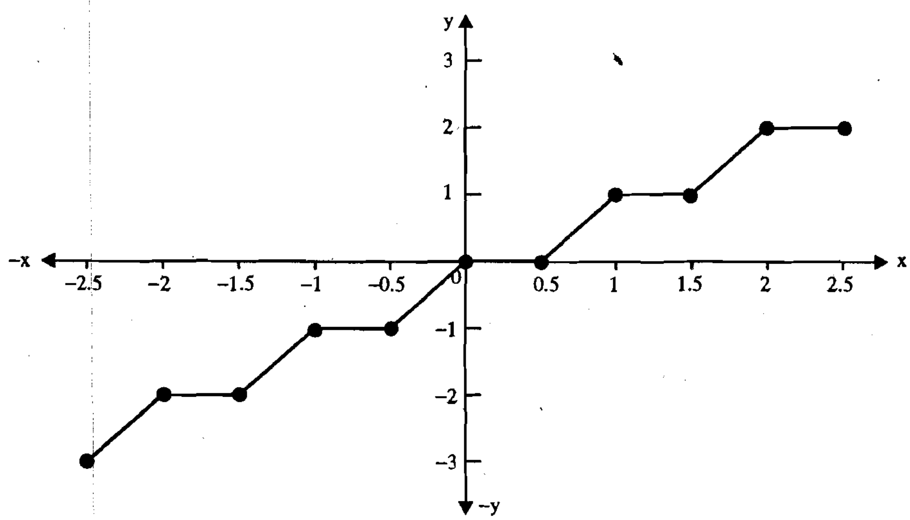


Fig. 12.4

Explain: The graph of the function y = [x] consists of a number of line segments all parallel to x-axis and has points of discontinuity at x = n, where n ∈ Z.

Methodology used: Students should be asked to work out the table of points to be plotted and then draw the graphs.

12.6.2 Graphs of Quadratic Functions

Main Teaching Point

To draw graph of $y = ax^2 + bx + c$

Teaching Learning Process

35. **Ask :** What is the graph of quadratic function $y = x^2$?

Bring out the table

x	− 4	−3	−2	−1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

Plot the graph as

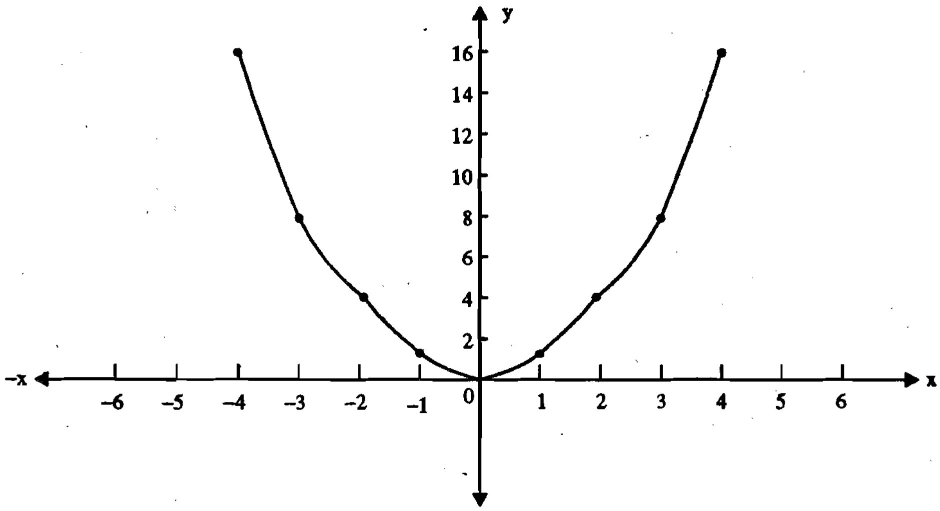


Fig. 12.5

Explain: The graph is called a parabola. When x is positive and as x increases, y increases rapidly. Explain the graph is symmetrical about y axis.

Explain if $y = -x^2$, graph is a parabola again, but in this case it opens downwards.

Ask: Think of graph of general quadratic function $y = ax^2 + bx + c$. Have students give their own arguments. If no good response is produced, bring out the graph.

Explain: The graph of quadratic function $y = ax^2 + bx + c$ is a parabola which opens upward if $a > 0$ and opens downward if $a < 0$.

36. **Ask:** What is the graph of the function $y = \frac{12}{x}$

Bring out the table

x	– 6	– 4	– 3	– 2	– 1	1	2	3	4	6
y	– 2	– 3	– 4	– 6	– 12	12	6	4	3	2

Plot the graph

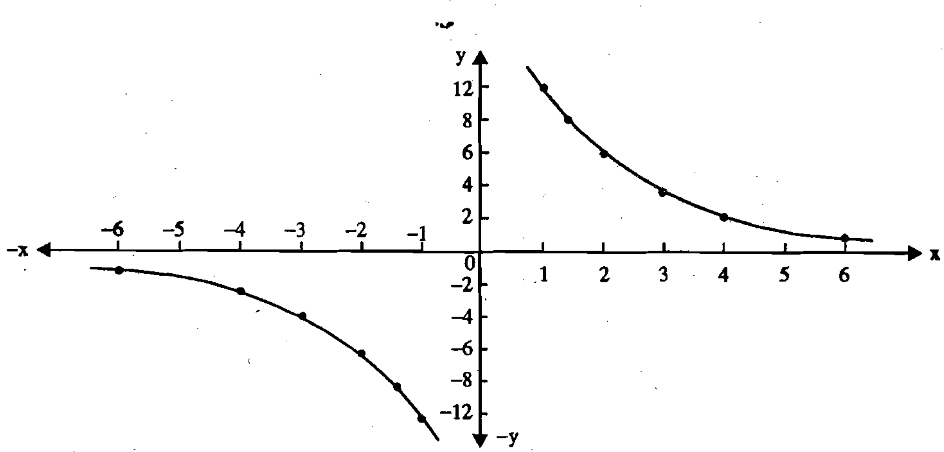


Fig. 12.6

Explain: The function is not defined at x = 0.

- As $x \rightarrow 0$, $Y \rightarrow \infty$. The graph consists of two branches, one in the first quardrant and the other in the third quadrant. No part of the curve lies in the second and fourth quardrants.

  The graph is called a rectangular hyperbola and arises whenever one quantity varies inversely as the other.

Methodology used: Skill can be developed only by practice or drill method.

Check Your Progress

Notes: a) Write your answer in the space given below.

b) Compare your answer with the one given at the end of the unit.

5. Draw the graphs of the following functions:

   i) $y = 2x$ ii) $y = 4x^2$ iii) $y = \sqrt{x}\ (x \geq 0)$

12.7 LET US SUM UP

This unit provides an opportunity to the teacher to demonstrate the importance of "logical way" of thinking as an essential feature of productive thinking. Logical way of thinking is a step-by-step analysis. To make a student think logically it is necessary that the teacher provides a logical development of the unit.

In this unit we have discussed many types of activities on sets, relations and functions. Some of these activities can be associated with daily life problems. Regardless of the ability level of the students, appropriate activities have been found. Teachers should constantly gather materials and ideas for their teaching of this unit.

12.8 UNIT-END ACTIVITIES

1. Which of the following collections are Sets?

   a) $\{x : x^2 - 5x + 6 = 0\}$

   b) {x : x is a person living on earth}

   c) {x : x is a good student in the class}

   d) {x : x is a capital city of India}

   e) {x : x is a river in India}

2. Write the following sets in Roster form

   a) $\{x : x^2 = 9\}$

   b) $\{x : x - 2 = 7)$

   c) {x . x is positive or x is negative integer}

   d) {x : x is a letter in the word 'collection'}

3. a) {x : x is an even number less than 10}

   b) {x : x is an odd number less than 10}

   c) {x : x is a prime number less than 10}

   d) {x : x is a divisor of 10}

4. Write the following sets in set builder form:

   a) {a, e, i, o, u}

   b) {1, 2, 3, 4, 5, ...}

   c) {1, 3, 5 ... }

   d) {2, 4, 6 8...}

   e) {..., –3, –2, –1, 0, 1, 2, 3, ...}

   f) {1, 9, 25, 49...}

5. Match each of the sets in column I described in the set builder form with the same set in column II described in the Roster form.

	Column I	**Column II**
i)	{x : x is a letter in the word College}	a) {1, 2, 4, 8}
ii)	{x : x is a natural number and divisor of 8}	b) {c, o, l, e, g}
iii)	{x : x is a prime number and divisor of 10}	c) {4, – 4}
iv)	{x : x is a root of the equation $x^2 – 16 = 0$)	d) {2, 5}

6. Which of the following sets are equal?

   a) {I, G, N, O, U} and {N, O, G, U, I}

   b) {1, 2, 3, 5} and {3, 5, 7, 9}

   c) {1, –1}, and {x : x is a solution of the equation $x^2 - 1 = 0$}

   d) {x : x is an odd natural number less than 10}
   and {x : x is a prime number less than 10}

7. Which of the following sets are equivalent?

   a) {a, b, c} and {5, 7, 9}

   b) {Pen, Pencil, Chalk} and {chair, table}

   c) {3, 6, 9, 12} and {9, 12, 6, 3}

8. Which of the following sets are finite sets?

   a) {x : x is an even natural number}

   b) {x : x is a natural number less than 100}

   c) {x : x is a day of the week}

   d) x : x is a day of the year}

9. Which of the following sets are null sets?

   a) {a : x x}

b) {x : x is past women President of India}

c) x : $x^2 = 4$}

d) x : $x^2 - 9 = 0$, x is even}

10. Write all subsets of the following sets.

    a) {e}

    b) {i, o}

    c) {2, 4, 6}

11. If A = {x : x is even natural number}

    B = { x : x is positive power of 2}

    Then which one is true?

    i) B A (ii) A B

12. If A = { x : x is a circle in a plane} and

    B = {x : x is a circle in the plane with radius 1.}

    Then which of the following is true?

    1) $A \subset B$. 2) $B \subset A$.

13. Examine whether the following statements are true or false.

    a) If ϕ_1 and ϕ_2 are two null sets, then $\phi_1 = \phi_2$

    b) If S is a set, then $S \in S$

    c) For a set A, A = {A}

    d) The empty set is a subset of every set.

14. If a statement given below is true, mark '√' and if false, mark '×' in the box provided:

    a) If $A \subset B$ and $B \subset C$, then $A \subset C$ ☐

    b) If $B \subset C$, and $C \subset B$, then B = C ☐

15. Which set is a Universal set in the study of Plane geometry.

16. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8}

    C = {3, 4, 5, 6}. Find

    a) $A \cup B$

    b) $A \cup C$

    c) $B \cup C$

    d) $B \cup B$

17. Let A, B, C be the sets described in problem 16. Find

    a) $(A \cup B) \cup C$, b) $A \cup (B \cup C)$

18. Prove

    a) $U \cup A = U$

    b) $\phi \cup A = A$

    c) $A \cup B = \phi$ implies $A = \phi$ and $B = \phi$

19. Find the union of the following sets

    a) A = {a, e, i, o, u}

       B = {a, o, p, q}

b) $\{x : x \text{ is a natural number less than } 5\}$

$\{x : x \text{ is a natural number and } 2 < x < 5\}$

c) $A = \{7, 8, 9\}, B = \phi$

20. If $A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$, find

a) $A \cap B$, b) $A \cap C$, c) $B \cap C$, d) $B \cap B$

21. If A, B and C are the sets described in problem 20, find

a) $(A \cap B) \cap C$, b) $A \cap (B \cap C)$

22. If $A = \{x : x \text{ is a rhombus}\}$

$B = \{x : x \text{ is a rectangle}\}$

$C = \{x : x \text{ is a square}\}$

State with reasons, whether the following statement is true or false:

a) $A \cap B = C$, b) $A \subset C$ c) $C \subset A$

d) $C \subsetneq B$, e) $B \subset C$

23. If $U = \{x / x \in N, x \leq 10\}$ is the universal set, $A = \{2, 4, 6, 8\}$ and $B = \{4, 5, 6, 7\}$. Verify whether the following statement is true or false:

a) $A \cap B = \{4, 6\}$

b) $A \cup B = \{2, 4, 5, 6, 7\}$

24 $A = \{x \in R : x^2 - 5x + 6 = 0\}$

and $B = \{x \in R : x^2 + 4x - 12 = 0\}$

Find

a) $A \cap B$, b) $A \cup B$

25. If $A = \{2, 4, 6\}, B = \{1, 2, 3\}$

Find:

a) $A \times A$, b) $B \times B$, c) $A \times B$, d) $B \times A$

26. Write true or false:

a) Two ordered pairs (a, b) and (c, d) are equal if a = c and b = d.

b) For any two sets A and B, $A \times B = B \times A$

c) For any two sets A and B, if R is a relation from A to B, then R is a relation from B to A.

27. Draw arrow diagrams to illustrate the relation R from set A to set B in the following cases.

a) $A = \{1, 2, 3, 4\}, B = \{4, 8, 12, 16, 20)$ and

$R = \{(1, 4), (2, 8), (3, 12)\}$

b) $A = \{1, 2, 3, 4\}, B = \{7, 8, 9\}$ and

$R = \{(1, 7), (2, 7), (3, 8), (4, 9), (4, 8), (2, 8)\}$

c) $A = \{20, 40. 60, 80\}, B = \{2, 4, 6, 8, 10\}$ and

$R = \{(20, 2), (40, 4), (60, 6), (80, 8)\}$

28. Draw arrow diagrams to illustrate the function f from set A to set B in the following cases:

a) $A = \{25, 16, 9\}$

$B = \{-5, -4, -3, -2, -1, 0 1, 2, 3, 4, 5\}$ and

$f(x) = \sqrt{x},\ x \in A$

b) $A = \{1/2, 1/3, 1/4\}$, $B = \{3, 4, 5\}$

$f(x) = \frac{1}{x} + 1,\ x \in A$

29. Sketch the graph of the following functions:

a) $f(x) = 5x - 7$

b) $f(x) = \frac{-3x}{2} - \frac{5}{2}$

c) $f(x) = x^2 - 5$

d) $f(x) = -x^2 + 6$

e) $f(x) = x$, for positive real x.

f) $f(x) = |x - 2|$

g) $f(x) = |x| - x$

12.9 ANSWERS TO CHECK YOUR PROGRESS

1. i) Not a set

   ii) A finite set

2. Set = {1, 2, 3}

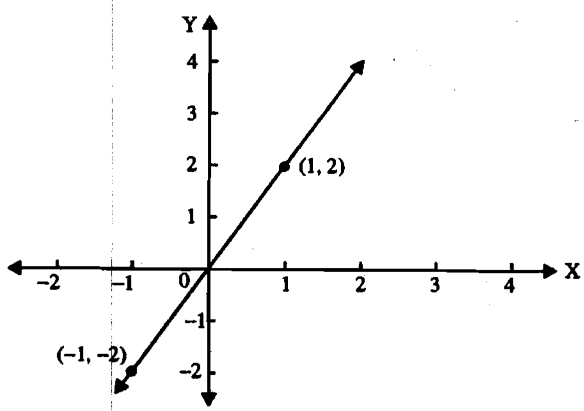
   Subsets: ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}.

   Universal set: {1, 2, 3}

3. i) $\sim A$, ii) $A \cap B$, iii) $A \cap (\sim B)$. i e. elements of A which are not in B

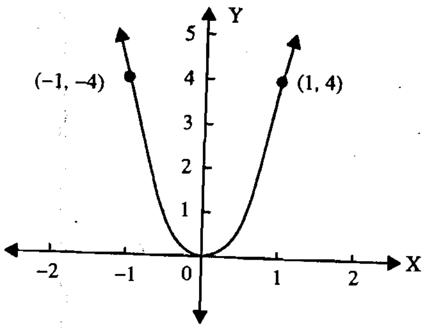
4. i), and ii) are functions.

5. i) $y = 2x$



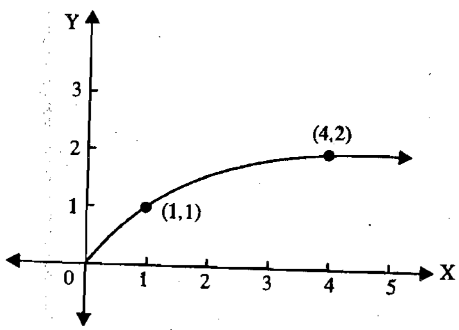
x	0	1	-1
y	0	2	-2

ii) $y = 4x^2$



x	0	1	−1
y	0	4	−4

iii) $y=\sqrt{x},(x\geq 0)$



x	0	1	4
y	0	1	2

12.10 SUGGESTED READINGS

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