UNIT 6 ELEMENTARY NUMBER THEORY

Structure

6.1 INTRODUCTION

Numbers have always fascinated man. The early history of mathematics is practically the history of numbers. The Greeks spent much of their energy on the discovery and exploitation of special and novel properties of numbers. We can discern two major motivations which encouraged their studies — one a rational motivation and the other the intuitive urge. Even today a strong intellectual urge exists to pursue rigorous logical research to fill in the gaps in the structure of mathematical systems. This is purely a rational motivation. Under this approach the structure of natural numbers is studied under the title "Number Theory", which is one of the most abstract branches of mathematics. An intuitive urge to sense and grasp mystic numbers lore has always gripped man. Number patterns, number sequences and their geometric representations help in discovering inner meanings and concepts which are brilliant examples of intuitive thinking. In some places this approach degenerated into forms of number worship under the heading "Numerology".

This unit will present problems of both the types, i.e., the problems which require reflective or analytical thinking and the problems which can be solved through intuitive thinking. In this unit we shall consider the set of natural numbers to be the universal set. We shall explore interesting aspects of some subsets of the set of natural numbers.

6.2 OBJECTIVES

At the end of the unit, you will be able to:

- state and interpret the meaning of different subsets of natural numbers such as odd, even, composite and prime numbers;
- demonstrate how number sequences are illustrated through geometric patterns;
- provide examples of different ways in which mathematical proof can be organized;
- encourage intuitive thinking to generalise important properties and principles;
- enumerate the various situations in which processes such as factorisation, L.C.M. and H.C.F. are applied;
- provide an insight into the "why" and "how" of processes such as "finding H.C.F. by the division method" and discourage mechanical application of steps;

- explain different types of numbers like pythagorean triplets, square numbers, triangular numbers, odd and even numbers;
- exhibit the relationships between these different types of numbers.

6.3 ELEMENTARY NUMBER THEORY

6.3.1 Basic Concepts

Main Teaching Points : a) What is a factor?
b) Even and odd numbers and their properties.

Teaching-Learning Process : The unit should be introduced by reviewing fundamental operations on natural numbers in such a manner that a pattern strikes the imagination of pupils and they are able to formulate a new generalisation which can be named as a new definition or property. The procedure to develop this ability is outlined in the unit.

A teacher should ensure that :

1. Induction is planned for and encouraged.
2. Deduction is urged Some additional reasoning is necessary to arrive at a generalisation. Also, there are no stereotype patterns of formulating "proofs". If the essential ideas which the student develops are correct and flawless in reasoning, his/her "proof" should be accepted as a good one.

   Some activities which set the stage for deductive arguments are given for a few selected topics.

Activities

1. Ask . What is 3×4 ?
   Write the answer "$3 \times 4 = 12$"

   Read it (as shown) in many ways.

   3 multiplied by 4 is 12
   3 is a divisor of 12
   4 is a divisor of 12
   3 is a factor of 12
   4 is a factor of 12
   12 is a multiple of 3
   12 is a multiple of 4

   Ask : What other numbers can be multiplied to get 12 ?

   Explain . In a product each of the numbers is called a factor. A factor is a divisor.

2. Ask : Repeat the above process for some other natural numbers, such as 24, 30, 36, 42, 48, 64, (If necessary students may refer to multiplication tables) 6×4, 8×3, 12×2, 24×1 are all factorizations of 24.

   $6 \times 4 = 24$, $8 \times 3 = 24$
   $12 \times 2 = 24$; $24 \times 1 = 24$
   $5 \times 6 = 30$; $10 \times 3 = 30$
   $15 \times 2 = 30$; $30 \times 1 = 30$

   Explain . This process of finding a factorization is called "factoring". Point out that factors of natural numbers may be found in several different ways. Give more exercises in factoring.

3. Ask · Does 3 have one as a factor ? Do 5, 7, 9 and 11 etc. have one as a factor ?

   What about 24. 30, 36 etc.?

   Bring out (by induction on the part of students) that every natural number has one as the factor

   Ask : What is the product of any natural number (say n) and 1? How can it be expressed in symbols? If n is any natural number what is $n \times 1$?

Demonstrate with examples that "$n \times 1 = n$" becomes true whenever a numeral for any natural number is put in place of n.

$30 \times 1 = 30$, $55 \times 1 = 55$, $82 \times 1 = 82$

Bring out that every natural number has itself as well as one as its factor.

4. Ask : Think of some number that has 2 as a factor. Write it on the blackboard. Bring out that this set of natural numbers is called the set of even numbers. An even number is one which has the factor 2. Write numerals for some large natural numbers, some even, some odd.

   Ask : Can you tell me which of these are even? Observe the digit at the unit's place.

   Bring out that those numbers which have the unit digit as 2, 4, 6, 8, or 0 are even and the others are odd.

   Ask : Suppose we multiply a natural number by an even number What do we get? Try a few examples and write them on the board. Help students to reach the conclusion (inductively) that such products are always even.

   Ask : Can we be sure that this is always true? We tried it for only a few cases. Can we reason this out for all natural numbers?

Let students give their own arguments. If none of the arguments produced is good enough, bring out the associative law and suggest they try using it: Any even number has 2 as a factor and so is $2 \times a$ where "a" represents any natural number. Then $(2 \times a) \times b = 2 \times (a \times b)$ for any natural number "b". We see that this has 2 as a factor.

This informal deduction should be encouraged by using any valid arguments proposed by students. It must be insisted that they argue in such a way that what they say is good for all natural numbers and not just a finite number of them

Point out that this shows that the set of even numbers is closed for multiplication

In the same manner encourage students to discover that the set of even numbers is closed under addition.

$$(2.a) + (2.b) = 2. (a+b).$$

Again use first induction, then informal deduction, to enable students to discover generalisations about operations with odd and even numbers so that the study of numbers becomes a more creative experience.

5. Addition of odd and even numbers :

   a) Even numbers added to even numbers

	2	6	2	230
+	4	4	4	512
	6	8	12	108
		18	18	850

   Elicit from pupils : It appears that the sum of any number of even numbers is even.

   b) The sum of two odd numbers

$$\begin{array}{r} 7 \\ +\,3 \\ \hline 10 \end{array} \qquad \begin{array}{r} 13 \\ +\,11 \\ \hline 24 \end{array}$$

   Ask pupils to find a proof

   Proof : Any odd number is an even number plus one.

$$\begin{array}{r} 7 = 6 + 1 \\ 3 = 2 + 1 \\ \hline 8 + 2 \end{array} \qquad \begin{array}{r} 13 = 12 + 1 \\ 11 + 10 + 1 \\ \hline 22 + 2 \end{array} \qquad 1 + 1 = 2$$

   (an even number)

Elicit from pupils : The sum of two odd natural numbers is an even number

In the same manner get the generalisation : **The sum of an odd and an even number is an odd number.**

6. Multiplication of an odd and an even number.
   a) Review the concept that multiplication is a short form of addition.
   b) Multiplication of an even number by an even number.

      Let pupils observe that this amounts to adding one even number to itself an even number of times. The answer should be even.
   c) Multiplication of an odd number by an even number.

      Let pupils observe that this amounts to adding an odd number to itself even number of times. Give examples

      Elicit : An odd number multiplied by an even number yields an even number.
   d) Multiplication of an even number by an odd number.

      Let pupils observe that this is the same as adding an even number to itself an odd number of times. Give examples

      Elicit : An even number multiplied by an odd number yields an even number.

      Similarly let pupils get the generalisation : **The product of two odd numbers is an odd number.**

      Methodology used : Mainly the inductive method is used. The deductive method is used to write down the proofs of some properties of even and odd numbers.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

1. A number x multiplied by any number y always gives product as y. What is the value of x?

   ....

2. Prove that the sum of two consecutive numbers always odd.

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3. Arguing as in the case of 'A number is even if its units place is even (either of the members 0 2, 4, 6, 8)' write and demonstrate the rule to check the divisibility of a given number by 4.

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4. Show that the product of two even numbers is divisible by 4.

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6.3.2 Prime Factorization: H.C.F., L.C.M.

Main Teaching Points : a) Prime factorization of a number.

b) To find H.C.F. and L.C.M. using prime factorization

Teaching-Learning Process

1. Ask : Write the factors of the numbers 1 to 10. Think of numbers that have exactly two different factors — 1 and the number itself

   Explain that these are called "prime numbers". Explain that numbers with more than two different factors are called "composite numbers".

   Ask : Have students examine prime numbers and find out how many of these are even

   Explain that 1 is not a prime number since it does not have two different factors

2. Write down numbers such as 36, 60 and 84. Have students write the various factorizations Use the factor tree and show how to complete the factorization by continuing to factorize composite factors.

   Ask students to repeat the process for several other numbers. Encourage them to tell what they notice.

   Bring out that the final prime factors are the same except for their order (induction). Then, ask for a deductive argument that this is true for all natural numbers.

   Explain : This is called the "unique factorization property of natural numbers" —that every composite natural number **can be factored into primes in only one way, except for order.**

3. Explain : If two or more numbers have a certain number as a factor, we say the factor is common to the numbers. For example, 12 and 18 have 1, 2, 3 and 6 as common factors Ask students to write common factors of different sets of numbers and pick out the "Highest or Greatest Common Factor". Explain that this is called the Highest (or Greatest) Common Factor (H.C.F.)

   Solve some more examples on finding common factors and picking out the highest common factor.

   Point out that we can use prime factorization of each number and find out H.C.F. by multiplying all the common factors.

   $60 = (2 \times 2 \times 3) \times 5$
   $84 = (2 \times 2 \times 3) \times 7$
   Common factors are (2, 2, 3)
   H.C.F $= 2 \times 2 \times 3 = 12$

4. Ask students to recall the factors of some natural numbers Explain that if one number is a factor of another, then the second is called a "multiple" of the first.

   Ask students to write "multiples" of several numbers, such as 3. 5, 7. 9, 12. Bring out the meaning of the common multiple of any two numbers.

   Given two (or more) numbers. any number which is a multiple of each of them is called a "common multiple" of them.

   Explain : Least Common Multiple (L.C.M.) of several numbers is the smallest number which is a multiple of each of them.

   Multiples of 14 = 14, 28, 42, 56

   Multiples of 21 = 21, 42, 63, 84

   Common multiples = 42. 84, 126

   L C M = 42

   Use prime factorization to find L.C.M. If the numbers are 8, 14 and 21 and N is their L.C.M. then N is divisible by all the three numbers. We find a numeral in place of ? mark, N must have 2×2 2 as a factor: it should

   $8 = 2 \times 2 \times 2$,
   $14 = 2 \times 7$ and
   $21 = 3 \times 7$.
   $N = 2 \times 2 \times 2 \times$?
   $N = 2 \times 7 \times$? $N = 2 \times 2 \times 2 \times 7 \times 3$
   $N = 3 \times 7 \times$?

also have 2×7 as a factor and 3×7 as a factor. We see $2 \times 2 \times 2 \times 7 \times 3$ satisfy these conditions Therefore, L.C.M. $= 2 \times 2 \times 2 \times 7 \times 3 = 168$

Methodology used: Inductive reasoning is used to illustrate the method of finding H.C.F and L.C.M using prime factorization.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

5. Find the H.C.F. and L.C.M of 28 and 98 by prime factorisation.

6. Find the product of H.C.F and L.C.M. in each of the following:

   i) 12 and 18 ii) 140 and 126.

7. Find the product of

   i) 12 and 18 ii) 140 and 126.

8. What is your observation on comparing the product obtained in Q.3 with those of the H.C.F. and L.C.M. in Q.2.

9. Generalise the result obtained in Q.4.

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6.3.3 Division Method for Finding H.C.F.; L.C.M.

Main Teaching Point : To find H.C.F. by the division method.

Teaching-Learning Process : Euclid developed a method of finding H.C.F that was based on the division relation. His process is called Euclidean Algorithm.

The process should be demonstrated through different examples

H.C.F (84, 270) =

H.C.F (18, 84) =

H.C.F (12, 18) =

H.C.F (6, 12)

Find H.C.F. of 84 and 270. Divide the smaller (84) into the larger (270). The remainder is 18. Now divide 18 into the first number 84. Repeat the process till the remainder is 0.

Explain the "why" of the process. H.C.F. is the largest number that divides each of the two numbers

$270 = 84 \times 3 + 18$

A divisor of both 270 and 84 must divide 18

$84 = 18 \times 4 + 12$

A divisor of 84 and 18 must divide 12

$18 = 12 \times 1 + 6$

A divisor of 12 and 18 must divide 6

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84) 270 (3
    252
    ----
    18) 84 (4
        72
        --
        12) 18 (1
            12
            --
             6) 12 (2
                12
                --
                 0
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The last divisor is the H.C.F. $\therefore$ H.C.F. = 6.

Methodology used : The demonstration-cum-lecture method is used. The reason at every step should be discussed.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

10. Define co-prime numbers and give three examples.

11. By division method show that two consecutive numbers are always coprime.

12. Using Euclid's method find the H.C.F. of 65 and 395.

6.3.4 Applications of H.C.F. and L.C.M

Main Teaching Point : To solve word problems.

Teching-Learning Process : Word problems are a device useful in teaching problem solving. These are used as practice material upon which a student can apply the generalization or the principle already learnt. A student's success is dependent on his possession and recall of generalizations that suit the situation under consideration.

Ask students to read the problem, analyse the information given in the problem to locate what is known and to understand what is to be found out. Encourage students to translate the verbal sentence into mathematical sentences to discover the procedure.

Example : A school had 851 boys and 629 girls. It divided the students into the largest possible number of classes which had an equal number of students so that each class of boys should have the same number of students as each class of girls. Find the number of classes.

Given : Number of boys = 851;
Number of Girls = 629

Analysis : Since the students are divided into largest possible equal classes, the size of each class is the H.C.F. of 851 and 629 i.e. 37

$$\text{Total number of classes} = \frac{851+629}{37} = \frac{1480}{37}$$

$$= 40$$

Example : Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 seconds respectively. When will they next toll together and how often will they toll together in 30 minutes?

Given : Six bells toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively.

Analysis : The timings of tolling for each bell separately will have to be the multiples of their respective intervals. Hence, the timing of their tolling together is the L.C.M. of 2, 4, 6, 8, 10 and 12.

i.e.

120 seconds

120 seconds = 2 minutes

No. of times they will toll together in 30 minutes

$$= \frac{30}{2} + 1 = 16 \text{ times}$$

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
3	1, 1, 3, 2, 5, 3
	1, 1, 1, 2, 5, 1

L.C.M. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$

(Discuss why 1 more)

Methodology used : The discussion method is used together with the Heuristic approach. It should be stressed that students translate the verbal sentence into a mathematical sentence themselves.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

13. Three cans of capacity 960 ml, 1*l* and 2*l* are to be filled with oil. What is the size of the largest container which will fill each of them a complete number of times?

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14. In a morning walk three persons step out together. If their steps measure 80 cm, 85 cm and 90 cm, at what distance will they again step together.

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6.3.5 Various Types of Numbers

Main Teaching Points : a) Polygonal numbers

b) Perfect numbers

c) Pythagorean triplets

Teaching-Learning Process : Some typical properties in the construction and shape of numbers have been discovered. These provide a fascinating experience and create interest in learning.

A. Polygonal numbers

The early Greeks represented the polygonal numbers by dots and named the numbers according to the resulting figure that the numbers .... represented. These are defined as sums of special arithmetic progression.

1. Triangular numbers

1	3	6	10	15
.	.	.	.	.
	..	..	..	..
		...	...	...
			....	....
				.....

$$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

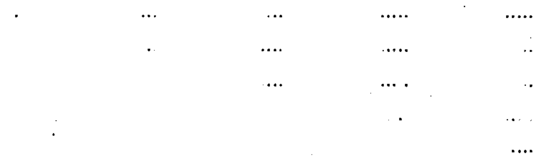
2. Square numbers

1	4	9	16	25
.	..	...	....	.....
	..	...	....	.....
		...	....	.....
			....	.....
				.....

$$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots + (2n - 1) = n^2$$

3. Rectangular numbers

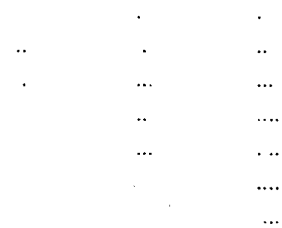
2 6 12 20 30



$$2 + 4 + 6 + 8 + 10 + \ldots\ldots\ldots + 2n = n(n+1)$$

4. Pentagonal numbers

1 5 12 22



$$1 + 4 + 7 + \ldots + (3n-2) = \frac{n(3n-1)}{2}$$

B. Perfect numbers

A perfect number is one that equals the sum of its proper divisors 6, 28, 496, 8128 are the examples of perfect numbers The smallest 6 and 28 were known to the early Hindu mathematicians.

$6 = 3 + 2 + 1 \qquad 28 = 14 + 7 + 4 + 2 + 1$

C. Pythagorean numbers

Pythagorean numbers are natural number triplets x,y,z, that satisfy the relation $x^2 + y^2 = z^2$

Pythagoras and his followers (c.570 – 497 B.C.) knew of this relation but it is not known whether they learned of it from others or discovered it.

Evidence has been found that Pre-pythagorean Hindu writings included a few pythagorean number sets. Perhaps Babylonians also had some knowledge of the pythagorean sets of numbers about 1000 years before Pythagoras.

Formulas for the pythagorean triplets

a) Pythagoras is said to have given the following formula

$x = 2n + 1$

$y = 2n^2 + 2n$

$z = 2n^2 + 2n + 1$ where $n \in N = \{1, 2, 3, 4, 5, 6.....\}$

b) Euclid gave the following formula in his elements

$x = 2mn$

$y = n^2 - m^2$

$z = n^2 + m^2$, where $n, m \in N = \{1, 2, 3, 4, 5, 6 .....\}$

Do the above formulae give all the possible triplets that satisfy the relations $x^2 + y^2 = z^2$?

A discussion on this point may be conducted. Put into simplified form, the general formulae are $y = p^2 - q^2$, $x = 2pq$ and $z = p^2 + q^2$ where p and q are relatively prime.

Methodology used : mainly the discussion method is used with inductive reasoning and reading of patterns

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

15. Give two example of triangular and rectangular numbers more than 50.

16. Give two examples of pythagorean triplets.

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6.4 ENRICHMENT MATERIALS

1. Sieve of Eratosthenes

The Sieve of Eratosthenes is an ingenious device for isolating the prime numbers below a certain number.

Give each student a chart of numbers such as:

2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	....

Ask students to circle 2(②) and cross out multiples of 2. Next circle 3 (③) and cross out multiples of 3. Next circle 5(⑤) and cross out multiples of 5. Continue till all numbers which are multiples of other numbers are crossed out.

The numbers 2, 3, 5, 7, 11, which are in the circles, are prime numbers.

Use this method to find the number of primes in each of the following intervals.

(a) 1 – 20 (b) 21 – 40 (c) 51 – 100

2. Goldbach's conjecture

Ask What is the sum of two primes 5 and 7 ? Ask for sums of several pairs of primes (always making sure not to include 2 as one of the primes) and have the students write the sums

Ask them what they notice about the numbers which are sums of two primes and hope for the conclusion that they are all even (or at least composite).

Then ask them to give a generalization or a deductive argument

Finally show an example like 2 + 3, or 2 + 11. Point out that the guessed argument (or generalization) was false

Thus the students may realise the limitation of the induction method. It requires only one example to show that the general principle arrived at is false.

3. How many are the primes?

Students may conjecture that really big numbers must necessarily be divisible by some smaller ones, thus there should be a greatest prime. This possibility occurred to Euclid. He ruled it out by providing that **there is no greatest prime**. The proof is an excellent example of use of logic in proving a conjecture. It illustrates **indirect proof** (or reduction and absurdum)

It p is to be proved assume **not p** ($\sim p$) is true and hence derive a contradiction. Then the assumption **not p** must be false. So **not not p** ($\sim(\sim p)$) is true.

Assume number of primes is finite

Then there is a largest prime K.

Let the primes be a b, c ...., k

Then $N = a \times b \times c \times \ldots \times k$ Now add 1 to this number

$N + 1 = a \times b \times c \times \ldots\ldots \times k + 1$

Since this new number exceeds the largest prime N, it must be composite, hence divisible by some number other than 1 and itself.

Let X be the smallest such number which divides N+1. If X were composite, some divisor of it would also divide N + 1, contradicting our provision that X is the smallest divisor of N + 1 (other than 1). Therefore X is prime. Since k is the largest prime, X is amongst the numbers a, b, c .... k. Consequently X divides N evenly. But then X cannot divide N + 1, a remainder of 1 is left when the division is attempted.

Thus our original supposition that there is a largest prime has led us to an impossible conclusion, a contradiction So the number of primes is infinite.

This means that the primes are distributed along the whole sequence of natural numbers although their number gradually thins out as the natural numbers mount in size

Ask students to find out the number of primes less than 100, and between 101 and 1000 to check this observation.

4. **Another unproved conjecture** is that every even number greater than two can be expressed as a sum of two primes (not necessarily different)

$4 = 2 + 2$; $6 = 3 + 3$; $8 + 5 + 3$; $10 = 7 + 3$ or $5 + 5$

No one as yet has a proof that this is possible for every even number.

5. Finding Primes

How can one test if a given number is prime? There is no known formula. One such formula known as Wilson's Theorem does exist, but its value is theoretical rather than practical. A general rule for finding a prime is as follows:

To find out if the number N is prime, test the primes in turn (2, 3, 5, 7, 11.........) for divisibility into N, but do not test beyond $\sqrt{N}$ (i e., beyond the larges natural number whose square is less than or equal to N).

6.5 LET US SUM UP

This unit provides an opportunity to the teacher to demonstrate the importance of "intuition" as an essential feature of productive thinking. "Intuition" is a mental act of teaching fromulations or ~~conclusions~~ without going through a step by step analysis. To make students think intuitively, it is necessary that the teacher himself thinks intuitively. He should be open to making mistakes in guessing the generalization as in this approach one does not have to be right first time. The teacher who poses an open-ended situation to the students and is then willing to examine it with them in the insecurity of not knowing the answer will be more effective. The repression of guessing inhibits the intuitive approach and throws the student back to entirely analytical procedures using known facts and consciously learned and remembered procedures. Intuitive thinking develops self-confidence and courage in the students since it permits willingness to make mistakes.

6.6 UNIT-END ACTIVITIES

1. Set up situations to show that :
   a) Each prime greater than 3 is either 1 less or 1 more than a multiple of 6.
   b) There is an infinite number of twin primes. (Twin primes are pairs of primes differing by 2 such as 3,5; 5,7; 11,13; and so on)
   c) From 2 onwards there is at least one prime number between any number and its double.

2. a) Ask pupils to examine the subtraction and division of odd and even numbers. Ask them to give generalisations by inspection and then work out a proof for the same.
   b) Have pupils test the effect of squaring on numbers.

3. Using the relation $84 = 60 + 24$ give an argument to show that any divisor of 60 and 84 must also be a divisor of 24.

4. Show that any common multiple of a pair of numbers is a multiple of their L.C.M.

5. Determine two numbers, knowing their H.C.F. and their sum as given in the following table:

Sum	72	360	552	420
H.C.F.	9	18	24	12

6. Given any counting number value for n such as 50, find a formula for the sum of the first n numbers.

   Sum = $1 + 2 + 3 + 4 + \ldots\ldots + n$.

   Give the analysis of the nature of the problem and examine it to discover a pattern which should help in arriving at a formulation.

7. Euclid proved that whenever the natural number n is such that $2^{n+1} - 1$ is a prime number, then multiplying this prime by 2^n produces a perfect number.

8. Explore Euler's polynomial : $n^2 - n + 41$ to get prime numbers.

9. Find sequences whose first differences are arithmetic progressions, but whose second differences are a constant.

$$\begin{array}{ccccccc} 11 & & 13 & & 17 & & 23\ldots\ldots \\ & 2 & & 4 & & 6 & \\ & & 2 & & 2 & & \end{array}$$

10. Explore the gaps or intervals between primes. Can you say that there are never any intervals of 8, 10, 12......

6.7 ANSWERS TO CHECK YOUR PROGRESS

1. $x = 1$

2. Let the two consecutive numbers be n and $n + 1$. sum $= n + (n+1) = 2n + 1$ = even number + one $\therefore$ sum is odd

3. A number is divisible by 4 if the number formed by its ten's and units place is divisible by 4.

   Example $348 = 300 + 48 = 3 \times 100 + 48$

   3×100 is divisible by 4 (because 100 is divisible by 4) & 48 is divisible by 4 $\Rightarrow$ 348 is divisible by 4.

4. Let the two even numbers be 2n and 2m (n, m $\in$ N).

   $\therefore$ Product $= (2n) \times (2m)$

   $= 2 \times (n \times 2) \times m$ (associativity of multiplication)

   $= 2 \times (2 \times n) \times m$ (commutativity of multiplication)

   $= (2 \times 2) \times n \times m$ (associative property)

   $= 4nm$

   Product of two even numbers is divisible by 4.

5. $98 = 2 \times 7 \times 7$ and $28 = 2 \times 2 \times 7$

   $\therefore$ H.C.F. $= 2 \times 7 = 14$ and L.C.M $= 2 \times 2 \times 7 \times 7 = 196$

6. i) H.C.F of 18 and 12 = 6, and L.C.M of 18 and 12 = 36

   H.C.F × L.C.M $= 6 \times 36 = 216$

   ii) H.C.F of 140 and 126 = 14, L.C.M = 1260

   H.C.F × L.C.M $= 14 \times 1260 = 17640$

7. i) $12 \times 18 = 216$ ii) $140 \times 126 = 17640$

8. i) $6 \times 36 = 12 \times 18$ ii) $14 \times 1260 = 140 \times 126$

9. Product of two numbers is equal to the product of their H.C.F. and L.C.M.

10. Numbers which have no common factor other than 1 are called coprimes. 5 and 7, 12 and 23, 4 and 9 are pairs of coprime numbers.

11. Let the number be n & n + 1

$$
\begin{array}{l}
n\,\overline{)\,n+1\,(}\,1 \\
\quad n \\
\quad 1\,\overline{)\,n\,(}\,n \\
\qquad n \\
\qquad \overline{0}
\end{array}
$$

$\therefore$ H.C.F. of n and $n + 1 = 1$

$\therefore$ n and $n + 1$ are coprime.

12.
$$
\begin{array}{l}
65\,\overline{)\,395\,(}\,6 \\
\quad 390 \\
\quad 5\,\overline{)\,65\,(}\,13 \\
\qquad 65 \\
\qquad \overline{0}
\end{array}
$$

$\therefore$ HCF = 5

13. Capacity of the first can = 960 ml

    Capacity of the second can = $1l$ = 1000 ml

    Capacity of the third can = $2l$ = 2000 ml

    The capacity of the largest container which can fill each one of them in complete number of times = H.C.F. of 960, 1000 & 2000

H.C.F. of 960 & 1000

$$
\begin{array}{l}
960\,)\,1000\,(\\
\quad 960\\
\quad 40\,)\,960\,(\,24\\
\qquad \underline{80}\\
\qquad 160\\
\qquad \underline{160}\\
\qquad \underline{0}
\end{array}
$$

H.C.F. of 40 and 2000 = 40

$$
\begin{array}{l}
40\,)\,2000\,(\,50\\
\quad \underline{2000}\\
\quad \underline{0}
\end{array}
$$

$\therefore$ Capacity of the container = 40 ml

14. They would again step together at a distance equal to the L.C.M of their steps = L.C.M. of 80, 85 & 90

= 12240 cm

= 122.40 m

15. Triangular number = $\frac{n(n+1)}{2}$

$\therefore \frac{n(n+1)}{2} > 50$

$\Rightarrow n(n+1) > 100$

$\Rightarrow n > 10$

For n = 10,

triangular number = $\frac{10 \times 11}{2} = 55$

For n = 11,

triangular number = $\frac{11 \times 12}{2} = 66$

i) 55

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ii) 66

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Rectangular number = n (n + 1)

$\therefore n(n+1) > 50$

when n = 7, Rectangular number = 7 × 8 = 56

when n = 8, Rectangular number = 8 × 9 = 72

2. 56

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(56 = 7 × 8)

72

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(72 = 8 × 9)

16. $x = 5$, $y = 12$, $z = 13$.

$\therefore x^2 + y^2 = z^2$

$\therefore$ (5, 12, 13) is a pythagorean triplet.

(3, 4, 5) is another pythagorean triplet. It has consecutive numbers.

6.8 SUGGESTED READINGS

Swain, Robert L. and Nichols, Ergene D. : *Understanding Arithmetic*, Holt, Rinehart and Winston Inc. New York.

Jones, Burton W : *Elementary Concepts of Mathematics*, The Macmillian Company, New York

Meyer, Jeromes *Fun with Mathematics*, Faweet Publications, New York.

Adler, Irving *Magic House of Numbers*, The John Day Company Inc., New York.

Dantizig, Tobias *Number, The Language of Science*, The Macmillan Company, New York.

Jones, Burton W . *The Theory of Numbers*, Holt, Rinehart and Winston Inc., New York