UNIT 3 CONDITIONAL PROOF AND INDIRECT PROOF

Contents

3.0 OBJECTIVE

In this unit I propose to introduce a new list of techniques of testing the validity of arguments. There are as many kinds of techniques as there are arguments. The main purpose of this unit is to make you understand that there is not a single technique which helps you to solve all kinds of problems.

It is not sufficient if you know the art of testing validity only. Therefore one of the aims is to introduce you to the art of testing invalidity also. To have a satisfactory knowledge of good argument you should also know what makes an argument bad. Therefore this unit introduces you to this aspect of the study of logic.

3.1 INTRODUCTION

The method of Conditional Proof (C.P.) is different in kind from the rules of inference or replacements. There are a certain types of arguments, which cannot be tested with any of the rules discussed in the previous chapters without further support.The rules discussed earlier are restricted only to those arguments, which have unconditional conclusions. So an argument, which has conditional conclusion, falls out of their purview. The most familiar example for conditional proposition is implicative proposition. Since implicative propositions have equivalent disjunctive and negation forms, they are also to be regarded as conditional propositions. Again, C.P is not a system of proof, which does away with the nineteen rules. Only, the number increases to twenty. Among them one rule is compulsorily used to test the validity when the conclusion is conditional. This rule is characteristic of C.P in the sense that nowhere else it is used. Hence this rule can be designated as the rule of C.P.

3.2 CONDITIONAL PROOF

Any deductive argument, whether valid or invalid, can be expressed in the form of a conditional proposition. What is more important to know is that the original argument is valid only when the corresponding conditional statement fulfills a condition known as 'tautology'. Otherwise the argument is invalid. Consider this example:

1). All A are B

All B are C / ∴ All A are C

Its corresponding conditional form is as follows:

"If all A are B and all A are C, then all A are C". (1).

Let the first premise be symbolized as P_1 and second as P_2. Conclusion is symbolized as C. Now (1) becomes:

$(P_1 \Lambda P_2) => C$ (2)

(2) is said to be tautologous because its corresponding proposition form is tautologous. A proposition form is said to be tautologous when it has only true substitution. No matter how many substitutions we make for proposition form, all of them must be true. In other words, if there are 'n' number of instances in which substitution is made to the proposition form, then in all these 'n' instances the proposition form must be true. There are two conditions to be satisfied if C. P. should show that the argument is valid.

1). Conclusion must be a conditional proposition.

2). It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments which satisfy the relevant rules of inference. That is, all premises in C.P. should be supported by these rules. The additional premise, which is a characteristic mark of C.P., is always the antecedent of the conclusion and the construction of proof always begins with antecedent of the conclusion as the premise. This premise itself is called C.P. An example of argument, which requires C.P., is given below.

(3) P => (A=> B)

When P stands for the conjunction of premises, one of the rules of replacement, i.e., exportation rule permits us to rewrite (3) as:

(4) (PΛ A) => B

It is obvious that the conclusion of (4) is the consequent of the conclusion of (3). Since we start with an assumed premise, the proof is known as C.P. Here is the difference. All other premises are taken as true. The assumption should not really matter. Even if the assumed premise is false, it is possible to deduce valid conclusion. If B can be validly drawn from P and A then not only (A) is valid its corresponding original argument (3) also must be valid because (3) and (4) are logically equivalent argument of this form.

1. 1. (A v B)=> (CΛ D)

2). (D v E) => F / ∴ A=>F

We should start from assuming A.

3). A /∴ F C. P.

In C. P. always the first line must have this structure. Slash against line 3 in, ∴ and (C.P) indicate that the method of conditional proof is being used.

4).	A v B	3,	Add.
5).	C Λ D	1, 4,	M.P.
6).	D	5,	Simp.
7).	D v E	6,	Add.
8).	∴ F	2, 7,	M.P.

If there is only one condition in the conclusion, then C.P is used once. If there are two conditions in the conclusion, then C.P. is used twice. In such cases the procedure to be followed is as follows.

2.

1).	A=> (B=> C)			
2).	B=> (C=> D)	/∴ A => (B =>D)		
3).	A	/∴ B => D		(C.P.)
4).	B	/∴ D		(C.P.)
5).	B => C		1, 4,	M.P.
6).	C		5, 4,	M.P.
7).	C => D		2, 4,	M.P.
8).	∴D		7, 6,	M.P.

3.3 INDIRECT PROOF (I.P.)

This method is also known as *reductio ad absurdum*, a method very common in the construction of proof of geometrical theorems. This method is characterized by a special feature. In order to prove a certain statement, its contradiction is assumed to be true from which the conclusion, which contradicts our assumption, is logically deduced. If A contradicts ⌐ B, then either A must be false or ⌐ B must be false. A cannot be false because it is logically deduced from what is purported to be true. Therefore ⌐B must be false, which means that B must be true. This is how a theorem in geometry or an argument in logic is, sometimes, proved.

This method has a distinct advantage. Sometimes the length of proof is too long. In logic it is important that we use the least number of steps. Second requirement is clarity. Combination of these two is what is most desired. In such circumstances, this method is most useful. The use of this method consists in beginning with the contradiction of what is to be proved. A point to be noted here is that, the contradiction of what has to be proved is marked by writing I.P. on the right hand side just adjacent to the assumption. In C.P. also we begin with assumption. The difference is that in the latter what is assumed is a part of the argument whereas in the case of former it is not. Consider this argument.

3.

1.	A => (BΛC)		
2.	(B v D) => E		
3.	D v A	/ ∴ E	
4.	⌐ E		I.P.
5.	⌐B Λ ⌐ D	2, 4,	M.T.
6.	⌐D	5,	Simp.

7.	A	3, 6,	D.S.
8.	BΛC	1, 7,	M.P.
9.	B	8,	Simp.
10.	B v D	9,	Add.
11.	E	2, 10,	M.P.
12.	E Λ ⌐ E	11, 4,	Conj.

Tenth Step can also be written and consequent step in this manner

13.	⌐ B	5,	Simp.
14.	B Λ ⌐ B	9, 13,	Conj.

Whether we get E Λ ⌐ E or B Λ ⌐ B, the result remains the same. In both the cases there are steps in the argument whose conjunction leads to contradiction. Wherever there is contradiction, one conjunct must be false so that the other one has to be true.

3.4 THE STRENGHTHENED RULE OF CONDITIONAL PROOF

In Conditional Proof method, the conclusion depends upon the antecedent of the conclusion. There is another method, which is called the strengthened rule of conditional proof. In this method, the construction of proof does not necessarily assume the antecedent of the conclusion. The structure of this method needs some elaboration. An assumption is made initially. There is no need to know the truth-status of the assumption because an assumption may be false, but the conclusion can still be true. Further, the assumption can be any component of any premise or conclusion. This method is called the strengthened rule because we enjoy more freedom in making assumption or assumptions, which means that plurality of assumptions is allowed. It strengthens our repertoire of testing equipments. In this sense, this method is called the strengthened rule of C.P. Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more than two assumptions in an argument, then there will be a distinct last step with respect to each assumption. This last step can be regarded as the conclusion relative to that particular assumption. It shows that the last step is deduced with the help of assumption in conjunction with the previous steps in such a way that the rules of inference permit such conjunction. Before the conclusion is reached the function of assumption also ceases. Then it will have no role to play. Then, automatically, the assumption is said to have been discharged. When the strengthened rule of C. P. is used adjacent to the line of assumption, the word assumption is not mentioned unlike in the case of C.P. here the head of the bent arrow points to ‘assumption’. In case of the strengthened rule of C.P., the conclusion is always a conditional statement which consists of statements from the sequence itself.

Thus the range of the application of condition is defined. In order to easily identify the range of its application, a slightly different method is used. An arrow is used to indicate what is assumed and with the help of the same arrow its range also is defined. The application of C.P. in restricted to the space covered by the arrows. All steps, which are outside this arrow, are also independent of the condition. While the head of the arrow marks the assumption, its terminus separates the lines, which depend upon the condition from the line, which does not depend on the condition. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first premise only. In this sense, it is a strengthened condition. In this case there is no reason to mention C.P. because the arrow helps us to identify the assumption. Consider this example:

1.	(A v B) =>{(C v D) => E} /∴ A=> [(C Λ D) => E]		
2.	A		
3.	A v B	2,	Add.
4.	(C v D) => E	1, 3,	M.P.
5.	(CΛD)		
6.	C	5,	Simp.
7.	C v D	6,	Add.
8.	E	4, 7,	M.P.
9.	(C v D) => E	5, 8,	C.P.
10.	A => [CΛD) => E]	2, 9,	C.P.

Rules mentioned on the RHS make it clear that all lines from 3 to 9 depend on A either directly or through bent arrows. In lines 9 and 10 implication makes them C.P.

One advantage of C.P. in its strengthened form is that it has an extended application. It can be used in all those cases where conclusions are conditional, but do not appear to be so.

3.5 PROVING INVALIDITY

Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation of a rule or rules makes arguments invalid. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. Therefore in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line as it happens in the case of truth-table. Let us consider some examples.

1. E=> (F v G)
2. G => (H Λ I)
3. ⌐ H /∴E => I

1	1	0	0	0	1
E	F	G	H	I	⌐H

While following this method ‘0’ should be assigned to the conclusion making the premises true. If this combination cannot be achieved, then the argument is valid, i.e., even after making the conclusion 0 if the premises cannot take the value 1, then the argument is valid. The components of conclusion and premises should be paired properly to carry out the test.

1. J => (K => L)
2. K=> (⌐ L=> M)
3. (L v M) => N /∴ J => N

1	0	1	0	0	0

J K L ¬L M N

Here the conclusion is '0' whereas the combination of premises is 1. Hence the argument is invalid.

1.

3.6 EXERCISES

I Here some arguments are given which are tested using the method of C. P.

1

1. P Λ (Q => R) /∴ (P => Q) => (P => R)
2. P => Q /∴ P => R C.P.
3. P /∴ R C.P.
4. (P=> Q) => R 1, Exp.
5. ∴R 4, 2, M.P.

2

1. P => (Q => R)/∴ Q => (P => R)
2. Q /∴ P => R C.P.
3. P /∴ R C.P.
4. Q => R 1, 3, M.P.
5. ∴ R 4, 2, M.P.

'

3 1 A=> B / ⋦ (B => C) => (A => C)
2 B => C /⋦ A => C C.P.
3 A /⋦ C C.P.
4 B 1, 3, M.P.
!!!6!⋦ C 2, 4, M.P.

5

1. (A => B) Λ (A => C) /⋦ A => (B v C)
2. A /⋦ B v C C.P.
3. A=> B 1, Simp.
4. B 3, 2, M.P.
5. A=> C 1, Simp.

6. C		5, 2,	M.P.
7. ≹ B v C		4,	Add.

6

1. (A=> B) Λ (A=> C)	/A=> (BΛC)		
2. A	/≹ B Λ C		C.P.
3. A => B		1,	Simp.
4. B		3, 2,	M.P.
5. A => C		1,	Simp.
6. C		5, 2,	M.P.
7. ≹ B Λ C		4, 6,	Conj.

7

1. (A => B)	/≹ (AΛC) => (BΛC)		
2. A Λ C	/≹ B Λ C		C.P.
3. A		2,	Simp.
4. B		1, 3,	M.P.
5. C		2,	Simp.
6. ≹ B Λ C		4, 5,	Conj.

8

1. B => C	/≹ (A v B) => (C v A)		
2. A v B	/≹ C v A		C.P.
3. ¬ A=> B		2,	Impl.
4. ¬ A=> C		3, 1,	H.S.
5. A v C		4,	Impl.
6. ≹ C v A		5,	Com.

9

1. (A v B) => C	/≹ [(C v D => E] => (A => E)		
2. (C v D) => E	/≹ A => E		C.P.
3. A	/≹ E		C.P.
4. A v B		3,	Add.
5. C		1, 4,	M.P.
6. C v D		5,	Add.
7. ≹ E		2, 6, M.P.	

II Here are some arguments, which can be proved using indirect method.

1.

1.	Av (B ΛC)		
2.	A => C /∴ C		
3.	⌐ C		I.P.
4.	⌐A	2, 3,	M.T.
5.	B Λ C	1, 4,	D.S.
6.	C	5,	Simp.
7.	C Λ ⌐ C	6, 3,	Conj.

Seventh step involves contradiction; therefore ⌐C is false which means that C is true.

2.

1.	(D v E) => (F => G)		
2.	(⌐ G v H) => (DΛF) /∴ G		
3.	⌐ G		I.P.
4.	⌐ G v H	3,	Add.
5.	D Λ F	2, 4,	M.P.
6.	D	5,	Simp.
7.	D v E	6,	Add.
8.	F=> G	1, 7,	M.P.
9.	⌐ F	8, 3,	M.T.
10.	F	5,	Simp.
11.	F Λ ⌐ F	10, 9,	Conj.

Eleventh step is contradiction. Therefore ⌐G is false; which means that G is true.

3.

1.	(H => I)Λ(J => K)		
2.	(I v K) => L		
3.	⌐ L /∴ ⌐ (H v J)		
4.	H v J		I.P.
5.	I v K	1, 4,	C.D.
6.	L	2, 5,	M.P.
7.	L Λ ⌐ L	6, 3,	Conj.

7th step involves contradiction. Therefore H v J is false; which means that ⌐ (H v J) is true.

4.

1.	(M v N) => (OΛP)		
2.	(O v Q) => (⌐ R Λ S)		
3.	(R v T) => (M Λ N) /∴ ⌐ R		
4.	R		I.P.
5.	R v T	4,	Add.
6.	M Λ N	3, 5,	M.P.
7.	O Λ P	1, 6,	M.P.
8.	O	7,	Simp.

9. O v Q	8,	Add.
10. (⌐ R Λ S)	2, 9,	M.P.
11. ⌐ R	10,	Simp.
12. R Λ ⌐ R	4, 11,	Conj.

Twelfth step involves contradiction. Therefore R is false which means that ⌐R is true.

5.

1. (V => ⌐ W) Λ (X =>Y)		
2. (⌐W => Z) Λ (Y => ⌐ A)		
3. (Z=> ⌐ B) Λ (⌐ A=> C)		
4. V Λ X / ≵ ⌐ B Λ C		
5. ⌐ (⌐ B Λ C)		I.P.
6. B v ⌐ C	5,	De.M.
7. ⌐ Z v A	3, 6,	D.D.
8. W v ⌐ Y	2, 7,	D.D.
9. ⌐V v ⌐X	1, 8,	D.D.
10. (V Λ X) Λ (⌐ V v ⌐ X)	4, 9,	Conj.

10th Step involves contradiction. Therefore ⌐ (⌐BΛC) is false, which mean that ⌐BΛC is true. We can also prove these arguments using formal proof of validity. Consider 3rd argument.

6.

1. (H => I) Λ (J => K)		
2. (I v K) => L		
3. ⌐L / ≵ !⌐ (H Λ J)		
4. ⌐I Λ ⌐ K	2, 3,	M.T.
5. ⌐ I	4,	Simp.
6. ⌐ I v ⌐ K	5,	Add.
7. ⌐ H v ⌐ J	1, 7,	D.D.
8. ≵ ⌐ (H Λ J)	8,	De.M.

When the 3rd argument was solved using IP method, it involved 7 steps, whereas formal proof required 8 steps. Therefore the former is shorter and preferable.

Now consider the fifth agreement.

7

1. (V => ⌐ W) Λ (X =>Y)		
2. (⌐W => Z) Λ (Y => ⌐A)		
3. (Z => ⌐ B) Λ (⌐A => C)		
4. V Λ X /≵ ⌐ B Λ C		
5. V=> ⌐ W	1,	Simp.
6. V	4,	Simp.
7. ⌐ W	5, 6,	M.P.
8. X => Y	1,	Simp.

9. X	4,	Simp.
10. Y	8, 9,	M.P.
11. ⌐ W => Z	2,	Simp.
12. Z	11, 7,	M.P.
13. Y => ⌐ A	2,	Simp.
14. ⌐ A	13, 10,	M.P.
15. Z => ⌐ B	3,	Simp.
16. ⌐ B	15, 12,	M.P.
17. ⌐A => C	3,	Simp.
18. C	17, 14,	M.P.
19. ∴ ⌐ B Λ C	16, 18,	Conj.

When the 5th argument was solved using I.P. method, it involved 10 steps; whereas formal proof required 19 steps. Therefore the former is shorter and preferable.

III Using the method of reductio ad absurdum (Indirect Proof) the following are proved to be tautologies.

1

1 (A => B) v (⌐ A => B)	
2 ⌐ {(A => B) v (⌐ A => B)}	1, I. P.
3 ⌐ (A => B) ∧ ⌐ (⌐ A => B)	2, De. M.
4 ⌐ (A => B)	3, Sim.
5 A ∧ ⌐ B	4, De. M.
6 ⌐ (⌐ A => B)	2, Simp.
7 ⌐ (A v B)	6,Impl.
8 A	5, Simp.
9 ⌐A ∧ ⌐ B	7, De. M.
10 ⌐A	9, Simpl.
11 A ∧ ⌐ A	8, 10., Conj.

Eleventh step involves contradiction which means that there is error in the second step, i.e., assumption. Therefore the given expression is a tautology.

2.

1 (A => B) v (B => A)	
2 ⌐ {(A => B) v (B => A)}	1, I. P.
3 ⌐ (A => B) ∧ ⌐ (B => A)	2, De. M.
4 ⌐ (A => B)	3, Simp.
5 ⌐(⌐ A v B)	4, Impl.
6 ⌐ (B => A)	3, Simp.
7 ⌐(⌐ B v A)	6, Impl.
8 A ∧ ⌐ B	5, De.M.
9 A	8, Simp.
10 B ∧ ⌐ A	7, De.M.
11 ⌐ A	10, Simp.
12 A ∧ ⌐ A	9,11, Cong.

Explanation for this argument is the same as the one given to the previous one.

3

1 (A => B) v (B => C)
2 ⌐ {(A => B) v (B => C)} 1, I. P.
3 ⌐ (A => B) ∧ ⌐ (B => C) 2, De. M.
4 ⌐ (⌐A v B) ∧ ⌐ (⌐ B v C) 3, Impl.
5 (A ∧ ⌐ B) ∧ (B ∧ ⌐ C) 4, De.M.
6 A ∧ ⌐ B 5, Simp.
7 ⌐ B 6, Simp.
8 B ∧ ⌐ C 5, Simp.
9 B 8, Simp.
10 B ∧ ⌐ B 9,7, Conj.

Since nineth step involves contradiction, there is error in the second step.
Therefore our assumption is wrong which means that the first step is a tautology.

4.

1 A v (A => B)
2 ⌐ {A v (A => B)} 1, I. P.
3 ⌐ A ∧ ⌐ (A => B) 2, De. M.
4 ⌐ A ∧ ⌐ (⌐ A v B) 3, Impl.
5 ⌐ A ∧ (A ∧ ⌐ B) 4, De. M.
6 (⌐ A ∧ A) ∧ ⌐ B 5, Ass.
7 ⌐ A ∧ A 6, Simp.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

5.

1 P ≡ ⌐ ⌐ P
2 ⌐ (P ≡ ⌐ ⌐ P) 1, I. P.
3 ⌐ {(P => ⌐ ⌐ P) ∧ (⌐ ⌐ P => P)} 2, Equiv.
4 ⌐ {(P => P) ∧ (P => P) } 3,D.N.
5 ⌐ {(⌐ P v P) v (⌐ P v P)} 4, Impl.
6 (P ∧ ⌐ P) ∧ (P ∧ ⌐ P) 5, De.M.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

6

1 ⌐ {(A => ⌐ A) ∧ (⌐ A => A)}
2 ⌐ [⌐ {(A => ⌐ A) ∧ (⌐ A => A)}] 1, I. P.
3 {(A => ⌐ A) ∧ (⌐ A => A)} 2, D. N.
4 (⌐ A v ⌐ A) ∧ (A v A) 3, Impl.
5 ⌐ A v ⌐ A ≡ ⌐ A By Taut.
6 A v A ≡ A By Taut.
7 A ∧ ⌐ A 6,5, Conj.

In this argument there is contradiction in the last step. Therefore the assumption is false. Therefore 1 is a tautology.

7. The next argument is very different.

1 ⌐ {(A => ⌐ A) v (⌐ A => A)}
2 (A => ⌐ A) v (⌐ A => A) 1, I. P.
3 (⌐ A v ⌐ A) v (A v A) 2, Impl.
4 ⌐ A v A By Taut.

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It is important to note that the fourth step is not a contradiction. On the other hand, it itself is a tautology. It means that the line no. 1 is itself a contradiction.

V. Truth-table technique and reductio ad absurdum method - a joint venture:

We can also prove the validity of an argument by integrating the method of reductio ad absurdum with the truth-table technique. We have to make certain assumptions before we use the combination of these two. These assumptions are as follows:

1. All premises are necessarily true. When the premises are truth-functionally compound, the truth-values of components should be such that the compound proposition is necessarily true.
2. The conclusion is necessarily taken to be false. When the conclusion is truth-functionally compound, the truth-values of components should be such that the conclusion is necessarily false.

While assigning the truth-values, in accordance with these assumptions, if we discover that any component takes the values 1 and 0 simultaneously, then it means that the path has led us to contradiction. Therefore the assumption that the argument is invalid is false. Hence it must be valid. What is important is that once a certain truth-value is assigned to a component, it becomes a permanent fixture of that component throughout the course of the argument. Let us consider this argument.

1

1. P1(A => B) => (C Λ⌐ D)
2. P2 (D => E) => F / ∴ ⌐ A => F
   Let us assume that (⌐ A => F) = 0
   (i.e., it is not the case that ⌐ A => F)
   This is possible only when ⌐ A=1 and F=0.
3. In P2 F= 0.
4. P2=1 iff (if and only if)
   (D => E) => F
   0 1 0
5. (D => E) = 0 iff (D => E)
   1 0 0
6. ⌐ D = 0 ∵ D = 1
7. (C Λ⌐ D) = 0 ∵ ⌐ D = 0; and if any one conjunct is false, then the whole conjunction is false.
8. When (CΛ⌐ D) = 0, which is the consequent, P1 can take the value 1 iff the antecedent (A => B) = 0 ∵ the consequent is false.

9. A = 0 ∵ ⌐ A = 1 (according to the law of contradiction, when A= 0, ⌐ A=1) (See2).
10. A => B necessarily takes the value 1 irrespective of the truth-value of B because A = 0 (See9).
11. 8 and 10 contradict.
12. (1), i.e., ⌐ (⌐ A => F) = 0 is false
13. ∴ ⌐ A => F

When P1, P2 and the conclusion are connected properly, it becomes a tautology. In order to get such an expression, implication should connect the conclusion to the premises which in turn are connected with conjunction.

Since the method of reductio ad absurdum demands that the conclusion must be assumed to be false when the given argument is valid, the truth-conditions of compound proposition must scrupulously be followed. Therefore if the conclusion is disjunctive, then both the components of the disjunction must be assigned 0-value. On the other hand, if the given conclusion is a conjunction, then it is sufficient if any one compound is assigned the 0-value. Thirdly, if the conclusion is the negation of conjunction, then the conjunction itself must be assigned the value-1, which means that both components of the conclusion must take the value-1.

Let us consider an argument in which conclusion is a conjunction.

2

P1 (B v ⌐ A) => (⌐ C Λ D)

P2 (D v E) => ⌐ F / ∴ (A Λ ⌐ F)

1. Let us assume that (A Λ ⌐ F) = 0
2. Out of three instances in which any conjunction is false, let us consider first instance.
3. The conclusion is false when A = 0 and ⌐ F = 0
4. P2 is true iff D v E is false
5. D v E = 0 iff D = 0 and E = 0
6. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
7. P1 is true iff (B v ⌐ A) = 0 (from 6)
8. ∴ ⌐ A = 0 (from 7)
9. 3 and 8 violate the law of contradiction because both A and ⌐ A cannot be false simultaneously.
10. ∴ A Λ ⌐ F

However, if we consider second instance in which we assume that A = 1 and ⌐ F = 0, then we get different result.

1. A = 1 and ⌐ F = 0
2. P2 = 1 iff D v E = 0
3. D v E = 0 iff D = 0 and E = 0
4. If D = 0 then (⌐ C Λ D) = 0 irrespective of the truth-value which ⌐ C takes
5. P1 = 1 iff (B v ⌐ A) = 0 (from 4)
6. ⌐ A must be 0
7. A = 1 if ⌐ A = 0
8. 7 and 1 are compatible ∵ when A =1, ⌐ A must be 0.
9. ∴ A and ⌐ F = 0

Since in one instance our assumption is wrong and in second instance it is correct, this argument is neither tautological nor contradictory. An argument is said to be contingent when in at least one instance it is true and in atleast one instance it is false. Therefore this argument is called contingent and to arrive at this conclusion we need not consider the result of third circumstance. Therefore it is invalid and to confirm the status of this type of argument at least two instances are necessary.

(The student is advised to consider the third instance in which the conclusion is assumed to be false and then work out the problem.)

It is evident that the method of reductio ad absurdum, when applied to conjunctive conclusion, makes the construction of proof lengthy which renders it the last choice. Secondly, this method succeeds in showing that the truth-table method is primitive because it can be easily shown that ultimately, any other method directly receives support from the truth-table method. It may be noted the rules of inference and replacement derive their authority from truth-table method only. Consider the rule of C.D. which is of the form

{(p => q) Λ (r => s) Λ (p v r)} => (q v s). We shall construct the truth-table to show that this is a tautology.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Sl. No.	p	q	r	s	**{(p => q)**	Λ	**(r => s)**	Λ	(p v r)}	**=>**	**(q v s)**
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	0	0	1	0
3	1	1	1	0	1	0	0	0	1	1	1
4	1	0	1	1	0	0	1	0	1	1	1
5	0	1	1	1	1	1	1	1	1	1	1
6	1	1	0	0	1	1	1	1	1	1	1
7	1	0	0	1	0	0	1	0	1	1	0
8	1	0	0	0	0	0	1	0	1	1	0
9	0	1	0	0	1	1	1	0	0	1	1
10	1	0	1	0	0	0	0	0	1	1	0
11	0	1	1	0	1	0	0	0	1	1	1
12	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	1	1	1	1	0	0	1	1
14	0	0	1	0	1	0	0	0	1	1	0
15	1	1	0	1	1	1	1	1	1	1	1

16	0	1	0	1	1	1	1	0	0	1	1

In the truth-table method the implication which precedes the consequence component is called the main column. In this table the 11th column is the main column. We notice that in this column, in all 16 instances the truth-value is 1. Therefore the rule is a tautology.

Reductio ad absurdum method makes another critical point more than obvious. If any argument is tautological, then it is logically impossible to assign the truth-values (without landing in self-contradiction) in such a way that the conjunction of premises takes the value 1 while the conclusion takes the value 0. It shows that the truth-values cannot be assigned in a random manner to the components of the statements which constitutes the argument.

Check Your Progress

Note: a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

1. What is the advantage of Indirect Proof? Substantiate your answers.

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2. Briefly explain the difference between the rule of conditioned proof and the strengthened rule.

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3. What is the specialty of combining truth-table method with reductio ad absurdum? Construct an argument using symbols and by applying the methods of truth-table and reductio ad absurdum show that it is a tautology.

3.7 LET US SUM UP

When the conclusion is conditional, formal method does not help. There are three types of conditional statements. There are two kinds of rules of C.P. Indirect Proof is not new to mathematics. Here we reason out in reverse direction. Strengthened rule makes the conclusion independent of assumption.

3.8 KEY WORDS

Reductio ad absurdum: *Reductio ad absurdum* (Latin: reduction to the absurd) is a form of argument in which a proposition is disproved by assuming the opposite of what is to be proved and deducing its implications to absurd , i.e., self-contradictory consequence.

Tautology: A tautology is a series of statements connected logically which is true in all instances. *Contradiction*: it is a form of statement which is false in all instances or whose truth table will have only false substitution instances. *Contingent statements* will have both true and false substitution instances in its truth table.

3.9 FURTHER READINGS AND REFERENCES

Alexander, P. *An Introduction to Logic & The Criticism of arguments*. London: Unwin University, 1969.
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3.10 ANSWERS TO CHECK YOUR PROGRESS

1. When there are more statements, the formal method of proof becomes unwieldy. In such circumstances I.P. provides a shorter route, sometimes providing proof in one line.

2. When the rule of C.P. is applied, always the antecedent of the conclusion is assumed. However when strengthened rule of C.P. is applied this restriction vanishes. Secondly when the antecedent of the conclusion is assumed invariably it has to be justified by writing C.P. on the R.H.S. adjacent to it. On the other hand, in the case of the strengthened rule a bent arrow is used, the extended part of which marks the limits of assumption. The arrow is a substitute for writing C.P.

3. to self-contradiction. When the truth-table method is applied, we proceed from premises to the conclusion. However, when it is combined with I.P. in order to show that the argument is valid, we proceed from the conclusion and assign '0' value and we proceed to show that it leads