
UNIT 21 BALANCING OF INLINE AND RADIAL ENGINES

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21.1 INTRODUCTION

The internal combustion engine of multi-cylinder type are in common use. Each cylinder carries piston which reciprocates and is responsible for creating disturbing force on foundation. Each cylinder will have primary and secondary disturbing forces which have to be balanced. The cylinders are arranged in a line with 2, 3 or more. Two cylinders or banks of four or six cylinders may be arranged with centre lines at angle resulting in a V-formation. The cylinders may be arranged on a circle with central lines along radii meeting at centre on a single crank pin giving configuration of a radial engine. In W-engine three cylinders or three rows of cylinder are arranged with their centre lines at some acute angle.

In each case the cylinders do not fire at the same time but two or more may fire at a time. Their firing order will not, however, affect unbalanced disturbing force or balancing process which are only affected by weight or mass of reciprocating and rotating parts.

Objectives

We will consider balancing – both primary and secondary – of several engines and when you have gone through this unit you would learn

- how to calculate balanced state in a multi-cylinder engine,
- how to balance multi-cylinder engine for primary and secondary disturbing forces, and
- how to use balancing machines.

21.2 PRIMARY BALANCE OF MULTI-CYLINDER IN-LANE ENGINE

In multi-cylinder in-line arrangement all the cylinder centre lines are parallel and in the same plane which is on the same side of the crank shaft centre line. This appears to be the most common configuration in use.

For the reciprocating parts weighing R and crank rotating at ω rad/s, the condition for balanced state will be defined by two statements. These statements are :

- Algebraic sum of all forces should be zero.
- Algebraic sum of all moments about any point in the plane of the forces should be zero.

These statements are expressed as

$$\sum \frac{R}{g} \omega^2 r \cos \theta = 0 \quad \dots (21.1)$$

$$\sum \frac{R}{g} \omega^2 r a \cos \theta = 0 \quad \dots (21.2)$$

Here a is the distance from reference plane to plane of rotation of any crank. We have already seen in earlier unit that a mass revolving at crank radius r will produce a force component along the line of stroke in Eqs. (21.1) and (21.2). r is the radius of crank and θ is the angle made by crank with cylinder line of centre. If the above equations are not satisfied then the right hand side will not be zero but have some magnitude giving unbalanced force and couple on the engine frame. Following example will explain how we can find if an engine is balanced.

Example 21.1

A 4-cylinder in-line engine has crank radius of 60 mm and connecting rod length of 240 mm. The engine crank shaft rotates at 1800 rpm. The centre lines of engine are spaced at 150 mm. If the cylinders are numbered 1 to 4 from one end the cranks appear at intervals of 90° in the end view in the order 1-4-2-3.

Reciprocating mass in each cylinder is 1.5 kg. Find

- unbalanced primary and secondary forces, and
- unbalanced primary and secondary couples with reference to central plane of the engine.

Solution

The in-line cylinder centre lines are shown in Figure 21.1 along with end view.

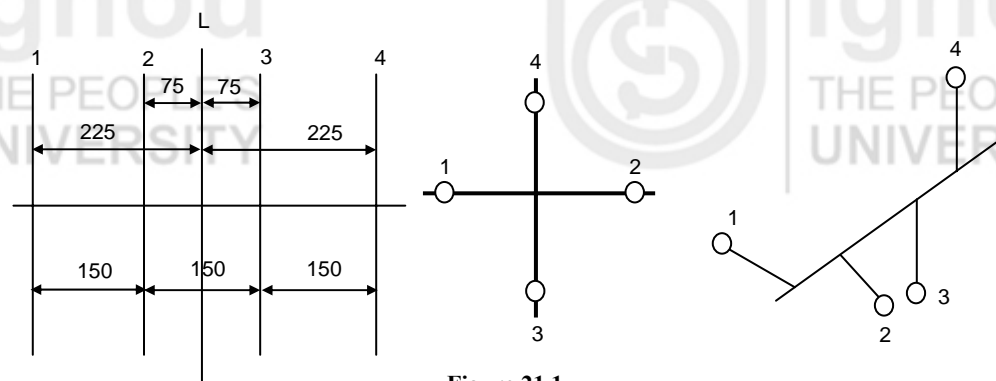


Figure 21.1

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

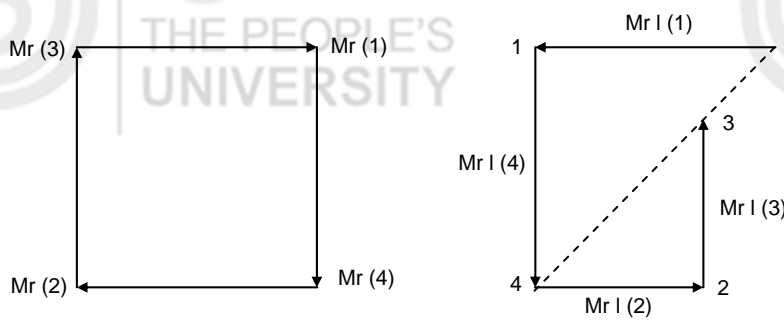
$$\frac{R}{g} = M = 1.5 \text{ g}$$

$$r = 60 \text{ mm}, l = 240 \text{ mm}, \frac{l}{r} = n = 4$$

Following table is prepared to see if there is balance among the forces.

Plane	Mass, M (kg)	Radius, r (mm)	Force/ ω^2 Mr	Distance from L l (mm)	Moment/ ω^2 $Mr l$ (kg mm ²)
1	1.5	60	90	225	2.025×10^4
2	1.5	60	90	75	0.675×10^4
3	1.5	60	90	75	0.675×10^4
4	1.5	60	90	225	2.025×10^4

The force polygon will be closed. We draw all forces toward the centre and a square is formed as in Figure 21.2(a).



(a) Balanced Force Polygon (Statically Balanced)

(b) Moment Diagram, 03 is the Balancing Moment

Figure 21.2

The last column represents moment about reference plane, L . The moments of forces in planes 1 and 2 are opposite to each other and both are in horizontal plane. The moments of forces due to forces in planes 3 and 4 are opposite to each other and they are in vertical plane. The polygon is drawn with 01, 14, 42 and 23. Thus, closing side 03 is the unbalanced moment or couple. It is apparently

$$\sqrt{[Mr l (1)]^2 + [Mr l (4)]^2} - \sqrt{[Mr l (2)]^2 + [Mr l (3)]^2}$$

∴ Unbalanced primary moment

$$\omega^2 = \sqrt{2 \times (2.025)^2} \times 10^4 - \sqrt{2 \times (0.675)^2} \times 10^4 \text{ kg mm}^2$$

$$= 28638 - 9546 = 19092 \text{ kg mm}^2$$

(i)

∴ Unbalanced primary moment

$$= 19092 (188.5)^2$$

$$= 678.4 \times 10^6 \text{ kg.m/s}^2 \cdot \text{m}$$

$$= 678.4 \text{ Nm}$$

(ii)

We have seen that the secondary force is given by

$$F_s = \frac{W}{g} \omega^2 r \frac{\cos 2\theta}{n}$$

$$= \frac{W}{g} (2\omega)^2 \frac{r}{4n} \cos 2\theta$$

(iii)

This is same as the centrifugal force of a weight W attached to crank of length $\frac{r}{4n}$ and revolving at a speed 2ω , i.e. at the speed which is twice the speed of actual crank. This explanation simply assumes an imaginary secondary crank. With inclination of θ and 2θ between actual and imaginary secondary crank 1 and 2 will join together while 3 and 4 will be together in the end view as shown in Figure 21.3.

The secondary forces are each equal as given by Eq. (iii) and shown in Figure 21.3(a). Apparently they are in equilibrium. The magnitude of each force

$$F_s = 1.5 (2 \times 188.5)^2 \times \frac{60 \times 10^{-3}}{4 \times 4} = 799.5 \text{ N} \quad \dots \text{(iv)}$$

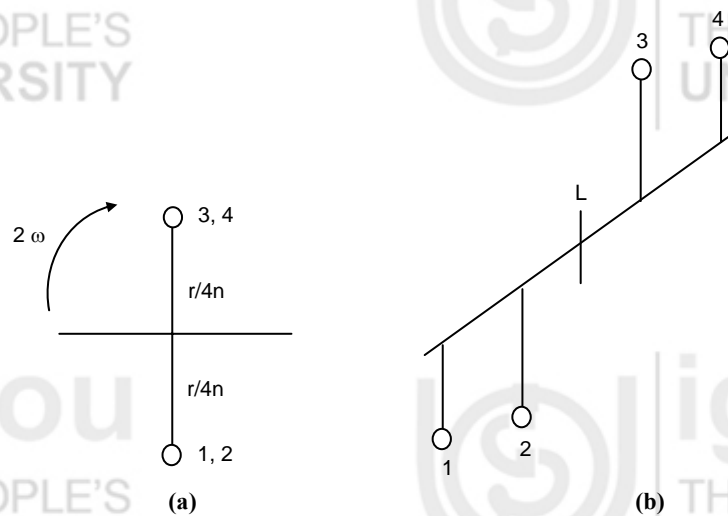


Figure 21.3

The secondary cranks are shown in Figure 21.3(b) with respect to crank shaft. The reference plane L is in the centre. It can be seen from this figure that moments about L due to secondary forces will add.

The unbalanced moment on shaft, M

$$\begin{aligned} M &= 799.5 (225) + 799.5 (75) + 799.5 (225) + 799.5 (75) \\ &= 799.5 (225 + 75 + 225 + 75) \\ &= 799.5 \times 600 \\ &= 479700 \text{ Nmm or } 478.7 \text{ Nm} \quad \dots \text{(v)} \end{aligned}$$

We thus see that without providing any balancing mass in any plane in a 4-cylinder engine the primary and secondary forces are balanced whereas moments due to both primary and secondary forces are not balanced. In general practice secondary moments are not balanced.

21.3 SECONDARY BALANCE OF MULTI-CYLINDER IN-LINE ENGINE

We have already seen the existence of secondary force in one cylinder and we considered several cylinders in last example. The conditions of equilibrium of secondary forces and

couples can be written in the same way as Eqs. (21.1) and (21.2) but with the imaginary secondary crank of length $\frac{r}{4n}$ where $n = \frac{l}{r}$. In this case the forces associated with all secondary cranks and their moments about a reference plane must vanish, i.e.

$$\sum \frac{R}{g} (2\omega)^2 \left(\frac{r}{4n} \right) \cos 2\theta = 0 \quad \dots (21.3)$$

$$\sum \frac{R}{g} (2\omega)^2 \left(\frac{r}{4n} \right) \cdot a \cdot \cos 2\theta = 0 \quad \dots (21.4)$$

As an example let us examine a 2-cylinder engine with cranks in the same plane and at 180° . Identical cylinders are placed on the same side. The engine is schematically depicted in Figure 21.4.

In such a configuration the primary forces which are equivalent to reciprocating masses placed at crank pin are equal for all angles of crank with the line of stroke. Hence, they are balanced.

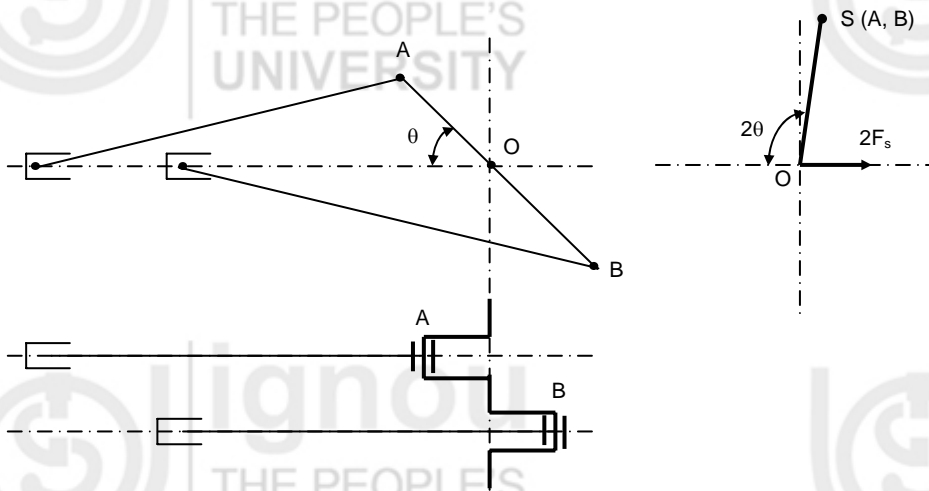


Figure 21.4

The secondary cranks make two times the angle at which actual crank is with the line of stroke. Thus, the secondary crank for crank with angle θ will be at 2θ and secondary crank for the other crank will make $2\theta + 2 \times 180 = 2\theta$ with the line of stroke. This is shown in Figure 20.4. The secondary force for both pistons will be added and a $2F_s$ unbalanced force will act on engine frame.

So far as moment due to primary forces are concerned they will not vanish as the equal and opposite forces are acting in two parallel planes. So moments are not balanced.

The secondary forces acting in the same direction and being equal will have their resultant pass through a point which is in the middle of two cranks and hence no moment about that point. However, about any other point moment will exist.

Example 21.2

In an opposed piston engine the strokes of top and bottom pistons are respectively 760 mm and 1040 mm and the reciprocating parts weigh respectively 54210 N and 39650 N. The connecting rod lengths are respectively 2080 mm and 3040 mm. The engine speed is 123 rpm. Find unbalanced forces.

Solution

The sketch of the engine is shown in Figure 21.5.

This type of engine, normally an internal combustion type, has a single cylinder in which two pistons move along the line of stroke. The pistons move towards or away from each other. The piston nearer crank shaft operates the central crank and other piston which is away from crank shaft operates two cranks on the two sides

Balancing

of the central crank. Thus the two pistons move inwards or outwards together as the side cranks are set at 180° with the central crank. The piston which is away from crank shaft transmits motion to the cranks through two sides rods connected to a crosshead. The rods are connected to the connecting rods of proper length. Thus the reciprocating parts of this piston weigh more than those of the first piston.

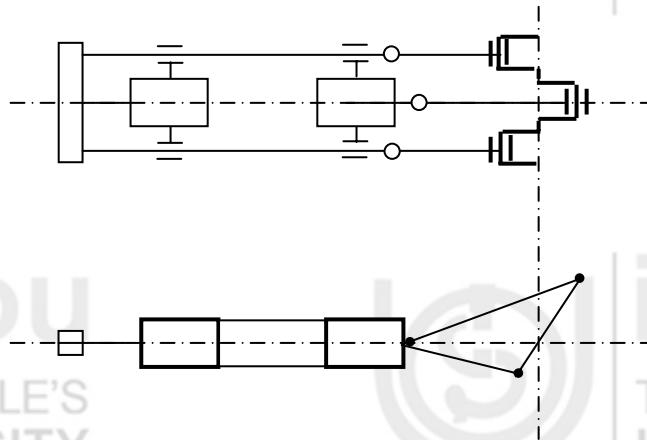


Figure 21.5

This type of engine has been designed as a slow marine engine and as in aircraft engine.

This engine offers the advantage of primary force and couple balancing. This will be achieved by making products of reciprocating mass and crank radius equal for central crank and side cranks. Since the reciprocating mass is less for central crank its radius must be larger.

Bottom piston is closer to crank shaft, $r_b = \frac{1040}{2} = 520 \text{ mm}$

Top piston is further away from crank shaft, $r_o = \frac{760}{2} = 380 \text{ mm}$

Reciprocating weight with top piston, $R_o = 54210 \text{ N}$

Reciprocating weight with bottom piston, $R_b = 39650 \text{ N}$

$$R_b r_b = 39650 \times 520 = 20.6 \times 10^6 \text{ N}$$

$$R_o r_o = 54210 \times 380 = 20.6 \times 10^6 \text{ N}$$

Thus, $R_b r_b = R_o r_o = W$ (say)

Hence, primary forces are balanced.

(i)

The three forces act on the crank shaft as shown in Figure 21.6.

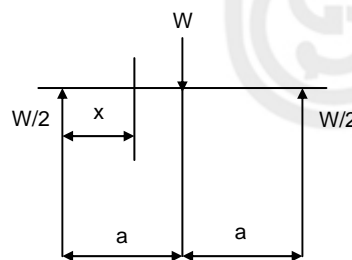


Figure 21.6

Choose any point at a distance x from left crank.

Take moment about this point.

$$\begin{aligned}
 M &= \frac{W}{2} x + W(a - x) - \frac{W}{2}(2a - x) \\
 &= \frac{W}{2} x + Wa - Wx - Wa + \frac{W}{2} x \\
 &= 0
 \end{aligned}$$

Hence, moments of primary forces are balanced.

(ii)

Thus, the advantage of balanced primary forces and moments is proved.

Let us now examine secondary forces.

The connecting rod lengths for top and bottom pistons $l_o = 2080$ mm and $l_b = 3040$ mm.

$$\therefore n_o = \frac{l_o}{r_o} = \frac{2080}{380} = 5.47$$

and
$$n_b = \frac{l_b}{r_b} = \frac{3040}{520} = 5.85$$

The secondary force due to two pistons

$$F_s = \frac{R_o}{g} \omega^2 r_o \frac{\cos 2\theta}{n_o} + \frac{R_b}{g} \omega^2 r_b \frac{\cos 2(\theta + 180)}{n_b}$$

But $R_o r_o = R_b r_b = W$

$$\begin{aligned}
 \therefore F_s &= \frac{W}{g} \omega^2 \left(\frac{\cos 2\theta}{n_o} + \frac{\cos 2\theta}{n_b} \right) \\
 &= \frac{20.6}{9.81} \left(\frac{2\pi \cdot 123}{60} \right)^2 \left(\frac{1}{5.47} + \frac{1}{5.85} \right) \cos 2\theta \times 10^6 \times 10^{-3} \\
 &= 2.1 \times 165.9 \times 0.354 \times 10^3 \times \cos 2\theta
 \end{aligned}$$

or $F_s = 123.245 \times 10^3 \cos 2\theta$ N

(iii)

The maximum value will occur for $\theta = 0$ and $\theta = \frac{\pi}{2}$, i.e. when cranks are in

horizontal or vertical plane. The resultant secondary force is apparently the sum of secondary forces due to two pistons. Hence, there will be moment upon the shaft.

21.4 DIRECT AND REVERSE CRANK METHOD

By this method, the reciprocating unbalancing can be converted into an equivalent rotating unbalance by using the direct and reverse crank for primary and secondary. This method is very useful for V-engines and radial engines.

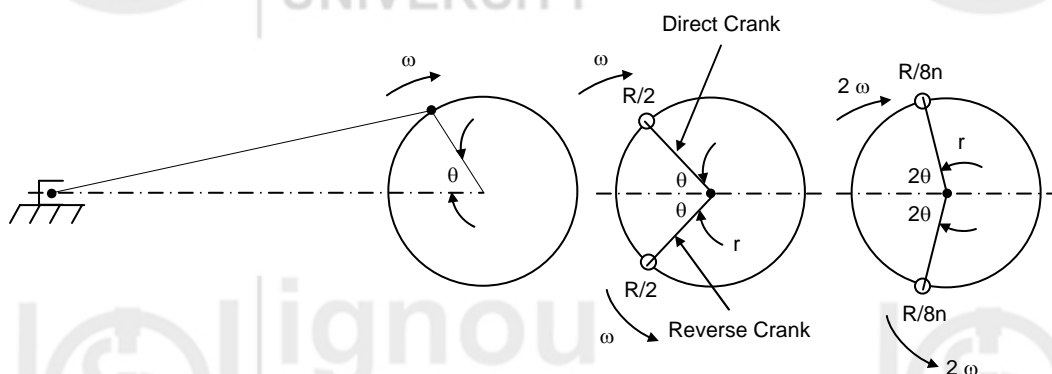


Figure 21.7

Let there be two cranks : one direct and other reverse crank as shown in Figure 21.7(b). The net unbalance in case of primary unbalance

$$\frac{\omega^2 r}{g} \left(\frac{R}{2} \cos \theta + \frac{R}{2} \cos 2\theta \right) = \frac{R}{g} r \omega^2 \cos \theta$$

It is same as in case of reciprocating engine. Thus, reciprocating primary unbalance is equivalent to two rotating masses each equal to $\frac{R}{2}$ each and rotating at radius 'r' and angular speed ' ω ' but in the direct and reverse sense. Figure 21.7(c) represents two rotating of magnitude $\frac{R}{8n}$ or $\frac{R}{2}$ at radius $\frac{r}{n}$ each rotating at angular speed ' 2ω ' in the direct and reverse sense. The net unbalance in this case is

$$= 2 \frac{R}{8n} (2\omega)^2 r \cos 2\theta = \frac{R}{n} \omega^2 r \cos 2\theta$$

It is same as secondary reciprocating unbalance.

21.5 COMPLETE BALANCE OF A SINGLE CYLINDER ENGINE

We can draw conclusion that complete balance, comprising balance of primary and secondary forces and primary and secondary moments, is normally not possible. In a single cylinder engine, however, we can create design that may satisfy the condition of complete balance. Our description below will show that such a proposal, though may be feasible, will increase the cost and make a single cylinder engine much bulkier than with partial balance.

The crank is assumed to make angle θ with the line of stroke. The reciprocating mass of weight R , can be assumed to be placed at crankpin (crank radius) in which case it will cause a centrifugal force equal to $\frac{R}{g} \omega^2 r \cos \theta$ along the line of stroke which acts on the foundation. It is the primary disturbing force. The secondary disturbing force will be $\frac{R}{g} \omega^2 r \frac{\cos 2\theta}{n} = \frac{R}{g} (2\omega)^2 \frac{r}{4n} \cos 2\theta$ which can be regarded to be produced by a weight

R , placed on a crankpin of crank of length $\frac{r}{4n}$ at a speed of two times the speed of actual crank.

The arrangement that will produce complete balance is shown in Figure 21.8. We will first describe the figure completely and then see if total balance is obtained.

On the crankshaft is mounted a gear A which will rotate with the speed of crank shaft and in the same direction.

A is in mesh with gear B of same number of teeth and mounted on shaft Q . A gear D of smaller number of teeth is mounted on Q . The gear D rotates at same speed on crank but in opposite direction to the crank. Gear D is so mounted that its pitch circle touches the line of stroke and another gear E of same number of teeth (identical) is mounted on other side of line of stroke and E will thus move in the same direction as the crank OC . A

weight $\frac{W}{2}$ is placed on the wheel E at a distance of r_1 on the line that makes angle θ with the line of stroke as shown. Another weight $\frac{W}{2}$ is placed at the radius r_1 on gear D at an

angle θ with the line of stroke as shown in Figure 21.8. The radial line from S on E is called **direct crank** and the radial line from Q on D is named **reverse crank**. The direct crank moves with the speed and in the direction of engine crank while the reverse crank moves with speed of the engine crank in the opposite direction.

The centrifugal force due to each of masses placed on direct and reverse cranks is $\frac{W}{2} \omega^2 r_1$ and its component on the line of stroke is $\frac{W}{2} \omega^2 r_1 \cos \theta$ towards right of O along the line of stroke. The sum of 2 components of both the forces, i.e. centrifugal forces of masses on direct and reverse crank is

$$2 \frac{W}{2g} \omega^2 r_1 \cos \theta = \frac{W}{g} \omega^2 r_1 \cos \theta \quad \dots (i)$$

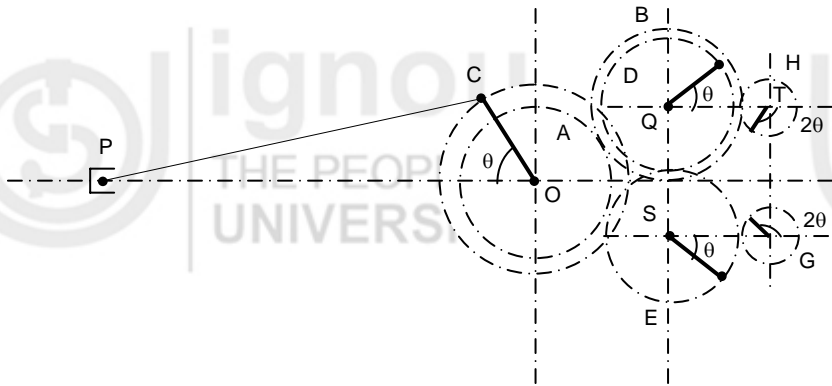


Figure 21.8

The sum of components of centrifugal forces due to $\frac{W}{2}$ on direct and reverse cranks in the direction perpendicular to stroke is

$$\frac{W}{2} \omega^2 r_1 \sin \theta - \frac{W}{2} \omega^2 r_1 \sin \theta = 0 \quad \dots (ii)$$

which means that the direct and reverse crank masses do not produce force component perpendicular to the line of stroke.

Due to weight of reciprocating masses, R , of the engine the disturbing force (primary) toward left on O along the line of stroke is

$$F_p = \frac{R}{g} \omega^2 r \cos \theta \quad \dots (iii)$$

For total balance of primary disturbing force the quantities on right hand side in (i) and (ii) should be equal

$$\text{i.e. } \frac{W}{g} \omega^2 r_1 \cos \theta = \frac{R}{g} \omega^2 r \cos \theta$$

$$\text{or } W r_1 = R r \quad \dots (iv)$$

$$\text{or } \frac{W}{2} r_1 = R \frac{r}{2} \quad \dots (v)$$

If this condition is satisfied the primary disturbing force is fully balanced along the line of the stroke but perpendicular to the line of stroke the force $\frac{R}{g} \omega^2 r \sin \theta$ remains and it

is maximum of the magnitude $\frac{R}{g} \omega^2 r$.

In case of partial balance as was discussed in last unit the force is higher than this, on the frame. But the design of the system requiring three gears is additional and alignment should be perfect so that the resultant force in (i) should be aligned with the line of the stroke, to avoid any unbalanced moment on the crank shaft.

The Figure 21.8 shows other two gears H and G meshing respectively with D and E and having number of teeth half of the mating gears. Thus the angular velocities of H and G are twice that of engine crank and they carry rotating masses on radial lines from shafts T and U , respectively at the same radius.

The secondary force (disturbing) due to piston acceleration on the engine frame along the line of stroke to the left is

$$F = \frac{R}{g} (2\omega)^2 \frac{r}{4n} \cos 2\theta \quad \dots (vi)$$

The components of forces due to centrifugal force of weights at radius of r_2 in each of H and G , along the line of stroke result in

$$\begin{aligned} F_1 &= \frac{W_2}{2g} (2\omega)^2 r_2 \cos 2\theta + \frac{W_2}{2g} (2\omega)^2 r_2 \cos 2\theta \\ &= \frac{W_2}{g} (2\omega)^2 r_2 \cos 2\theta \quad \dots \end{aligned}$$

(vii)

For balance of secondary force in (vi)

$$\frac{W_2}{g} (2\omega)^2 \cos 2\theta = \frac{R}{g} (2\omega)^2 \frac{r}{4n} \cos 2\theta$$

$$\text{or } W_2 r_2 = R \frac{r}{4n}$$

$\frac{W_2}{2}$ is the weight of mass attached to gears H and G at radius of r_2 in each of H and G .

Practically the method is very cumbersome but the arrangement has been used for secondary force balance in some engines.

21.6 BALANCING OF THREE-CYLINDER RADIAL ENGINE

A three-cylinder may have in-line or radial configuration. A radial configuration in which cylinders are placed at 120° angles is shown in Figure 21.9. All the connecting rods (three in number) are directly coupled to a single crank. Thus, there are two bearings in which the crankshaft is supported. For convenience we represent three centre lines as OX , OY and OZ . We assume that reciprocating parts of all three cylinders are identical. The lengths of cranks and connecting rods are same. In the position shown in Figure 20.8 the crank of cylinder 1 makes an angle zero with the line of stroke whereas in the same position cranks of 2 and 3 makes angle of 120° and 240° with respective lines of stroke. Thus, the primary direct cranks coincide but reverse crank of 2 will be at 120° from OY and reverse crank of 1 remains in the same position. (For finding the position of reverse crank you may take the mirror image by imagining a mirror placed along the line of stroke.)

The secondary direct cranks make angle 2θ with the line of stroke. Thus, the secondary direct crank of cylinder 1 is along OX . The secondary direct crank of cylinder 2 makes angle of 2×240 with the line of stroke OY . The secondary direct crank of cylinder 3 makes an angle of 2×120 with the line of stroke OZ . The secondary reverse cranks are the mirror images of the direct cranks, hence they coincide along OX .

Both primary and secondary cranks are shown on the side of engine in Figure 21.9.

In Figure 21.9 note that cranks have been drawn as they exist and imagined (the reverse cranks are imagined). Each crank carries half of reciprocating mass at the crank pin. The radius of the primary crank is r (radius of actual crank) and radius of secondary cranks is $\frac{r}{4n}$ where n is the ratio of crank length to crank radius. While primary cranks rotate at ω rad/s which is the speed of engine the secondary cranks rotate at 2ω . As angles are measured from line of stroke in clockwise direction, i.e. the direction of rotation of the engine crank.

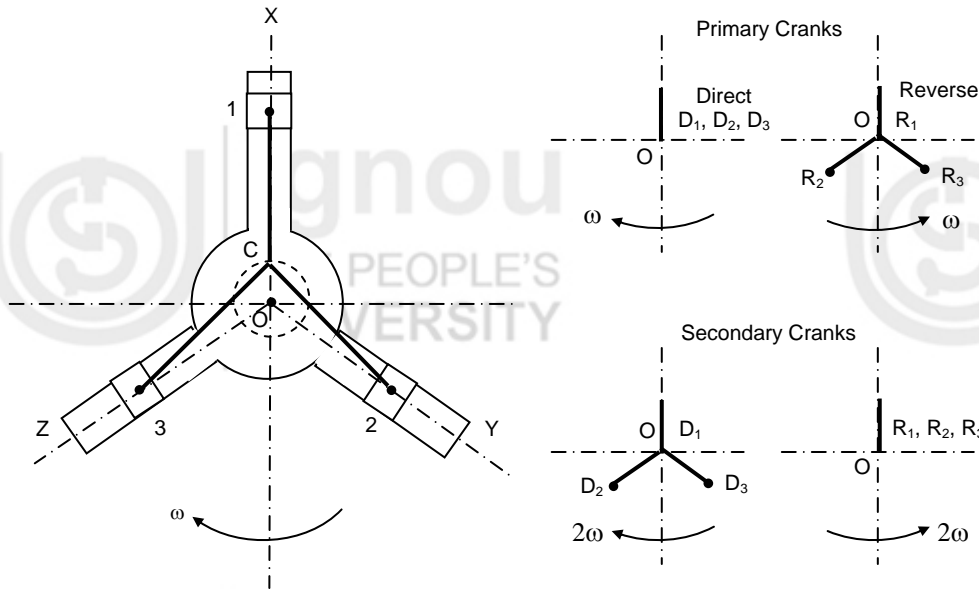


Figure 21.9

For examining the balancing of the engine (without any addition from outside) we can see that primary reverse cranks present balanced system whereas primary direct cranks give addition of three forces upward, i.e. along OX .

The unbalanced force is

$$F_p = 3 \times \frac{R}{2g} \omega^2 r$$

(i)

Looking at the secondary cranks we see that the direct cranks present balanced system but the reverse cranks cause unbalanced force again in the direction of OX .

The unbalanced secondary force is

$$F_s = 3 \times \frac{R}{2g} (2\omega)^2 \frac{r}{4n}$$

(ii)

F_s revolves opposite to crank with speed of 2ω . Whenever the crank is along the line of stroke with piston at top, the direct primary (F_p) and reverse secondary forces will add up.

For example in a three-cylinder radial engine of the type shown in Figure 21.9, the three connecting rods are coupled directly to a single crank which has a radius of 62.5 mm and length of each connecting rod is 225 mm. The weight of reciprocating mass of each cylinder is 20 N. The crank is rotating at 1500 rpm. We have to calculate resultant unbalanced primary and secondary forces.

$$\omega = \frac{2 \times \pi \times 1500}{60} = 157.08 \text{ rad/s}$$

$$n = \frac{l}{r} = \frac{225}{62.5} = 3.6$$

$$R = 20 \text{ N}$$

$$\therefore F_p = 3 \times \frac{20}{2 \times 9.81} \times (157.08)^2 \times 62.5 \times 10^{-3} = 4716 \text{ N}$$

$$\text{and } F_s = 3 \times \frac{20}{2 \times 9.81} (2 \times 157.08)^2 \frac{62.5 \times 10^{-3}}{4 \times 3.6} = 1310 \text{ N}$$

Balancing of primary force can be achieved by placing a balance mass opposite to the crank at a radius, r_1 . We can choose r_1 conveniently, i.e. one which can be accommodated within engine. Let us choose $r_1 = 87.5 \text{ mm}$, the balance weight of W_1 .

$$\text{Then } W_1 r_1 = \frac{3}{2} R r$$

$$\therefore W_1 = \frac{3}{2} \frac{R r}{r_1} = \frac{3}{2} \frac{20 \times 62.5}{87.5} = 21.45 \text{ N}$$

It will be difficult to balance secondary force as was explained in Section 21.4.

21.7 BALANCING OF IN-LINE ENGINES WITH IDENTICAL RECIPROCATING PARTS FOR EACH CYLINDER

The high speed multi-cylinder engines (in-line cylinders) are widely used and they are internal combustion engines, hence the fuel is fired in different cylinders at different times. It is not difficult to understand that firing will occur when piston will be at top dead centre with crank making zero angle with line of stroke. However, it must be understood that in four stroke engine the piston will be at top dead centre two times during a cycle in which cylinder will fire once. On the other hand in two stroke engine the piston will be at top dead centre only once during a cycle in which cylinder will fire once.

If $N_c =$ Number of cylinders, and

$\alpha =$ Angular spacing of the cranks round the shaft.

Then in four stroke engines the uniform firing strokes will be achieved if

$$\alpha = \frac{4\pi}{N_c}$$

The inertia forces of primary and secondary nature for reciprocating parts of each cylinder may be calculated by application of direct and reverse primary and secondary cranks. The inertia force (both primary and secondary) can still be expressed as

$$F_1 = \frac{R}{g} \omega^2 r \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

$$F_2 = \frac{R}{g} \omega^2 r \left[\cos (\theta + \alpha) + \frac{\cos (\theta + \alpha)}{n} \right] \quad \dots$$

(i)

$$\dots \dots \dots$$

$$F_{N_c} = \frac{R}{g} \omega^2 r \left[\cos (\theta + \overline{N_c - 1} \alpha) + \frac{\cos 2 (\theta + \overline{N_c - 1} \alpha)}{n} \right]$$

Providing uniform firing order ensures that F_1, F_2 , etc. will be balanced if the cylinders fire in the order 1, 2, 4, 6, etc. Apparently $N_c = 3$ or 5 will leave some unbalanced force and external balancing may be required.

The balancing of moments can be achieved if the crank shaft is symmetrical about some plane. The cranks can be arranged in pairs if number is 4, 6, 8, etc. The pairs will be formed by parallel cranks which are situated at equal distances from plane of symmetry. This condition will be satisfied only if engines consist of even number of cylinders.

For example, consider a four cylinder engine of four stroke type. Uniform firing interval will result if cranks are spaced at $\alpha = \frac{4\pi}{N_c} = \pi$. This will make two pairs of cranks set at 180° , as shown in Figure 21.10.

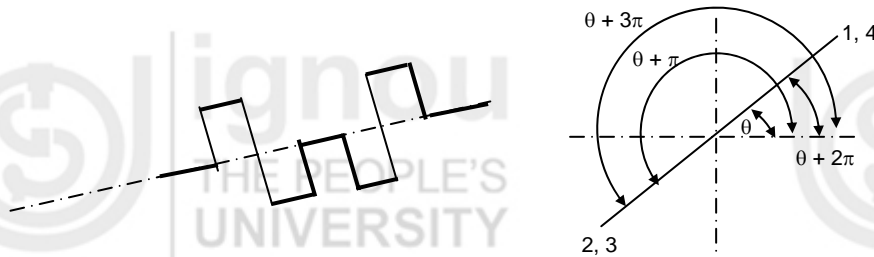


Figure 21.10

If we calculate inertia forces due to identical reciprocating masses by separating forces F_1, F_2 , etc. (Eq. (i)) into primary and secondary forces we see that

$$F_{p1} = \frac{R}{g} \omega^2 r \cos \theta$$

$$F_{p2} = \frac{R}{g} \omega^2 r \cos (\theta + \pi) = -\frac{R}{g} \omega^2 r \cos \theta$$

$$F_{p4} = \frac{R}{g} \omega^2 r \cos (\theta + 2\pi) = \frac{R}{g} \omega^2 r \cos \theta$$

$$F_{p3} = \frac{R}{g} \omega^2 r \cos (\theta + 3\pi) = -\frac{R}{g} \omega^2 r \cos \theta$$

$$\therefore \sum F_p = \frac{R}{g} \omega^2 r (\cos \theta - \cos \theta + \cos \theta - \cos \theta) = 0$$

which shows that primary forces are balanced.

Again from Eq. (i) it is not difficult to see that secondary forces are not balanced.

$$\text{Let } \frac{R}{g} (2\omega)^2 \cdot \frac{r}{4n} = K_s$$

$$F_{s1} = K_s \cos 2\theta$$

$$F_{s2} = K_s \cos 2(\theta + \pi) = K_s \cos 2\theta$$

$$F_{s4} = K_s \cos 2(\theta + 2\pi) = K_s \cos 2\theta$$

$$F_{s3} = K_s \cos 2(\theta + 3\pi) = K_s \cos 2\theta$$

$$\therefore \sum F_s = 4K_s \cos 2\theta$$

It can be seen that the moments of primary forces about the central plane will vanish if the distance between cylinders 1 and 2 should be same as that between 3 and 4. The

Balancing

firing order in such an engine will be 1, 2, 4, 3. If we start cycle with piston 1 at top of the cylinder when firing occurs in 1, then after half a revolution the cylinder 2 fires, after another half revolution cylinder 4 fires, followed by firing in cylinder 3 after yet another $\frac{1}{2}$ revolution. Another half revolution and cycle repeats.

In two stroke engine for uniform firing intervals $\alpha = \frac{2\pi}{N_c}$ and for four cylinders, $\alpha = \frac{\pi}{2}$.

The firing order will be 1, 2, 4, 3. The crank arrangements on the shaft is shown in Figure 21.11 along with forces of unbalance. 2 and 3 may interchange depending upon the shaft rotation direction (clockwise or counter clockwise).

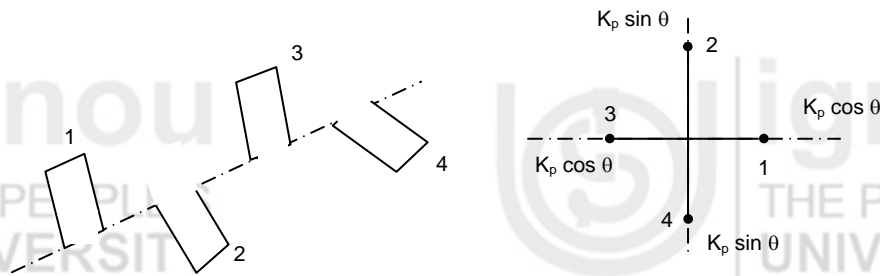


Figure 21.11

The primary forces and moments about central plane are balanced but secondary forces are not.

For a six cylinder 4-stroke engine with cylinders in line and reciprocating parts and connecting rods identical, the cylinders must be spaced at $\alpha = \frac{4\pi}{6} = 120^\circ$. The cranks are arranged in three pairs of angular spacing of 120° . Such a shaft and crank arrangement is shown in Figure 21.12.

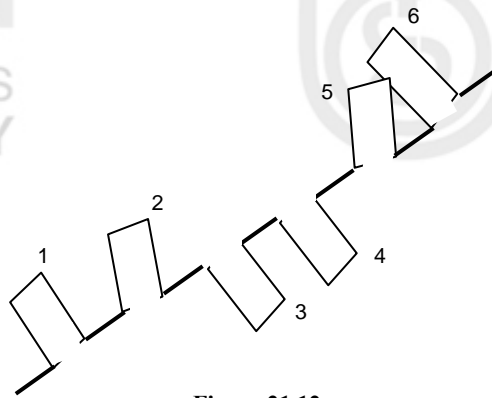


Figure 21.12

The primary forces and moments are balanced. Centre line pairs of 1, 6 ; 2, 5 and 3, 4 are parallel. The firing order will be 1, 2, 3, 6, 5, 4.

Example 21.3

A four cylinder vertical engine has 300 mm long crank. The planes of rotation of the first, third and fourth cranks are 750, 1050 and 1650 mm from that of the second crank and their reciprocating masses weigh 1500 N, 4000 N and 2500 N, respectively. Find the weight of the reciprocating parts for the second cylinder and the relative angular positions of the cranks so that the engine is in complete primary balance. Each connecting rod is 1350 mm long and the speed is 150 rpm.

Solution

The problem of multi-cylinder in-line engine is same as that solved in Unit 19 for several masses revolving in parallel planes. The reciprocating masses can be transferred to crank pin (assumed as transferred) for analysis. Thus, we have situation of cranks rotating in parallel planes placed at specified distances (such as b_A = distance of plane A from plane B) from reference planes. Here we shall

assume four planes *A*, *B*, *C* and *D* in which cranks are rotating and all cranks have same length as 300 mm. So we insert the data in following table and calculate forces as proportional to Rr and moments about reference planes (*B* in this case) as proportional to Rrb .

Figure 21.13

Table 21.1

Plane	Weight, R (N)	Radius, r (mm)	Force $\frac{\omega^2}{g}, Rr$	Distance from Plane B, b (mm)	Moment $\frac{\omega^2}{g}, Rrb$
A	1500	300	45×10^4	- 750	$- 33.75 \times 10^7$
B	R_b	300	$0.03 \times R_b \times 10^4$	0	0
C	4000	300	120×10^4	1050	126×10^7
D	2500	300	75×10^4	1650	123.75×10^7

From moment polygon the angular placement of cranks can be determined. The polygon of moments is drawn by representing a moment by the side of the polygon in the direction of the crank. If moment vector points from axis of the shaft to the crankpin, the moment is positive and if it points from crankpin to the axis the moment is negative. As can be seen from above Table 21.1 the moment of centrifugal force due to mass in plane *A* taken about plane *B* is negative. Hence, the moment 33.75×10^7 will be represented by a line which will point to the axis from crankpin. The other two sides will represent line positive moments. To draw the moment polygon in Figure 21.14, we should depend upon the fact that three sides will close the polygon.

We start with drawing crank *OA* of plane *A* and side *oa* of polygon parallel to *AO*. The side *oa* of the polygon of moments in Figure 20.14 is thus obtained being equal to 33.75×10^7 . With *a* and *o* as centers draw arcs at radii of drawn equal to 126×10^7 and 123.75×10^7 , respectively with the help of the compass. The two arcs intersect *a* with *c* to complete polygon. The cranks *OC* and *OD* are thus found in direction by drawing lines *OC* and *OD* parallel to *ac* and *cd*, respectively and thus angular position of cranks in planes *C* and *D* is determined. The angles α and β can be determined by measurement. Note that the angular position of crank in plane *B* is yet to be determined.

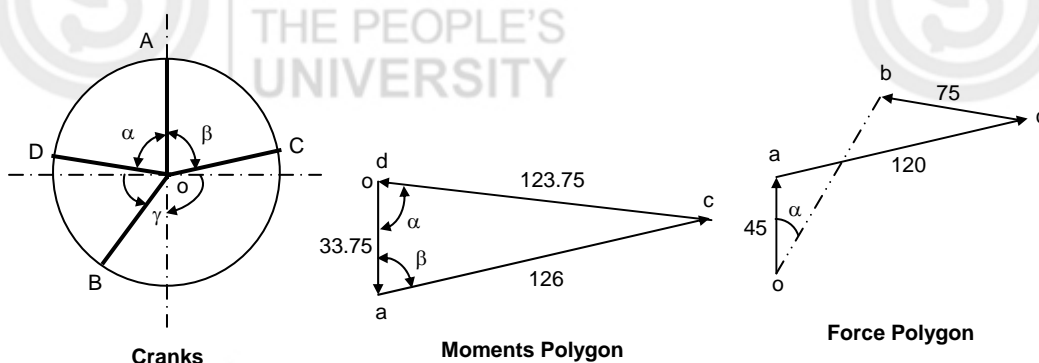


Figure 21.14

Balancing

For determining the angular position of crank in plane B we draw the force polygon in Figure 21.14 which should close for primary forces are balanced. The closing side will be representing the centrifugal force of mass at B , others being known from column 4 of the above Table 21.1. Remember in this case forces will point away from axis toward crankpin.

By measurement $\alpha = 85^\circ$, $\beta = 79^\circ$, $\gamma = 30^\circ$

(i)

(Note : When we draw polygon of moments we assumed C to be on right hand side of A in crank arrangement. We could have taken D on the right hand side and C on left of A . In the second case B should turn out to be on right of vertical line in crank arrangement.)

The length ob measures 85.5×10^4 .

$$\therefore 0.03 R_b \times 10^4 = 85.5 \times 10^4$$

$$\therefore R_b = 2850 \text{ N}$$

(ii)

The relative positions of the cranks are shown in Figure 21.14.

SAQ 1

- Explain reverse crank in a single cylinder reciprocating engine.
- Sketch a 3-cylinder engine arranged radially.
- Discuss balancing of a 2-cylinder engine with parallel strokes and crank at 180° .
- An air compressor has four vertical cylinders A , B , C and D . The driving cranks which are placed at 90° angles in succession have 150 mm radius each. The cylinder centre lines are placed at intervals of 400 mm and each connecting rod is 500 mm long. The reciprocating mass in each cylinder weighs 220 N. The compressor operates at 400 rpm. Determine if compressor is balanced for primary and secondary effects. Choose central plane for reference.
- A V-twin engine has the cylinder axes at 90° and connecting rods are connected to a common crank. Each cylinder carries a reciprocating mass of weight 100 N. The crank radius is 75 mm and both connecting rods are 250 mm long. Find if the engine has any imbalance and suggest how to balance the engine. The crank makes 500 rpm.

21.8 BALANCING MACHINES

You have been exposed to the calculation of disturbing forces that arise due to unbalanced system of forces in machines and machine parts. The possible suggestions for design have also been offered for complete or partial balancing.

The designer is likely to take all measures in making rotating parts of machines perfectly balanced so that out-of-balance force or moment are eliminated, yet either due to slight variation in material density or inaccuracies in casting and machining residual errors may exist. Such residual out-of-balance force and couple have to be determined for deciding

and executing practical measures for achieving 100% balanced system. For this balancing machines are needed. The need for determining actual state of machine part (whether static or dynamic) cannot be overlooked even if such imbalances are small. We must remember that the centrifugal forces and moments due to these forces are proportional to square of angular speed and hence small increase of angular speed may result in considerable increase in unbalanced forces and moments even if the inaccuracy in machine part is very small.

In such cases where a rotating part is of large diameter and comparatively of narrow width the static balancing may be sufficient. Dynamic couples in such cases will be so small that their effects may not cause much disturbance. However, for parts which are long axially, dynamic balancing may be necessary. The requirement of balancing has translated into existence of several balancing machines of static and dynamic types to help determine the balanced state of rotating parts. There are machines which can measure both static and dynamic unbalance.

21.8.1 Static Balancing Machines

A very simple balancing machine to find out-of-balance disc and ring is shown in Figure 21.15. It essentially consists of a heavy base extended in a column to sufficient height. It carries a scale on which a needle indicator moves. The scale may read unbalance in units of Nm. A beam or an arm is supported on a knife edge near the middle of the height. When left to itself the beam will be horizontal. At one end of the machine a mandrel can be carried in a matching groove. On the other end similar mandrel will carry a hanger to support dead weight.

Rotating part, whose balance is to be checked is placed on mandrel which is placed in groove as shown. Dead weights are placed in hanger to bring the indicator near zero on the scale to indicate that the dead weight can balance the weight of part. The part is then rotated or displaced on mandrel either by hand or by a motor to see if the beam remains horizontal in all positions of the part to be balanced. If there is additional mass in the part as shown by small circle at a distance r from the centre then the beam will tilt towards the dead weight as shown in Figure 21.15. If the additional mass moves to far off position then the beam will tilt in the direction of the part. The additional mass being anywhere between extreme horizontal to extreme vertical positions will cause the beam to occupy some intermediate position.

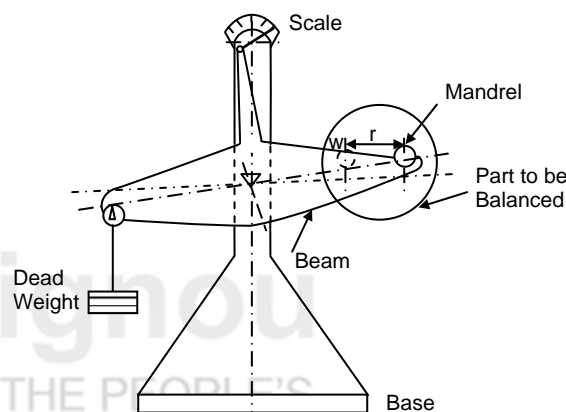


Figure 21.15 : Static Balancing Machine

If unbalance is detected then attempt is made to find its highest value which can be read on scale. Suppose the observed value is C which can be regarded as $W r$.

The balance can be achieved by removing a mass W at the radius r on left of mandrel or adding a mass W at a radius r on right of mandrel. This type of machine is used for wheels and discs of small width like automobile tyres which can be balanced by adding small weights on rubber tyre. The static balance will result in stationary indicator when the disc or tyre is rotated in the mandrel.

Yet another arrangement for determining static unbalance is shown in Figure 21.16. This machine has much higher sensitivity as the condition of resonance is used. The machine

consists of a cast iron base on the top of which a cradle is supported on knife edge supports Q, Q on the central line. On either side of the knife edge support springs S, S are placed between the cradle and the base. An electric motor M , placed on the cradle is connected to the part P through a flexible coupling. The part to be balanced which is in shape of cylinder is supported in bearing B at the other end. The whole system will have a specific natural frequency. The part P to be balanced may have a small unbalance due

to tiny mass of weight w at a radius r . the centrifugal force of w , i.e. $\frac{W}{g} \omega^2 r$ acts in

radial direction, making an angle θ with the horizontal. Its horizontal component is $\frac{W}{g} \omega^2 r \cos \theta$ and it will be $\frac{W}{g} \omega^2 r$ when w is on horizontal radius. The force will tend

to cause the cradle to rock about Q, Q which will be damped by the springs S, S . If the speed of motor is gradually increased it may coincide with the natural frequency of cradle and then resonance will occur causing cradle to shake violently. Thus, even smallest static unbalance will be detected. The dynamic unbalance will not be detected because the moment will be in the plane of the knife edges which will not allow the cradle to oscillate.

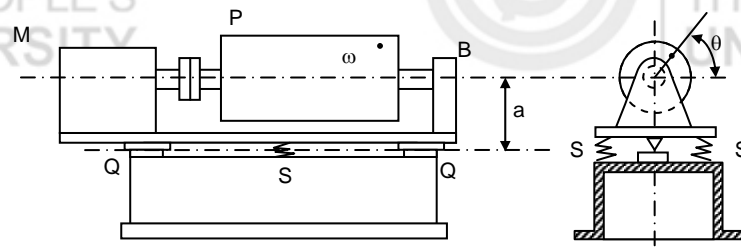


Figure 21.16

21.8.2 Dynamic Balancing Machine

The cradle in machine of Figure 21.16 may be made to rock about knife edge if it is placed in the centre as shown in Figure 21.17. The axis of knife edge, QQ is perpendicular to axis of rotation, MB . Presence of unbalanced force or an unbalanced moment will cause the cradle to rock about the knife edge axis, QQ . This machine (Figure 21.3), thus is a dynamic balancing machine. Before any part is tested in this machine it should be statically balanced so that only effect of unbalanced moment is observed. If the long part (say a cylinder) is balanced statically it may still be in dynamic

unbalanced due to the moment $\frac{W}{g} \omega^2 r l \sin \theta$. Note that other component of moment

$\frac{W}{g} \omega^2 r l \cos \theta$ acts in horizontal plane and hence does not cause cradle to vibrate.

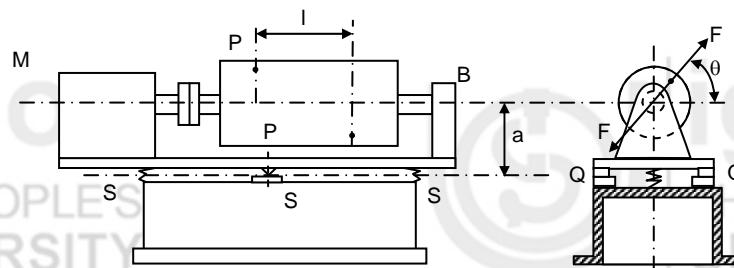


Figure 21.17 : Dynamic Balancing Machine

A machine similar to one in Figure 21.17 is obtained after small modification to balance the part without first statically balancing it. The principle is based on the fact that masses rotating in several parallel plane can be balanced by providing balance masses in two arbitrarily chosen parallel planes. This was illustrated in Unit 18. The part P to be balanced is to mounted that plane L , one of the two parallel planes in which balance weight is to be placed, coincides with the knife edge as shown in Figure 21.18.

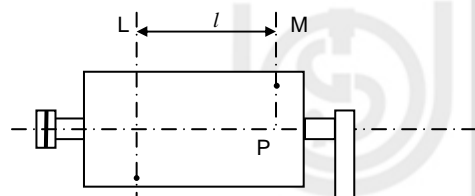


Figure 21.18

In this case the unbalanced moment will be caused by the unbalance effect in plane M which can be measured and correction applied. The part is readjusted in the machine either by removing and refixing or by changing position of the cradle along slides (guides) provided for the purpose. With the readjustment the plane M is now made to coincide with the knife edge axis. The out-of-balance effect will thus cause a moment about knife edge axis which can be measured and correction applied.

21.8.3 Measurement of Unbalanced Force and Moment

In dynamic balancing machine as separate device is used for measurement of force and couple which are not balanced. The basic principle is to apply known force and couple on the cradle in a direction to oppose the effect of disturbance. This is achieved by rotating masses which are statically balanced on vertical shaft but being in different planes (parallel planes) exert a moment in vertical plane which is transferred to cradle. This imposed moment can be measured and varied by changing the distance between planes of rotating masses and also by changing their relative position in the parallel planes. Thus, the device requires rotation of shaft, change of distance between the planes of rotation of masses and the angular positions of the masses in parallel planes. Yet another requirement will be to effect the changes when the part to be balanced (P in Figures 21.17 and 21.18) is rotating on the cradle.

The construction and working of the device is described with the help of Figure 21.19.

The cradle is free to slide and long enough to accommodate the measuring device whose shaft AA is placed in line with the axis of the part to be balanced.

Placed on cradle to shaft AA will be rotated by the motor. AA carries a spiral gear, G , which meshes with similar spiral gear on vertical shaft DD . The gear G is much wider and can slide on AA . When it slides on AA , it causes spiral gear on vertical shaft DD to rotate. The vertical shaft carries to eccentrically placed masses B_1 and B_2 which are equal in weight and rotate at the same radius. They are placed in static balanced. The mass B_1 is fixed in position but the arm of B_2 can slide on shaft DD whereby the distance between the planes of B_1 and B_2 (shown d can change). For sliding arm of B_2 the screw H is rotated which rotates in a nut which carries an arm supporting the arm of B_2 . Its downward slide is under its own weight.

Remember that the vertical shaft DD rotates at the same speed as the part to be balanced which is coupled directly to the shaft AA . The equal masses B_1 and B_2 (equal to B) will create centrifugal couple equal to

$$\frac{B}{g} \omega^2 b \cdot d$$

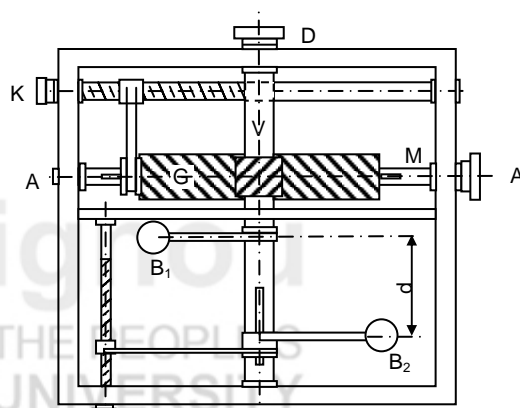


Figure 21.19

This couple may be resolved into two components. These are :

$$\frac{B}{g} \omega^2 b \cdot d \cos \phi$$

(i)

in a vertical plane at right angles to the axis of shaft AA and

$$\frac{B}{g} \omega^2 b \cdot d \sin \phi$$

(ii)

in a vertical plane parallel to the axis of rotation of AA.

If the axis of knife edge AA is parallel to AA, as in static balance machine of Figure 21.2 the effective component to neutralise the cradle oscillations is (i). If the knife edge axis QQ is perpendicular to the axis AA the effective balancing couple is (ii). This moment should neutralise disturbing couple $\frac{W}{g} \omega^2 r l \sin \theta$.

$$\therefore \frac{B}{g} \omega^2 b \cdot d \sin \phi = \frac{W}{g} \omega^2 r l \sin \theta$$

(iii)

is the condition that the cradle will not rock about the knife edge axis. This requires that

$$\sin \phi = \sin \theta \quad \text{or} \quad \phi = \theta \quad \dots \text{(iv)}$$

This condition is achieved by rotating K and ϕ can be read.

From Eq. (iii)

$$B b d = \omega r l \quad \dots \text{(v)}$$

when the cradle is not vibrating.

All quantities on left hand side of Eq. (v) are read from measuring device. l is found by adjusting the position of part to be balanced as depicted in Figure 21.4. Thus, $w r$ is determined.

SAQ 2

- Describe the how you can determine if a part is statically balanced? What is the significance of part being narrow in width?
- Sketch a dynamic balancing machine and describe how it is used?

- (c) Discuss the difference between static and dynamic balancing machines. Describe how the distance between two parallel planes of disturbance in a rotating part is determined.
- (d) Describe the device that is used for measuring the balancing requirement of a part.
- (e) Describe the functioning of a device which measures unbalanced couple in a cylindrical part.

21.9 SUMMARY

The methods of calculating the balanced or unbalanced situations in in-line and V-engines have been brought out. The IC engines are often used in multi-cylinder configuration. Many configurations are balanced by properly placing the cranks round the crank shaft. However, no single method can be used to determine balancing. The methods deal individual cylinders and then unbalance is found in terms of forces and couples. The method of reverse crank has been discussed to find residual unbalance. Providing balance for secondary forces is much cumbersome as it needs rotating masses which would revolve at twice the speed of crank of the engine. Such masses will be required to rotate at very small distance from the axis.

Machines that can measure unbalance force in a rotating part have been designed and made. They are used practically. Narrow parts that do not create significant moment or some parts are balanced by static balancing machines which consist of balanced beam. One end of the beam carries the part to be balanced and other end carries the dead weight. The swing of balanced beam measured on the machine indicates the out-of-balance force which can be removed by adding or removing material from the part to be balanced.

The dynamic balancing machine is built up on the principle that a component of unbalanced couple may cause the supporting platform (called cradle) to vibrate since the unbalanced component of couple will rotate. At certain value the speed of rotation will coincide with the natural frequency of the cradle on its knife edge support. When such a condition is reached even the smallest unbalance will be detected.

Device consisting of rotating masses on a vertical shaft and rotating at the same speed as the shaft of dynamic balancing machine has been made to measure the out-of-balance masses and distance between planes of such out-of-balance forces. The method is used to determine the planes and then adding or subtracting masses to the part to be balanced.

21.10 ANSWERS TO SAQs

SAQ 4

$$(d) \quad r = 150 \text{ mm}, \quad l = 500 \text{ mm}, \quad n = \frac{l}{r} = \frac{500}{150} = 3.33$$

$$R = 220 \text{ N}, \quad \omega = \frac{2\pi \cdot 400}{60} = 41.9 \text{ rad/s}$$

Balancing

$$\frac{\omega^2}{g} = \frac{(41.9)^2}{9.81} = 178.9$$

The cylinder planes and cranks around the shaft are shown in Figure 21.20. The data are inserted in the Table 21.2 and calculated values are further tabulated.

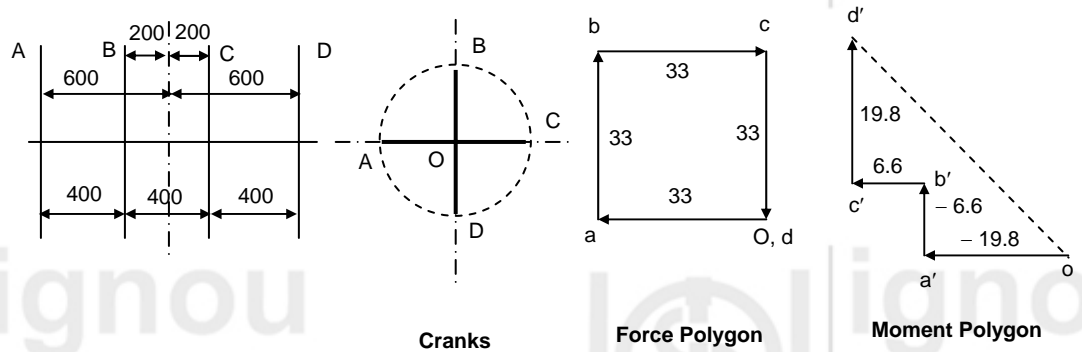


Figure 21.20

Table 21.2

Plane	Weight, R (N)	Radius, r (mm)	Force $\frac{\omega^2}{g}, Rr$	Distance from Plane Ref. Plane, l (m)	Moment $\frac{\omega^2}{g} Rrl$ (Nm^2)
A	220	150	33	-0.6	-19.8
B	220	150	33	-0.2	-6.6
C	220	150	33	0.2	6.6
D	220	150	33	0.6	19.8

The primary forces are apparently balanced and also shown by closed force polygon. The couple polygon is drawn. Negative couples drawn from shaft axis to crankpin and positive moment from crankpin to the axis. The polygon is not balanced and unbalanced couple is

$$od' = 37.4 \times \frac{\omega^2}{g} = 37.4 \times 178.9 = 6691 \text{ Nm} \quad \dots$$

(i)

Taking secondary crank in plane A as one making zero angle with line of stroke the secondary crank in plane B will make angle of $2 \times 90 = 180^\circ$ with line of stroke, with the secondary crank in plane C will make an angle $2 \times 180 = 360^\circ$ with line of stroke and crank in plane D will make an angle of $2 \times 270 = 540^\circ$ with line of stroke. Thus, the secondary crank configuration is drawn in Figure 20.21.

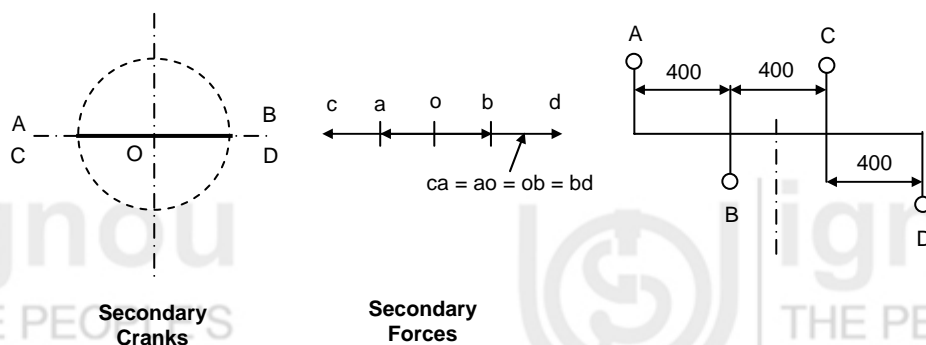


Figure 20.21

Secondary forces act along line of stroke, with those of cylinders *B* and *D* acting opposite to the secondary forces of cylinders *A* and *C*. Each cylinder has same magnitude of forces. This is also shown by force polygon. The secondary forces with respect to crank shaft are also shown in Figure 20.16.

Secondary force of each cylinder is proportional to $\frac{R}{g} (2\omega)^2 \frac{r}{4n}$, i.e.

proportional to $\frac{220}{9.81} (2 \times 41.9)^2 \times 0.15$, i.e. 23623 N.

Secondary force = 23623 \times cos 2 θ

$$2\theta = 0 \text{ for } A \text{ and } C, \cos 2\theta = 1$$

$$2\theta = 180 \text{ for } B \text{ and } D, \cos 2\theta = -1$$

\therefore Secondary force of *A* and *C* = 23623 N,

and Secondary force of *B* and *D* = - 23623 N.

\therefore Moment about central plane due to secondary forces of cylinders *A* and *B*,

$$M_1 = 23623 (0.6 - 0.2) = 9449.2 \text{ Nm}$$

and moment about central plane due to secondary forces of cylinders *C* and *D*,

$$M_2 = 23623 (0.6 - 0.2) = 9449.2 \text{ Nm}$$

Both M_1 and M_2 are clockwise and hence a net moment due to secondary forces about central plane exists.

The unbalanced moment is $2 \times 9449.2 = 18898.4 \text{ Nm}$.



FURTHER READING

Amitabha Gosh and Ashok Kumar Mallik, *Theory of Mechanisms and Machines*,
Affiliated East-West Press Pvt. Ltd., New Delhi.

J. S. Rao and R. V. Dukkipati, *Mechanisms and Machine Theory*, New Age International
(P) Limited Publications, New Delhi.

V. P. Singh, *Theory of Machines*, Dhanpat Rai and Company (Pvt.) Ltd., Delhi.



BALANCING

This block has four units. First unit (Unit 18) is on Introduction to Balancing. The high speed engines and machines are common now-a-days. Complete balancing of all the rotating and reciprocating parts is very essential. If these parts are not properly balanced, the dynamic force setup will produce unpleasant and dangerous vibrations. In the first unit of the block, force on shaft due to single revolving mass, balancing of single revolving mass, procedure for balancing and balancing of several masses revolving in the same transverse plane have been explained to you.

Unit 19 introduces Static, Dynamic and Field Balancing. In this unit, descriptions about static and dynamic balancing have been provided. Under field balancing, methods of balancing a thin disc and a large rotor has been described. Balancing of Reciprocating Engine is illustrated in the Unit 20. To and fro motion of a piston in a reciprocating engine causes reaction on the engine frame. This reaction varies from maximum to minimum and has deleterious effect on bolts, engine and foundation. In this unit, you will learn basic concepts about velocity and acceleration of piston in reciprocating engine. How unbalanced is caused by piston is also explained. Procedure of balancing reciprocating mass is illustrated in this unit.

Unit 21 describes balancing of Inline and Radial Engines. The internal combustion engine to multi-cylinders carries more than one reciprocating piston in cylinder. Each cylinder causes primary and secondary disturbing forces which needs to be balanced. This unit covers primary balance of multi-cylinder inline engine, complete balance of single cylinder engine, balancing of three cylinder radial engine, balancing of inline engines with reciprocating parts for each cylinder and balancing of machines.

With these last four units, you will be able to have in-depth knowledge on Kinematics and Dynamics of Mechanisms. You are advised to refer the books mentioned under “Further Reading”. You are also requested to provide feedback to us so that we may be able to improve the course material.