UNIT 8 COMPUTATION OF WATER SURFACE PROFILES - II

Structure

8.1 INTRODUCTION

We discussed in Unit 7 various methods of computation of gradually varied flow profiles. It was seen that the differential equation, describing a **gradually varied flow**, cannot be integrated explicitly except for channels of geometrically simple cross-section. As an example in point, the Bresse Method is based on the integration of the equation for a wide rectangular channel. It is, in fact, only by introducing some assumptions that integration becomes easier to perform; of such procedures, following two methods are explained :

i) The Bakhmeteff Method, and

ii) The Chow Method.

It was mentioned in Unit 7 that the computation of flow profile in a natural channel involves numerical integration based on trial and error procedure, as is the case with standard step method that is commonly adopted, vis-a-vis, natural channels.

Objectives

In this unit also we continue with the computational procedures regarding gradually varied flow profiles; and, we shall be learning about the following important aspects in this regard :

- hydraulic exponents for critical flow computations, as well as for uniform flow computations,
- two methods, namely, Bakhmeteff's and Chow's under Direct Integration Procedure as applied to GVF profiles, and
- Standard Step Method, as applied to natural channels.

8.2 HYDRAULIC EXPONENTS FOR A PRISMATIC CHANNEL

Recalling one particular form of gradually varied flow equation, such as :

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{K_n}{K}\right)^2}{1 - \left(\frac{Z_c}{Z}\right)^2} \qquad ...(5.17)$$

where, K_n is the conveyance of the channel when discharge Q occurs at normal depth, K is the conveyance for a depth y, Z_c is the section factor when discharge Q occurs in its critical state and Z is the section factor for the depth of flow y. It is preferable to express K_n, K, Z_c and Z as functions of depth of flow, and as such use of exponents of depth of flow become necessary. The exponents used for conveyance of channel section, and the section factor comprise its **hydraulic exponents**.

8.2.1 Hydraulic Exponent for Critical-flow Computation

At critical state of flow, section factor for a channel is defined as :

$$Z = A\,(D)^{1/2} = \left(\frac{A^3}{T}\right)^{1/2} \qquad \text{...(8.1)}$$

where, D is the hydraulic depth, given by (A/T). It is clear, from this equation, that Z is a function of the depth of flow (y) only, and, so may be assumed to have the following relationship with y :

$$Z^2 = C\,y^M \qquad \text{...(8.2)}$$

where, C is a co-efficient, and M is a parameter called **hydraulic exponent for critical-flow computation.**

Taking logarithm of both the sides of equation (8.2), and then differentiating the result w.r.t. y :

$$\frac{d}{dy}(\ln Z) = \frac{M}{2g} \qquad \text{...(8.3)}$$

Similarly, from equation (8.1) we get :

$$\frac{d}{dy}(\ln Z) = \frac{3}{2A}\left(\frac{dA}{dy}\right) - \frac{1}{2T}\frac{dT}{dy} \qquad \text{...(8.4)}$$

As $dA = T\,dy$, i.e., $\frac{dA}{dy} = T$, equation (8.4) becomes :

$$\frac{d}{dy}(\ln Z) = \frac{3T}{2A} - \frac{1}{2T}\frac{dT}{dy} \qquad \text{...(8.5)}$$

Equating the R.H.S. of equations (8.3) and (8.5), and solving for M :

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right) \qquad \text{...(8.6)}$$

which is the general equation for the exponent M for any given channel section. It is seen that M is a function of the channel section and the depth of flow obtaining in it. Now, for a trapezoidal section, substituting appropriately for A and T, and simplifying, equation (8.6) becomes :

$$M = \frac{3\left[1 + 2z\left(\frac{y}{b}\right)\right]^2 - 2z\left(\frac{y}{b}\right)\left[1 + z\left(\frac{y}{b}\right)\right]}{\left[1 + 2z\left(\frac{y}{b}\right)\right]\left[1 + z\left(\frac{y}{b}\right)\right]} \qquad \text{...(8.7)}$$

where, z is the side slope of the channel section. Equation (8.7) indicates that the value of M for a trapezoidal section is a function of z and $\left(\frac{y}{b}\right)$. For normal values of z and $\left(\frac{y}{b}\right)$, M varies in the range of 3 to 5. For a rectangular channel (z = 0), M works out to be equal to 3.

8.2.2 Hydraulic Exponent for Uniform-flow Computation

The conveyance of a channel section, for a uniform-flow computation, using Chezy's formula, can be written as :

$$K = CAR^{1/2} \qquad \text{...(8.8)}$$

and, using Manning's formula, it can be expressed as :

$$K = \left(\frac{1}{n}\right) AR^{2/3} \qquad ...(8.9)$$

Both these equations lead to the conclusion that K is a function of y; and, it can be assumed that :

$$K^2 = C\, y^N \qquad ...(8.10)$$

where, C is a co-efficient (as in equation (8.2)), and N is a parameter called **hydraulic exponent for uniform-flow computation**. Dealing with equations (8.1) and (8.9) in the same manner as was done with equations (8.2) and (8.1), we get :

$$\frac{d}{dy}(\ln K) = \frac{N}{2y} \qquad ...(8.11)$$

$$\frac{d}{dy}(\ln K) = \frac{1}{A}\frac{dA}{dy} + \frac{2}{3}\left(\frac{1}{R}\right)\frac{dR}{dy} \qquad ...(8.12)$$

Putting $\frac{dA}{dy} = T$ and $R = \frac{A}{p}$, equation (8.12) becomes :

$$\frac{d}{dy}(\ln K) = \frac{1}{3A}\left(5T - 2R\frac{dP}{dy}\right) \qquad ...(8.13)$$

Equations (8.11) and (8.13) give :

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \qquad ...(8.14)$$

This is the general expression for the exponent N, for any shape of the section. It is noted that N too is a function of y. For a trapezoidal channel section, equation (8.14) becomes :

$$N = \frac{10}{3}\left[\frac{1 + 2z\left(\frac{y}{b}\right)}{1 + z\left(\frac{y}{b}\right)}\right] - \frac{8}{3}\left[\frac{(1+z^2)^{1/2}\left(\frac{y}{b}\right)}{1 + 2(1+z^2)^{1/2}\left(\frac{y}{b}\right)}\right] \qquad ...(8.15)$$

Equation (8.15) indicates, as in the case of M, that the value of N, for a trapezoidal section is a function of z and $\left(\frac{y}{b}\right)$. For z varying from 0.5 to 4.0, the value of N varies in the range of 2.0 to 5.3. For a rectangular channel ($z = 0$) N, in practice varies from 2.0 to 3.3.

8.3 COMPUTATION OF WATER SURFACE PROFILE BY DIRECT INTEGRATION METHOD

In this Section of computation of flow profile in a prismatic channel, by the integration of the GVF equation, is explained for other than a wide rectangular channel (which was discussed in the Bresse Method). For such channels, certain simplifications are called for – many investigators have tried various approaches in this regard; and, we will consider only the procedures adopted by Bakhmeteff (2, 3), and Ven Te Chow (1).

8.3.1 The Bakhmeteff Method

Recalling the Bresse method where we had an integral,

$$\phi(u) = \int_0^u \frac{1}{1-u^3}\, du$$

which was intended to be integrated. Integration of an equation of the form :

$$F(u, N) = \int_0^u \frac{1}{1-u^N}\, du \qquad ...(8.16)$$

is possible numerically even if N is not a whole number. Bakhmeteff suggested a method to write the gradually varied flow equation in the form of equation (7.4) and then integrate the equation.

Equation (5.17), we noted, can be written as :

$$\frac{dx}{dy} = \frac{1}{S_o} \frac{\left[1 - \left(\frac{Z_c}{Z}\right)^2\right]}{\left[1 - \left(\frac{K_n}{K}\right)^2\right]} \qquad ...(7.2)$$

Substitution for $\left(\frac{Z_c}{Z}\right)^2$ from equation (8.2), and for $\left(\frac{K_n}{K}\right)$ from equation (8.10), equation (7.2) is written as :

$$\frac{dx}{dy} = \frac{1}{S_o} \frac{1 - \left(\frac{y_c}{y}\right)^M}{1 - \left(\frac{y_n}{y}\right)^N} \qquad ...(8.17)$$

When M is not equal to N, equation (8.17) does not lead to a result that would be wholly in terms of integrals such as the one in equation (8.16). Now, noting the following definitions :

$$Q = A\left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right) = K_n S_o^{1/2}$$

also, $$Q = K S_f^{1/2}$$

i.e., $$\left(\frac{K_n}{K}\right)^2 = \left(\frac{S_f}{S_o}\right)$$

Further, noting that at critical flow

$$\frac{V^2}{2g} = \frac{D}{2}$$

and, $Z = A\sqrt{D}$, we have,

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{AV}{\sqrt{g}}$$

and, dividing Z_c by $Z = A\sqrt{D}$ and squaring we get:

$$\left(\frac{Z_c}{Z}\right)^2 = \frac{V^2}{gD} = (F_r)^2$$

We can put equation (5.17) as :

$$\frac{dx}{dy} = \frac{1}{S_o}\left[\frac{(1 - F_r^2)}{\left\{1 - \left(\frac{S_f}{S_o}\right)\right\}}\right] \qquad ...(8.18)$$

where, F_r is the Froude Number. Bakhmeteff set $F_r^2 = \beta\left(\frac{S_f}{S_o}\right)$, and then considered the behaviour of β :

$$\beta = F^2 \frac{S_o}{S_f} = \frac{Q^2 T}{g A^3} S_o \left(\frac{A^2 C^2 R}{Q^2}\right) = \frac{C^2 S_o}{g} \frac{T}{P} \qquad \text{...(8.19)}$$

where, C is Chezy's resistance factor. For a moderately wide channel the term $\frac{T}{P}$ does not vary appreciably and hence β can be taken as a constant for a given channel. Bakhmeteff treated β as a constant, allowing for any variation by dividing the channel length into necessary number of reaches – short enough so that β could be assumed constant over each reach. Under this assumption equation (8.18) becomes [keeping also in mind that $Q = \frac{K}{n} (S_f)^{1/2} = \frac{1}{n} (\sqrt{C}\, y^{N/2}) (S_f)^{1/2}$; and $Q = \frac{1}{n} (\sqrt{C}\, y_n^{N/2}) \cdot (S_o)^{1/2}$, where, C is a coefficient in equations 8.2 and 8.10]:

$$\frac{dx}{dy} = \frac{1}{S_o} \frac{\left(1 - \beta \left(\frac{y_n}{y}\right)^N\right)}{\left\{1 - \left(\frac{y_n}{y}\right)^N\right\}} \qquad \text{...(8.20)}$$

Substituting $u = \left(\frac{y}{y_n}\right)$, equation (8.20) becomes (after adding and subtracting 1 from its R.H.S.) :

$$dx = \frac{y_n}{S_o}\left[1 - (1 - \beta) \frac{1}{1 - u^N}\right] du \qquad \text{...(8.21)}$$

and, its integration leads to :

$$x = \frac{y_n}{S_o} [u - (1 - \beta) F(u, N)] + \text{Constant} \qquad \text{...(8.22)}$$

where, $$F(u, N) = \int_0^u \frac{1}{1 - u^N} du$$

By using equation (8.22) the length of a flow profile, between any two consecutive sections, 1 and 2, is equal to

$$x = x_2 - x_1$$

or, $$x = \frac{y_n}{S_o}\left[(u_2 - u_1) - (1 - \beta) \left(F(u_2, N) - F(u_1, N)\right)\right] \qquad \text{...(8.23)}$$

where, the subscripts 1 and 2 refer to sections 1 and 2, respectively, and $F(u, N)$ denotes the varied flow function. Varied-flow-function values are available in the form of a table (Appendix 7.1) for values of N varying from 2.2 to 9.8. Equation (8.22) developed by Bakhmeteff, is used to compute the flow profile in a prismatic channel as determined by the underlying assumption of β being a constant. Equation (8.14) is used for the computation of N for each section of a channel, knowing the geometry of the channel. Application of Bakhmeteff method for computation of flow profile to a prismatic channel section is demonstrated in Example 8.1.

Example 8.1

With reference to the channel described in Example 7.1, compute the length of the backwater profile extending from the dam site to an upstream section at 1% greater than the normal depth.

Solution

With the data given in Example 7.1, the Backmeteff method of computation is carried out as shown in Table 8.1. The values in each column of the table are explained as follows.

col. 1 Depth of flow (m), arbitrarily assigned from 2.0 m to 1.01 m,

col. 2 Water area (m^2) corresponding to the depth y in col. 1,

col. 3 Wetted perimeter (m) corresponding to y in col. 1,

Table 8.1: Computation of Flow Profile by Bakhmeteff Method for the Problem Given in Example 7.1

$Q = 12.26\ m^3/s$, $n = 0.025$, $S_o = 0.0016$, $\alpha = 1.00$, $y_c = 0.64$ m, and $y_n = 1.00$ m.

Note : Flow profile is to be computed from the depths 1.01 m through 2.00 m.

y	A	P	R	T	u	C	β	N	F (uN)	Δx	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
2.00	22.00	15.944	1.380	15.00	2.00	42.20	0.2733	3.80	0.053	-	-
1.60	16.32	14.155	1.153	13.40	1.60	40.96	0.2590	3.71	0.112	277+	277
1.20	11.28	12.367	0.912	11.80	1.20	39.40	0.2415	3.61	0.318	347++	624
1.01	9.11	11.517	0.791	11.04	1.01	38.50	0.2317	3.56	1.108	496	1120

col. 4 Hydraulic radius (m) corresponding to y in col. 1

col. 5 Top width (m) corresponding to y in col. 1

col. 6 The ratio of depth y in col. 1 to y_n

col. 7 Chezy's coefficient, $C = \frac{1}{n} R^{1/6}$, where R is given in col. 4. However, if the value of C is given directly it can be tabulated as such

col. 8 $\beta = \left(\frac{C^2 S_o}{g}\right)\left(\frac{T}{P}\right)$

col. 9 N, as computed by equation (8.14)*

col. 10 F (u, N) as obtained from Appendix 7.1 for given u, and N (i.e., col. 6 and col. 9)

col. 11 $\Delta x(m)$ is the distance between two consecutive locations–computed by using equation (8.23), where the value of β is taken as the average of the two consecutive values.

col. 12 $x = \Sigma(\Delta x)$, i.e., cumulative value of items in col. 11.

\+ Neglecting the –ve sign, for the upstream direction in this problem is reckoned +ve with reference to the location where y = 2.00 m.

++ Rounding off the figure.

* $P = 7 + (2\sqrt{5})\,y$; hence, $\frac{dP}{dy} = 2\sqrt{5}$

8.3.2 The Chow Method

The method described here is due to the study made by Ven Te Chow (1). In order to integrate the gradually varied flow equation, we noted that Bakhmeteff, in effect, replaced $\left(\frac{y_c}{y}\right)^M$ in equation (8.17) by $\beta\left(\frac{y_n}{y}\right)^N$ and assumed β as a constant and obtained equation (8.20) which after integration led to equation (8.22). But in the Chow Method no such assumption is considered; and equation (8.17) is written as :

$$dx = \frac{1}{S_o} \frac{1 - \left(\frac{y_c}{y}\right)^M}{1 - \left(\frac{y_n}{y}\right)^N} dy \qquad ...(8.24)$$

Now, as before, letting $u = y / y_n$, equation (8.24) becomes :

$$dx = \frac{y_n}{S_o} \frac{\left[1 - \left(\frac{y_c}{y_n}\right)^M \left(\frac{1}{u}\right)^M\right]}{\left[1 - \frac{1}{u^N}\right]} du \qquad ...(8.25)$$

which can be expressed in a convenient form (after adding and subtracting 1 to its R.H.S.) such as :

$$dx = \frac{y_n}{S_o}\left[1 - \frac{1}{1 - u^N} + \left(\frac{y_c}{y_n}\right)^M \frac{u^{(N-M)}}{1 - u^N}\right] du \qquad ...(8.26)$$

Equation (8.26) can be integrated for a length x of the flow profile. Since the change in the depth of gradually varied flow is generally small, the hydraulic exponents may be assumed constant within the range of the limits of integration. However, in a case where the hydraulic exponents are noticeably varying within the limits of a given reach, the reach should accordingly be subdivided for in egration, then the hydraulic exponents in each subdivided reach may be assumed constant. Integrating equation (8.26), we have :

$$x = \frac{y_n}{S_o}\left[u - \int_0^u \frac{1}{1 - u^N} du + \left(\frac{y_c}{y_n}\right)^M \int_0^u \frac{u^{N-M}}{1 - u^N} du\right] + C_1 \qquad ...(8.27)$$

where, C_1 is a constant of integration. The first integral on the right-hand side of the above equation is, as already mentioned, designated by F(u, N), and the second integral in the equation may also be expressed in the form of a varied flow function by the following substitutions :

$$v = (u)^{N/J}, \text{ and } J = \frac{N}{(N - M + 1)}$$

such that, $$\int_0^u \frac{u^{N-M}}{1 - u^N} du = \frac{J}{N}\int_0^v \frac{dv}{1 - (v)^J} = \frac{J}{N} F(v, J) \qquad ...(8.28)$$

where, $$F(v, J) = \int_0^u \frac{dv}{1 - (v)^J}$$

This is a varied-flow function like $F(u, N)$, except that the variables u and N are replaced by v and J, respectively. Using the notation of varied flow function, equation (8.27) may be written as :

$$x = \frac{y_n}{S_o}\left[u - F(u, N) + \left(\frac{y_c}{y_n}\right)^M \frac{J}{N} F(v, J)\right] + C_1 \qquad ...(8.29)$$

or, $$x = A\,[u - F(u, N) + B \times F(v, J)] + C_1 \qquad ...(8.30)$$

where, $$A = \frac{y_n}{S_o}, \quad B = \left(\frac{y_c}{y_n}\right)^M \left(\frac{J}{N}\right)$$

By using equation (8.30) the length of a flow profile (Δx) between any two consecutive sections 1 and 2 is given by :

$$\Delta x = x_2 - x_1$$

$$\Delta x = A\left\{(u_2 - u_1) - [F(u_2, N) - F(u_1, N)] + B\,[F(v_2, J) - F(V_1, J)]\right\} \qquad ...(8.31)$$

where, the subscripts 1 and 2 refer to sections 1 and 2, respectively.

Equation (8.31) contains varied flow functions and its solution can be simplified by the use of the varied flow function table (given in Appendix 7.1).

Example 8.2

With reference to the channel described in Example 7.1, compute the length of the backwater profile extending from the dam site to an upstream section where the depth of flow is 1% greater than the normal depth.

Solution

It is given that $Q = 12.26\ m^3/s$, $b = 7$ m, $z = 2$, $S_o = 0.0016$, $n = 0.025$, $y_n = 1.00$ m, and $y_c = 0.64$ m.

Depth at the downstream end, $y_2 = 2.00$ m

Depth at the upstream end, $y_1 = 1.01$ m

The average depth of flow in the reach

$$= \frac{(1.01 + 2.0)}{2} = 1.505 \text{ m}$$

Values of M and N can either be computed (from equations (8.6) and (8.14), respectively) on the basis of this average value of depth, or the relevant curves (Open Channel Hydraulic by Ven Te Chow (1)) can yield the values directly though approximately.

Thus, $M = 3.44$

and, $N = 3.68$

$$\therefore \quad J = \frac{N}{(N - M + 1)} = 2.97$$

The required values of u and v can be computed, using Appendix 7.1 (and, resorting to interpolation wherever necessary), Table 8.2 can be constructed.

Table 8.2 : Chow's Method of Computation for Flow Profile for Example 8.2

y	u	v	$F(u, N)$	$F(v, J)$
2.00	2.00	2.360	0.0608	0.0926
1.01	1.01	1.012	1.0562	1.3658
Difference in the values between sections 1 and 2	0.99	–	– 0.9954	–1.2732

For equation (8.27),

$$A = \frac{y_n}{S_o} = \frac{1}{0.0016} = 625$$

and,

$$B = \left(\frac{J}{N}\right)\left(\frac{y_c}{y_n}\right)^M$$

$$= \left(\frac{2.97}{3.68}\right)\left(\frac{0.64}{1.0}\right)^{3.44}$$

$$= 0.1738$$

Therefore, the approximate length of the backwater profile (because the intermediate values of y were skipped over in the above procedure) is given by :

$$x = 625\,[0.99 - (-0.9954) + 0.1738\,(-1.2732)]$$

$$= 1102.60 \text{ m}$$

As the solution is obtained, taking the entire length as one reach, the values of M and N are computed for the average value of the two end depths. If the channel is divided into subreaches, and flow profile is computed for each subreach, the computed solution will be more accurate than what is obtained adopting the entire length as a single reach.

Table 8.3 shows the application of Chow method for the above mentioned problem, dividing the length into three subreaches. The depths chosen for the computation are the same as adopted in Table 8.1.

Table 8.3 : Computation of Flow Profile Using Chow's Method for Example 8.2

$Q = 12.26\ m^3/s$, $n = 0.025$, $S_o = 0.0016$, $y_0 = 1.00$ m, and $y_c = 0.64$ m.

y	*u*	*N*	*M*	*J*	*v*	*B*	*F (u, N)*	*F (v , J)*	Δ*x*	*x*
1	2	3	4	5	6	7	8	9	10	11
2.00	2.00	3.80	3.558	3.06	2.362	-	0.053	0.0856	-	-
1.60	1.60	3.705	3.464	2.985	1.792	0.168	0.113	0.1660	279	279
1.2	1.2	3.606	3.359	2.892	1.255	0.175	0.320	0.4460	349	628
1.01	1.01	3.557	3.306	2.843	1.013	0.181	1.108	1.428	500	1128

Here, the values of B are computed foi each subreach, taking the average values of N, M and J between the two depths under consideration. A comparison of the results obtained in Tables (8.1) and (8.3) shows that the two methods lead to almost the same result as far as the example under consideration is concerned.

8.4 STANDARD STEP METHOD

This method is applicable to both prismatic and non-prismatic channels. In non-prismatic channels, the hydraulic elements are no longer independent of the distance along the channel. Thus, in natural channels, it is necessary to conduct a field survey to collect the data required at all sections considered in the computation. The computation is carried out in steps from station to station where the hydraulic characteristics have already been determined. In such cases the distance between the stations is given, and the procedure is to determine the depths of flow at these stations by trial and error.

In standard step method it is convenient to refer the position of the water surface to a horizontal datum, such as, in Figure (7.4), and the water surface elevation at the two end sections can be expressed as :

$$Z_1 = S_o\,\Delta x + y_1 + z_2 \qquad \text{...(8.32)}$$

$$Z_2 = y_2 + z_2 \qquad \text{...(8.33)}$$

The friction loss between the two sections is given by :

$$h_f = (S_{f_a})\,\Delta x = \frac{1}{2}(S_{f_1} + S_{f_2})\,\Delta x \qquad \text{...(8.34)}$$

where, the friction slope S_{f_a} is taken as the average of the friction slopes corresponding to the two sections. Substituting the above expressions in equation (7.9), the following may be written, with reference to the chosen datum :

$$Z_1 + \alpha_1\left(\frac{V_1^2}{2g}\right) = Z_2 + \alpha_2\left(\frac{V_2^2}{2g}\right)^2 + h_f + h_e \qquad \text{...(8.35)}$$

where, h_e is the eddy loss which may be appreciable in non-prismatic channels. No rational method of evaluating eddy loss is available; it, however, depends mainly on the change in the velocity head and may be expressed as a fraction of the same, such as $k\left(\frac{\Delta(\alpha V)}{2g}\right)$, where k is a coefficient. For gradually converging and diverging reaches k varies from 0 to 0.1, and 0 to 0.2, respectively. For abrupt expansions and contractions k is requal to about 0.5. For prismatic channels $k = 0$.

For convenience in computation, h_e is sometimes clubbed with h_f, and so, the value of Manning's n is appropriately adjusted (i.e., increased) while computing h_f.

The total head at each section can be written as:

$$H_1 = Z_1 + \alpha_1 \left(\frac{V_1^2}{2g}\right) \quad ...(8.36)$$

and,

$$H_2 = Z_2 + \alpha_2 \left(\frac{V_2^2}{2g}\right) \quad ...(8.37)$$

Therefore, equation (8.35) becomes:

$$H_1 = H_2 + h_f + h_e \quad ...(8.38)$$

This is the basic equation that defines the procedure comprising the standard step method.

The computation of a flow profile, by this method, is generally carried towards upstream in the case of a subcritical flow, and downstream in the case of a supercritical flow. For example, in a subcritical flow the total energy computed, for the assumed water surface elevation at section 1 is compared with the total energy at the section (after accounting for the energy loss between the two sections); if the two energies are equal (with acceptable variation), the assumed water surface elevation at section 1 is taken as relevant, otherwise a new water surface elevation is assumed and computations repeated till the energy balance consideration between the two sections is satisfied. The standard step method is best suited to computations for natural channels.

For the sake of simplicity, a prismatic channel is taken to illustrate the application of this method.

Example 8.3

Compute the flow profile for a channel section as given in Example 7.1 by standard step method, taking the elevation of the channel bed at the dam site as + 100 m.

Solution

The step computations, arranged in a tabular form in Table 8.4, are explained as follows :

col. 1 Section identified by station number or the distance (x) from the first station (m),

col. 2 Water surface elevation at the station: A trial value is first entered in this column to be verified or rejected on the basis of the computations made in the remaining columns of the table. For the first step, this elevation must be given or can be assumed. Since the elevation of the dam site is 100 m (above M.S.L.) and the depth of water at dam site is 2 m, the elevation of the water surface is 102 m. When the trial value used in the second step has been verified, it becomes the basis for the verification of the trial value in the next step, and so on,

col. 3 Depth of flow in metres corresponding to the water surface elevation in col. 2 : for instance, the depth of flow at a station is equal to water surface elevation minus the elevation of the bed,

col. 4 Water area corresponding to y in col. 3,

col. 5 Mean velocity of flow equal to the given discharge divided by the water area in col. 4,

col. 6 Velocity head corresponding to the velocity in col.5,

col. 7 Total head computed by equation (8.36) equal to the sum of Z in col.2 and the velocity head in col. 6,

col. 8 Hydraulic radius, corresponding to y in col.3,

col. 9 Four-thirds power of the hydraulic radius, col. 10 Friction slope $\left(S_f = \dfrac{n^2 V^2}{R^{4/3}}\right)$ computed using the values of n, V & $R^{4/3}$,

col. 11 Average friction slope through the reach between the two consecutive sections (in each step) : approximately equal to the arithmetic mean of the friction slopes (computed in col 10) of this and previous step,

col. 12 Length of reach between the sections as determined on the basis of site survey,

col. 13 Friction loss in the reach given by the product of the values in col. 11 and 12,

col. 14 Eddy loss in the reach (assumed zero in the present problem),

col. 15 Evaluation of the total head (m) : computed by equation (8.38), i.e., adding col. 13 and 14 values to the elevation of the total head corresponding to the lower end of the reach. If the value so obtained does not agree closely with that entered in col. 7, a new trial value of the water surface elevation is assumed and so on till the required agreement is obtained. After, thus, arriving at the appropriate water surface elevation, the computation may then proceed to the next location, and

col. 16 Bed level at the chosen location, as per site survey carried out earlier.

The standard step method has an advantage while applying to natural channels. The water surface elevation at the initial section, where a flow-profile computation starts, may not be known; and, if the computation proceeds with an assumed elevation that is incorrect (for a given discharge) the resulting flow profile will get more and more correctly adjusted after every step, provided the computation is carried in the right direction. Therefore, if no elevation is known within or near the reach under consideration, an arbitrary elevation may be assumed at a distant location quite far away (upstream or downstream) from the initial location. By the time the step computation is carried to the initial section the assumed elevation would stand corrected to the appropriate value. Now, a check may be made by performing the same computation with another assumed elevation at the distant section. The computed elevation at the initial section would be taken as the relevant elevation if the second computed value agrees with the first computed value. The two values usually agree if the distant location is sufficiently away from the initial section.

To sum up, the following information is required to proceed with the computations:

1) The discharge for which the flow profile is required,

2) The water surface elevation at the so called control section. If this is not available the computation may start from an assumed elevation at a location quite far away from the initial section through which the profile is desired, and

3) The geometric elements at various channel sections along the reach for all depths of flow within the expected range. Hence, a hydrographic survey or the use of a contour map of the channel is called for. Other data include the length of the reach, and value of Manning's n (or Chezy's C).

SAQ 1

Compute the flow profile for Example 7.2 using Bakhmeteff, Chow, and Standard Step Methods.

SAQ 2

Use Standard Step Method for the solution of Example 7.1, assuming an eddy loss to be equal to half the loss due to friction.

Table 8.4 : Computation of Flow Profile for Example 8.3 by Standard Step Method

$Q = 12.26\ m^3/s$, $n = 0.025$, $S_0 = 0.0016$, $b = 7$ m, $z = 2.00$, $\alpha = 1.00$, $h_e \doteq 0$, $y_n = 1.0$ m, and $y_c = 0.64$ m

Note : Let the datum be set at the dam site; and assume $h_e = 0$.

Distance x	Z	y	A	V	$\frac{V^2}{2g}$	H	R	$(R)^{4/3}$	S_f	S_{fa}	Δx	h_f	h_e	H	Bed Level
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10) $\times 10^{-4}$	(11) $\times 10^{-4}$	(12)	(13)	(14)	(15)	(16)
0	102.00	2.00	22.00	0.56	0.02	102.02	1.38	1.54	1.27	-	-	-	-	102.02	100.00
150	102.04	1.80	19.08	0.64	0.02	102.06	1.27	1.38	1.86	1.57	150.00	0.023	0.0	102.043	100.24 (= 100 + 0.0016 × 150)
200	102.06	1.74	18.24	0.67	0.02	102.08	1.23	1.32	2.13	2.00	50.00	0.010	0.0	102.053	100.32
250	102.075	1.675	17.34	0.69	0.02	102.095	1.20	1.28	2.32	2.23	50.00	0.011	0.0	102.064	100.40
300	102.08	1.60	16.32	0.75	0.03	102.11	1.15	1.20	2.93	2.63	50.00	0.013	0.0	102.077	100.48
350	102.12	1.56	15.79	0.78	0.03	102.15	1.13	1.18	3.22	3.08	50.00	0.015	0.0	102.092	100.56
400	102.196	1.556	15.73	0.78	0.03	102.226	1.13	1.18	3.22	3.22	50.00	0.016	0.0	102.108	100.64
450	102.268	1.548	15.63	0.78	0.03	102.298	1.12	1.16	3.40	3.31	50.00	0.017	0.0	102.125	100.72

8.5 SUMMARY

We learnt about the hydraulic exponents which are the parameters much useful in the computation of a gradually-varied-flow profile, particularly with reference to Bakhmeteff Method and also the Chow Method. These computational procedures were illustrated by numerical examples.

Moreover, we learnt about standard step method which is a very useful (or more appropriately a powerful method) in dealing with non-prismatic channels. This method is based on the principle of conservation of energy.

8.6 ANSWERS TO SAQs

Check your answers of all SAQs with respective preceding text.

FURTHER READING

1) Chow, V.T., 1959, *Open Channel Hydraulics*, McGraw Hill Book Company, New York.

2) French, R.H. 1985, *Open Channel Hydraulics*, McGraw Hill Book Company, New York.

3) Henderson, F.M., 1966, *Open Channel Flow*, The MacMillan Company, New York.

4) Streeter, L. and E.B. Wyile, 1983, *Fluid Mechanics*, McGraw Hill Book Company, Tokyo.

5) Subramanya, K., *Flow in Open Channels*, Tata McGraw Hill Publishing Co. Pvt. Ltd., 1990.

6) Ranga Raju K.G., *Flow Through Open Channel*, Tata McGraw Hill Publishing Co. Pvt. Ltd., 1993.