UNIT 7 COMPUTATION OF WATER SURFACE PROFILES - I

Structure

7.1 INTRODUCTION

We derived the dynamic equation for the gradually varied flow in sub-sections 5.2.2 and 5.2.3, and noted that this gradually varied flow equation can be written in different forms. The equation (5.9) gives the slope of the water surface with reference to the channel bottom, and it can take either a positive or a negative value. A positive value indicates that the depth of flow increases in the direction of flow while a negative value indicates that the depth of flow decreases in the direction of flow. In Section 5.4, we considered the classification of flow profiles, giving the nomenclature based on the slope of the channel and the zone in which the profiles lie. Figures 5.2 and 5.3 show the shape of the different flow profiles and the zones in which they occur.

In Unit 6 we discussed in further detail the occurrence of such profiles under complex practical situations, which enables one to learn a priori the possible flow profile that may occur in a given channel layout. We also introduced, through the solution of a problem, a procedure – based on **varied flow functions**. In fact, many methods are available for computing the GVF profiles, which shall be discussed in this unit, as well as in Unit 8. Computation of flow profile gives its length and shape, and its position or location with reference to the channel bottom. The location of water surface becomes essential in many cases, for example, in the provision of a free board in channel design, determining the water spread area behind a dam, and deciding the length of the stilling basin where a hydraulic jump is to occur. In Units 7 and 8, as mentioned above, we will consider the methods of computation of flow profile applicable to prismatic and non-prismatic channels.

Objectives

This unit shall define a **control section**, and emphasize its importance. And, then introduce following computational methods for the determination of GVF profiles :

- Graphical Integration Method – it is applicable to both prismatic and non-prismatic channels of any shape,
- Analytical Method, or Method of Direct Integration (Bresse Method) – for wide rectangular channels, and
- Numerical Method (Step Method) – applicable under a variety of differing practical conditions.

respectively, from a chosen origin, with the corresponding depths y_1 and y_2. The distance along the channel floor is given by :

$$x = x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} \frac{dx}{dy}\, dy \qquad \text{...(7.1)}$$

where, $\frac{dx}{dy}$ is the reciprocal of the RHS - term of the dynamic equation derived in Section 5.2 or 5.3. Now, from equation (5.17):

$$\frac{dx}{dy} = \frac{1}{S_0}\left[\frac{1 - \left(\frac{Z_c}{Z}\right)^2}{1 - \left(\frac{K_n}{K}\right)^2}\right] \qquad \text{...(7.2)}$$

Once the type of water surface profile for a given flow is envisaged and the depths of flow at the upstream and downstream ends of the profile are known, the computation of the flow profile between the two depths can be started. We can assume several values of y between the two depths and the corresponding values of $\frac{dx}{dy}$ are computed from equation (7.2). A curve of $\frac{dx}{dy}$ against y is then constructed (Figure 7.1(b)). According to equation (7.1), it is apparent that the value of x is equal to the shaded area under this curve, and bounded by y – axis and the ordinates of $\frac{dx}{dy}$ corresponding to y_1 and y_2. This area can be measured and the value of x determined.

This method has broad application. It applies to flow in prismatic as well as non-prismatic channel of any shape. The procedure is straight forward and easy to follow. A simple example is given below to illustrate the application of the graphical integration method.

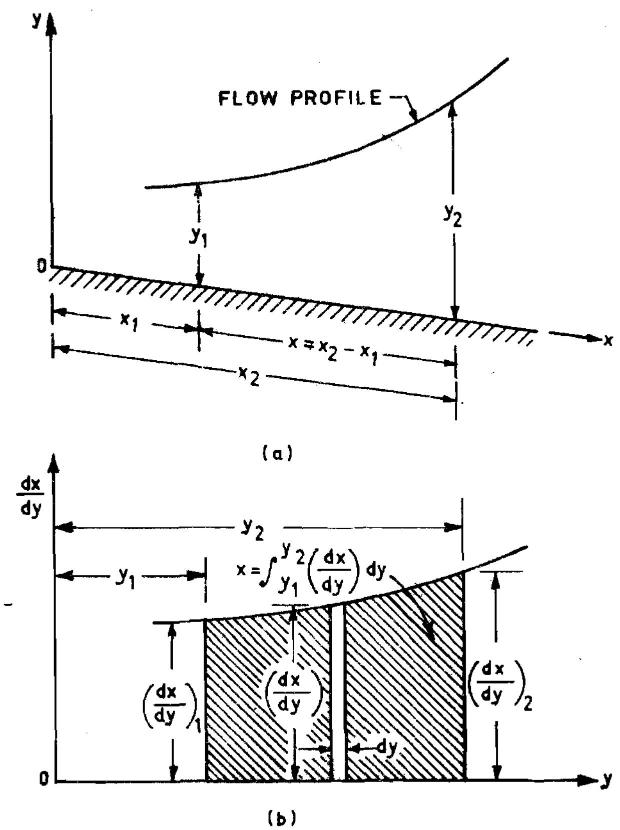

Figure 7.1 : Graphical Integration Method

Example 7.1

A trapezoidal channel having a bed width $b = 7$ m, side slope $z = 2$, bed slope $S_o = 0.0016$ and Manning's $n = 0.025$ carries a discharge of 12.26 m^3/s. Compute the back water profile created by a dam which backs up the water to a depth of 2 m immediately behind the dam. The upstream end of the profile is assumed at a depth equal to 1 % greater than the normal depth. Assume the energy coefficient = 1.0.

Solution

For the given data normal depth $y_n = 1.0$ m, and the critical depth $y_c = 0.64$ m. As $y_c < y_n$, the channel slope is mild. As the depth of flow behind the dam is 2.0 m which is greater than y_n, the flow profile occurs in Zone 1 in this mild channel, and hence the flow profile is MI type. The flow profile from 2.0 depth (at the downstream end) onwards is to be computed to an upstream depth of 1% greater than the normal depth, i.e., 1(1.01) = 1.01 m. **The value $\frac{dx}{dy}$ when $y = y_n$ becomes infinite. Hence the computation of flow profile is made upto a depth 1% greater or less than normal depth depending on the nature of the flow profile.**

For simplicity, the channel bottom at the site of the dam is chosen as origin and the x value in the upstream direction is taken as positive. The values of Z_c and K_n are computed as follows:

$$Z_c = \left(\frac{A^3}{T}\right)^{1/2}$$

where, A and T are values corresponding to the critical depth of flow ($y_c = 0.64$ m), and we have :

$$A = (b + zy)\, y = (7 + 2 \times 0.64) \times 0.64 = 5.30 \text{ m}^2$$

$$T = (b + 2zy) = 7 + 2 \times 2 \times 0.64 = 9.56 \text{ m}$$

$$Z_c = \left(\frac{A^3}{T}\right)^{1/2}$$

$$= \left(\frac{5.3^3}{9.56}\right)^{1/2} = 3.946 \text{ m}^{5/2}$$

$$K_n = \left(\frac{1}{n}\right) AR^{2/3}$$

where, A and R values correspond to the normal depth of flow ($y_n = 1.0$ m), and we have :

$$A = (b + zy_n) \times y_n = (7.0 + 2 \times 1.0) \times 1.0 = 9.0 \text{ m}^2$$

$$P = (b + 2 \times y_n\, (1 + z^2)^{1/2}) = (7.0 + 2 \times 1.0 \times (1 + 2^2)^{1/2}) = 11.47 \text{ m}$$

$$R = \left(\frac{A}{P}\right) = \left(\frac{9.0}{11.47}\right) = 0.785 \text{ m}$$

$$\therefore \quad K_n = \left(\frac{1}{n}\right) AR^{2/3} = \left(\frac{1}{0.025}\right) \times 9.0 \times (0.785)^{2/3} = 306.35 \text{ m}^{8/3}$$

The other values in Table 7.1 are computed using appropriate equations.

Table 7.1 : Computation of the Flow Profile for Example 7.1 by Graphical Integration

$Q = 12.26\ m^3/s$ $n = 0.025$ $S_0 = 0.0016$ $y_c = 0.64\ m$

$y_n = 1.0\ m$ $\alpha = 1.00$ $Z_c = 3.946$ $K_n = 306.4$

y	*T*	*A*	*P*	*R*	*K*	*Z*	$\frac{dx}{dy}$ (equation (7.2))	*dA* (elementary area under the curve)	*x*
2.00	15.00	22.00	15.94	1.38	1091.0	26.64	664	—	0
1.80	14.20	19.08	15.05	1.27	895.0	22.12	685	135	135
1.60	13.40	16.32	14.16	1.15	717.0	18.01	728	141	276 (=135 +141)
1.50	13.00	15.00	13.71	1.09	635.0	16.11	765	75	351
1.40	12.60	13.72	13.26	1.03	560.0	14.32	824	79	430
1.30	12.20	12.48	12.81	0.97	489.0	12.62	928	88	518
1.25	12.00	11.88	12.59	0.94	456.0	11.82	1012	49	567
1.20	11.80	11.28	12.37	0.91	424.0	11.02	1140	54	621
1.15	11.60	10.70	12.14	0.88	393.0	10.27	1358	62	683
1.10	11.40	10.12	11.92	0.85	363.0	9.53	1800	79	762
1.07	11.28	9.78	11.79	0.83	346.0	9.11	2353	62	824
1.05	11.20	9.56	11.70	0.82	335.0	8.83	3058	54	878
1.03	11.12	9.33	11.61	0.804	323.0	8.55	4912	80	958
1.01	11.04	9.11	11.52	0.79	311.0	8.27	16439	213	1171

For instance, when $y = 2.00$ m, the values in the columns of the Table are: $T = 15.00$ m, $A = 22.0\ m^2$, $P = 15.94$ m, and $R = 1.38$ m. Further, $K = \left(\frac{AR^{2/3}}{n}\right)$ where, the values A and R correspond to the depth of flow, under consideration at the concerned section; $Z = \left(\frac{A^3}{T}\right)^{1/2}$ where, the values A and T correspond to the depth of flow at that concerned section. Therefore, from equation (7.2),

$$\frac{dx}{dy} = \left(\frac{1}{0.0016}\right) \times \left[\frac{1 - \left(\frac{3.946}{26.64}\right)^2}{1 - \left(\frac{306.4}{1091}\right)^2}\right]$$

$$= 664$$

Hence, the elementary area under the curve (dA)

$$= \left(\frac{dx}{dy}\right) \times dy = \left[\frac{664 + 685}{2}\right] (2-1.8)$$

$$= 134.9 \simeq 135\ m$$

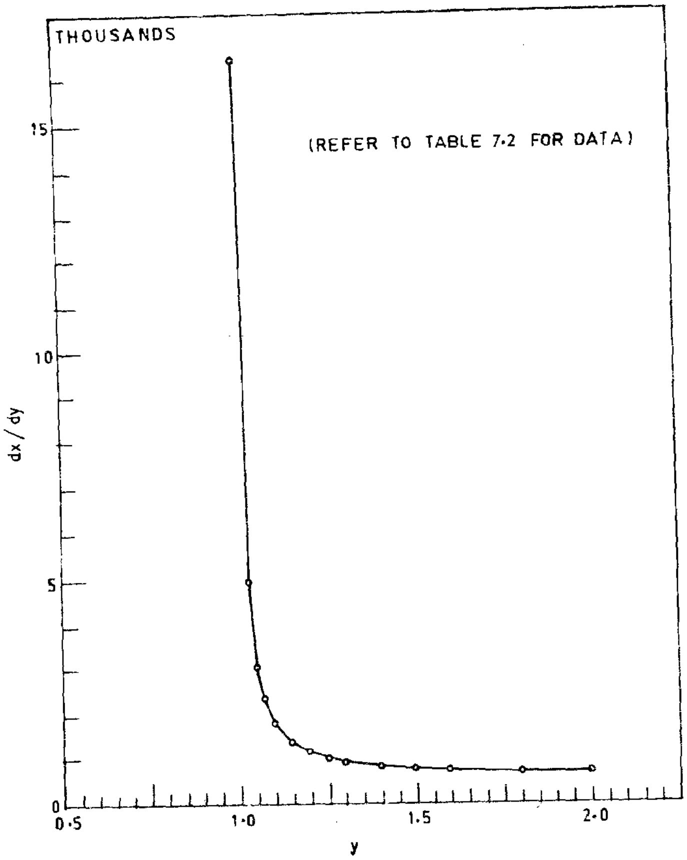

Figure 7.2 : A Curve of *dx/dy* versus *y* (Graphical Integration Method)

Values of $\frac{dx}{dy}$ are then plotted against the corresponding values of y (Figure 7.2). The increments in area (dA) under the curve can be planimetered and listed in a table. According to equation (7.1), the cumulative value of dA, upto the point of interest, should give the length x of the flow profile. The backwater profile, thus computed, is presented as a plot of y against x (Figure 7.3).

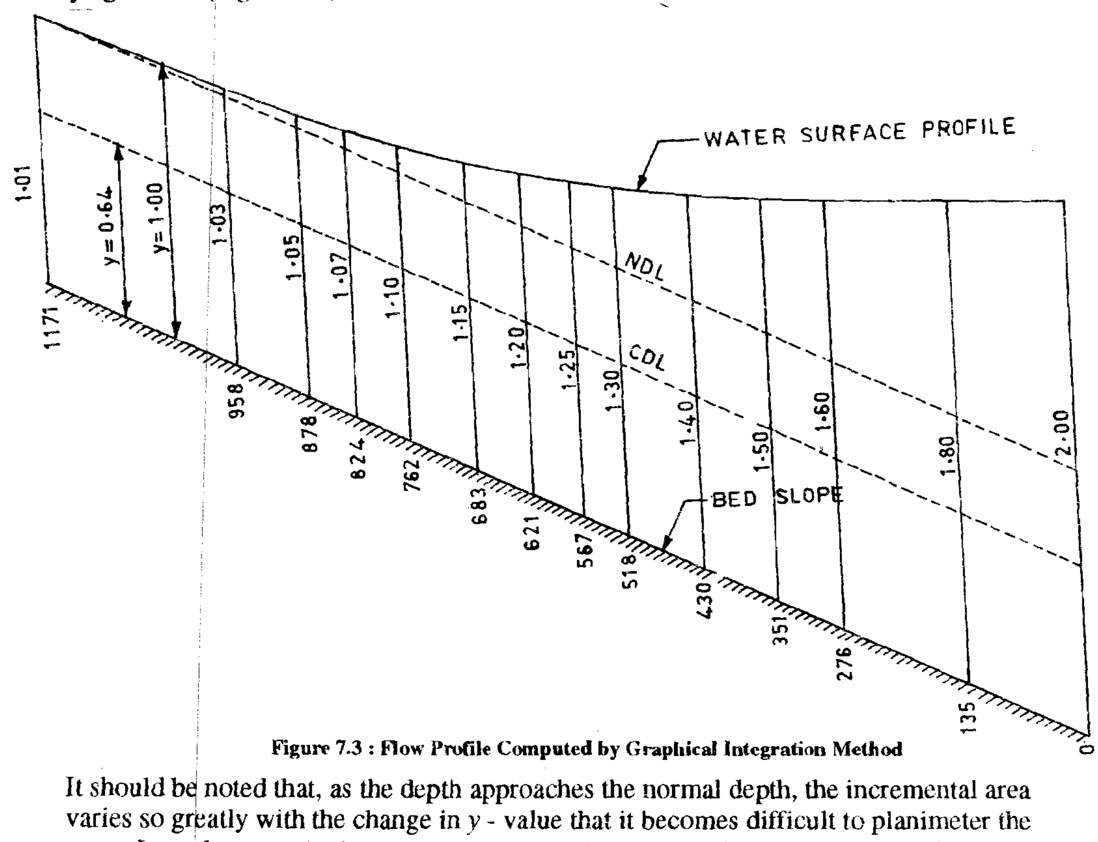

Figure 7.3 : Flow Profile Computed by Graphical Integration Method

It should be noted that, as the depth approaches the normal depth, the incremental area varies so greatly with the change in y - value that it becomes difficult to planimeter the same. In such cases, the incremental area may be computed by assuming it to be a trapezoid.

7.5 METHOD OF DIRECT INTEGRATION (BRESSE METHOD)

The differential equation of gradually varied flow cannot be expressed explicitly in terms of y for all types of channel cross-section and hence a direct and exact integration of the equation is practically impossible. Many attempts have been made either to solve the equation for a few special cases or to introduce assumptions that make the equation amenable to mathematical integration. Bresse in 1860 solved the gradually varied flow equation for wide rectangular channel, with reference to which, the equation can be expressed explicitly in terms of y and the method is known as BRESSE Method.

We know, that, for a wide rectangular channel (i.e., where $R \approx y$), the gradually varied flow profile can be expressed as :

$$\left(\frac{dy}{dx}\right) = S_o \frac{\left[1 - \left(\frac{y_n}{y}\right)^3\right]}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]} \qquad \text{...(5.19)}$$

$$dx = \frac{\left[1 - \left(\frac{y_c}{y}\right)^3\right]}{S_o\left[1 - \left(\frac{y_n}{y}\right)^3\right]} dy \qquad \text{...(7.3)}$$

or,

$$dx = \frac{1}{S_o}\left[\frac{y^3 - y_c^3}{y^3 - y_n^3}\right] dy$$

$$= \frac{1}{S_o}\left[1 - \left\{\frac{1 - \left(\frac{y_c}{y_n}\right)^3}{1 - \left(\frac{y}{y_n}\right)^3}\right\}\right] dy$$

(i.e., through simple division of the numerator by the denominator inside the bracket)

Put

$$u = \frac{y}{y_n}$$

$$\therefore \quad du = \frac{1}{y_n} dy$$

$$\therefore \quad dx = \frac{y_n}{S_o}\left[1 - \left\{1 - \left(\frac{y_c}{y_n}\right)^3\right\}\frac{1}{1 - u^3}\right] du \qquad \text{...(7.4)}$$

Now, noting that $y_c = \left(\frac{q^2}{g}\right)^{1/3}$ where, q is the discharge intensity, which is given by $(q^2 = C^2 S_o y_n^3)$ if Chezy's equation is used in the analysis; and, thus $\left(\frac{y_c}{y_n}\right)^3 = C^2 \frac{S_o}{g}$, equation (7.4) can be written as :

$$dx = \left(\frac{y_n}{S_o}\right)\left[1 - \left\{1 - \frac{C^2 S_o}{g}\right\}\frac{1}{1 - u^3}\right] du$$

Further, putting $\frac{y_n}{S_o} = A$, and $1 - \left(\frac{C^2 S_o}{g}\right) = B$, we get :

$$dx = A\left[1 - \frac{B}{1 - u^3}\right] du \qquad \text{...(7.5)}$$

which is easily integrable by established procedures, such that :

$$x = A\,[u - B\,\phi\,(u)] + \text{constant} \qquad ...(7.6)$$

where, $\phi(u) = \int_0^u \frac{1}{(1-u^3)}\,du$

$$= \frac{1}{6}\int \frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3}}{2u+1}\right] \qquad ...(7.7)$$

and, x gives the required length of the profile upto the desired value of y. $\phi\,(u)$ is known as **Bresse Back Water Function**. The values of $\phi(u)$, for different values of u (for $N = 3$), have been computed and are available in the form of a table (Refer Appendix 7.1) known as **Bresse Varied Flow Function.**

Now, the length of the profile between two consecutive sections (or, say the distance between the two sections), having the depths of flow y_1 and y_2, respectively, is given by :

$$\Delta x = x_2 - x_1 = A\left[(u_2 - u_1) - B\left\{\phi(u_2) - \phi(u_1)\right\}\right] \qquad ...(7.8)$$

Application of the Bresse method which is relevant only for a wide rectangular channel, is illustrated by an example given below.

Example 7.2

A wide rectangular channel carries a discharge of 4.0 m^3/s per metre width of the channel on a bed slope of 0.001, and with Manning's $n = 0.025$. A weir across the channel raises the water depth at the weir site to 4.0 m. Compute the flow profile from the weir site to an upstream section where depth of flow is 5% greater than the normal depth.

Solution

Computation of normal depth y_n, critical depth y_c and Chezy's C.

Discharge/unit width, $q = \frac{1}{n}\,y_n\left(y_n\right)^{2/3}\left(S_o\right)^{1/2}$

$$4 = \left(\frac{1}{0.025}\right)\left(y_n\right)^{5/3}(0.001)^{1/2}$$

$\therefore$ Normal depth, $y_n = 1.995 \simeq 2.00\ m^2$

Calculations are meant to be carried out upto $y = 1.05\,y_n = 2.10$ m

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$= \left(\frac{4^2}{9.81}\right)^{1/3} = 1.18\text{ m}$$

Average depth of flow $\frac{1}{2}(2+4) = 3$ m

Hence, $R \simeq 3$ m

Chezy's $C = \left(\frac{1}{n}\right)R^{1/6}$

$$C = \left(\frac{1}{0.025}\right)(3)^{1/6} = 48$$

or,

$$A = \frac{y_n}{S}$$

$$= \frac{2.0}{0.001} = 2000$$

$$B = \left[1 - \frac{C^2\,S_o}{g}\right] = 0.765$$

As $y > y_n > y_c$, the flow profile is M1 type. Applying equation (7.8), the flow profile computations are carried out and presented in Table 7.2.

In Table 7.2, the values of $\phi(u)$ are obtained from Bresse varied flow function table (Appendix 7.1). Δx gives the distance between adjacent depths of flow taken for computation, and the cumulative distance, x. The distance between depths 2.1m and 4.0 m can also be computed in one step by substituting the values for u_2, u_1, $\phi(u_2)$ and $\phi(u_1)$, corresponding to downstream depth of 4 m and upstream depth of 2.1 m in equation (7.8). Thus the total length is :

$$x = 2000\,[(2.00 - 1.05) - 0.765\,(0.132 - 0.896)] = 3069 \text{ m}$$

where, $u_1 = \dfrac{2.1}{2} = 1.05$ and $u_2 = \dfrac{4}{2} = 2$

Table 7.2 : Computation of the Flow Profile for Example 7.2 by Bresse Method

$q = 4$ m³/s/m, $n = 0.025$, $S_o = 0.001$, $y_n = 2.0$ m,

$y_c = 1.18$m, $C = 48$, $A = 2000$, $B = 0.765$

y (m)	u	$\phi(u)$	Δx (m)	x (m)
2.1	1.05	0.896		
2.5	1.25	0.420	1128	1128
3.0	1.50	0.255	753	1881 (= 1128 + 753)
3.5	1.75	0.177	619	2500
4.0	2.00	0.132	569	3069

7.6 NUMERICAL METHOD

The gradually varied flow equation is not, in general, explicitly solvable. If the channel geometry and the form of the resistance equation are both particularly simple, the equation may be solved explicitly, but in most cases we must seek a solution by numerical integration. The characteristic of a numerical method is that it can account for the differing circumstances of particular problems requiring special methods of solution, although the basic equation itself remains unchanged. It will be found, for example, that methods suitable for prismatic channels are not suitable for natural rivers. However, all numerical solutions of gradually varied flow equations have one thing in common that the calculations must start at the control section and proceed in the direction towards which control is being exercised. Commonly known method of computation under numerical procedure is the **step method**.

7.6.1 Step Method

In general a step method is characterised by dividing the channel into short reaches and carrying the computation step by step from one end of the reach to other. There is a great variety of step methods. Some methods appear superior to others in certain respects, but no one method has been found to be the best in all applications. The step methods are classified as :

i) Direct step method

ii) Standard step method

The **direct step method** does not involve any trial and error tedium, and is applicable to a prismatic channel only. Its application is explained herein by means of an illustrative example.

However, the **standard step method** involves a trial and error procedure, and is applicable to both the prismatic as well as non-prismatic channels. The application of this method, viz-a-vis, a natural channel (i.e., non-prismatic channel) is explained in Unit 8.

The Direct Step Method

The direct step method, as usual, computes the distance between two chosen depths of flow. Figure 7.4 illustrates a short channel reach of length Δx. Equating the total heads at the two end sections 1 and 2, the following may be written:

$$S_o(\Delta x) + y_1 + \alpha_1\left(\frac{V_1^2}{2g}\right) = y_2 + \alpha_2\left(\frac{V_2^2}{2g}\right) + S_f\,\Delta x \qquad ...(7.9)$$

$$\therefore \quad S_o\Delta x + E_1 = S_f\Delta x + E_2 \qquad ...(7.10)$$

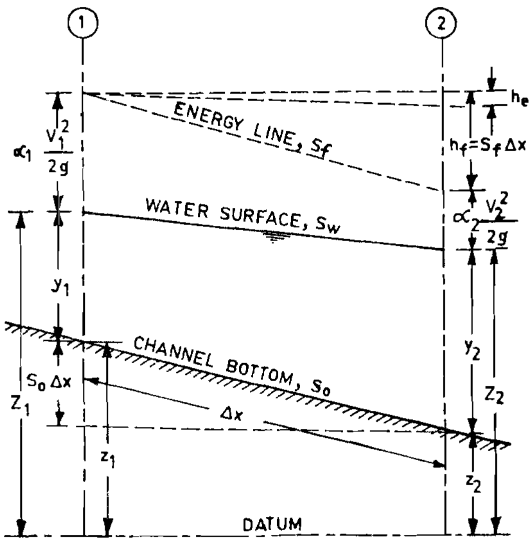

Figure 7.4 : A Channel Reach for the Derivation of Step Methods

Solving for the incremental distance, Δx, between the depths y_1 and y_2,

$$\Delta x = \frac{E_2 - E_1}{S_o - S_{fa}} \qquad ...(7.11)$$

In the above equations, y is the depth of flow, V is the mean velocity, α is the energy coefficient, S_o is the bed slope, S_f is the friction slope, S_{fa} is the average friction slope (energy slope) with reference to end sections (i.e., the average value of energy slopes at depths y_1 and y_2), and is equal to $\frac{1}{2}(S_{f_1} + S_{f_2})$ – where S_{f_1} and S_{f_2} are friction slopes at sections 1 and 2, respectively – and E is the specific energy. S_f at a section is computed using either Manning's equation or Chezy's equation. Equation (7.11) is written as :

$$\Delta x = \frac{\Delta E}{S_o - S_{fa}} \qquad ..(7.12)$$

where, ΔE is $(E_2 - E_1)$. Equation (7.12) gives the distance between the sections 1 and 2 where depths of flow are y_1 and y_2, respectively. The application of **direct step method**, based on equation (7:12), is illustrated by considering Example 7.1; and Table 7.3 gives the details of these computation. Entry in each column of the table is explained as follows:

col. 1. Depth of flow (m), arbitrarily assigned from 2.00 to 1.01 m.

col. 2. Water area (m^2), corresponding to the depth y in col.1.

col.3. Hydraulic radius (m) corresponding to y in col.1.

col. 4. Four - thirds power of the hydraulic radius.

col. 5. Mean velocity (m/s) obtained by dividing 12.26 m^3/s by the water area in col.2.

col. 6. Velocity head (m) – Energy Coefficient is taken as unity.

col. 7. Specific energy (m) obtained by adding the velocity head in col.6 to the depth of flow in col.1.

col. 8. Change of specific energy (m), equal to the difference between the two consecutive E–values in col. 7.

Table 7.3 : Computation of Flow Profile by the Direct Step Method for the Problem Given in Example 7.1

$Q = 12.26\ m^3/s,\ n = 0.025,\ S_o = 0.0016,\ \alpha = 1.00,\ y_c = 0.64\ m,\ y_n = 1.0\ m$

y	A	R	$R^{4/3}$	V	$\frac{V^2}{2g}$	E	ΔE	S_f ($\times 10^{-4}$)	S_{f_a} ($\times 10^{-4}$)	$S_o - S_{f_a}$ ($\times 10^{-3}$)	Δx	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
2.00	22.00	1.38	1.54	0.557	0.016	2.016	—	1.259	—	—	—	—
1.80	19.08	1.28	1.37	0.643	0.021	1.821	0.195	1.886	1.572	1.443	135.00	135
1.60	16.32	1.15	1.20	0.751	0.029	1.629	0.192	2.938	2.412	1.360	141.18	276
1.50	15.00	1.09	1.12	0.817	0.034	1.534	0.095	3.725	3.331	1.269	74.86	351
1.40	13.72	1.03	1.04	0.894	0.041	1.441	0.093	4.803	4.264	1.14	79.21	430
1.30	12.48	0.97	0.96	0.982	0.049	1.347	0.092	6.278	5.540	1.046	87.95	518
1.25	11.88	0.94	0.92	1.032	0.054	1.304	0.045	7.235	6.756	0.924	48.70	567
1.2	11.28	0.91	0.88	1.087	0.060	1.260	0.044	8.392	7.813	0.818	53.79	620
1.15	10.70	0.88	0.84	1.146	0.067	1.27	0.043	9.772	9.082	0.691	62.22	683
1.10	10.12	0.85	0.81	1.211	0.075	1.175	0.042	11.316	10.544	0.545	77.06	760
1.05	9.56	0.82	0.77	1.282	0.084	1.134	0.041	13.340	12.328	0.367	111.71	871
1.01	9.11	0.79	0.73	1.346	0.092	1.102	0.032	15.511	14.425	0.157	203.82	1075

col. 9. Friction slope S_f which is equal to $(nV)^2/(R^{(4/3)})$ where, $n = 0.025$ and V is the mean velocity obtained in col.5.

col. 10. Average friction slope equal to the arithmetic mean of the friction slopes, considering the two consecutive sections.

col. 11. Difference between the bottom slope S_o (= 0.0016) and the average friction slope S_{fa}.

col. 12. Length of the reach in between the consecutive steps, computed by dividing the value of ΔE in col 8 by the corresponding value in col.11.

col.13. Distance from the section under consideration to the dam site given by the cumulative sum of the values in col. 12.

Note : Refer to Table 7.3

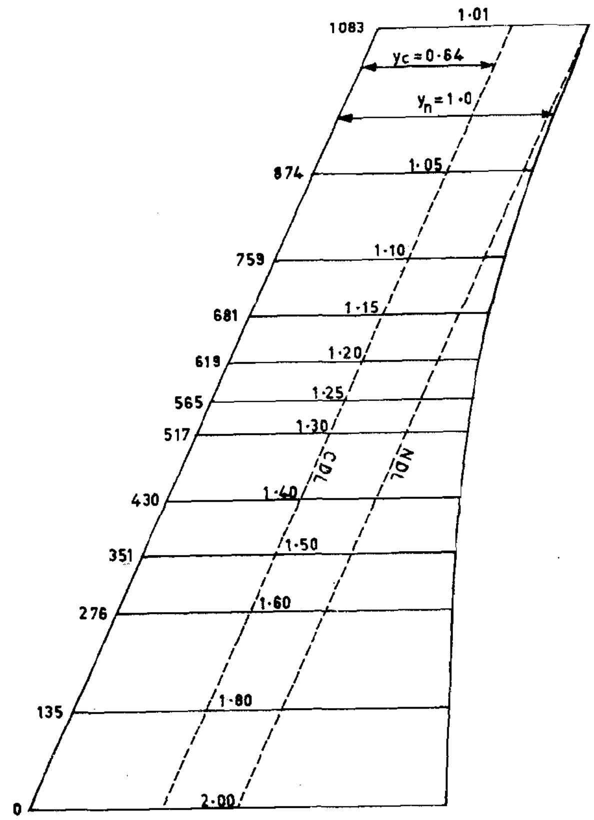

Figure 7.5 : Flow Profile Computed by Direct Step Method

Figure 7.5 gives the flow profile computed by the direct step method. A comparison of Figures 7.3 and 7.5 indicates that the flow profiles computed by the graphical integration method and the direct step method are almost identical.

SAQ 1

A trapezoidal channel of base width 6.0 m, and side slope $z = 1.5$ is laid on a bed slope of 0.001; it carries a discharge of 28 m^3/s. The channel terminates in a free overfall. Compute the flow profile from the free-overfall point to a section where the depth of flow is 2% less than the normal depth by Graphical Integration Method, as well as by the Direct Step Method. Take Manning's $n = 0.025$.

SAQ 2

A wide rectangular channel carries a discharge of 3.083 m^3/s/m on a slope of 0.0008. Plot the flow profile for a distance of 1000 m upstream from a weir which maintains 3 m depth behind itself if n = 0.02. Use Bresse Method.

SAQ 3

Water flows from under a sluice into a trapezoidal channel having a bed width (b) of 6 m, side slope (z) of 2, bed slope (S_o) of 0.036, energy co-efficient (α) = 1.0, and n = 0.025. The sluice gate is regulated to discharge 11.5 m^3/s with a depth of 0.016 m at the vena contracta. If a hydraulic jump occurs somewhere at the downstream of the sluice gate, compute the flow profile between the vena contracta and the jump. Use Graphical Integration and Direct Step Methods.

7.7 SUMMARY

In this unit we have indicated the importance, and the underlying principles of the procedures used to compute the GVF profiles. Each procedure basically involves integration of the dynamic equation. However, as the equation is not easily expressible as an explicit function of y, it is difficult to proceed with the integration unless the channel geometry is simplified.

Every computation of a flow profile starts from a known depth of flow in the channel. Existence of a control section (where a definitive relationship exists between the stage and discharge) influences the flow either in the upstream or downstream direction, depending on whether the state of flow is subcritical or supercritical. Thus, a control section has its own importance in the computational strategy.

The methods of computation were classified as : (i) Graphical Integration Method, (ii) Direct Integration Method, and (iii) Numerical Method. Under the second method, we derived the Bresse equation for a wide rectangular channel. Illustrative examples were used to demonstrate the application of these methods.

7.8 KEY WORD

Control Section : A section in an open channel were a definitive relationship between stage and discharge exists.

7.9 ANSWERS TO SAQs

Check your answers of all SAQs with respective preceding text.

APPENDIX 7.1 : Table of the Varied Flow Functions

$$F(u, N) = \int_0^u \frac{du}{1 - u^N} \quad \text{and} \quad F(u, N)_{-s_0} = \int_0^u \frac{du}{1 + u^N}$$

The Varied Flow Function for Positive Slopes, $F(u, N)$

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
0.16	0.161	0.161	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
0.18	0.181	0.181	0.181	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.20	0.202	0.201	0.201	0.201	0.200	0.200	0.200	0.200	0.200	0.200
0.22	0.223	0.222	0.221	0.221	0.221	0.220	0.220	0.220	0.220	0.220
0.24	0.244	0.243	0.242	0.241	0.241	0.241	0.240	0.240	0.240	0.240
0.26	0.265	0.263	0.262	0.262	0.261	0.261	0.261	0.260	0.260	0.260
0.28	0.286	0.284	0.283	0.282	0.282	0.281	0.281	0.281	0.280	0.280
0.30	0.307	0.305	0.304	0.303	0.302	0.302	0.301	0.301	0.301	0.300
0.32	0.329	0.326	0.325	0.324	0.323	0.322	0.322	0.321	0.321	0.321
0.34	0.351	0.348	0.346	0.344	0.343	0.343	0.342	0.342	0.341	0.341
0.36	0.372	0.369	0.367	0.366	0.364	0.363	0.363	0.362	0.362	0.361
0.38	0.395	0.392	0.389	0.387	0.385	0.384	0.383	0.383	0.382	0.382
0.40	0.418	0.414	0.411	0.408	0.407	0.405	0.404	0.403	0.403	0.402
0.42	0.442	0.437	0.433	0.430	0.428	0.426	0.425	0.424	0.423	0.423
0.44	0.465	0.460	0.456	0.452	0.450	0.448	0.446	0.445	0.444	0.443
0.46	0.489	0.483	0.479	0.475	0.472	0.470	0.468	0.466	0.465	0.464
0.48	0.514	0.507	0.502	0.497	0.494	0.492	0.489	0.488	0.486	0.485
0.50	0.539	0.531	0.525	0.521	0.517	0.514	0.511	0.509	0.508	0.506
0.52	0.565	0.557	0.550	0.544	0.540	0.536	0.534	0.531	0.529	0.528
0.54	0.592	0.582	0.574	0.568	0.563	0.559	0.556	0.554	0.551	0.550
0.56	0.619	0.608	0.599	0.593	0.587	0.583	0.579	0.576	0.574	0.572
0.58	0.648	0.635	0.626	0.618	0.612	0.607	0.603	0.599	0.596	0.594
0.60	0.676	0.663	0.653	0.644	0.637	0.631	0.627	0.623	0.620	0.617
0.61	0.691	0.678	0.667	0.657	0.650	0.644	0.639	0.635	0.631	0.628
0.62	0.706	0.692	0.680	0.671	0.663	0.657	0.651	0.647	0.643	0.640
0.63	0.722	0.707	0.694	0.684	0.676	0.669	0.664	0.659	0.655	0.652
0.64	0.738	0.722	0.709	0.698	0.690	0.683	0.677	0.672	0.667	0.664
0.65	0.754	0.737	0.724	0.712	0.703	0.696	0.689	0.684	0.680	0.676
0.66	0.771	0.753	0.738	0.727	0.717	0.709	0.703	0.697	0.692	0.688
0.67	0.787	0.769	0.754	0.742	0.731	0.723	0.716	0.710	0.705	0.701
0.68	0.804	0.785	0.769	0.757	0.746	0.737	0.729	0.723	0.718	0.713
0.69	0.822	0.804	0.785	0.772	0.761	0.751	0.743	0.737	0.731	0.726

* The table of the varied-flow function for positive slopes $F(u,N)$ is reproduced from Ven Te Chow, Integrating the equation of gradually varied flow, *Proceedings, American Society of Civil Engineers*, vol. 81, paper no. 838, pp. 1–32, November, 1955.

The Varied Flow Function for Positive Slopes, *F* (*u*, *N*) (continued)

u \ *N*	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0.70	0.840	0.819	0.802	0.787	0.776	0.766	0.757	0.750	0.744	0.739
0.71	0.858	0.836	0.819	0.804	0.791	0.781	0.772	0.764	0.758	0.752
0.72	0.878	0.855	0.836	0.820	0.807	0.796	0.786	0.779	0.772	0.766
0.73	0.898	0.874	0.854	0.837	0.823	0.811	0.802	0.793	0.786	0.780
0.74	0.918	0.892	0.868	0.854	0.840	0.827	0.817	0.808	0.800	0.794
0.75	0.940	0.913	0.890	0.872	0.857	0.844	0.833	0.823	0.815	0.808
0.76	0.961	0.933	0.909	0.890	0.874	0.861	0.849	0.839	0.830	0.823
0.77	0.985	0.954	0.930	0.909	0.892	0.878	0.866	0.855	0.846	0.838
0.78	1.007	0.976	0.950	0.929	0.911	0.896	0.883	0.872	0.862	0.854
0.79	1.031	0.998	0.971	0.949	0.930	0.914	0.901	0.889	0.879	0.870
0.80	1.056	1.022	0.994	0.970	0.950	0.934	0.919	0.907	0.896	0.887
0.81	1.083	1.046	1.017	0.992	0.971	0.954	0.938	0.925	0.914	0.904
0.82	1.110	1.072	1.041	1.015	0.993	0.974	0.958	0.945	0.932	0.922
0.83	1.139	1.099	1.067	1.039	1.016	0.996	0.979	0.965	0.952	0.940
0.84	1.171	1.129	1.094	1.064	1.040	1.019	1.001	0.985	0.972	0.960
0.85	1.201	1.157	1.121	1.091	1.065	1.043	1.024	1.007	0.993	0.980
0.86	1.238	1.192	1.153	1.119	1.092	1.068	1.048	1.031	1.015	1.002
0.87	1.272	1.223	1.182	1.149	1.120	1.095	1.074	1.055	1.039	1.025
0.88	1.314	1.262	1.228	1.181	1.151	1.124	1.101	1.081	1.064	1.049
0.89	1.357	1.302	1.255	1.216	1.183	1.155	1.131	1.110	1.091	1.075
0.90	1.401	1.343	1.294	1.253	1.218	1.189	1.163	1.140	1.120	1.103
0.91	1.452	1.389	1.338	1.294	1.257	1.225	1.197	1.173	1.152	1.133
0.92	1.505	1.438	1.351	1.340	1.300	1.266	1.236	1.210	1.187	1.166
0.93	1.564	1.493	1.435	1.391	1.348	1.311	1.279	1.251	1.226	1.204
0.94	1.645	1.568	1.504	1.449	1.403	1.363	1.328	1.297	1.270	1.246
0.950	1.737	1.652	1.582	1.518	1.467	1.423	1.385	1.352	1.322	1.296
0.960	1.833	1.741	1.665	1.601	1.545	1.497	1.454	1.417	1.385	1.355
0.970	1.969	1.866	1.780	1.707	1.644	1.590	1.543	1.501	1.464	1.431
0.975	2.055	1.945	1.853	1.773	1.707	1.649	1.598	1.554	1.514	1.479
0.980	2.164	2.045	1.946	1.855	1.783	1.720	1.666	1.617	1.575	1.536
0.985	2.294	2.165	2.056	1.959	1.880	1.812	1.752	1.699	1.652	1.610
0.990	2.477	2.333	2.212	2.106	2.017	1.940	1.873	1.814	1.761	1.714
0.995	2.792	2.621	2.478	2.355	2.250	2.159	2.079	2.008	1.945	1.889
0.999	3.523	3.292	3.097	2.931	2.788	2.663	2.554	2.457	2.370	2.293
1.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
1.001	3.317	2.931	2.640	2.399	2.184	2.008	1.856	1.725	1.610	1.508
1.005	2.587	2.266	2.022	1.818	1.649	1.506	1.384	1.279	1.188	1.107
1.010	2.273	1.977	1.757	1.572	1.419	1.291	1.182	1.089	1.007	0.936
1.015	2.090	1.807	1.602	1.428	1.286	1.166	1.065	0.978	0.902	0.836
1.020	1.961	1.711	1.493	1.327	1.191	1.078	0.982	0.900	0.828	0.766
1.03	1.779	1.531	1.340	1.186	1.060	0.955	0.866	0.790	0.725	0.668
1.04	1.651	1.410	1.232	1.086	0.967	0.868	0.785	0.714	0.653	0.600
1.05	1.552	1.334	1.150	1.010	0.896	0.802	0.723	0.656	0.598	0.548
1.06	1.472	1.250	1.082	0.948	0.838	0.748	0.672	0.608	0.553	0.506
1.07	1.404	1.195	1.026	0.896	0.790	0.703	0.630	0.569	0.516	0.471
1.08	1.346	1.139	0.978	0.851	0.749	0.665	0.595	0.535	0.485	0.441
1.09	1.295	1.089	0.935	0.812	0.713	0.631	0.563	0.506	0.457	0.415
1.10	1.250	1.050	0.897	0.777	0.681	0.601	0.536	0.480	0.433	0.392
1.11	1.209	1.014	0.864	0.746	0.652	0.575	0.511	0.457	0.411	0.372
1.12	1.172	0.981	0.833	0.718	0.626	0.551	0.488	0.436	0.392	0.354

The Varied Flow Function for Positive Slopes, $F(u, N)$ (continued)

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
1.13	1.138	0.950	0.805	0.692	0.602	0.529	0.468	0.417	0.374	0.337
1.14	1.107	0.921	0.780	0.669	0.581	0.509	0.450	0.400	0.358	0.322
1.15	1.078	0.892	0.756	0.647	0.561	0.490	0.432	0.384	0.343	0.308
1.16	1.052	0.870	0.734	0.627	0.542	0.473	0.417	0.369	0.329	0.295
1.17	1.027	0.850	0.713	0.608	0.525	0.458	0.402	0.356	0.317	0.283
1.18	1.003	0.825	0.694	0.591	0.509	0.443	0.388	0.343	0.305	0.272
1.19	0.981	0.810	0.676	0.574	0.494	0.429	0.375	0.331	0.294	0.262
1.20	0.960	0.787	0.659	0.559	0.480	0.416	0.363	0.320	0.283	0.252
1.22	0.922	0.755	0.628	0.531	0.454	0.392	0.341	0.299	0.264	0.235
1.24	0.887	0.725	0.600	0.505	0.431	0.371	0.322	0.281	0.248	0.219
1.26	0.855	0.692	0.574	0.482	0.410	0.351	0.304	0.265	0.233	0.205
1.28	0.827	0.666	0.551	0.461	0.391	0.334	0.288	0.250	0.219	0.193
1.30	0.800	0.644	0.530	0.442	0.373	0.318	0.274	0.237	0.207	0.181
1.32	0.775	0.625	0.510	0.424	0.357	0.304	0.260	0.225	0.196	0.171
1.34	0.752	0.605	0.492	0.408	0.342	0.290	0.248	0.214	0.185	0.162
1.36	0.731	0.588	0.475	0.393	0.329	0.278	0.237	0.204	0.176	0.153
1.38	0.711	0.567	0.459	0.378	0.316	0.266	0.226	0.194	0.167	0.145
1.40	0.692	0.548	0.444	0.365	0.304	0.256	0.217	0.185	0.159	0.138
1.42	0.674	0.533	0.431	0.353	0.293	0.246	0.208	0.177	0.152	0.131
1.44	0.658	0.517	0.417	0.341	0.282	0.236	0.199	0.169	0.145	0.125
1.46	0.642	0.505	0.405	0.330	0.273	0.227	0.191	0.162	0.139	0.119
1.48	0.627	0.493	0.394	0.320	0.263	0.219	0.184	0.156	0.133	0.113
1.50	0.613	0.480	0.383	0.310	0.255	0.211	0.177	0.149	0.127	0.108
1.55	0.580	0.451	0.358	0.288	0.235	0.194	0.161	0.135	0.114	0.097
1.60	0.551	0.425	0.335	0.269	0.218	0.179	0.148	0.123	0.103	0.087
1.65	0.525	0.402	0.316	0.251	0.203	0.165	0.136	0.113	0.094	0.079
1.70	0.501	0.381	0.298	0.236	0.189	0.153	0.125	0.103	0.086	0.072
1.75	0.480	0.362	0.282	0.222	0.177	0.143	0.116	0.095	0.079	0.065
1.80	0.460	0.349	0.267	0.209	0.166	0.133	0.108	0.088	0.072	0.060
1.85	0.442	0.332	0.254	0.198	0.156	0.125	0.100	0.082	0.067	0.055
1.90	0.425	0.315	0.242	0.188	0.147	0.117	0.094	0.076	0.062	0.050
1.95	0.409	0.304	0.231	0.178	0.139	0.110	0.088	0.070	0.057	0.046
2.00	0.395	0.292	0.221	0.169	0.132	0.104	0.082	0.066	0.053	0.043
2.10	0.369	0.273	0.202	0.154	0.119	0.092	0.073	0.058	0.046	0.037
2.20	0.346	0.253	0.186	0.141	0.107	0.083	0.065	0.051	0.040	0.032
2.3	0.326	0.235	0.173	0.129	0.098	0.075	0.058	0.045	0.035	0.028
2.4	0.308	0.220	0.160	0.119	0.089	0.068	0.052	0.040	0.031	0.024
2.5	0.292	0.207	0.150	0.110	0.082	0.062	0.047	0.036	0.028	0.022
2.6	0.277	0.197	0.140	0.102	0.076	0.057	0.043	0.033	0.025	0.019
2.7	0.264	0.188	0.131	0.095	0.070	0.052	0.039	0.029	0.022	0.017
2.8	0.252	0.176	0.124	0.089	0.065	0.048	0.036	0.027	0.020	0.015
2.9	0.241	0.166	0.117	0.083	0.060	0.044	0.033	0.024	0.018	0.014
3.0	0.230	0.159	0.110	0.078	0.056	0.041	0.030	0.022	0.017	0.012
3.5	0.190	0.126	0.085	0.059	0.041	0.029	0.021	0.015	0.011	0.008
4.0	0.161	0.104	0.069	0.046	0.031	0.022	0.015	0.010	0.007	0.005
4.5	0.139	0.087	0.057	0.037	0.025	0.017	0.011	0.008	0.005	0.004
5.0	0.122	0.076	0.048	0.031	0.020	0.013	0.009	0.006	0.004	0.003
6.0	0.098	0.060	0.036	0.022	0.014	0.009	0.006	0.004	0.002	0.002
7.0	0.081	0.048	0.028	0.017	0.010	0.006	0.004	0.002	0.002	0.001
8.0	0.069	0.040	0.022	0.013	0.008	0.005	0.003	0.002	0.001	0.001
9.0	0.060	0.034	0.019	0.011	0.006	0.004	0.002	0.001	0.001	0.000
10.0	0.053	0.028	0.016	0.009	0.005	0.003	0.002	0.001	0.001	0.000
20.0	0.023	0.018	0.011	0.006	0.002	0.001	0.001	0.000	0.000	0.000

The Varied Flow Function for Positive Slopes, *F* (*u*, *N*) (continued)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
0.16	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
0.18	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.20	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
0.22	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220
0.24	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240
0.26	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260
0.28	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280
0.30	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300
0.32	0.321	0.320	0.320	0.320	0.320	0.320	0.320	0.320	0.320	0.320
0.34	0.341	0.340	0.340	0.340	0.340	0.340	0.340	0.340	0.340	0.340
0.36	0.361	0.361	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360
0.38	0.381	0.381	0.381	0.380	0.380	0.380	0.380	0.380	0.380	0.380
0.40	0.402	0.401	0.401	0.400	0.400	0.400	0.400	0.400	0.400	0.400
0.42	0.422	0.421	0.421	0.421	0.420	0.420	0.420	0.420	0.420	0.420
0.44	0.443	0.442	0.441	0.441	0.441	0.441	0.440	0.440	0.440	0.440
0.46	0.463	0.462	0.462	0.461	0.461	0.461	0.460	0.460	0.460	0.460
0.48	0.484	0.483	0.482	0.481	0.481	0.481	0.480	0.480	0.480	0.480
0.50	0.505	0.504	0.503	0.502	0.501	0.501	0.501	0.500	0.500	0.500
0.52	0.527	0.525	0.523	0.522	0.522	0.521	0.521	0.521	0.520	0.520
0.54	0.548	0.546	0.544	0.543	0.542	0.542	0.541	0.541	0.541	0.541
0.56	0.570	0.567	0.565	0.564	0.563	0.562	0.562	0.561	0.561	0.561
0.58	0.592	0.589	0.587	0.585	0.583	0.583	0.582	0.582	0.581	0.581
0.60	0.614	0.611	0.608	0.606	0.605	0.604	0.603	0.602	0.602	0.601
0.61	0.626	0.622	0.619	0.617	0.615	0.614	0.613	0.612	0.612	0.611
0.62	0.637	0.633	0.630	0.628	0.626	0.625	0.624	0.623	0.622	0.622
0.63	0.649	0.644	0.641	0.638	0.636	0.635	0.634	0.633	0.632	0.632
0.64	0.661	0.656	0.652	0.649	0.647	0.646	0.645	0.644	0.643	0.642
0.65	0.673	0.667	0.663	0.660	0.658	0.656	0.655	0.654	0.653	0.653
0.66	0.685	0.679	0.675	0.672	0.669	0.667	0.666	0.665	0.664	0.663
0.67	0.697	0.691	0.686	0.683	0.680	0.678	0.676	0.675	0.674	0.673
0.68	0.709	0.703	0.698	0.694	0.691	0.689	0.687	0.686	0.685	0.684
0.69	0.722	0.715	0.710	0.706	0.703	0.700	0.698	0.696	0.695	0.694
0.70	0.735	0.727	0.722	0.717	0.714	0.712	0.710	0.708	0.706	0.705
0.71	0.748	0.740	0.734	0.729	0.726	0.723	0.721	0.719	0.717	0.716
0.72	0.761	0.752	0.746	0.741	0.737	0.734	0.732	0.730	0.728	0.727
0.73	0.774	0.765	0.759	0.753	0.749	0.746	0.743	0.741	0.739	0.737
0.74	0.788	0.779	0.771	0.766	0.761	0.757	0.754	0.752	0.750	0.748
0.75	0.802	0.792	0.784	0.778	0.773	0.769	0.766	0.763	0.761	0.759
0.76	0.817	0.806	0.798	0.791	0.786	0.782	0.778	0.775	0.773	0.771
0.77	0.831	0.820	0.811	0.804	0.798	0.794	0.790	0.787	0.784	0.782
0.78	0.847	0.834	0.825	0.817	0.811	0.806	0.802	0.799	0.796	0.794
0.79	0.862	0.849	0.839	0.831	0.824	0.819	0.815	0.811	0.808	0.805

The Varied Flow Function for Positive Slopes, $F(u, N)$ (continued)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
0.80	0.878	0.865	0.854	0.845	0.838	0.832	0.828	0.823	0.820	0.818
0.81	0.895	0.881	0.869	0.860	0.852	0.846	0.841	0.836	0.833	0.830
0.82	0.913	0.897	0.885	0.875	0.866	0.860	0.854	0.850	0.846	0.842
0.83	0.931	0.914	0.901	0.890	0.881	0.874	0.868	0.863	0.859	0.855
0.84	0.949	0.932	0.918	0.906	0.897	0.889	0.882	0.877	0.872	0.868
0.85	0.969	0.950	0.935	0.923	0.912	0.905	0.898	0.891	0.887	0.882
0.86	0.990	0.970	0.954	0.940	0.930	0.921	0.913	0.906	0.901	0.896
0.87	1.012	0.990	0.973	0.959	0.947	0.937	0.929	0.922	0.916	0.911
0.88	1.035	1.012	0.994	0.978	0.966	0.955	0.946	0.938	0.932	0.927
0.89	1.060	1.035	1.015	0.999	0.986	0.974	0.964	0.956	0.949	0.943
0.90	1.087	1.060	1.039	1.021	1.007	0.994	0.984	0.974	0.967	0.960
0.91	1.116	1.088	1.064	1.045	1.029	1.016	1.003	0.995	0.986	0.979
0.92	1.148	1.117	1.092	1.072	1.054	1.039	1.027	1.016	1.006	0.999
0.93	1.184	1.151	1.123	1.101	1.081	1.065	1.050	1.040	1.029	1.021
0.94	1.225	1.188	1.158	1.134	1.113	1.095	1.080	1.066	1.054	1.044
0.950	1.272	1.232	1.199	1.172	1.148	1.128	1.111	1.097	1.084	1.073
0.960	1.329	1.285	1.248	1.217	1.188	1.167	1.149	1.133	1.119	1.106
0.970	1.402	1.351	1.310	1.275	1.246	1.319	1.197	1.179	1.162	1.148
0.975	1.447	1.393	1.348	1.311	1.280	1.250	1.227	1.207	1.190	1.173
0.980	1.502	1.443	1.395	1.354	1.339	1.288	1.262	1.241	1.221	1.204
0.985	1.573	1.508	1.454	1.409	1.372	1.337	1.309	1.284	1.263	1.243
0.990	1.671	1.598	1.537	1.487	1.444	1.404	1.373	1.344	1.319	1.297
0.995	1.838	1.751	1.678	1.617	1.565	1.519	1.479	1.451	1.416	1.388
0.999	2.223	2.102	2.002	1.917	1.845	1.780	1.725	1.678	1.635	1.596
1.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
1.001	1.417	1.264	1.138	1.033	0.951	0.870	0.803	0.746	0.697	0.651
1.005	1.036	0.915	0.817	0.737	0.669	0.612	0.553	0.526	0.481	0.447
1.010	0.873	0.766	0.681	0.610	0.551	0.502	0.459	0.422	0.389	0.360
1.015	0.778	0.680	0.602	0.537	0.483	0.440	0.399	0.366	0.336	0.310
1.02	0.711	0.620	0.546	0.486	0.436	0.394	0.358	0.327	0.300	0.276
1.03	0.618	0.535	0.469	0.415	0.370	0.333	0.300	0.272	0.249	0.228
1.04	0.554	0.477	0.415	0.365	0.324	0.290	0.262	0.236	0.214	0.195
1.05	0.504	0.432	0.374	0.328	0.289	0.259	0.231	0.208	0.189	0.174
1.06	0.464	0.396	0.342	0.298	0.262	0.233	0.209	0.187	0.170	0.154
1.07	0.431	0.366	0.315	0.273	0.239	0.212	0.191	0.168	0.151	0.136
1.08	0.403	0.341	0.292	0.252	0.220	0.194	0.172	0.153	0.137	0.123
1.09	0.379	0.319	0.272	0.234	0.204	0.179	0.158	0.140	0.125	0.112
1.10	0.357	0.299	0.254	0.218	0.189	0.165	0.146	0.129	0.114	0.102
1.11	0.338	0.282	0.239	0.204	0.176	0.154	0.135	0.119	0.105	0.094
1.12	0.321	0.267	0.225	0.192	0.165	0.143	0.125	0.110	0.097	0.086
1.13	0.305	0.253	0.212	0.181	0.155	0.135	0.117	0.102	0.090	0.080
1.14	0.291	0.240	0.201	0.170	0.146	0.126	0.109	0.095	0.084	0.074
1.15	0.278	0.229	0.191	0.161	0.137	0.118	0.102	0.089	0.078	0.068
1.16	0.266	0.218	0.181	0.153	0.130	0.111	0.096	0.084	0.072	0.064
1.17	0.255	0.208	0.173	0.145	0.123	0.105	0.090	0.078	0.068	0.060
1.18	0.244	0.199	0.165	0.138	0.116	0.099	0.085	0.073	0.063	0.055
1.19	0.235	0.191	0.157	0.131	0.110	0.094	0.080	0.068	0.059	0.051
1.20	0.226	0.183	0.150	0.215	0.105	0.088	0.076	0.064	0.056	0.048
1.22	0.209	0.168	0.138	0.114	0.095	0.080	0.068	0.057	0.049	0.042
1.24	0.195	0.156	0.127	0.104	0.086	0.072	0.060	0.051	0.044	0.038

The Varied Flow Function for Positive Slopes, *F* (*u*, *N*) (continued)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
1.26	0.182	0.145	0.117	0.095	0.079	0.065	0.055	0.046	0.039	0.033
1.28	0.170	0.135	0.108	0.088	0.072	0.060	0.050	0.041	0.035	0.030
1.30	0.160	0.126	0.100	0.081	0.066	0.054	0.045	0.037	0.031	0.026
1.32	0.150	0.118	0.093	0.075	0.061	0.050	0.041	0.034	0.028	0.024
1.34	0.142	0.110	0.087	0.069	0.056	0.045	0.037	0.030	0.025	0.021
1.36	0.134	0.103	0.081	0.064	0.052	0.042	0.034	0.028	0.023	0.019
1.38	0.127	0.097	0.076	0.060	0.048	0.038	0.032	0.026	0.021	0.017
1.40	0.120	0.092	0.071	0.056	0.044	0.036	0.028	0.023	0.019	0.016
1.42	0.114	0.087	0.067	0.052	0.041	0.033	0.026	0.021	0.017	0.014
1.44	0.108	0.082	0.063	0.049	0.038	0.030	0.024	0.019	0.016	0.013
1.46	0.103	0.077	0.059	0.046	0.036	0.028	0.022	0.018	0.014	0.012
1.48	0.098	0.073	0.056	0.043	0.033	0.026	0.021	0.017	0.013	0.010
1.50	0.093	0.069	0.053	0.040	0.031	0.024	0.020	0.015	0.012	0.009
1.55	0.083	0.061	0.046	0.035	0.026	0.020	0.016	0.012	0.010	0.008
1.60	0.074	0.054	0.040	0.030	0.023	0.017	0.013	0.010	0.008	0.006
1.65	0.067	0.048	0.035	0.026	0.019	0.014	0.011	0.008	0.006	0.005
1.70	0.060	0.043	0.031	0.023	0.016	0.012	0.009	0.007	0.005	0.004
1.75	0.054	0.038	0.027	0.020	0.014	0.010	0.008	0.006	0.004	0.003
1.80	0.049	0.034	0.024	0.017	0.012	0.009	0.007	0.005	0.004	0.003
1.85	0.045	0.031	0.022	0.015	0.011	0.008	0.006	0.004	0.003	0.002
1.90	0.041	0.028	0.020	0.014	0.010	0.007	0.005	0.004	0.003	0.002
1.95	0.038	0.026	0.018	0.012	0.008	0.006	0.004	0.003	0.002	0.002
2.00	0.035	0.023	0.016	0.011	0.007	0.005	0.004	0.003	0.002	0.001
2.10	0.030	0.019	0.013	0.009	0.006	0.004	0.003	0.002	0.001	0.001
2.20	0.025	0.016	0.011	0.007	0.005	0.004	0.002	0.001	0.001	0.001
2.3	0.022	0.014	0.009	0.006	0.004	0.003	0.002	0.001	0.001	0.001
2.4	0.019	0.012	0.008	0.005	0.003	0.002	0.001	0.001	0.001	0.001
2.5	0.017	0.010	0.006	0.004	0.003	0.002	0.001	0.001	0.000	0.000
2.6	0.015	0.009	0.005	0.003	0.002	0.001	0.001	0.001	0.000	0.000
2.7	0.013	0.008	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000
2.8	0.012	0.007	0.004	0.002	0.001	0.001	0.001	0.000	0.000	0.000
2.9	0.010	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000
3.0	0.009	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000
3.5	0.006	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000
4.0	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The Varied Flow Function for Positive Slopes, $F(u, N)$ (continued)

u \ N	8.2	8.6	9.0	9.4	9.8
0.00	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140
0.16	0.160	0.160	0.160	0.160	0.160
0.18	0.180	0.180	0.180	0.180	0.180
0.20	0.200	0.200	0.200	0.200	0.200
0.22	0.220	0.220	0.220	0.220	0.220
0.24	0.240	0.240	0.240	0.240	0.240
0.26	0.260	0.260	0.260	0.260	0.260
0.28	0.280	0.280	0.280	0.280	0.280
0.30	0.300	0.300	0.300	0.300	0.300
0.32	0.320	0.320	0.320	0.320	0.320
0.34	0.340	0.340	0.340	0.340	0.340
0.36	0.360	0.360	0.360	0.360	0.360
0.38	0.380	0.380	0.380	0.380	0.380
0.40	0.400	0.400	0.400	0.400	0.400
0.42	0.420	0.420	0.420	0.420	0.420
0.44	0.440	0.440	0.440	0.440	0.440
0.46	0.460	0.460	0.460	0.460	0.460
0.48	0.480	0.480	0.480	0.480	0.480
0.50	0.500	0.500	0.500	0.500	0.500
0.52	0.520	0.520	0.520	0.520	0.520
0.54	0.540	0.540	0.540	0.540	0.540
0.56	0.561	0.560	0.560	0.560	0.560
0.58	0.581	0.581	0.580	0.580	0.580
0.60	0.601	0.601	0.601	0.600	0.600
0.61	0.611	0.611	0.611	0.611	0.610
0.62	0.621	0.621	0.621	0.621	0.621
0.63	0.632	0.631	0.631	0.631	0.631
0.64	0.642	0.641	0.641	0.641	0.641
0.65	0.652	0.652	0.651	0.651	0.651
0.66	0.662	0.662	0.662	0.661	0.661
0.67	0.673	0.672	0.672	0.672	0.671
0.68	0.683	0.683	0.682	0.682	0.681
0.69	0.694	0.693	0.692	0.692	0.692
0.70	0.704	0.704	0.703	0.702	0.702
0.71	0.715	0.714	0.713	0.713	0.712
0.72	0.726	0.725	0.724	0.723	0.723
0.73	0.736	0.735	0.734	0.734	0.733
0.74	0.747	0.746	0.745	0.744	0.744
0.75	0.758	0.757	0.756	0.755	0.754
0.76	0.769	0.768	0.767	0.766	0.765
0.77	0.780	0.779	0.778	0.777	0.776
0.78	0.792	0.790	0.789	0.788	0.787
0.79	0.804	0.802	0.800	0.799	0.798

The Varied Flow Function for Positive Slopes, *F* (*u*, *N*) (continued)

u \ *N*	8.2	8.6	9.0	9.4	9.8
0.80	0.815	0.813	0.811	0.810	0.809
0.81	0.827	0.825	0.823	0.822	0.820
0.82	0.839	0.837	0.835	0.833	0.831
0.83	0.852	0.849	0.847	0.845	0.844
0.84	0.865	0.862	0.860	0.858	0.856
0.85	0.878	0.875	0.873	0.870	0.868
0.86	0.892	0.889	0.886	0.883	0.881
0.87	0.907	0.903	0.900	0.897	0.894
0.88	0.921	0.918	0.914	0.911	0.908
0.89	0.937	0.933	0.929	0.925	0.922
0.90	0.954	0.949	0.944	0.940	0.937
0.91	0.972	0.967	0.961	0.957	0.953
0.92	0.991	0.986	0.980	0.975	0.970
0.93	1.012	1.006	0.999	0.994	0.989
0.94	1.036	1.029	1.022	1.016	1.010
0.950	1.062	1.055	1.047	1.040	1.033
0.960	1.097	1.085	1.074	1.063	1.053
0.970	1.136	1.124	1.112	1.100	1.087
0.975	1.157	1.147	1.134	1.122	1.108
0.980	1.187	1.175	1.160	1.150	1.132
0.985	1.224	1.210	1.196	1.183	1.165
0.990	1.275	1.260	1.243	1.228	1.208
0.995	1.363	1.342	1.320	1.302	1.280
0.999	1.560	1.530	1.500	1.476	1.447
1.000	∞	∞	∞	∞	∞
1.001	0.614	0.577	0.546	0.519	0.494
1.005	0.420	0.391	0.368	0.350	0.331
1.010	0.337	0.313	0.294	0.278	0.262
1.015	0.289	0.269	0.255	0.237	0.223
1.020	0.257	0.237	0.221	0.209	0.196
1.03	0.212	0.195	0.181	0.170	0.159
1.04	0.173	0.165	0.152	0.143	0.134
1.05	0.158	0.143	0.132	0.124	0.115
1.06	0.140	0.127	0.116	0.106	0.098
1.07	0.123	0.112	0.102	0.094	0.086
1.08	0.111	0.101	0.092	0.084	0.077
1.09	0.101	0.091	0.082	0.075	0.069
1.10	0.092	0.083	0.074	0.067	0.062
1.11	0.084	0.075	0.067	0.060	0.055
1.12	0.077	0.069	0.062	0.055	0.050
1.13	0.071	0.063	0.056	0.050	0.045
1.14	0.065	0.058	0.052	0.046	0.041
1.15	0.061	0.054	0.048	0.043	0.038
1.16	0.056	0.050	0.045	0.040	0.035
1.17	0.052	0.046	0.041	0.036	0.032
1.18	0.048	0.042	0.037	0.033	0.029
1.19	0.045	0.039	0.034	0.030	0.027
1.20	0.043	0.037	0.032	0.028	0.025
1.22	0.037	0.032	0.028	0.024	0.021
1.24	0.032	0.028	0.024	0.021	0.018

The Varied Flow Function for Positive Slopes, $F(u, N)$ (continued)

u \ N	8.2	8.6	9.0	9.4	9.8
1.26	0.028	0.024	0.021	0.018	0.016
1.28	0.025	0.021	0.018	0.016	0.014
1.30	0.022	0.019	0.016	0.014	0.012
1.32	0.020	0.017	0.014	0.012	0.010
1.34	0.018	0.015	0.012	0.010	0.009
1.36	0.016	0.013	0.011	0.009	0.008
1.38	0.014	0.012	0.010	0.008	0.007
1.40	0.013	0.011	0.009	0.007	0.006
1.42	0.011	0.009	0.008	0.006	0.005
1.44	0.010	0.008	0.007	0.006	0.005
1.46	0.009	0.008	0.006	0.005	0.004
1.48	0.009	0.007	0.005	0.004	0.004
1.50	0.008	0.006	0.005	0.004	0.003
1.55	0.006	0.005	0.004	0.003	0.003
1.60	0.005	0.004	0.003	0.002	0.002
1.65	0.004	0.003	0.002	0.002	0.001
1.70	0.003	0.002	0.002	0.001	0.001
1.75	0.002	0.002	0.002	0.001	0.001
1.80	0.002	0.001	0.001	0.001	0.001
1.85	0.002	0.001	0.001	0.001	0.001
1.90	0.001	0.001	0.001	0.001	0.000
1.95	0.001	0.001	0.001	0.000	0.000
2.00	0.001	0.001	0.000	0.000	0.000
2.10	0.001	0.000	0.000	0.000	0.000
2.20	0.000	0.000	0.000	0.000	0.000
2.3	0.000	0.000	0.000	0.000	0.000
2.4	0.000	0.000	0.000	0.000	0.000
2.5	0.000	0.000	0.000	0.000	0.000
2.6	0.000	0.000	0.000	0.000	0.000
2.7	0.000	0.000	0.000	0.000	0.000
2.8	0.000	0.000	0.000	0.000	0.000
2.9	0.000	0.000	0.000	0.000	0.000
3.0	0.000	0.000	0.000	0.000	0.000
3.5	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000
4.5	0.000	0.000	0.000	0.000	0.000
5.0	0.000	0.000	0.000	0.000	0.000
6.0	0.000	0.000	0.000	0.000	0.000
7.0	0.000	0.000	0.000	0.000	0.000
8.0	0.000	0.000	0.000	0.000	0.000
9.0	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000
20.0	0.000	0.000	0.000	0.000	0.000