UNIT 6 STUDY OF FLOW PROFILES

Structure

6.1 INTRODUCTION

This unit attempts to help you to understand the use of flow profiles in solving some of the field problems, such as delivery of a canal under different conditions, locating a hydraulic jump, etc. We shall now begin by recapitulating the definitions and the classification of different flow profiles you learnt in Unit 5 for the sake of developing clarity in the concepts developed so far about gradually varied flows. We shall conclude with an outline of the procedure to locate a hydraulic jump under different situations, and then introduce, through the solution of problems, some methods of computing the shape of GVF profiles.

Objectives

After learning the material presented in this unit, you should be able to :

- deal with the problem of flow through a canal delivering from one reservoir to the other,
- trace the profile of a hydraulic drop, and locate a hydraulic jump in an open channel, and
- understand the formation of flow profiles on multi-sloped channels, and those formed downstream of a sluice; and compute the necessary data for sketching these profiles.

6.2 SURFACE PROFILES AND DELIVERY OF A CANAL

We shall recall that the dynamic equation of gradually varied flow, developed in the earlier unit, expresses the longitudinal slope of the water surface with respect to the channel bottom. Obviously, this differential equation comes handy in describing the profile of the water surface under different flow conditions.

We shall also recall that for a given discharge and channel conditions, the normal depth (y_n) and critical depth (y_c) lines (i.e., NDL and CDL, respectively) divide the space, above the bed of the channel, into three zones: the space above the top most line being Zone 1; the space in between the two lines being Zone 2; and, the space in between the lower line and channel bottom being Zone 3. Considering the channel slope and the zone of occurrence, the profiles may be classified into thirteen different types, such as, H2, H3 (horizontal slope);

M1, M2, M3 (mild slope); C1, C2, C3 (critical slope); S1, S2, S3 (steep slope); and, A2, A3 (adverse slope). A recapitulation of the picture that we developed, in Unit 5, about these profiles, is presented in Table 6.1 under sub-section 6.1.2

6.2.1 Surface Profiles for Practical Problems

In practical engineering it is invariably true that an open channel can never run, for longer distances, as a prismatic channel — existence of control and regulatory structures along the alignment of the channel or sudden changes in bed slope or changes in cross-sectional shape or dimensions induce the formàtion of gradualy-varied flow profiles. For the sake of developing an overall picture in this regard, following discussion in a concise form should prove helpful.

GVF Surface Profiles Under Practical Conditions

M1-Profile

Occurrence in the Field
In a long mild channel ending in a reservoir whose water surface is above the NDL; behind a dam across a natural river (Figure 6.1 (a)); in a canal that joins two reservoirs (Figures 6.1 and 6.6); at the junction of a mild slope with a comparatively milder slope (Figure 6.14-1); and, at the junction of a mild to adverse slope (Figure 6.14-9).

Behaviour

Lies above NDL (Zone 1). It is tangential to NDL (since $\frac{dy}{dx} = 0$ for $y = y_n$) at the very start, as well as at the end as the case may be; and meets the horizontal pool surface also tangentially (since, $\frac{dy}{dx} = S_0$ as $y \to \infty$, i.e., to a depth $> y_n$)

M2 - Profile

Occurrence in the Field
At the upstream of a sudden enlargement in the canal section (Figure 6.1 (c)); at the downstream end of the channel bed that submerges under a pool whose water surface (W.S.) is less than y_n (Figure 6.1 (d) and 6.7); while a mild slope meets a critical slope (Figure 6.14-5); and, while a mild slope meets a steep slope (Figure 6.14-7)

Behaviour

Lies below NDL (Zone 2). Upstream end is tangential to NDL (since, $\frac{dy}{dx} = 0$ for $y \le y_n$). The downstream end may be a hydraulic drop if the depth of submergence $< y_c$ (since, $\frac{dy}{dx} \to \infty$ at $y = y_c$).

M3 - Profile

Occurrence in the Field
On the downstream side of a sluice (Figure 6.1 (e)); and, after a steep slope meets a mild slope (Figures 6.1 (f) and 6.14-2) — i.e., a supercritical flow entering a mild channel.

Behaviour
Lies below CDL (Zone 3); and, theoretically it starts from the channel bed, and terminates into a hydraulic jump. However, as $y \to 0$, i.e., at the bed, $V \to \infty$, thus, there is no physical existence of this portion of M3 profile therefore, always shown dotted).

S1 Profile

Occurrence in the Field
On the upstream of a pool ($y > y_n$) like: behind a dam (built on a steep channel) — Figure 6.2 (a); on the downstream of a steep slope meeting a mild slope (Figure 6.2 (b), and 6.14-2), or a steep slope emptying into a pool of high elevation; and, a steep slope meeting an advese slope (Figure 6.14-10).

Behaviour
Lies above CDL (Zone 1). It is preceded by a hydraulic jump, and meets the pool level ($y > y_n$) tangentially at the downstream end.

S2 Profile

Occurrence in the Field
On the downstream of a channel enlargement (Figure 6.2 (c)); on the steeper channel while

a mild slope meets a steep slope (Figure 6.2 (d), and 6.14-7); on the steeper-sloped channel when a less steep slope comes to meet the former (Figure 6.14-8); on the steeper channel when a critical slope comes to meet the former (Figure 6.14-13); and, on the steep channel after an adverse slope approaches to meet the former (Figure 6.14-16).

Behaviour

Lies below CDL (Zone 2). Begins with a vertical slope at the critical depth, and meets the downstream NDL tangentially. The upstream end may be preceded by a hydraulic drop (occurring at the end of the preceding slope); this drop is tangential to the vertical line at the CDL.

S3 Profile

Occurrence in the Field

On a milder steep channel when a steeper slope joins the former at its upstream end (Figures 6.2 (e), and 6.14-6); and, on the downstream side of a sluice gate that is situated on a steep channel (Figure 6.2 (f)), i.e., a supercritical flow meeting another supercritical flow with greater F_r.

Behaviour

Lies below NDL (which in turn lies below CDL) — Zone 3. It, theoretically starts from the channel bed with a positive slope—but, obviously, this portion of the curve has no physical existence, because velocity $\rightarrow \infty$, as $y \rightarrow 0$ — and, meets the downstream NDL tangentially.

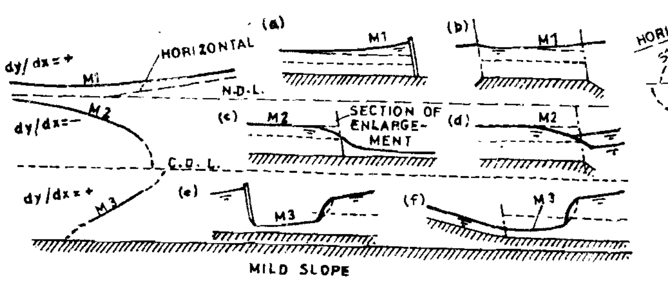

Figure 6.1 : Profiles in Mild Channels

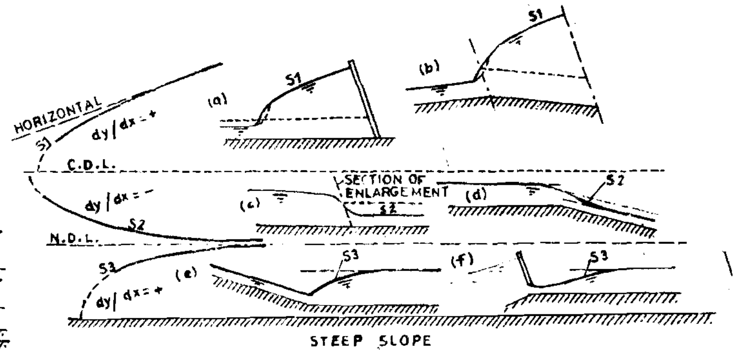

Figure 6.2 : Profiles in Steep Channels

C Profiles

These profiles are formed in channels of critical slope (i.e., $S_0 = S_c$) where NDL and CDL coincide with each other ($y_n = y_c$). The **C1 profile** is asymptotic to a horizontal line (Figures 6.3 (a), 6.14-3, and 6.14-15); **C2 profile** represents the case of a uniform critical flow; and, **C3 profile** can be formed on the downstream side of a sluice gate in a critical channel (Figure. 6.3 (b)), and after a steep slope meets a critical slope (Figure 6.14-4).

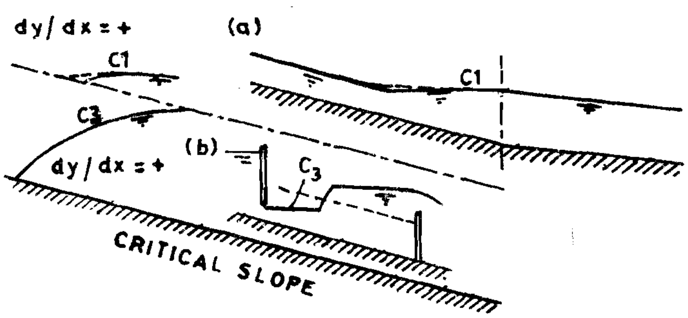

Figure 6.3 : Profiles in Critical Channels

H Profiles

These are profiles formed in horizontal channels (i.e., $S_0 = 0$). The NDL is located at infinite depth (i.e., $y_n = \infty$) and therefore, no H1 profile can actually be established. These profiles are actually the limiting cases of M profiles as the channel bottom tends to become horizontal. H2 and H3 profiles correspond to M2 and M3 profiles, examples of which are indicated in Figure 6.4.

A Profiles

These profiles are formed in channel of adverse slope (i.e., $S_0 < 0$). In these channels, y_n (and hence the NDL) is not a real physical entity. Therefore, A1 profiles are impossible. A2 and A3 profiles are similar to H2 and H3 profiles, respectively. In general, therefore, A profiles are very unlikely to occur (Figure 6.5).

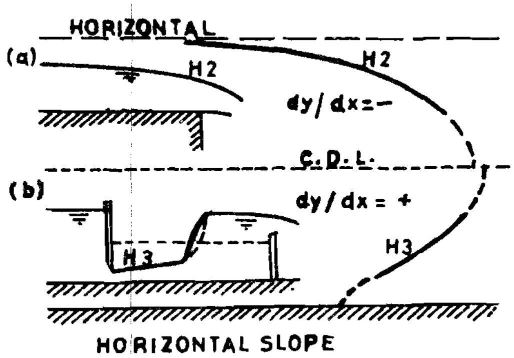

Figure 6.4 : Profiles in Horizontal Channels

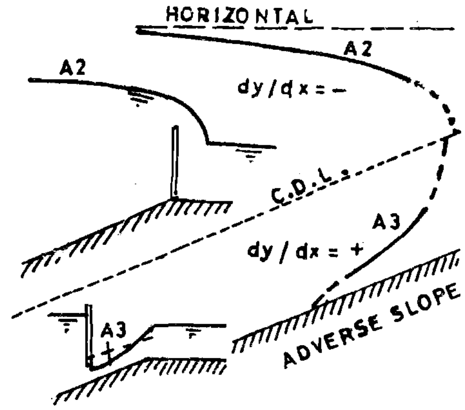

Figure 6.5 : Profiles in Adverse Channels

6.2.2 Delivery of a Canal for Subcritical Flow

When two reservoirs having varying levels are connected by a canal, the discharge of the canal under different conditions of levels in the reservoirs is termed as the **delivery of canal**. If the flow is subcritical, it can be treated under three general cases according to the conditions of following three variables :

(i) the depth of flow at the upstream end of the canal (y_1);

(ii) the depth of flow at the downstream end of the canal (y_2); and,

(iii) the discharge of the canal (Q).

Case of Constant y_1

This is the case when the upstream pool level A is constant, and hence the depth of flow at the upstream end of the canal y_1, is constant (Figure 6.6). The depth of flow at the downstream end, y_2, fluctuates and is determined by pool level B; and thus, the discharge of the canal, Q, will also vary accordingly. The flow profiles under different conditions of y_2 and the relationship between y_2 and Q (termed as 'delivery curve', $Q = f(y_2)$) are also shown in Figure 6.6. Various flow conditions under this case are discussed as under :

1) ***Uniform flow*** : The flow is uniform if $y_1 = y_2 = y_n$. The corresponding water surface, represented by a straight line, is parallel to the channel bottom and the discharge, Q_n, indicated on the delivery curve is equal to $K_n \sqrt{S_0}$, where K_n is the conveyance of the channel (for $y_1 = y_2$) and S_0 is the bed slope.

2) ***Maximum discharge*** : The discharge will reach its maximum possible value when y_2 is equal to the critical depth, y_c, at section 2, since y_2 cannot be less than y_c and thus the head is at its maximum. The maximum discharge ($Q_{\max}$) is equal to the critical discharge, Q_c, at section 2, which is given by $Z_c \sqrt{g}$, where Z_c is the section factor of section 2. A free overfall will occur if the downstream pool level B falls below y_c.

   The maximum discharge can be estimated by a trial calculation taking a series of discharges, starting from Q_n and increasing the value in steps, and making $y_2 = y_c$, in

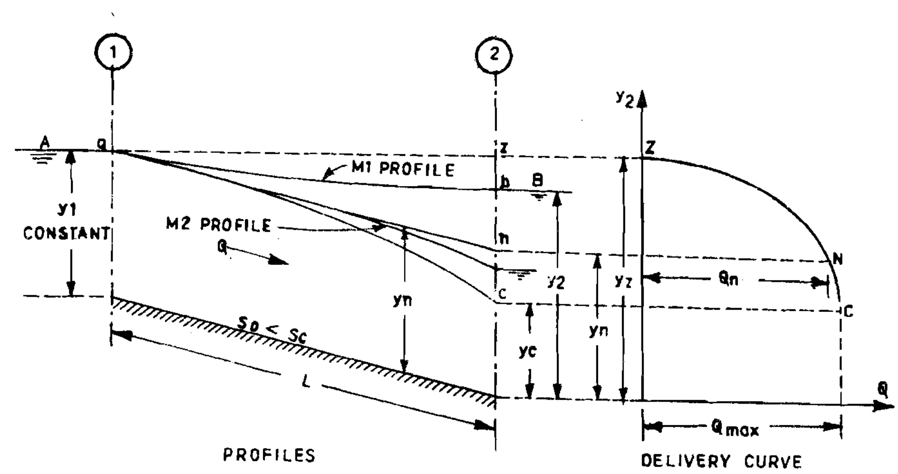

Figure 6.6 : Delivery of a Canal (Subcritical Flow with Upstream Depth Constant)

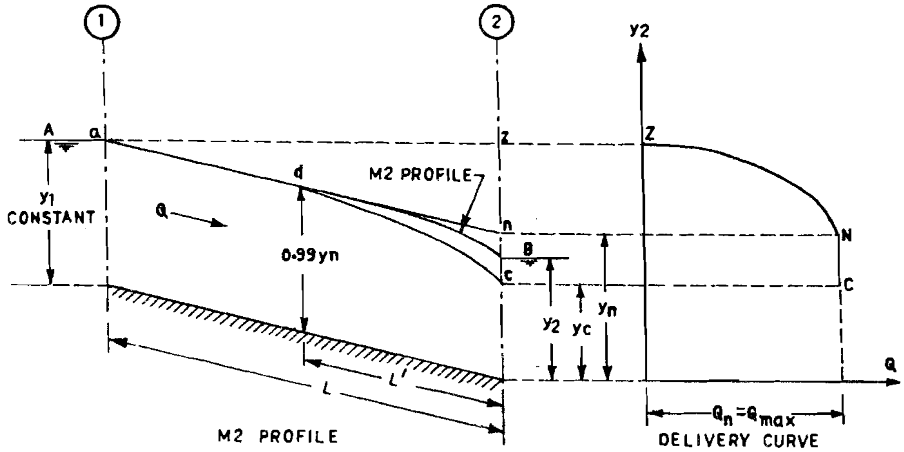

Figure 6.7 : Delivery of a Canal (Length of Canal more than Length of M2 Profile)

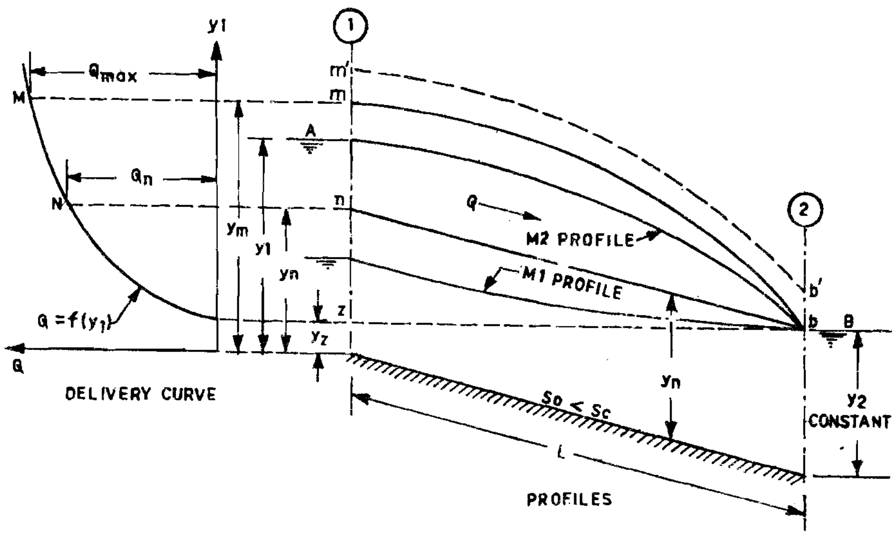

Figure 6.8 : Delivery of a Canal (Subcritical Flow with Downstream Depth Constant)

each case and then calculating the corresponding y_1. That discharge which makes y_1 equal to the given depth at the upstream end is the required maximum discharge, $Q_{\max}$.

3) ***M1 profile*** : The flow profile is of M1 type if $y_2 > y_n$. The upper limit of this curve, which is a horizontal level *az*, will occur when the downstream pool level B rises up to the upstream pool level A, making the head causing the flow and hence the discharge both equal to zero. The lower limit of the M1 profile is, obviously, the uniform flow surface *an* . The depth y_2 and the corresponding discharge, for any intermediate flow situation, can be determined by a trial calculation — assuming a discharge less than Q_n, and computing the corresponding y_2, and then drawing the delivery curve. It is to be understood that for any $y_2 > y_z$, the flow direction will get reversed.

4) ***M2 profile*** : The flow profile is of M2 type if $y_c \leq y_2 < y_n$. The lower limit is, of course, the critical flow surface *ac*. The relationship between y_2 and Q can be determined as in the case of M1 profile (discussed above).

You may notice, in the delivery curve, that the portion NC is very steep, such that Q_{max} may exceed Q_n by only a small amount. This is true in many practical cases except in cases of very short or very flat canals. We shall now discuss this in detail with reference to Figure 6.7.

Let us assume that the upstream end point *d* of the limiting M2 curve, measuring from the downstream end, can be located at a depth 0.99 y_n. If the length of the canal (L) is greater than the limiting length of the M2 profile (L'), then any change in y_2 between y_c and y_n will not affect the condition upstream of point *d*. This means that the discharge will remain the same, and hence Q_{max} will be practically equal to Q_n as long as $L > L'$. Thus, the portion NC of the corresponding delivery curve will be practically vertical. You may recall from the flow-profile equation, you studied in Unit 5, that the length of a flow profile is inversely proportional to the bottom slope of the canal ($L' \propto 1/S_0$). Thus, smaller the slope, the longer is the flow profile, and vice-versa; and therefore, reducing the slope will have an effect similar to that of making the canal shorter. For practical cases, therefore, you may assume that **the maximum possible discharge, in a long canal or in a canal of not-too-small a slope, is equal to the normal discharge.**

Case of Constant y_2

We shall now discuss the case in which the downstream water level, or the depth y_2 is constant, while the upstream depth y_1 fluctuates, and hence the discharge, Q, is a function of y_1 as shown in Figure 6.8. Again the following flow conditions are of interest :

1) ***Uniform flow:*** The flow is uniform if $y_1 = y_2 = y_n$. The corresponding flow profile *nb* is parallel to the channel bottom; and the discharge Q_n, corresponding to point *N* on the delivery curve, is equal to $K_n \sqrt{S_0}$, where K_n is the conveyance at section 1 with depth $y_1 = y_2$, and S_0 is the bed slope.

2) ***Maximum discharge :*** As we have seen earlier, maximum discharge will occur when the flow at section 2 is critical. For this to happen, the upstream depth y_1 should reach a depth y_m to yield a critical discharge as indicated in the Figure. Thus, the maximum discharge, Q_{max} (equal to the critical flow discharge), at section 2 is given by $Z_c \sqrt{g}$, where Z_c is the section factor at section 2 for y_2. Any depth $y_1 > y_m$ is out of consideration since it would result in a situation requiring an increase in the downstream depth y_2 as in *m'b'* in the Figure.

3) ***M1 profile :*** The flow profile is of M1 type for any depth $y_1 < y_n$, and the discharge, $Q < Q_n$. The lowest possible limit for the profile is being horizontal, because y_z is the lowest limit of y_1, and the discharge is zero.

4) ***M2 profile :*** For any upstream depth y_1 varying between y_n and y_m (i.e., $y_n < y_1 < y_m$), the flow profile is of M2 type, and the discharge is in the range of $Q_n < Q < Q_{max}$.

Case of Constant Q

We shall now make an attempt to understand the delivery of a canal with the discharge remaining constant while the pool levels at both upstream and downstream ends of the canal fluctuate (Figure 6.9). Following cases are of importance in this regard :

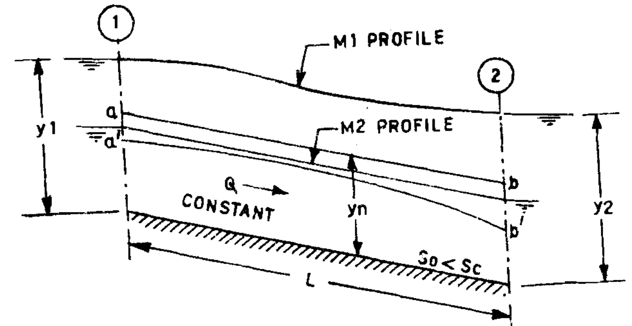

Figure 6.9 : Delivery of a Canal (Subcritical Flow with Constant Discharge)

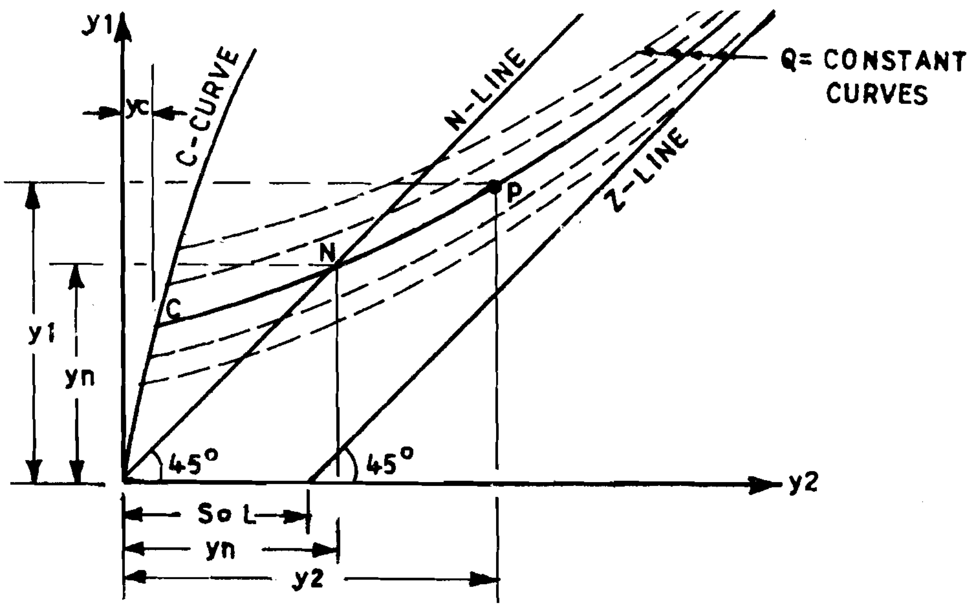

Figure 6.10 : Delivery of a Canal (Q - Constant Curves)

1) ***Uniform flow*** : The flow is uniform if $y_1 = y_2 = y_n$ and the surface is a straight line *ab* parallel to the channel bottom. This normal depth can be easily determined for the given constant discharge by Manning's formula.

2) ***M1 profile :*** The flow profile is of M1 type for all positions above the line *ab*. When y_2 reaches $y_1 + S_0 L$, at its upper limit, the surface becomes horizontal. As this condition is approached, the difference between the upstream and downstream pool levels and hence the head or velocity of flow decreases.

3) ***M2 profile*** : The flow profile is of M2 type for all positions below the line *ab*. The lowest possible position for M2 profile is $a'b'$ which would occur when the downstream depth y_2 is equal to the critical depth corresponding to the given discharge Q .

Q - Constant Curve

The Q – constant curve (such as, CNP in Figure 6.10) is obtained as a result of plotting the relationship between y_1 and y_2 for a given constant discharge, Q. Many such curves are indicated in the Figure to make you understand certain characteristic features of the Q-constant curve.

The *N-line*, which is the locus of the normal depth for all discharges, is a straight line drawn from the origin at an angle of 45^0 with the coordinate axes. You will easily see that for any point on this line $y_1 = y_2 = y_n$; and also, the Q-constant curve intersects the N-line at point N where $y_1 = y_2 = y_n$, which is the normal depth for the given discharge Q .

The C - curve is a curve on which y_2 is equal to the critical depth y_c at section 2 for the given discharge, and y_1 is the corresponding depth at section 1. It should be obvious to you that y_2 can never be less than y_c of section 2 for the given discharge Q ; and hence the Q-constant curve should terminate at the point C on this curve that makes $y_2 = y_c$.

The *Z-line* represents the condition that $y_2 = y_1 + S_0 L$, which is the upper limit of M1 profile. It is a straight line drawn parallel to N – line from a point on the y_2-axis at a distance $S_0 L$ from the origin. When both y_1 and y_2 become very large, the Q-constant curve approaches the Z - line asymptotically from the left .

For any point P on the Q – constant curve, for a given discharge Q, the coordinates y_1 and y_2 can be determined by a flow-profile computation you shall be studying in this and the following units. When points C and N and, in addition, a couple of points more are located on the diagram, the Q-constant curve can smoothly be drawn. In order to represent all possible flow situations in a given canal, you may construct a general chart, as shown in Figure 6.10, by plotting a series of Q - constant curves for different discharges.

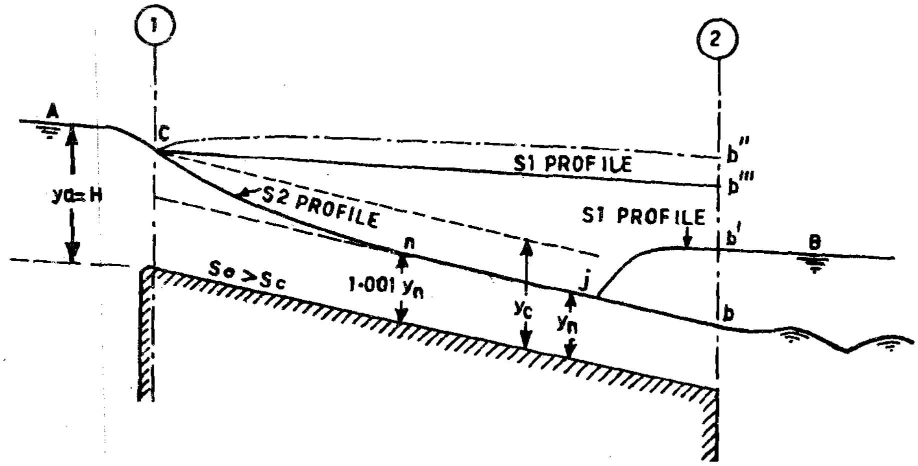

Figure 6.11 : Delivery of a Canal (Supercritical Flow)

6.2.3 Delivery of a Canal for Supercritical Flow

You studied the delivery of a canal for subcritical flow in the earlier Section. Let us now take the case of supercritical flow due to the steep slope of the channel — CDL lies above the NDL (Figure 6.11). In the field, steep channels are usually short, like log chutes and raft (used as spillways). However, in very steep channels the flow will be unsteady and not amenable to analysis given in this Section. Unlike subcritical flow, this one presents only very simple flow situations, you know why?

Discharge : The control section of supercritical flow in a channel being at the upstream end, the delivery of the canal is completely controlled by the discharge at the critical section at 1. This discharge can be simply calculated as the discharge over a weir — the brink depth there being equal to the critical depth.

Flow Profile: The tailwater situation at B determines the flow profile in a steep canal. When the tailwater level B is lower than the outlet depth at section 2, the flow in the channel is not affected by the tail water, and the flow is a smooth drawdown curve of S2 type passing through the critical depth at c. This curve is of convex nature on the upstream side of c, changing to concave on the downstream side of c, approaching the NDL very smoothly.

When the tailwater level B is higher than the outlet depth, an S1 profile between j and b' will develop as the tailwater level rises. The upstream end of this profile is a hydraulic jump at j . The flow profile on the upstream side of the jump remains unaffected by the tailwater level.

The hydraulic jump will move upstream as the tailwater level rises further. Its height and form are maintained in the uniform flow zone nb until it reaches the point n . From then on, the height of the jump will gradually decrease as the jump moves up on the curve cn . When it reaches the critical depth point at c, the height of the jump becomes zero, and the flow profile reaches its theoretical limit cb'' of S1 type. For practical cases, the horizontal line cb'' may be taken as the limit of the tailwater stage which will avoid the computation of $b''' \, b''$.

You should have, by now, developed a fair idea about the problem of delivery of a canal under different conditions of subcritical and supercritical flow situations.

6.3 FLOW BELOW A SLUICE

You may recall that sluice is a hydraulic structure used to let in water into a channel or to let out water from a reservoir. Generally, the depth of water on the upstream side of a sluice is much greater than that on the downstream side. The discharge coming out of the sluice is a function of the head of water acting on the sluice from the upstream side. The type of flow below a sluice and the consequent flow profile on the downstream side are influenced by the bottom slope of the channel receiving the water. Having studied the different types of flow profiles, it should be simple for you to understand the flow situations discussed below.

6.3.1 Profile Below a Sluice Under Different Conditions

The profiles that are possible below a sluice gate, under different conditions, are illustrated in Figure 6.12. You may note that, whatever may be the discharge and whatever may be the bottom slope of the channel, the profiles will occur only in Zone 3 since the depth of flow under the sluice is, in general, very small for the discharges passing under it.

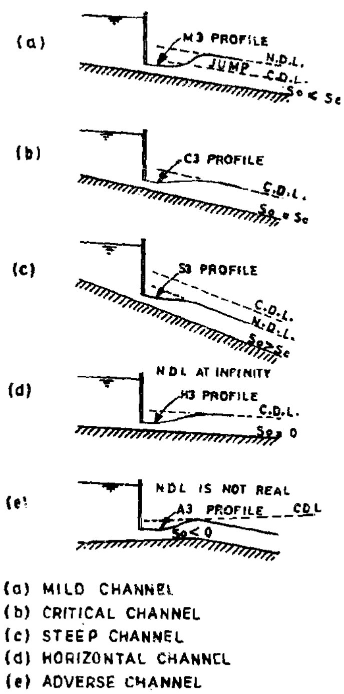

Figure 6.12 : Flow below a Sluice

M3 profile : If a mild channel happens to receive the issuing jet of water from a sluice, you will see an M3 profile formed in the zone between the CDL and the channel bottom (Figure 6.12 (a)). Since in a mild channel the NDL is above the CDL and the flow would stabilize at normal depth, the profile will end up with a hydraulic jump. The length of the profile and the height of the jump both depend on the amount of discharge. The higher the discharge, the longer is the length and the higher is the jump.

C3 profile: A C3 profile will occur when a sluice discharges into a critical channel. The profile will smoothly merge with the CDL as the flow tends to become critical, its normal flow depth (Figure 6.12 (b)).

S3 profile: If the sluice discharges into a steep channel, an S3 profile will be formed in the zone between NDL and the channel bottom (Figure 6.12 (c)). There is no possibility of a hydraulic jump here (You should know why?).

H3 profile: You should recall here that the normal depth in a horizontal channel tends to be infinity and hence also the NDL. An H3 profile will emerge if a sluice discharges into a horizontal channel (Figure 6.12 (d)). What happens after the profile formation depends on the extension of the channel and the downstream conditions. For instance, if the channel discharges its flow into a reservoir of high depth, a hydraulic jump will occur after the H3 profile.

A3 profile: If a sluice is located in the portion of a channel having an adverse slope, an A3 profile will occur (Figure 6.12 (e)). Continuation of the profile depends on the condition downstream of the adverse slope.

6.4 FLOW PROFILE AT A DROP

A channel may end up with a drop and if the drop is deep, a nappe — free falling sheet of water — is formed. Do you know what a drop is? Profiles at drops of different channels are presented in Figure 6.13. In the cases of steep and critical channels, the surfaces will simply drop down. But in other cases, they are well defined to occur in Zone 2, in between the NDL and CDL as indicated in the Figure. Discussion on the subject of nappe is beyond the scope of this unit. However, you should find it easy to reason out why *M2, H2 and A2 profiles* form as shown in the Figure.

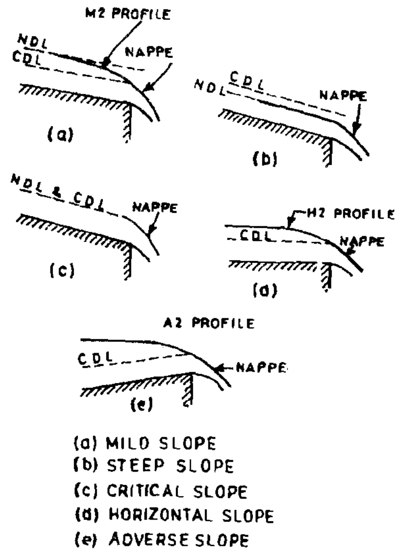

Figure 6.13 : Flow Profiles at Drops

6.5 FLOW PROFILE ON A MULTIPLE SLOPED CHANNEL

Multiple sloped channel is a pair of prismatic channels with the same cross-section but different slopes connected in series. You can easily recall that as the slope changes, the depth of flow also changes, giving rise to the occurrence of surface profiles. Several such cases are presented in Figure 6.14 which can easily be understood. However, you should be careful to make note of the following points:

1) The flow profile generally varies rapidly near the critical depth and hence cannot be predicted accurately,

2) The profile should theoretically, have a vertical face while crossing the CDL and, hence, again the actual slope of the profile there cannot be predicted accurately, and

3) In the case of a steep slope changing to mild or adverse slope, a hydraulic jump may occur either in the upper channel or in the lower channel; the location depends on the normal depth relevant in respect of the lower channel.

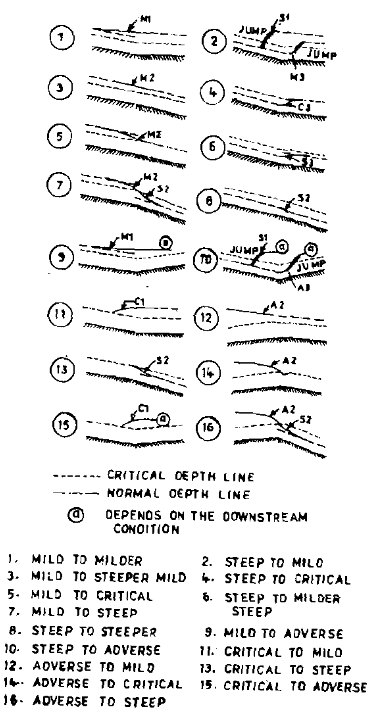

Figure 6.14 : Profiles in Channels with Break in Bed Slope

6.6 LOCATION OF A HYDRAULIC JUMP

We shall start this Section by recalling the definition of a hydraulic jump. It is a rapidly varied flow phenomenon where the depth changes from a low stage, i.e., supercritical (shooting or torrential flow) to a high stage, i.e., subcritical (tranquil or streaming flow) which results in an abrupt rise in the water surface. This local phenomenon is termed a *hydraulic jump*. Problems such as designing the free board for the retaining walls, and the horizontal floor where the jump takes place require a knowledge of the surface profile of the jump.

Generally, a hydraulic jump can occur in a horizontal rectangular channel if the initial and sequent depths and the approaching supercritical Froude Number satisfy the equation given as:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8(F_{r_1})^2} - 1\right) \qquad \text{...(6.1)}$$

where, y_1 is the depth before the jump, y_2 is the depth after the jump, and F_{r_1} is the Froude Number of flow before the jump.

The above equation is normally used to locate the position of a jump. If a detailed estimate is needed, the length of the jump should be considered. We shall examine certain specific cases of the problem, as given in following sub-sections :

6.6.1 Location of a Jump in a Mild Channel

In the case of a mild channel, a jump occurs when it receives water from a sluice (Figure 6.1 (e) and 6.12 (a)) or from a steep channel (Figure 6.1 (f), 6.14-2, and 6.14-10)

Figure 6.15 presents the case of a jump forming below a regulating sluice in a mild channel. You can easily identify that the curve AB is of M3 type and line CD is the normal depth line. You have already studied the methods by which CD can be drawn; moreover, methods are available to draw curve AB (Units 6 and 7). The curve $A'B$ is a plot of the sequent depth to AB (each pair of points lying on a vertical intersecting the two limbs of curve $AB - BA'$) which can be easily calculated with the help of equation 6.1. With the curve ABA' and the line CD known, the problem now is to fix the location of G (the point at which the jump starts) on AB and F (the point at which the jump profile terminates) on CD.

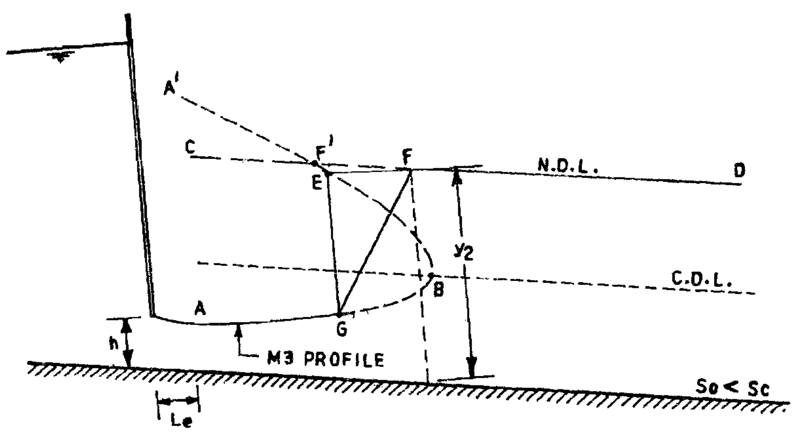

Figure 6.15 : Location of a Hydraulic Jump in a Mild Channel

The discharge through the sluice is known since the head of water on the sluice is known and hence the normal depth of flow can be calculated. The flow will become uniform after the jump, and therefore, the final depth y_2 is also known. Through a trial and error (see why?) calculation using equation (6.1), the initial depth y_1 can be determined. This y_1 will help to fix G on the M3 curve and hence its sequent depth point *E*. The location of *F* depends on the length of the jump, which is defined as the distance from the front face of the jump to a point on the surface immediately downstream from the roller as shown in Figure 6.16. This Figure presents a relationship between the length of the jump (*L*) and the Froude Number (F_{r_1}) which is based on the recommendations of the U.S. Bureau of Reclamation. With the length taken from Figure 6.16, *F* can be located from *E*. The jump will form between G and F, since the depth at F is sequent to depth at G and the distance EF is equal to the length of the jump.

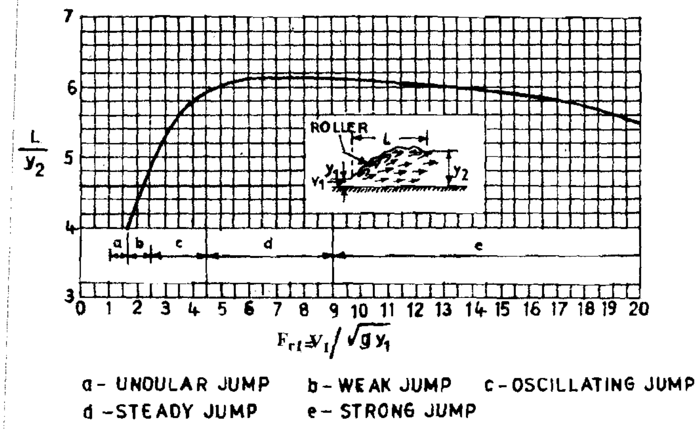

Figure 6.16 : Length, in terms of Sequent Depth of Jumps, in Horizontal Channels (Based on Data and Recommendations of U.S.B.R.)

6.6.2 Location of a Jump at a Junction of Steep and Mild Slopes

Figure 6.17 presents the case of a jump in a steep channel changing to a mild slope. The jump may occur in either the steep channel or the mild channel (see Figure 6.14). This depends on whether the normal depth in the mild channel (y_2) is greater than or less than the depth y_1' which is sequent to the normal depth in the steep channel (y_1). If y_2 is greater than y_1', the jump will occur in the steep channel. It will occur in the mild channel if y_2 is less than y_1' and the analysis of the case proceeds exactly on the same lines as was followed in sub-section 6.5.1.

The surface curve CO is S1 type of profile. The line AR is the NDL of the steep channel and $A'P$ is the sequent depth line of AR. The normal depths in the steep and mild channels are respectively the initial and final depths of the jump. With these depths, the length of the jump can be determined with the help of the curve in Figure 6.16. By determining a horizontal intercept IJ between $A'P$ and CO, which is equal to the length of the jump, it can be located at HJ. The jump will begin at H, the section containing I.

6.6.3 Use of Specific Energy and Specific Force Curves in Locating a Jump

The concept of specific energy and specific force as well as their variations with depth of flow are presented in Block 1. One of the flow situations where the use of specific energy and specific force curves finds a prominent place is the analysis of a hydraulic jump If the principles of specific energy and specific force are not very clear to you, it is strongly advised that you should go back to the above units and make sure you are thorough with them.

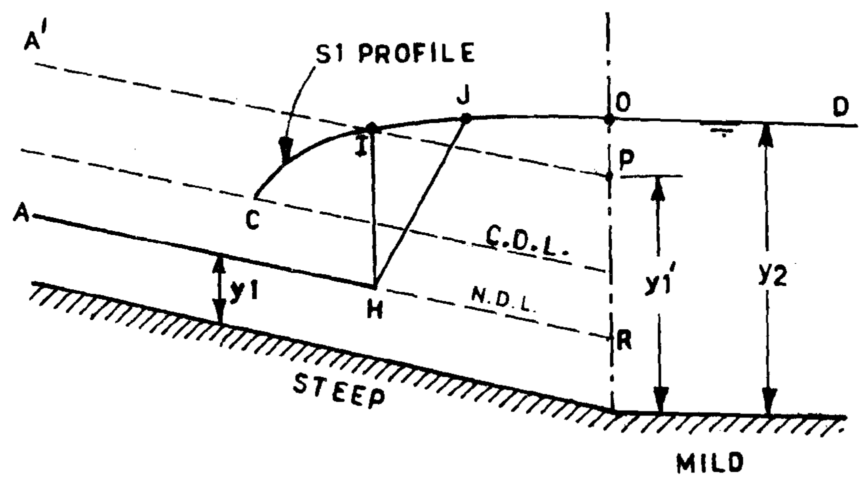

Figure 6.17 : Location of a Hydraulic Jump at a Junction of Steep and Mild Channels

However, let us recall that for a given depth its sequent depth is smaller than its alternate depth. This means that in the case of a jump, while the specific force remains the same, the specific energy undergoes a loss indicating a high internal friction.

In any problem of locating a jump in a given channel, the discharge is known, and therefore, the specific energy and specific force curves can be constructed. If we consider the case presented in Figure 6.15, the curve *A'F'B* is sequent to AGB which is a part of an M3 profile. *A'F'B* can be easily constructed with the help of the specific force curve. Also, we stated that a trial and error solution of equation 6.1 is needed to find out the initial depth for the given final depth of the jump. This can be altogether eliminated if the specific force curve is available.

The specific energy curve does not play a direct role in locating a jump, but would help one to find out the energy loss that is taking place in the jump.

6.7 SKETCHING FLOW PROFILES

Sketching the flow profiles theoretically for a given situation will help us in identifying the various types of profiles that might occur along the course of a flow. This will also lead us to selecting an approximate method of computation in respect of water surface profiles. The sketches presented in Figures 6.1 to 6.5 and 6.12 to 6.14 should guide you in sketching the flow profiles. While sketching, it may be summed up that, one must keep the following points in mind:

1) The upstream and downstream boundary conditions — for instance, whether the flow is taking place from a sluice, whether the flow enters a reservoir or ends in a drop, etc.,
2) The nature of the channel bottom slope and presence of any break in the slope,
3) The crossing of the critical depth line–whether it is downward or upward,
4) Recognition of the zone of occurrence of the profile, and
5) Recognition of the possibility of occurrence of a hydraulic jump and its approximate location.

You would appreciate the importance of the above points when you actually solve a problem.

6.8 ILLUSTRATIVE EXAMPLES

Example 6.1

Sketch the possible flow profiles in the channel shown in Figure 6.18.

Solution

Data given

There are four consecutive portions of the channel; and, the NDLs and CDLs in each of these parts are available. It is seen that the flow in general changes from mild to steep.

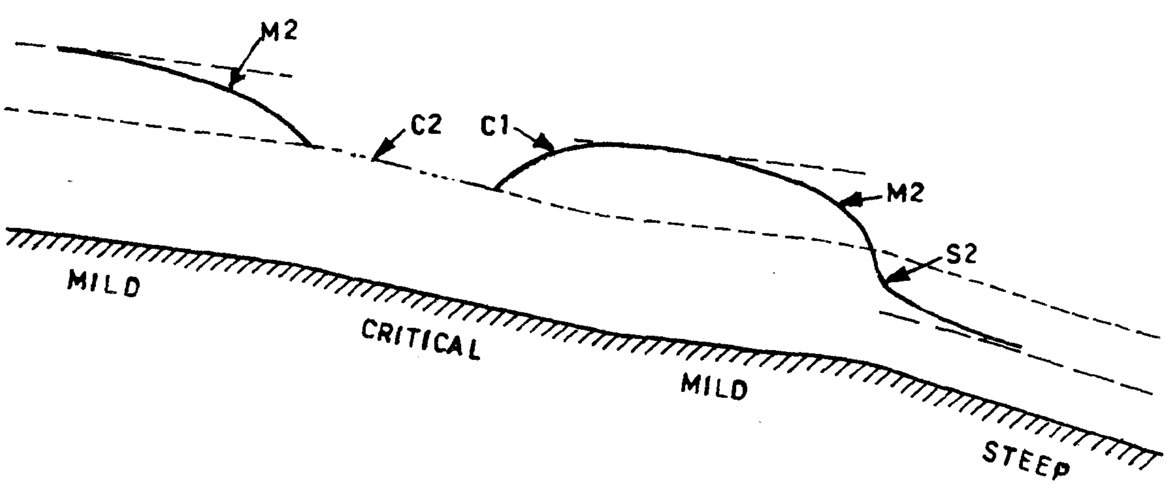

Figure 6.18 : Channels for Example 6.1

Points to be noted

1) The upstream condition is that of a uniform flow in a mild channel,
2) The downstream condition also pertains to a uniform flow, but in a steep channel,
3) There is no possibility of a jump at all, and
4) The flow crosses the CDL, at the last break in the channel slope (from mild to steep bed).

Profiles

It is an M2 profile on the first mild channel. This is followed by C2 profile representing the case of a uniform critical flow over the upper reaches of the critical channel. Before the change over of the critical to the second mild slope, the profile is C1 type. At the break of the mild to steep channel, the profiles are M2 and S2 types on the mild and steep channels, respectively. The profile will cross CDL downwardly at this break in the slope, with a vertical face.

The approximate profiles shapes along the channels are indicated in Figure 6.18.

Example 6.2

Water flows from under a sluice into a rectangular channel having a width of 5 m, bed slope 0.004 and Manning's friction factor 0.025. The sluice gate is regulated to discharge 50 m^3/s with a depth equal to 0.25 m at the vena contracta. Compute and locate the flow profile.

Solution

Here, in this solution, the computation of the flow profile is introduced (basically dealt with in Unit 7) through the use of 'varied flow functions'.

Data given

The channel is rectangular,

Base width of the channel, $b = 5$ m,

Bottom slope of the channel, $S_0 = 0.004$,

Manning's friction factor, $n = 0.025$,

Discharge, $Q = 50\ m^3/s$, and

Depth of flow at vena contracta = 0.25 m

The solution would comprise the following steps:

1) Calculate the critical depth, y_c.
2) Calculate the normal depth, y_n.
3) Comparing y_c with y_n, identify whether the channel is mild or steep and hence whether a hydraulic jump in possible or not.
4) If a jump occurs, compute the M3 profile and then locate it.
5) If there is no jump, compute the S3 profile.

Critical depth

The condition for critical flow is,

$$F_{r_1} = \frac{V}{\sqrt{gD}} = 1, \text{ or } \frac{V^2}{2g} = \frac{D}{2}$$

where, F_{r_1} is the upstream Froude Number, V is the mean velocity of flow, D is the hydraulic depth and g is the acceleration due to gravity.

Substituting the appropriate values (i.e., $Q = A_c V$; $A_c = by_c$ and $D = A_c / b$) :

$$\frac{Q^2}{(by_c)^2} \times \frac{1}{2g} = \frac{b \times y_c}{2b}$$

or, $$y_c^3 = \frac{1}{g} \times \frac{Q^2}{b^2}$$

or, $$y_c = \left[\frac{1}{g} \times \frac{Q^2}{b^2}\right]^{1/3}$$

$$\therefore \quad y_c = \left[\frac{1}{9.81} \times \frac{502}{(5)^2}\right]^{1/3} = 2.168 \text{ m}$$

According to Manning's formula, the discharge Q is given by :

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

where, R is hydraulic radius (given by $R = A/P$), P is the wetted perimeter, and S_0 is channel bottom slope.

$$AR^{2/3} = \frac{(A)^{5/3}}{P^{2/3}} = \frac{nQ}{S_0^{1/2}}$$

If y_n is the normal depth, then :

$$\frac{(by_n)^{5/3}}{(b + 2y_n)^{2/3}} = \frac{nQ}{S_0^{1/2}}$$

or , $$= \frac{(5\, y_n)^{5/3}}{(5 + 2\, y_n)^{2/3}}$$

$$= \frac{0.025 \times 50}{(0.004)^{1/2}}$$

or , $$\frac{(5\, y_n)^{5/3}}{(5 + 2\, y_n)^{2/3}} = 19.7642 \text{ m}^{8/3}$$

Solving this equation by trial and error :

Trial value of y_n	**$(5y_n)^{5/3}$**	**$(5+2y_n)^{2/3}$**	**(2)/(3)**
(1)	(2)	(3)	(4)
3.000	91.233	4.946	18.446
3.100	96.358	5.006	19.249
3.200	101.594	5.065	20.057
3.150	98.962	5.036	19.652
3.160	99.486	5.042	19.733
3.165	99.749	5.046	19.774
3.164	99.696	5.044	19.765 ≈ 19.764

Therefore, the normal depth of flow (y_n) is 3.164 m.

Note : The normal depth (y_n = 3.164 m) is greater than the critical depth (y_c = 2.168 m). Therefore, the channel is treated as mild for the given discharge.

The depth of flow at vena contracta of the issuing jet is 0.25 m which is less than the critical depth. Therefore, a hydraulic jump will occur, which can be located after determining the M3 profile.

Specific energy and specific force curves

$$E = y + \frac{V^2}{2g} = y + \frac{1}{2g}\frac{Q^2}{(by)^2}$$

$$= y + \frac{50^2}{2 \times 9.81 \times 5^2 \times y^2} = y + \frac{5.0968}{y^2}$$

$$F = A\,\bar{z} + \frac{Q^2}{gA}$$

$$= (by)\frac{y}{2} + \frac{Q^2}{gby}$$

$$= \frac{5}{2}y^2 + \frac{50^2}{9.81 \times 5}\left(\frac{1}{y}\right) = 2.5y^2 + \frac{50.9684}{y}$$

Computing E and **F** for various values of y :

Depth y	**Specific Energy E**	**Specific Force F**	**Depth y**	**Specific Energy E**	**Specific Force F**
1.0	6.10	53.47	2.6	3.35	36.50
1.2	4.74	46.07	2.8	3.45	37.80
1.4	4.00	41.31	3.0	3.57	39.49
1.6	3.59	38.26	3.2	3.70	41.53
1.8	3.37	36.42	3.4	3.84	43.89
2.0	3.27	35.48	3.6	3.99	46.56
2.2	3.25	35.26	3.8	4.15	49.51
2.4	3.29	35.64	4.0	4.32	52.74

The specific energy and specific force curves are presented in Figure 6.19.

Computation of M3 profile

Using the concept of **varied flow functions**, the distance (x) from the origin to a specific depth (y) of the M3 profile is given by

$$x = A\,[u - F(u, N) + BF(v, J)] + \text{constant}$$

$$\text{where,} \quad A = \frac{y_n}{S_0}\;;\; B = \left[\frac{y_c}{y_n}\right]^M \left(\frac{J}{N}\right);\; u = \frac{y}{y_n}$$

$$v = (u)^{N/J} \text{ and } J = N/(N - M + 1)$$

Length between any two depths y_2 and y_1 is given by :

$$L = x_2 - x_1$$

N and M are the hydraulic exponents for the uniform flow and critical flow computations, respectively.

The normal depth of flow is 3.164 m. The following calculations are made with respect to an average depth of flow $\left[\frac{(0.25 + 3.164)}{2}\right]$ of 1.707 m.

Hydraulic exponent, M, for critical flow computation is given by :

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right)$$

Since the channel is rectangular, $T = B$, and hence $dT/dY = 0$.

$$M = \frac{y}{A}(3T) = \frac{1.707 \times 3 \times 5}{5 \times 1.707} = 3$$

Hydraulic exponent for uniform flow computation is given by:

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right)$$

$$P = 5 + 2y_1, \text{ and hence } \frac{dp}{dy} = 2$$

$$N = \frac{2 \times 1.707}{3 \times 5 \times 1.707}\left[5 \times 5 - \frac{2 \times 5 \times 1.707}{(5 + 2 \times 1.707)} \times 2\right]$$

$$= 2.792$$

$$J = N/(N - M + 1)$$

$$= 2.792/(2.792 - 3.0 + 1)$$

$$\therefore \quad J = 3.525$$

$$A = \frac{y_n}{S_0} = \frac{3.164}{0.004} = 791{:}0$$

$$B = \left[\frac{y_c}{y_n}\right]^M \left(\frac{J}{N}\right)$$

$$= \left[\frac{2.168}{3.164}\right]^3 \frac{3.525}{2.792} = 0.406$$

$$u = \frac{y}{y_n}$$

$$v = (u)^{N/J} = (u)^{2.792/3.525} = (u)^{0.792}$$

$$x = 791\ [u - F(u, 2.792) + 0.406\ F(v, 3.525)]$$

The constant can be droped out since we will be taking finally the difference between the two values of x. The calculations of the length of M3 profile are shown in a tabular form.

Depth of flow after the jump is the normal depth $y_n = 3.164$ m. Depth before the jump is the depth sequent to 3.614 m. From the specific force curve and by checking with equation 6.1, the initial depth is found to be 1.41 m and the Froude Number is 1.91. The formation is a weak jump (Figure 6.16), and from the same Figure (for $F_{r_1} = 1.91$), the ratio between the length of the jump and its final depth is found to be 4.3.

But, the final depth of the jump $= 3.164$ m

$\therefore$ The length of the jump $= 4.3 \times 3.614 = 13.61$ m

The computed flow profile is plotted as shown in Figure 6.19. The distance from the jump to the gate would be about 127.0 m and the jump will occur at an initial depth of 1.41 m. Refer the following table for above mentioned calculations.

Depth y	u	v	F(u, 2.792)	F (v, 3.525)	x	L
	(y/y_n)	$(u)^{0.792}$	(From standard tables)*			
2.168	0.685	0.741	0.766	0.813	197.02	0.00
2.000	0.632	0.695	0.687	0.746	196.07	0.95
1.800	0.569	0.640	0.604	0.674	188.77	8.25
1.600	0.506	0.583	0.523	0.604	180.53	16.49
1.400	0.443	0.524	0.456	0.537	162.17	34.85
1.200	0.379	0.464	0.386	0.471	145.72	51.30
1.000	0.316	0.402	0.320	0.405	126.90	70.12
0.800	0.253	0.337	0.255	0.339	107.29	89.73
0.600	0.190	0.268	0.191	0.269	85.60	111.42
0.400	0.126	0.194	0.126	0.194	62.30	134.72
0.200	0.063	0.112	0.063	0.112	35.97	161.05
0.000	0.000	0.000	0.000	0.000	0.00	197.02

* Refer Appendix 7.1 of Unit 7

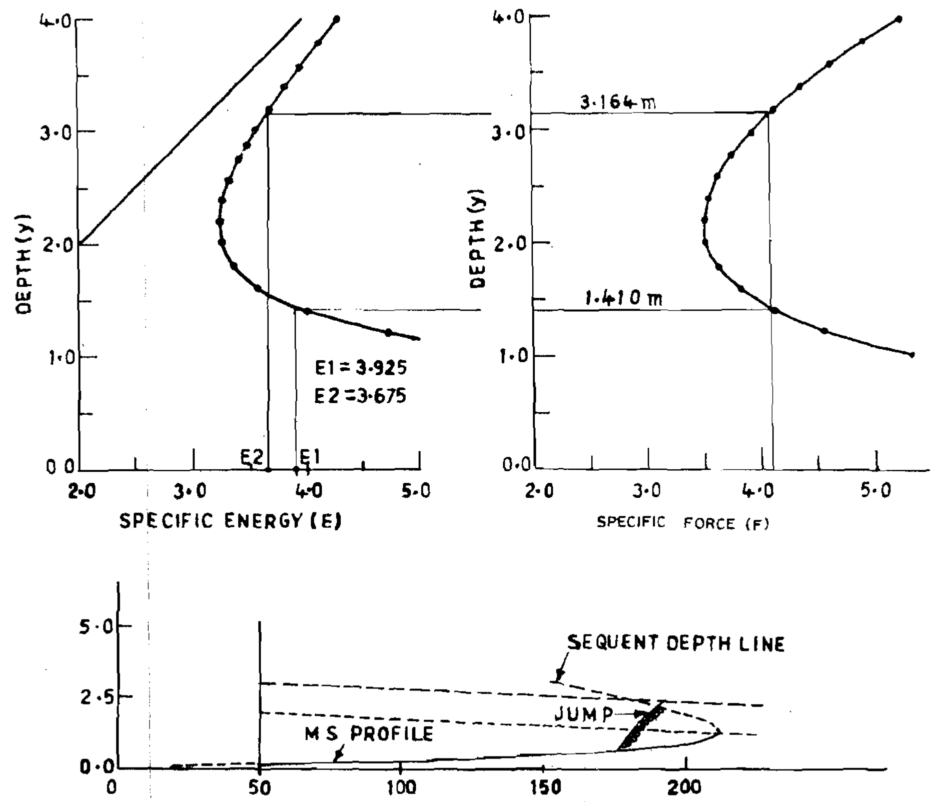

Figure 6.19 : Flow Profile for Example 6.2

SAQ 1

Recognise the flow profiles for the following cases of flow situations:

i) A mild channel discharging into a reservoir.

ii) A steep channel discharging into a reservoir.

iii) A mild channel breaking into a steep channel.

iv) A sluice discharging into a mild channel.

v) A mild channel ending with a drop.

vi) A steep channel breaking into a mild channel.

vii) A mild channel enlarging its section.

viii) An adverse channel discharging into a mild channel.

ix) A critical channel breaking into a mild channel.

x) A steep channel breaking in sectional dimensions.

SAQ 2

In a problem of delivery of a canal, indicate the conditions under which M1 and M2 profile will occur. (Refer relevant Figures)

SAQ 3

A long channel has three different prismatic reaches. The first and the last are mild slopes with the middle one being steep. The channel ends with a drop. Sketch the approximate profiles along the channel and identify them, reasoning out your conclusions (check with reference to relevant Figures).

SAQ 4

A rectangular channel with a base of 5 m carries a discharge of 20 m^3/s. The bed slope is 0.05 and the Manning's friction coefficient is 0.03. After some distance the bed slope changes to 0.001. Compute the flow profile and locate the hydraulic jump. (Initially we have a slope S_{0_1} and latter S_{0_2}.)

6.9 SUMMARY

Let us conclude with a summary of what we have attempted to learn in this unit of study about flow profiles. We have:

- recalled the basis of classification of flow profiles in different channels,
- recognised the types of flow profiles associated with some of the practical situations,
- investigated the problem of *delivery of a canal* for subcritical and supercritical flow situations,
- analysed the profiles that might be formed below a sluice under different conditions of channel bottom slope,
- studied the formation of profiles at a drop,
- learnt in detail the formation of profiles on multiple-sloped channels,
- attempted to locate a hydraulic jump when it occurs in a mild channel,
- also attempted to locate a hydraulic jump when it occurs at a junction of steep and mild slopes,
- learnt the use of specific energy and specific force curves in locating a jump, and
- worked out a few examples.

6.10 KEY WORDS

Delivery of a canal : Flow passing through a canal that connects two reservoirs having different water levels.

Multiple-sloped channel : A channel with sudden changes in its bed slope

6.11 ANSWERS TO SAQs

Check your answers for all SAQs with respective preceding text.