UNIT 5 DIFFERENTIAL EQUATION OF GRADUALLY VARIED FLOW AND CLASSIFICATION OF FLOW PROFILES

Structure

5.1 INTRODUCTION

In the beginning of the course we classified the steady open-channel flows as :

i) uniform flow, and

ii) varied flow.

Varied flows are further classified as: (a) gradually varied flow, and (b) rapidly varied flow. In uniform flow, you know, the depth of flow remains constant along the length of the channel and the flow surface is parallel to the channel bottom. On the other hand, in varied flow the depth of flow varies along the length of the channel; and hence, the slope of the flow surface differs from the slope of the channel bed. The variation in the depth of flow can occur abruptly over a comparatively short distance, as in a hydraulic jump or flow over a weir, and this type of flow is known as **rapidly varied flow**. If variation in depth of flow occurs gradually over a long distance as in the case of flow behind a dam, the flow is known as **gradually varied flow**. Occurrences of gradually varied flow are common in field situations; for example, flow behind a dam, flow behind a free overfall and flow below a sluice are all instances of gradually varied flows.

With reference to a uniform flow we studied that the channel bed, water surface, and total energy lines are all parallel to each other; and, therefore, the slopes of channel bed, S_0, water surface, S_w, and energy gradient, S_f, are all equal. As the depth of flow remains constant along the length of the channel, the water surface slope, with respect to channel bed, (dy/dx), is zero. In gradually varied flow, as the depth of flow varies along the length of the channel, the slope of the water surface with reference to the channel bed, (dy/dx) has a non-zero value, which may be positive or negative, depending on the increase or decrease in the depth of flow along the length of the channel. In this unit, using the energy principle, equation for the slope of the water surface is derived. This equation, known as **dynamic equation for gradually varied flow**, or simply **gradually varied flow equation (GVF)**, is written in different forms to suit easy application to various flow situations, as well as, easy interpretation/prediction of various flow profiles. Finally, the flow profiles that occur in channels, under different conditions, are classified.

Objectives

After studying this unit, you should be able to :

- determine the slope of the water surface profile at a given depth for a given discharge, channel section and bed slope,
- understand the basic features of various types of water surface profiles, and
- classify a given water surface profile.

5.2 DYNAMIC EQUATION OF GRADUALLY VARIED FLOW

As emphasized above, a gradually varied flow is a steady flow whose depth varies gradually along the length of the channel. This definition signifies that: (1) the flow, being steady, the hydraulic characteristics of flow remain constant for the time interval under consideration; and, (2) the streamlines are practically parallel to each other, and thus, the hydrostatic distribution of pressure prevails over the channel section. The basic differential equation, describing a gradually varied flow profile, is based on energy principle; and, it expresses the slope of water surface profile, at any given location (i.e., section) in terms of easily known parameters.

5.2.1 Basic Assumptions

Flow of water in an open channel is associated with the loss of energy from one location to the other . The Manning's equation or Chezy's equation, you used in uniform flow computations, relates the bed slope with the hydraulic radius and velocity of flow. The computation of energy gradient, in gradually varied flow, involves the assumption that the rate of head loss at a section is same as for a uniform flow having the same velocity and hydraulic radius as prevail at the section under consideration.

According to this assumption, the uniform flow formulae may be used to evaluate the energy slope of a gradually varied flow at a given channel section, and the corresponding coefficient of roughness developed primarily for uniform flow is also applicable to the varied flow.

In addition to the above given basic assumption, the following assumptions are also made :

1) The slope of the channel is so small that :
   a) the depth of flow is the same whether the vertical or normal to-the-channel-bed direction is considered,
   b) the pressure-correction factor, $\cos\theta$ (applied to the depth of flow at a section) is equal to unity (θ = bed slope), and
   c) no air entrainment occurs in the flow field.
2) The channel is prismatic (i.e., the channel has a constant alignment and shape) ;
3) The velocity distribution in the channel section is constant. Hence, the velocity-distribution coefficients are constant;
4) The conveyance $K = \left(\frac{Q}{\sqrt{S}}\right)$ and section factor Z are exponential functions of the depth of flow; and
5) The roughness coefficient is independent of the depth of flow and is constant throughout the channel reach under consideration.

5.2.2 Gradually Varied Flow Equation

Consider the profile of a gradually varied flow in an elementary length dx of an open channel, shown in Figure 5.1. The total head, H, at the upstream section 1 (with reference to the datum) is given by :

$$H = z + d\cos\theta + \alpha\frac{V^2}{2g} \qquad ...(5.1)$$

where, H is the total head (m); z is the vertical distance of the channel bottom above the datum (m); d is the depth (m) of flow in the section (perpendicular to the bed); θ is the bottom-slope angle; α is the energy coefficient; and, V is the mean velocity of flow through the section (m/s). It is assumed that, θ and α are constant throughout the channel reach under consideration.

Differentiation of the total head, H, with respect to x (an axis taken along the channel bottom) gives :

$$\frac{dH}{dx} = \frac{dz}{dx} + \cos\theta \frac{dd}{dx} + \alpha \frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad ...(5.2)$$

The term $\alpha \frac{d}{dx}\left(\frac{V^2}{2g}\right)$ can be written as $\alpha \frac{d}{dd}\left(\frac{V^2}{2g}\right)\frac{dd}{dx}$

and hence, equation (5.2) becomes :

$$\frac{dH}{dx} = \frac{dz}{dx} + \cos\theta \frac{dd}{dx} + \alpha \frac{d}{dd}\left(\frac{V^2}{2g}\right)\frac{dd}{dx} \qquad ...(5.3)$$

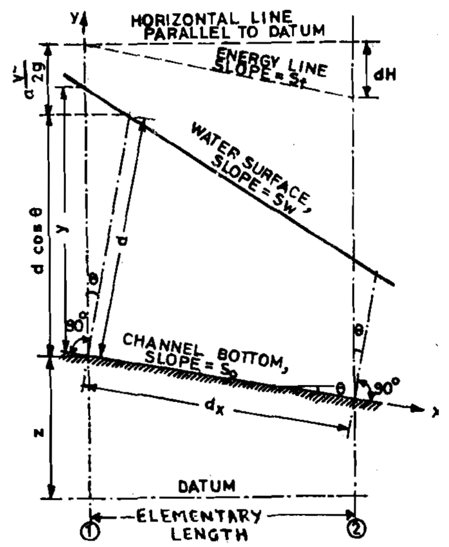


Figure 5.1 : Derivation of the Gradually Varied Flow Equation

where, $\frac{dH}{dx} = -S_f$, the energy gradient; the negative sign accounting for H decreasing while x increases; and $\frac{dz}{dx} = -S_o$, the bed slope, z decreasing as x increases. Thus, substituting for $\frac{dH}{dx}$ and $\frac{dz}{dx}$, equation (5.3) becomes :

$$\frac{dd}{dx} = \frac{S_o - S_f}{\cos\theta + \alpha \frac{d}{dd}\left(\frac{V^2}{2g}\right)} \qquad ...(5.4)$$

When θ is small, $\cos\theta \approx 1$, $y \approx d$, and $\frac{d}{dx} \equiv \frac{d}{dy}$ which reduces equation (5.4) to :

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 + \alpha \frac{d}{dd}\left(\frac{V^2}{2g}\right)} \qquad ...(5.5)$$

where, y is the vertical depth of flow.

As $$V = \frac{Q}{A}, \quad \frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = -\frac{Q^2 T}{g A^3}$$

$$\text{or,} \quad \frac{d}{dd}\left(\frac{V^2}{2g}\right) = -\frac{Q^2 T}{g A^3}$$

where, T = channel width at free surface. Substituting for $\frac{d}{dd}\left(\frac{V^2}{2g}\right)$, equation (5.5) becomes :

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \alpha \left(\frac{Q^2 T}{g A^3}\right)} \quad \text{...(5.6)}$$

In equation (5.6), $\frac{dy}{dx}$ gives the slope of the water surface with reference to the channel bed. Equation (5.6) can now also be written as :

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \alpha F_r^2} \quad \text{...(5.7)}$$

where, Fr is Froude Number at the section under consideration. For a rectangular channel section, when $\alpha = 1$, equation (5.6) becomes :

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \left(\frac{V^2}{g y}\right)} \quad \text{...(5.8)}$$

Example 5.1

A rectangular channel, 6 m wide, carries a discharge of 36.16 m^3/s, with a bed slope, $S_0 = 0.0016$, and Manning's $n = 0.015$. Compute the water surface slopes at sections where depths of flow are 2.5 m and 1.8 m, respectively.

Assume following data to be applicable to the channel under consideration :

The normal depth, y_n = 2.00 m,

The critical depth, y_c = 1.55 m,

Uniform flow velocity, V_n = 3.01 m/s,

$\cos\theta$ =1, and

Velocity coefficient α = 1.0,

Solution

a) for y = 2.5 m.

Area of flow section, $A = 6 \times 2.5 = 15 \text{ m}^2$

Wetted perimeter, $P = 6 + 2 \times 2.5 = 11$ m

Hydraulic radius, $R = \frac{A}{P} = \frac{15}{11} = 1.364$ m

Velocity of flow, $V = \frac{Q}{A} = \frac{36.16}{15} = 2.41$ m/s

Using Manning's equation, we have (where, as mentioned in Block 1, $S = S_f$):

$$S_f = \left(\frac{n V}{R^{2/3}}\right)^2 = \left(\frac{0.015 \times 2.41}{1.364^{2/3}}\right)^2 = 0.00086$$

$$\therefore \quad \frac{dy}{dx} = \frac{S_o - S_f}{1 - \left(\frac{V^2}{g \times y}\right)} = \frac{0.0016 - 0.00086}{1 - \left(\frac{2.41^2}{9.8 \times 2.5}\right)} = 0.00097$$

b) For $y = 1.8$ m

$A = 10.8\ m^2, P = 9.6$ m, $R = 1.125$ m, $V = 3.35$ m/s

$$\therefore \quad S_f = \left(\frac{nV}{R^{2/3}}\right)^2 = \left(\frac{0.015 \times 3.35}{1.125^{2/3}}\right)^2 = 0.0022$$

$$\text{Hence,} \quad \frac{dy}{dx} = \frac{S_o - S_f}{1 - \left(\frac{V^2}{g \times y}\right)} = \frac{0.0016 - 0.0022}{1 - \left(\frac{3.35^2}{9.8 \times 1.8}\right)} = -\ 0.00165$$

Water surface slopes for the different depths of flow in Example 5.1 are tabulated in Table 5.1.

Table 5.1 : Comparison of Water Surface Slope for Depths of Flow of 2.5 m, 1.80 m, Normal Depth and Critical Depth

y (m)	*V* (m/s)	*Fr*	S_f	$\frac{dy}{dx}$	**Remarks**
2.00	3.01	0.68	0.0016	0	$S_0 = 0.0016$; Uniform flow, $S_o = S_f$
2.50	2.41	0.49	0.00086	9.7×10^{-4}	$y > y_n$, $S_f < S_0$
1.80	3.35	0.80	0.0022	-1.65×10^{-3}	$y < y_n$, $S_f > S_0$
1.55	3.89	1.00	0.0033	∞	$y = y_c$, $S_f > S_0$; Critical state of flow

Table 5.1 shows that, for the given discharge and channel section when the depth of flow is greater than the normal depth, $S_f < S_0$, and the water surface slope at the given depth is positive, i.e.. the depth of flow increases with the length of the channel. On the other hand, when the depth of flow is less than the normal depth, the water surface slope is negative, a condition indicating that the depth of flow decreases in the direction of flow. When the flow is equal to critical depth, the denominator of equation (5.8) tends to zero, and the water surface slope tends to infinity. It should now be possible to determine the slope of water surface profile at a given depth of flow using equation (5.6) if the discharge and channel section are given.

5.2.3 Different Forms of Gradually Varied Flow Equation

The GVF equation (5.6) can be written in different forms, and when α is not equal to 1.00, it can be written as :

$$\left(\frac{dy}{dx}\right) = \frac{S_o\left(1 - \frac{S_f}{S_o}\right)}{1 - \alpha\left(\frac{Q^2 T}{g A^3}\right)} \qquad ...(5.9)$$

For uniform flow ($S_f = S_0$), the equation for discharge, using Manning's equation (in which, in fact, the slope (S) refers to the friction slope, namely, S_f), can be put as:

$$Q = \left(\frac{1}{n}\right) A R^{2/3} S_0^{1/2} = K_n S_0^{1/2} \qquad ...(5.10)$$

where, $K_n = \left(\frac{1}{n}\right) A R^{2/3}$, and is known as the **conveyance of the channel** when Q occurs at normal depth (refer sub-section 5.2.1). Now, from equation (5.10) :

$$S_0 = \left(\frac{Q^2}{K_n^2}\right) \qquad ...(5.11)$$

In a non-uniform flow ($S_f \neq S_o$), we can write :

$$Q = \left(\frac{1}{n}\right) A R^{2/3} S_f^{1/2} = K S_f^{1/2} \qquad ...(5.12)$$

where, K (already defined) is the conveyance of the channel for the depth of flow, y. From equation (5.12), we can write :

$$S_f = \left(\frac{Q^2}{K^2}\right) \qquad ...(5.13)$$

Assuming Z_c ($= A\sqrt{D}$) to be the section factor which is computed for the given Q at depth y_c, i.e., when the flow is critical (i.e., $\alpha V^2/2g = D/2$, and, so, $\alpha \frac{Q^2}{g} = A^2 D = Z_c$), we can write :

$$Q = Z_c \left(\frac{g}{\alpha}\right)^{1/2} \qquad ...(5.14)$$

In this context, Z simply represents the numerical value of $\left(\frac{A^3}{T}\right)^{1/2}$, i.e., $A\sqrt{D}$ which is computed for the discharge Q at the existing depth y (at the section under consideration) of the given gradually varied flow, i.e., A and D would be calculated on the basis of existing y only. Now from equations (5.9), (5.11), (5.13) and (5.14) we have :

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{K_n}{K}\right)^2}{1 - \alpha\left(\frac{Q^2 T}{g A^3}\right)} \qquad ...(5.15)$$

But,

$$\alpha\left(\frac{Q^2 T}{g A^3}\right) = \alpha \frac{Q^2}{g D A^2}$$

$$\text{or, } \alpha\left(\frac{Q^2 T}{g A^3}\right) = \left(\frac{Z_c}{Z}\right)^2 \qquad ...(5.16)$$

Hence, equation (5.15) gives:

$$\frac{dy}{dx} = S_0 \frac{1 - \left(K_n / K\right)^2}{1 - \left(Z_c / Z\right)^2} \qquad (5.17)$$

For a wide rectangular section (i.e., $R \approx y$), using Manning's equation, we can write :

$$1 - \left(\frac{K_n}{K}\right)^2 = 1 - \left(\frac{b\, y_n\, y_n^{2/3}}{b\, y\, y^{2/3}}\right)^2 = 1 - \left(\frac{y_n}{y}\right)^{10/3}$$

and,

$$1 - \left(\frac{Z_c}{Z}\right)^2 = 1 - \left(\frac{b y_c \sqrt{\left(\frac{b y_c}{b}\right)}}{b y \times \sqrt{\left(\frac{b y}{b}\right)}}\right)^2$$

$$= 1 - \left(\frac{y_c}{y}\right)^3$$

Therefore, equation (5.17) becomes :

$$\frac{dy}{dx} = S_o \frac{\left[1-\left(\frac{y_n}{y}\right)^{10/3}\right]}{1-\left[\left(\frac{y_c}{y}\right)^3\right]} \qquad ...(5.18)$$

Similarly, for wide rectangular channel, using Chezy's equation (where, $K = AC\sqrt{R}$) we obtain :

$$\frac{dy}{dx} = S_o \frac{\left[1-\left(\frac{y_n}{y}\right)^{3}\right]}{\left[1-\left(\frac{y_c}{y}\right)^3\right]} \qquad ...(5.19)$$

It should now be easy to recognise other forms of GVF equations, such as :

$$\frac{dy}{dx} = S_o \frac{\left[1-\left(\frac{Q}{Q_n}\right)^{2}\right]}{\left[1-\left(\frac{Q}{Q_c}\right)^2\right]} \qquad ...(5.20)$$

where, Q is the given discharge of the gradually varied flow at the actual depth y; Q_n is the normal discharge at the depth equal to y ; and Q_c is the critical discharge at the depth equal to y ,

and,

$$\frac{dy}{dx} = \frac{S_0 - \left(\frac{Q^2}{CA^2 R}\right)}{1-\alpha\left(\frac{Q^2}{gA^2 D}\right)} \qquad ...(5.21)$$

where, C is Chezy's resistance factor, and D is the hydraulic depth (A/T).

5.2.4 Water Surface Slope

The dynamic equation for gradually varied flow, equation (5.6), as already pointed out, gives the slope of the water surface at any given depth y; and it is obvious that the water surface slope (dy/dx) can take either a positive or a negative value depending upon the value and sign of various quantities composing the gradually varied flow equation. As already pointed out, a positive value of dy/dx indicates that the depth of flow increases in the direction of flow, while a negative value indicates that the depth of flow decreases in the direction of flow In fact, a GVF equation helps in determining the shape of a water surface profile that obtains under given channel and flow conditions; this knowledge is of great importance in the solution of practical problems.

Example 5.2

Water flows from under a sluice into a trapezoidal channel having a bed width = 6.0 m, side slope of 2 horizontal to 1 vertical, bed slope, $S_0 = 0.0036$, energy coefficient $\alpha = 1.10$, and Manning's $n = 0.025$. The sluice gate is regulated to discharge 11 m^3/s with a depth of 0.2 m at the vena contracta. Determine the behaviour of flow profile at a section where the depth of flow is 0.40 m.

Solution

Bed width = 6.0 m

Side slope, $z = 2.00$

Bed slope, $S_0 = 0.0036$

Manning's $n = 0.025$

Discharge, $Q = 11.0\ m^3/s$.

Computing the normal depth y_n and critical depth y_c for the given discharge and channel section, we have :

$y_n = 0.81$ m and, $y_c = 0.67$ m.

The given channel slope, therefore, sustains a uniform flow (for $Q = 11\ m^3/s$) with a depth of flow 0.2 m, as occurring at the vena contracta. The depth of flow has to attain the normal depth value (as computed above for $Q = 11.0\ m^3/s$) as the flow progresses beyond vena contracta so as the depth of flow increases from 0.20 m onwards, and a hydraulic jump must occur once the flow meets a subcritical state of flow.

For $y = 0.4$ m, we have :

$A = 2.72\ m^2$

$T = 7.60$ m

$P = 7.79$ m

$R = 0.349$ m

Using $Q = \frac{1}{n} AR^{2/3} S_f^{1/2}$, we get :

$$11.0 = \left(\frac{1}{0.025}\right) \times 2.72 \times (0.349)^{2/3} \times \left(S_f\right)^{1/2}$$

Therefore, $S_f = 0.042$

Now, from equation (5.6) we get :

$$\frac{dy}{dx} = \frac{(0.0036 - 0.042)}{1 - 1.10\left(\frac{11.0^2 \times 7.60}{9.81 \times 2.72^3}\right)}$$

$$= 0.0093$$

Therefore, the slope of the flow profile at depth of flow $y = 0.4$ m is positive, and = 0.0093.

Thus, the depth of flow keeps increasing, in the direction of flow, beyond the location where $y = 0.4$ m.

5.3 WATER SURFACE PROFILES

A flow in a prismatic channel will always tend to attain to a uniform-flow condition, and will ultimately sustain itself at the relevant normal depth of flow, provided the available length of the channel is sufficient for the purpose. This will happen irrespective of any initial depth of flow whose value may be other than the normal depth. It is understood by now that the pertinent change in the depth of flow has either to be a negative or a positive increment in the succeeding depths of flow till y_n is attained.

Based on the behaviour of the available bed gradient, the slope (for a given Q) is considered as a **sustaining slope** if it sustains a uniform flow in the channel, otherwise the slope is said to be **non-sustaining.** A sustaining slope can be a mild, critical or a steep slope (discussed in the following Sections) for different discharges. It is, however, to be understood that a horizontal bed, and an adverse slope (i.e., a bed rising in the direction of flow) are non-sustaining slopes.

In equation (5.6), S_0 takes a positive value for being a sustaining slope; is equal to zero for a bed which is horizontal; and, takes a negative value in the case of an adverse slope. As will be observed, in the following Sections, the flow profiles are appropriately classified according to the bed slope, and the actual depth of flow (y) as compared to the normal depth, y_n, and the critical depth, y_c.

5.3.1 Characteristics of Flow Profiles

Characteristics of a flow profile, corresponding to a given depth of flow (in other words, a given discharge), are influenced by the nature of bed slope of the channel. A channel is said to have a **mild slope** (or a subcritical slope) if a subcritical flow (i.e., $y_n > y_c$) exists in the channel; the slope is said to be a **steep slope** (i.e., a supercritical slope), or the channel is said to be a steep channel if a supercritical flow (i.e., $y_c > y_n$) exists. It is, therefore, clear, as a corollary, that the channel is said to have a **critical slope** if it admits the given flow as a critical flow (i.e., when $y_n = y_c$).

If $\frac{dy}{dx}$ is positive (i.e., y increasing in the direction of flow), the flow profile is known as a **backwater curve,** and if (dy/dx) is negative, the flow profile is known as a **drawdown curve.** For a prismatic channel, the conditions for backwater curve and drawdown curve are analysed in this Section. For this analysis any one of the GVF equations can be used; and for an easy explanation, equation (5.19), applicable to a wide rectangular channel, is used in the following discussion. Therefore, for a channel, having a positive sustaining slope, (S_0), $\left(\frac{dy}{dx}\right)$ will be positive if :

1) $\left[1 - \left(\frac{y_n}{y}\right)^3\right] > 0$, and $\left[1 - \left(\frac{y_c}{y}\right)^3\right] < 0$; or

2) $\left[1 - \left(\frac{y_n}{y}\right)^3\right] < 0$, and $\left[1 - \left(\frac{y_c}{y}\right)^3\right] > 0$

For case (1) to be possible : y has to be $>y_n$, and $y > y_c$; this gives rise to two possible flow situations, such as :

a) $y > y_n > y_c$, or

b) $y > y_c > y_n$

It is assumed that the student understands that y refers to any depth of flow, that is of interest.

In each of these flows [(a), and (b)] we observe that $y > y_c$, indicating a sub-critical state of flow. If $y > y_n > y_c$, a sub-critical flow is said to occur in a mild channel. On the other hand, if $y > y_c > y_n$, a sub-critical flow is said to occur in a steep channel.

For case (2) to be possible : $y_n > y$, and $y_c > y$. Thus, the possible flow conditions can be written as:

a) $y_n > y_c > y$, or

b) $y_c > y_n > y$

As $y < y_c$, the flow is supercritical in both these sub-cases. If $y < y_c < y_n$ [i.e., sub-case (a)], a supercritical flow must occur in a mild channel. On the other hand, if $y < y_n < y_c$ [i.e., sub-case (b)], a supercritical flow occurs in a steep channel.

For a drawdown curve, (dy/dx) is negative, and equation (5.19) again gives two possible cases:

a) $\left[1 - \left(\frac{y_n}{y}\right)^3\right] > 0$, and $\left[1 - \left(\frac{y_c}{y}\right)^3\right] < 0$; or

b) $\left[1 - \left(\frac{y_n}{y}\right)^3\right] < 0$, and $\left[1 - \left(\frac{y_c}{y}\right)^3\right] > 0$

For case (1), $y > y_n$ and $y < y_c$, that is $y_c > y > y_n$. As $y < y_c$, the flow is in a supercritical state; and, also, as $y_n < y < y_c$, the supercritical flow occurs in a steep channel. For case (2), $y < y_n$ and $y > y_c$, which means $y_n > y > y_c$, implying a subcritical flow occurring on a mild slope.

When the water surface is parallel to the bed of the channel, $(dy/dx) = 0$, and equation (5.19) gives $\left(1 - \frac{y_n^3}{y^3}\right) = 0$, i.e., $y = y_n$, it is the characteristic of a uniform flow. The flow is uniform and subcritical if $y = y_n > y_c$; uniform and critical if $y = y_n = y_c$; and, uniform and supercritical if $y = y_n < y_c$.

Considering a horizontal slope (a non-sustaining slope), $S_0 = 0$, where $y_n = \infty$, equation (5.19) can alternatively be written, for a wide rectangular channel (noting that $1/y_n \to 0$, $R \approx y_n$, and $S_0 = S_f$ for a uniform flow), as :

$$\frac{dy}{dx} = -\left(\frac{Q^2}{b^2 C^2 y^3}\right)\left[\frac{1}{1 - \left(\frac{y_c}{y}\right)^3}\right]$$

or,

$$\frac{dy}{dx} = \frac{-\frac{q^2}{C^2 y^3}}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]} \qquad \text{...(5.22)}$$

where, b is the width of the channel, q is the discharge per unit width (discharge intensity) of the channel, and C is the Chezy's constant. As $y_n = \infty$, equation (5.22), in a straight forward manner, indicates two possible conditions of flow :

1) $y_n > y > y_c$, or
2) $y_n > y_c > y$

Situation (1) represents a subcritical flow with a drawdown curve as (dy/dx) remains negative.

In a channel with an adverse slope ($S_0 < 0$), equation (5.11) indicates that K_n^2 must be negative, i.e., K_n must be imaginary $\left[K_n = \left(\frac{1}{n}\right)\left(AR^{2/3}\right) = \left(\frac{1}{n}\right) b\, y_n \times y_n^{2/3}\right]$, for a wide rectangular channel, and hence y_n (and also y_n^3) should be negative. Thus, in equation (5.19), the term $\left(1 - \frac{y_n^3}{y^3}\right)$ is positive, but S_0 is having a negative value giving rise to following possible flow situations :

1) $y > y_c$, or
2) $y < y_c$

Condition (1) indicates a subcritical flow with a drawdown curve; and condition (2) indicates a supercritical flow with a backwater curve.

Considering Example 5.1 – sub-section 5.2.2 – the normal depth for the given discharge and channel section is 2.0 m. If the flow becomes gradually varied and the depth of flow at a given section is 2.50 m, then $(dy/dx) = 0.00097$, i.e., it is positive, and thus the flow profile is a backwater curve; and $y > y_n > y_c$, i.e., a subcritical flow occurring in a mild channel. Similarly, when in this gradually varied flow $y = 1.80$ m, $dy/dx = -0.00165$ (i.e., assumes a negative sign), a drawdown curve is indicated; and $y_n > y > y_c$ shows a subcritical flow occurring in a mild channel. If $y = 1.55$ m, which, is the critical depth for the given case, (dy/dx) tends to infinity, implying a hydraulic drop as discussed in sub-section 5.3.3.

In Example 5.2 ($y_n = 0.81$m, $y_c = 0.67$m), $(dy/dx) = 0.0093$ for $y = 0.4$ m – a supercritical flow occurring in a mild channel; and, as (dy/dx) is positive, the flow profile is a backwater curve.

5.3.2 Channel Slopes and Types of Curves

In Section 5.3, we discussed the various sets of conditions that generate a backwater curve, or a drawdown curve, as the case may be. These discussions are summarized in Table 5.2, and Figures 5.2 and 5.3; and also the behaviour of non-sustaining slopes is highlighted.

5.3.3 Special Features of Theoretical Flow Profiles

A) Discontinuity in the Profile

When $y = y_c$, equation (5.19) giving $dy/dx = \infty$, the flow profile will become vertical while intersecting the critical-depth line (CDL). Therefore, under these conditions, if the depth of flow changes suddenly from a low stage to a high stage crossing the CDL, a hydraulic jump will occur, representing a discontinuity in the flow profile. But, if the depth registers a change from high stage to a low stage, it is obvious that, a hydraulic drop will occur. It must be emphasized that when the depth of flow in the channel approaches CDL in the development of a flow profile, there occurs a sizeable curvature in the stream lines, and the pressure distribution in the flow field deviates significantly from hydrostatic law (hydrostatic distribution assumed to be prevailing under normal conditions). Thus, the flow becomes a rapidly varied flow (RVF), and the description of the profile, by GVF equation is no longer admissible.

B) Behaviour of Flow Profile at Specific Depths

It is important to recognize the behaviour of a flow profile at specific depths as per the theoretical considerations. This understanding helps envisage the measures necessary to optimise the training of a given flow with a view to achieve an efficient conveyance of water along the given channel.

When $y = \infty$, equation (5.19) shows that $(dy/dx) = S_0$, that is, the profile is horizontal. When $y = y_c$, as pointed out above, a hydraulic jump or a drop in the profile must occur under suitable circumstances. And, when $y = y_n$, $(dy/dx) = 0$, that is, the water surface is parallel to the bed of the channel, signifying a uniform flow; and, when $y = y_n = y_c$, the flow is uniform as well as critical.

Table 5.2 : Types of Flow Profiles in Prismatic Channels

Channel Slope	Designation			Relation of y to y_n and y_c			General Type of Curve	Type of Flow
	Zone 1	Zone 2	Zone 3	Zone 1	Zone 2	Zone 3		
Horizontal $S_o = 0$	None			$y > y_n > y_c$			None	None
		H2		$y_n > y > y_c$			Drawdown	Subcritical
			H3	$y_n > y_c > y$			Backwater	Supercritical
Mild $0 < S_0 < S_c$	M1			$y > y_n > y_c$			Backwater	Subcritical
		M2		$y_n > y > y_c$			Drawdown	Subcritical
			M3	$y_n > y_c > y$			Backwater	Supercritical
Critical $S_o = S_c > 0$	C1			$y > y_c = y_n$			Backwater	Subcritical
		C2		$y_c = y = y_n$			Parallel to channel bottom	Uniform-critical
			C3	$y_c = y_n > y$			Backwater	Supercritical
Steep $S_o > S_c > 0$	S1			$y > y_c > y_n$			Backwater	Subcritical
		S2		$y_c > y > y_n$			Drawdown	Supercritical
			S3	$y_c > y_n > y$			Backwater	Supercritical
Adverse $S_o < 0$	None			$y > (y_n)^* > y_c$			None	None
		A2		$(y_n)^* > y > y_c$			Drawdown	Subcritical
			A3	$(y_n)^* > y_c > y$			Backwater	Supercritical

* yn is assumed to be positive

C) Flow Profile at $y = 0$

When $y = 0$, equation (5.19) seems to assume an indeterminate form, i.e., $dy/dx = \infty/\infty$. It is interesting to note that the theoretical behaviour of the flow profile, at or near $y = 0$ is influenced by the particular resistance formula (i.e., uniform flow formula) that is chosen to be used. Equation (5.18) gives $(dy/dx) = \infty$ when $y = 0$ – leading to the result that the flow profile is vertical at the channel bottom (or, more precisely, is perpendicular to the bed), while equation (5.19) gives

$$\frac{dy}{dx} = S_0 \left(\frac{y_n}{y_c}\right)^3 \text{ for } y = 0\text{, i.e., the flow profile, at the bottom } (y = 0),$$

assumes a definite inclination $\left[\left\{ = S_0 \left(\frac{y_n}{y_c}\right)^3 \right\}\right.$, which is other than being vertical$\left.\right]$ with respect to the channel bed.

5.4 CLASSIFICATION OF WATER SURFACE PROFILES

We have seen that a gradually varied flow profile may, broadly speaking, either be a backwater curve or a drawdown curve. However, each particular profile, as shown in Table 5.2 is given a definite nomenclature for the sake of appropriate designation or identification. These identifications are based on the parameters as discussed below.

5.4.1 Basis of Classification of Flow Profiles

In a given channel, normal-depth and critical-depth lines, corresponding to a given discharge, can be determined dividing the space in the channel into three zones [Figures (5.2) and (5.3)], such as :

	Description	*Nomenclature*
1)	Space above the top most line	Zone 1
2)	Space between the two lines	Zone 2
3)	Space between the bed and the lowest line	Zone 3

It is clear, therefore, that thirteen flow profiles (Table 5.2), depending on the bed slope and the zone in which the profile lies, are possible to be identified, such as: H2, H3; M1, M2, M3; Cl, C2, C3: S1, S2, S3; and A2, A3 — the letter being descriptive of the type of slope (horizontal, mild, critical, supercritical, and adverse), and the numeral designating the zone in which the profile falls. It is to be noted that, out of the these water profiles, twelve correspond to gradually varied flow profiles, while one, namely, C2 represents a uniform flow condition. As mentioned earlier, Table 5.2 delineates the general features of these profiles, while the Figure 5.2 or 5.3 presents their respective shapes. Since the portions of profiles near the critical depth and the channel bed cannot be accurately defined by the theory of gradually varied flow, these are shown with short dashed or dotted lines.

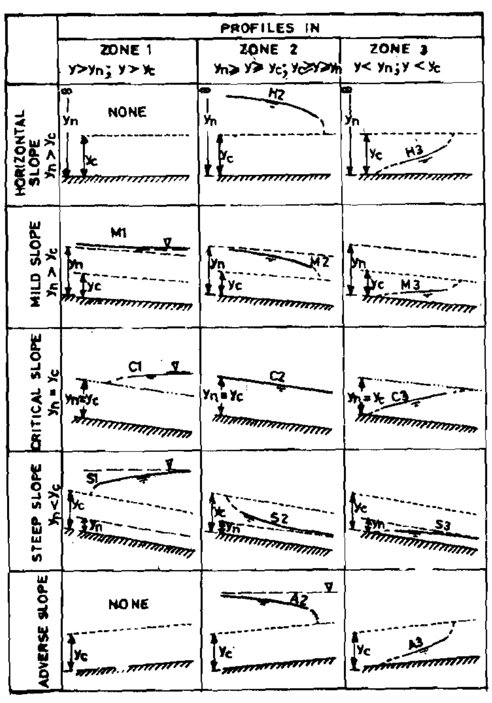


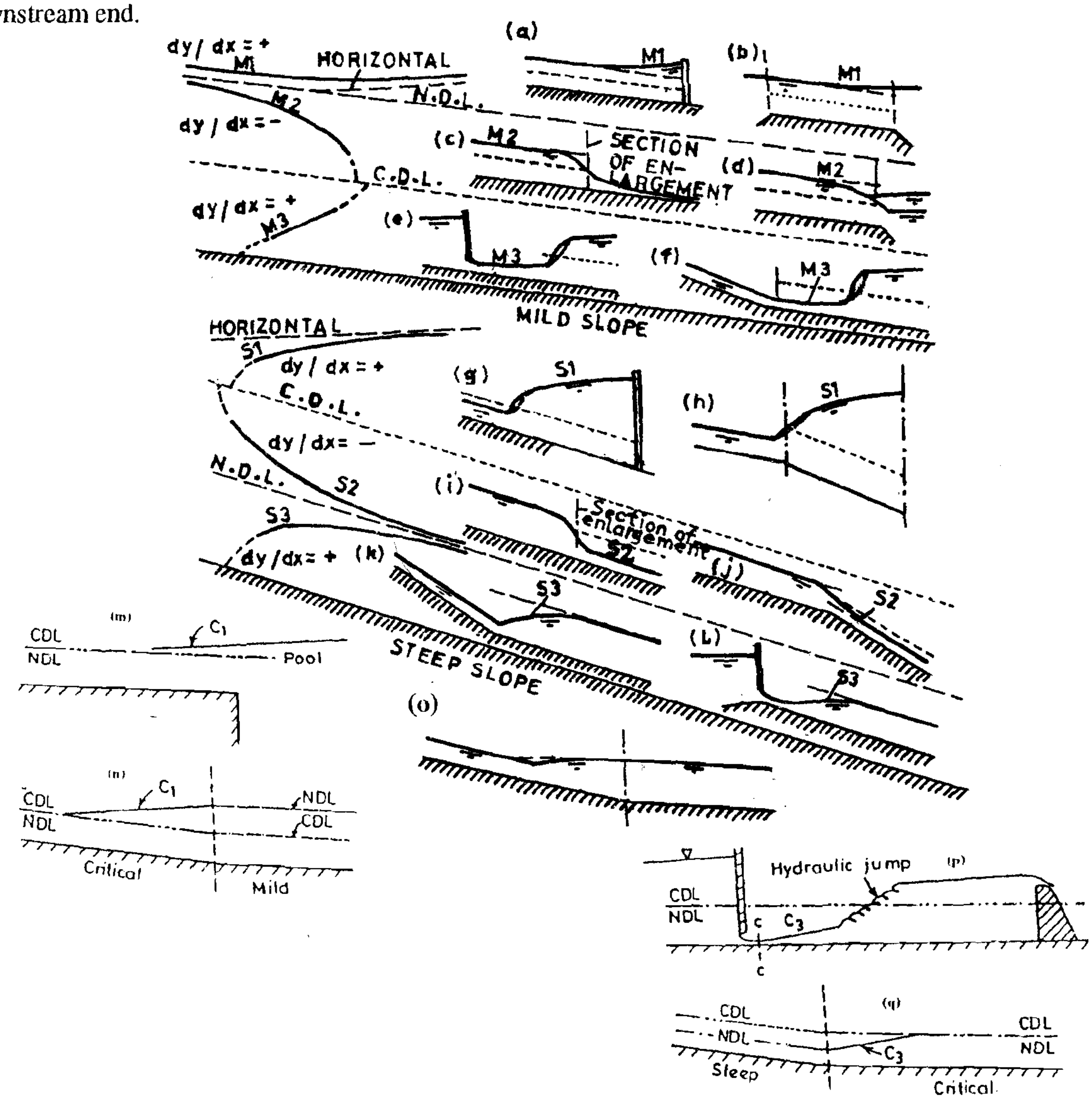
Figure 5.2 : Classification of Flow Profiles of Gradually Varied Flow

5.4.2 Flow Profiles, vis-a-vis, Practical Situations

A) **M–Profiles** ($S_0 < S_c$, and $y_n > y_c$)

The M1 profile represents the well-known backwater curve, and it has its own importance from a practical point of view. Water surface assumes this profiles, e.g., when the downstream end of a long mild-sloped channel is submerged into a reservoir to a depth greater than the normal depth of flow in the given channel. This flow profile lies in Zone 1, with the upstream end of the curve being tangential to the NDL (since $dy/dx = 0$ for $y = y_n$), and the downstream end being tangential to the given pool surface (generated by a dam, or any other similar obstruction to flow) because $dy/dx = S_0$ at $y = \infty$. Typical examples of M1 profile are water surface formations behind a dam (as mentioned above) on a natural river (Figure 5.3 (a)), and the profile in a canal joining two reservoirs (Figure 5.3 (b)). The overall shape of the profile is influenced by the slope of the profile at the limiting values of the depth of flow, namely, upstream and downstream depths of flow.

An M2 profile occurs when there is a sudden enlargement of a canal section (Figure 5.2 (c)), or when the downstream end of the channel bed is submerged in a reservoir to a depth less than the normal depth (Figure 5.2 (d)). Here also, as usual, the upstream end of the profile is tangential to the normal - depth line (NDL) — i.e., $dy/dx = 0$ at $y = y_n$. If the reservoir water level is less than y_c (corresponding to the given discharge), the profile will terminate abruptly at a depth equal to y_c (because, the bed slope cannot sustain a supercritical flow), meeting the limiting depth, at the downstream end, non-tangentially, but being tangential to a vertical line (Figures 5.3 (d) and 5.3 (e)) — since, $dy/dx = \infty$ at $y = y_c$. The student, by now, will easily interpret the situation as a case involving the formation of a hydraulic drop. However, in case the downstream limiting depth (i.e., the depth of submergence) is greater than the critical depth (y_c), then only as much of the M2 profile will appear as lies above the reservoir water- surface level. It can, also, be stated that, in fact, the extreme limiting values of depth in respect of M2 profile are : y_n at the upstream end, and y_c at the downstream end.



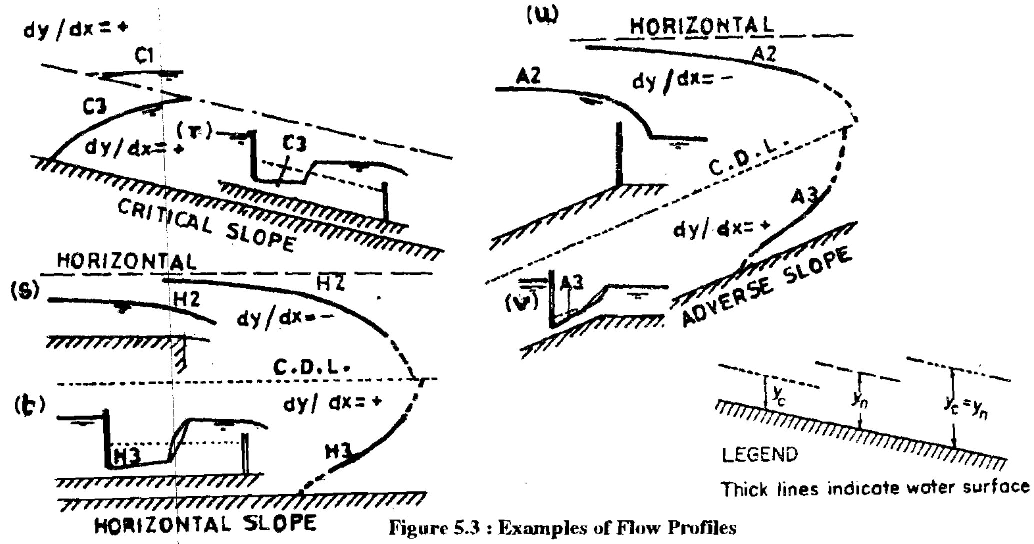


Figure 5.3 : Examples of Flow Profiles

The M3 profile, theoretically speaking, takes off from the upstream channel bottom at either a vertical or an acute angle, depending on the uniform flow formula used (sub-section 5.3.3) and terminates with a hydraulic jump at the downstream end. M3 profiles are usually exhibited when a supercritical flow enters a mild channel (i.e., a mild-sloped channel for the given Q) — Figures 5.3 (e), and 5.3 (f). The trace of the curve, at its commencement cannot be defined precisely, but all the same it is dependent on the initial issuing velocity of water with which it enters the channel — the greater the velocity, the farther downstream the profile commences itself to form. Theoretically, the upstream end of the profile begins at $y = 0$, and thus it is obvious that this geometry of the trace can never exist in reality. Examples of M3 profile are : the profile in a stream downstream of a sluice (Figure 5.3 (e)), and the profile that comes into being after the channel bed undergoes a change in slope from steep to mild (Figure 5.3 (f)).

B) **S–Profile** ($S_0 > S_c$ **and** $y_n < y_c$

The S1 profile is a backwater curve. It starts with a hydraulic jump at the upstream end of the profile, becoming tangential to the horizontal pool level at the downstream end. Examples of this class of profiles are observed behind a dam on a steep channel (Figure 5.3 (g)), and in a steep channel emptying into a pool of high water surface elevation (Figure 5.3 (h)).

S2 profile is a drawdown curve. It is usually short in length, like a transition between a uniform flow and a hydraulic drop, starting with a vertical slope at the critical depth and progressing towards being tangential to the NDL at the downstream end. Profiles formed on the downstream side of an enlargement in the channel (Figure 5.3 (i)), and also on the steeper-slope side of the channel as the bed slope changes from steep to steeper one (Figure 5.3 (f)) furnish the relevant examples of S1 profile.

S3 Profile is a backwater curve, being a transition curve connecting tangentially a supercritical flow to the NDL. Profiles on the steep-slope side where the channel slope changes from steep to less-steep slope (Figure 5.3 (k)), and those formed downstream of a sluice with the depth of the entering flow being less than the normal depth on a steep slope (Figure 5.3 (l)) fall under this classification.

C) **C–Profile** ($S_o = S_{c\,i}$ **and** $y_n = y_c$)

C1 and C3 profiles are both backwater curves, while C2 represents a uniform critical flow. These profiles are identified as representing GVF conditions that lie between mild- and steep-channel profiles.

Bed slope of a given channel becomes the critical slope for one particular discharge; thus, occurrence of critical-slope profiles is rare in nature. C1 profile is formed if the downstream depth of flow is greater than the critical depth, as shown in Figures 5.3 (m), 5.3 (n) and 5.3 (o). C3 profiles occur on a critical slope when a shooting flow meets the critical or subcritical depth through a jump — Figures 5.3 (p), 5.3 (q), 5.3 (r). However, it is to be noted that C3 curve (meeting the subcritical depth through a jump) will come up only if the subcritical depth, prior to the jump formation, is less than the conjugate depth corresponding to the approaching supercritical flow.

If Chezy's coefficient, C, is assumed to be constant, for a wide rectangular channel (with reference to critical slope), we have from equation (5.9) :

$$\frac{dy}{dx} = S_c \left[\frac{1 - S_f/S_c}{1 - F_r^2} \right] \qquad \text{...(5.23)}$$

But,

$$q = y_c \, (C \sqrt{(y_c S_c)}\,) \qquad \text{...(5.24)}$$

also,

$$q = y\,(\sqrt{(y S_c)}\,) \qquad \text{...(5.25)}$$

Dividing (5.25) by (5.24) :

$$S_f/S_c = \left(\frac{y_c}{y} \right)^3 = \left[\frac{q^2/g}{y^3} \right]$$

$$= \frac{q^2}{g\,y^3} = \frac{V^2}{gy} = F_r^2$$

Therefore, equation (5.23) reduces to :

$$\frac{dy}{dx} = S_c = \text{a constant} \qquad \text{...(5.26)}$$

This shows that C1 and C3 water surface profiles are straight, and horizontal curves; put in other words, we can say that since the variation of C in the range of depths obtained at the two ends of the profile is generally small, C1 and C3 profiles are practically straight. However, using Manning's formula, equation (5.18) leads to the conclusion that C1 and C3 profiles are curved, with C1 being asymptotic to the horizontal.

D) **H–Profiles ($S_0 = 0$ and $y_n = \infty$)**

H-profiles are the limiting manifestations of M-profiles with the channel bottom becoming horizontal. H2 and H3 profiles correspond to M2 and M3 profiles, respectively; but the concept of H1 profile is far from being near reality, since $y_n = \infty$ is an unreal situation. Figures 5.3 (s) and 5.3 (u) present examples of H profiles.

E) **A–Profiles ($S_0 < 0$)**

The occurrence of A1 profile is not possible because y_n is obviously not a real entity in an adverse slope which is evident from any uniform flow formula that may be considered. A2 and A3 profiles are similar to H2 and H3 profiles, respectively. Figures 5.3 (u) and 5.3 (v) present some examples of A profiles.

It may be mentioned here, in passing, that the flow profiles in conduits of gradually closing top (circular, etc.) have not been discussed at this stage of the course development. The student may take up this exercise on his own, if he so desires.

SAQ 1

With reference to Example 5.2, designate the water surface profile obtaining in the channel.

SAQ 2

A rectangular channel 4.5 m wide having a bed slope of 0.00009 carries water at a depth of 1.4 m. If the channel terminates in an abrupt drop in the bed, what type of flow profile, upstream of the drop, is expected to form ? Assume $n = 0.016$.

SAQ 3

A long rectangular channel, of 15.5 m width and a bed slope of 10^{-4}, connects two reservoirs of water. If depth of water at the upstream reservoir is 1.6 m, and at the downstream reservoir it is maintained at 2.1m, what type of water surface is expected to form in the channel ? Take $n = 0.016$.

SAQ 4

A discharge of 10.85 m^3/s is issued, from under a sluice gate of a reservoir into a trapezoidal channel of side slope 1:1 which has its bed width equal 5.75 m. The depth of flow at vena contracta was measured to be 0.16 m. If the channel bed slope is $(10)^{-2}$ and $n = 0.014$, determine which type of flow profile will form downstream of the vena contracta.

5.5 SUMMARY

In this unit we started with the definition of gradually varied flow and derived the dynamic equation for this type of flow. This equation gives the slope of the water surface (dy/dx) at any desired location along the length of the channel. When (dy/dx) is positive, the flow profile is said to be a backwater curve, indicating increasing depth of flow along the channel length; and, when $\left(\frac{dy}{dx}\right)$ is negative, the flow is said to be a drawdown curve, implying decreasing depth of flow along the channel length. We know that any depth of flow, however, will tend to approach the uniform flow depth (appropriate to the given discharge, bed slope, roughness condition) if the conditions are favourable. The shape of a flow profile is basically determined by the value of y_n and y_c, as well as the initial and final existing depths, i.e., the given end conditions. You have, thus, learnt about the gradually varied flow, classification of flow profiles and the shape of all the possible profiles.

We also considered examples to demonstrate the computation of the slope of water surface at any particular depth and identify the profiles so formed in the channel.

5.6 KEY WORDS

Gradually Varied Flow (GVF) : A gradually varied flow in an open channel is characterised by the depth of flow varying along the flow direction over a long distance.

Rapidly Varied Flow (RVF) : The flow is rapidly varied if the depth of flow changes abruptly over a comparatively short distance in an open channel. It is, therefore, a local phenomenon unlike a GVF.

Wide Rectangular Channel : It is a rectangular channel, whose width is greater than 5 to 10 times the depth of flow because of the existence of a central region where the influence of wall roughness is negligible (Refer sub-section 2.2.6 of Block 1 of Open Channel Flow); moreover, y and even 2y are negligible compared to the bed width (b).

Normal – Depth Line (NDL) : A line parallel to the bed of the channel, indicating the water surface profile at the normal depth of flow (for a particular discharge) when the channel is prismatic, and without any obstruction to flow.

Critical – Depth Line (CDL) : A line parallel to the bed of the channel, indicating the water surface profile at critical depth of flow (for a particular discharge) when the channel is prismatic. This line will coincide with NDL if the bed slope is critical otherwise it is simply a conceptual line.

5.7 ANSWERS TO SAQs

SAQ 1

y_n (= 0.81m) > y_c (= 0.67 m); thus, the channel slope is mild. Considering the depth of flow y = 0.4 m, we have:

$$y_n > y_c > y$$

Thus, the profile lies in Zone 3, and, hence, a formation of an M3 profile takes place.

SAQ 2

From Manning's equation,

$$V = \frac{1}{n} R^{2/3} \; S_0^{1/2}$$

$$= \frac{1}{0.016} \left[\frac{4.5 \times 1.4}{(4.5 + 2 \times 1.4)}\right]^{2/3} (0.00009)^{1/2}$$

$$= 0.54 \text{ m/s}$$

$$q = Vy = 0.54 \times 1.4$$

$$= 0.76 \text{ m}^3/\text{s}/\text{m}$$

$$F_r = \frac{V}{\sqrt{(gD)}} = \frac{0.54}{\sqrt{(9.81 \times 1.4)}}$$

$$= 0.146 < 1$$

As the flow is subcritical, the bed slope of the channel is taken to be mild for the given discharge; hence, from the knowledge of the principles of hydraulics we know that the depth of flow at the overfall will be critical, such that :

$$y_c = \sqrt[3]{q^2/g} = \left[\frac{0.76^2}{9.81}\right]^{1/3} = 0.39 \text{ m}$$

Thus, the profile lies in Zone 2 of a mild slope, i.e., it will be an M2 profile.

SAQ 3

$$Q = \frac{A}{n} R^{2/3} \; S_0^{1/2} = \frac{15.5 \times 1.6}{0.016} \left[\frac{15.5 \times 1.6}{15.5 + 2 \times 1.6}\right]^{2/3} \left(10^{-4}\right)^{1/2}$$

$$= 18.71 \text{ m}^3/\text{s}$$

Now, $y_C = \sqrt[3]{(q^2/g)} = \sqrt[3]{[(18.71/15.5)^2 / 9.81]}$

$$= 0.53 \text{ m}$$

$$\therefore \quad y_2 \; (= 2.1 \text{ m}) > y_1 \; (= 1.6 \text{ m}) > y_c \; (= 0.53 \text{ m})$$

Thus, it is clear that the slope is mild for the given discharge; hence, an M1 profile formation will take place.

SAQ 4

For the determination of y_c in the trapezoidal channel we proceed by trial and error method, starting with an approximate value of

$$y_c = \sqrt[3]{(Q^2/b^2\,g)} = \left[\frac{(10.85)^2}{(5.75)^2 \times 9.81}\right]^{1/3} = 0.71 \text{ m}$$

(however, the actual value of y_c should be less than this approximate value because of the influence of side slopes) :

Trial No	y (m)	T (m)	A (m^2)	$D=A/T$ (m)	$V = Q/A$ (m/s)	Fr $= V/\sqrt{gD}$
1	0.69	7.13	4.44	0.623	2.44	0.987
2	0.68	7.11	4.37	0.615	2.48	1.0096 ≈ 1.00

Therefore, taking $y_c = 0.68$ m, which is greater than the depth at the vena contracta (0.16 m), the flow is supercritical immediately downstream of the sluice.

Now, for the determination of y_n we proceed as follows :

$$Q = \frac{A}{n}\, R^{2/3}\, S^{1/2}$$

$$\text{or } 10.85 = \frac{(5.75 + y)\,y}{0.014}\left[\frac{(5.75 + y)\,y}{(5.75 + 2\sqrt{2}y)}\right]^{2/3} (10^{-2})^{1/2}$$

Again, by trial and error procedure we get :

$$y_n = 0.452 \text{ m}$$

However, it may be pointed out that using the relevant curves, as applicable to trapezoidal channels (namely, (zy_c/b) vs $(z^{1.5}\,Q/g^{0.5}\,b^{2.5})$, and (y_n/b) vs $(Z_1/b^{8/3})$, respectively) one can easily determine the required values of y_c and y_n ; and, here we also get $y_n = 0.452$ m,

Thus, we observe :

$y_n < y_c$, and, therefore, the channel slope is steep for the given discharge. The depth at the vena contracta (0.16 m) $< y_n < y_c$. Therefore, S3 profile will form downstream of the vena contracta.