# UNIT 4 MOMENTUM PRINCIPLE

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## 4.1 INTRODUCTION

Amongst various rapidly varying flow situations (an overfall,a hydraulic drop, and a hydraulic jump) a hydraulic jump has its own importance in an open channel flow. Sometimes occurrence of a jump is engineered in water conveyance systems, in order to dissipate excessive kinetic energy in the approaching flow field, or raise the downstream water level to meet the utility requirement in a water distribution system.

In the analysis of a hydraulic jump formation,that occurs in a short reach of the channel it is difficult to directly account for the external forces such as channel boundary friction, and weight effect of water. Therefore, obviating this difficulty, the application of momentum principle comes handy.

The momentum principle is very often applied to open channel flow problems, and is described in this unit; also the concept of specific force is introduced. The formation of hydraulic jump is described in some detail, and the characteristics of a hydraulic jump are explained with the help of problems.

### Objectives

After studying this unit, you should be able to

- describe the concept of momentum as applied to open channel flow,
- elaborate the requirements for a hydraulic jump to form,
- distinguish the relationship between the sequent depths, and
- calculate the energy loss due to the formation of a hydraulic jump.

## 4.2 CONCEPT OF SPECIFIC MOMENTUM/SPECIFIC FORCE

Applying Newton second law of motion and using the control volume concept from elementary fluid mechanics, we can easily derive an expression for specific momentum (also known as specific force function) in steady channel flow. We realise that there is an unknown energy loss between any two sections under consideration. The result is a change in the linear momentum of the flow. In many cases this change in momentum is accompanied by a change in the depth of flow. With the assumptions that the channel slope (angle $\theta$) is small and hence, $\sin\theta = 0$ and $\cos\theta = 1$, and the momentum correction coefficients, $\beta$'s, are equal to unity we can show (Try yourself !) :

$$M = \frac{Q^2}{gA} + \bar{z}A \qquad \dots (4.1)$$

where, $M$ is known as the specific momentum or specific force, or force function and $\bar{z}$ is the distance to the centroid of flow area $A$ below the free water surface.

Plotting the depth of flow y versus specific momentum $M$, for a given $Q$ produces a specific momentum curve having two limbs (Figure 4.1) The lower limb CA approaches the specific momentum axis ($x$ axis) asymptotically to the right. The upper limb CB rises upward and extends indefinitely to the right. Thus, in analogy with the concept of specific energy for a given value of $M$, the $M$-$y$ curve predicts two possible depths of flow ( $y_1$ and $y_2$). These depths shown in Figure 4.1, are termed as sequent depths of a hydraulic jump.

The minimum value of $M$ ( $M_c$ ) can be found under the assumptions of parallel flow and uniform velocity distribution. Taking the first derivative of $M$ with respect to $y$ and setting the resulting expression to zero ( Try yourself !) rearranging the terms and simplifying, we can easily show that

$$\frac{V^2}{2g} = \frac{D}{2} \qquad \text{...(4.2)}$$

which is the same criterion as developed for the minimum value of specific energy, i.e.,the critical state of flow. Thus for a given discharge, minimum specific momentum (i.e., minimum specific force ) occurs at minimum specific energy or critical depth.

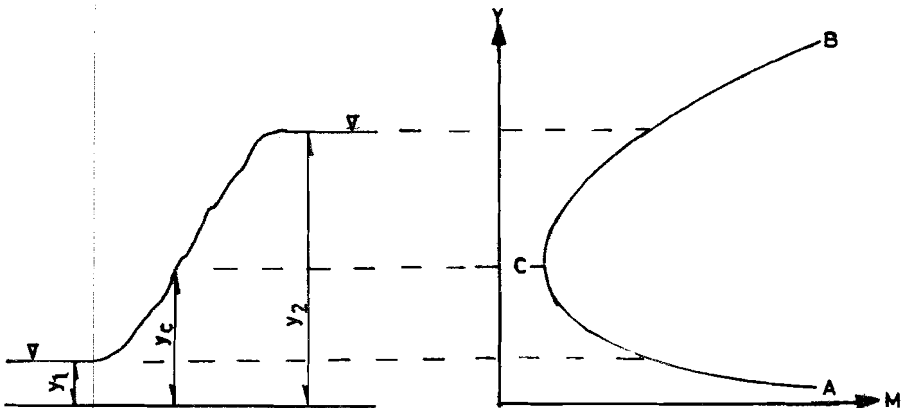


**Figure 4.1 : Specific Momentum Curve and Sequent Depths $y_1$ and $y_2$ of a Hydraulic Jump**

Let us compare the specific energy curve with the specific force curve (Figure 4.2). The low stage depth $y_1$ in supercritical flow and the high stage depth $y_2$ in subcritial flow are alternate depths on the $E - y$ curve. The specific force curve also indicates two possible depths known as sequent depths, the initial depth and the final depth. Let us assume for comparison that the low stage depth in $E - y$ curve and initial depth in $M - y$ curve are both equal to $y_1$.

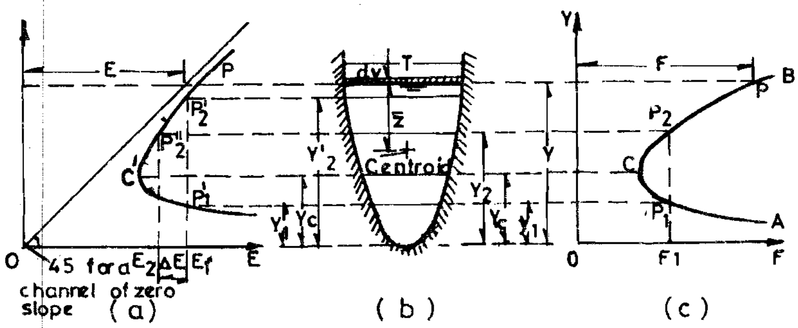


**Figure 4.2 : Specific Force Curve in Comparision with Specific Energy Curve**
**[ (a) Specific Energy Curve (b) Channel Section (c) Specific Force Curve ]**

We can see that the two curves jointly indicate that the final sequent depth $y_2$ is always less than the high stage alternate depth $y_2$. Furthermore, the $E - y$ curve shows that the energy content $E_2$ for the depth $y_2$ in momentum curve is less than the energy content $E_1$ for the depth $y_1$. Therefore, in order to maintain a constant value of $M$ the depth of flow has to be changed from $y_1$ to $y_2$ at the price of losing a certain amount of energy equal to $\Delta E$ given by :

$$\Delta E = E_1 - E_2 \qquad ...(4.3)$$

An example of such an energy change is afforded by the phenomena of a hydraulic jump.

A similarity between the applications of the energy and momentum principles need not confuse us while a clear understanding of the basic difference is kept in mind . The inherent distinction lies in the fact that energy is a scaler quantity whereas momentum is a vector quantity. Also, the energy equation contains a term for internal loss,whereas the momentum equation contains a term for external resistance!

Though, in the first instance, the energy principle seems to offer a clear explanation,but it is the momentum principle alone that accounts for the high internal energy change taking place in a specific situation like that of a long hydraulic jump (Figure 4.3). Hence, the importance of momentum principle in analysing such flow situation cannot be over emphasised.

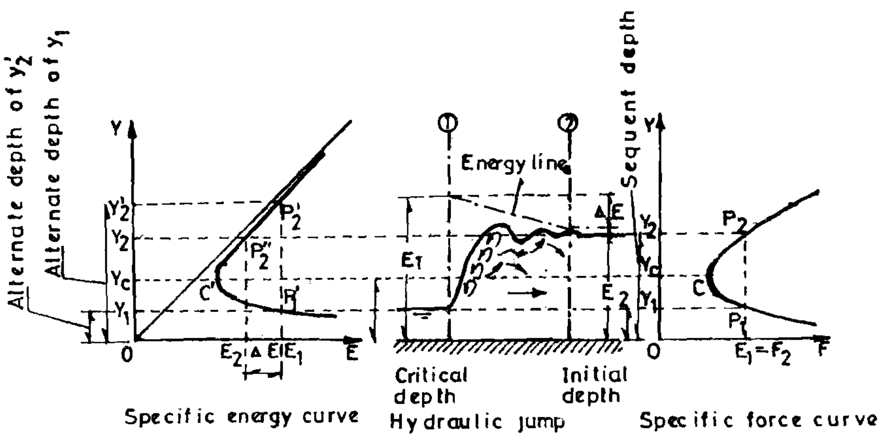


**Figure 4.3 : Hydraulic Jump Interpreted by Specific Energy and Specific Force Curves**

## 4.3 HYDRAULIC JUMP

A hydraulic jump consists of a sudden (i.e., over a short distance) rise in water level – i.e., R.V.F. – from a supercritical flow ( shooting flow) at the upstream side, to a subcritical flow at the downstream side. This situation may be caused due to the existence of various flow control devices; change of channel section, slope, or bed profile in an open channel.

Experimental evidence suggests that flow changes, from a supercritical to a subcritical state occurring through a hydraulic jump, are characterised by significant turbulence and therefore, energy dissipation. The applications of hydraulic jump are many, such as :

1. Discipation of energy in flow over dams, weirs and other hydraulic structures,
2. Maintenance of high water levels in channels for water distribution purposes,
3. Increase of the discharge of a sluice gate by repelling the downstream tailwater and thus increasing the effective head across the gate,
4. Reduction of uplift pressure under structures by raising the water depth on the apron of the structure,
5. Mixing of chemicals used for water purification or waste water treatment, or for manufacturing various chemicals,
6. Aeration of flows and the dechlorination of waste waters,
7. Removal of air pockets from flows in circular channels, and
8. Identification of special flow conditions such as the existence of supercritical flow for locating a control section for cost effective measurement of flow.

### 4.3.1 Sequent Depths

The specific momentum equation is the basis for analysis and computation of elements of a hydraulic jump. If the jump occurs in a channel with a horizontal bed (and ignoring horizontal component of any unknown force), we can write

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \qquad ...(4.4)$$

where, suffixes 1 and 2 refer to the upstream and downstream sections respectively.

**Rectangular Sections**

For a rectangular channel of width $b$, carrying a discharge intensity ( i.e., discharge per unit width) of $q$, the sequent depths $y_1$ and $y_2$, can be shown to be related as follows (Try yourself !) :

$$\frac{q^2}{g\left[\frac{1}{y_1} - \frac{1}{y_2}\right]} = \frac{1}{2}\left[ y_2^2 - y_1^2 \right] \qquad ...(4.5)$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[ \sqrt{(1 + 8 F_{r1}^2)} - 1 \right] \qquad ...(4.6)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left[ \sqrt{(1 + 8 F_{r2}^2)} - 1 \right] \qquad ...(4.7)$$

Here, we have three independent variables $y_1$, $y_2$ and $F_{r1}$. Two of them must be known before the third can be found. We understand that the downstream depth $y_2$ is not the result of upstream conditions but is the result of a downstream control. Thus, if the downstream control produces the depth $y_2$, then a jump will form.

At the risk of belabouring the obvious again, it may be stated that the use of these equations in solving hydraulic jump problems in rectangular channels is simple and straightforward, but significant computational difficulties can arise, when $F_{r2}^2$ is very small with the term $[\sqrt{1 + 8 F_{r2}^2} - 1]$ approaching zero. This difficulty can be avoided by expressing this term as a binomial series expansion. Substitution and simplification leads to :

$$\frac{y_1}{y_2} = 2 F_{r2}^2 - 4 F_{r2}^4 + 16 F_{r2}^6 - \ldots \qquad ...(4.8)$$

This expansion is valid only when $F_{r2}$ is very small and $F_{r2}^2 < 0.05$.

### 4.3.2 Submerged Jump

We discussed for a specified supercritical upstream condition, the determination of the required downstream depth of flow if a hydraulic jump is to occur. If the actual downstream depth $y_4$ is less than the sequent depth $y_2$ (i.e., the depth required for the formation of a jump) then a jump will not form and the flow will continue to be supercritical. If the downstream depth is greater than $y_2$ then a submerged jump (drowned jump) is formed which remains enveloped under the existing downstream water surface. Such jumps are commonly encountered downstream of gates in irrigation systems. The crucial unknown in such situations is the depth of submergence $y_3$.

Govinda Rao (1963) using the principles of conservation of momentum and mass demonstrated that in horizontal rectangular channels :

$$\frac{y_3}{y_1} = \left[ (1 + S)^2 \Phi^2 - 2 F_{r1}^2 + \frac{2 F_{r1}^2}{(1 + S)\Phi} \right]^{1/2} \qquad ...(4.9)$$

where $S$, the submergence factor $= \dfrac{y_4 - y_2}{y_2}$,

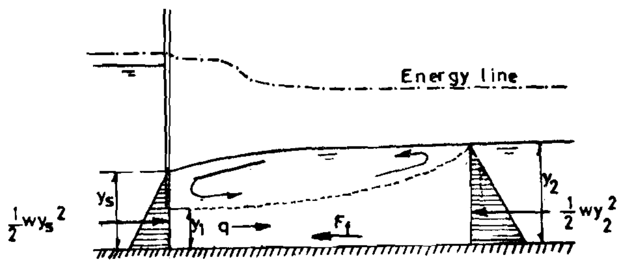


**Figure 4.4 : Definition Sketch of a Submerged Hydraulic Jump**

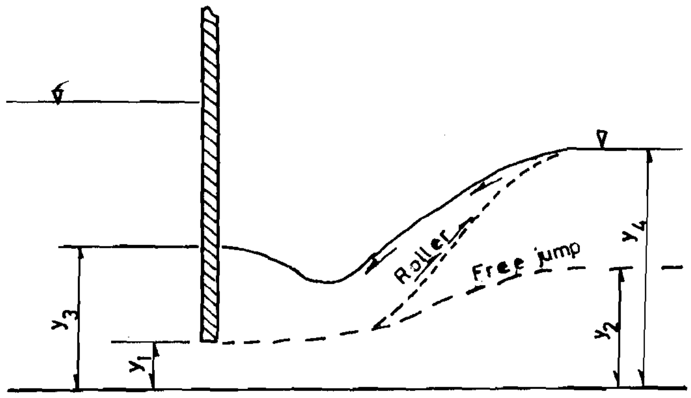


**Figure 4.5 : A Submerged Hydraulic Jump at Sluice Outlet**

$$\Phi = \frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{(1 + 8\,F_{r1}^2)} - 1\right)$$

and, $y_2$, as usual, is the subcritical sequent depth of the free jump corresponding to $y_1$ and $F_{r1}$. Govinda Rao verified this equation with the help of laboratory experiments. For the same situation Chow (1959) gave the following equation:

$$\frac{y_3}{y_4} = \left[1 + 2\,F_{r4}^2\left(1 - \frac{y_4}{y_1}\right)\right]^{1/2} \qquad \text{...(4.10)}$$

where, $F_{r4}$ is the Froude Number corresponding to depth $y_4$.

Estimates of $y_3$ from both these equations (4.9) and (4.10) are comparable.

### 4.3.3 Energy Loss

The primary utility of a hydraulic jump, in engineering problems, is to cause an intense dissipation of energy, where ever deemed necessary. In a horizontal channel, the change in energy (i.e., loss of energy ) across a jump is given by :

$$\Delta E = E_1 - E_2 \qquad \text{...(4.3)}$$

where,

$\Delta E$ = change in energy from section 1 to 2,

$E_1$ = specific energy at section 1, and

$E_2$ = specific energy at section 2.

In the case of a horizontal rectangular channel it can be shown that

$$\Delta E = \frac{\left(y_2 - y_1\right)^3}{4\,y_1\,y_2} \qquad \text{...(4.11)}$$

and,

$$\frac{E_2}{E_1} = \frac{\left[(8\,F_{r1}^2 + 1)^{3/2} - 4\,F_{r1}^2 + 1\right]}{\left[8\,F_{r1}^2\left(2 + F_{r1}^2\right)\right]} \qquad \text{...(4.12)}$$

### 4.3.4 Length of Hydraulic Jump

The length of a hydraulic jump ($L_j$ ) is a crucial parameter while designing the channel flows downstream of high flow velocity fields (spillways, control gates, etc.). It is not possible to estimate its length from theoretical considerations. $L_j$ is defined as the distance from the upstream edge (i.e., the start of the jump) to a point immediately downstream of the roller associated with the jump (i.e., where the jump ends).

Silvester presented a formula to estimate the length of a free jump, on a horizontal channel having a rectangular section,

$$\frac{L_j}{y_1} = 9.75\,(F_{r1} - 1)^{1.01} \qquad ...(4.13)$$

Other procedures, based on experimental data (due to USBR, and Rajaratnam), are also available for the estimation of $L_j$. The length of a roller ($L_r$) in a submerged jump can be estimated by (Stepanov, 1959) :

$$\frac{L_r}{y_c} = \frac{3.31}{\left[\dfrac{y_4 - y_3}{y_3 F_{r1}}\right]^{0.895}} \qquad ...(4.14)$$

This equation was shown to be valid for $S \le 2$ and $1 \le F_{r1} \le 8$ (Rajaratnam, 1967).

## 4.4 ILLUSTRATIVE PROBLEMS

**Example 4.1**

In a horizontal rectangular channel, 1.2 m wide, determine whether a well defined and free jump will be formed if the flow in the channel is 0.14 $m^3/s$; and $y_1 = 0.018$, and $y_2 = 0.39$ m.

**Solution**

$$A_1 = b \times y_1 = 1.2 \times 0.018 = 0.022 \text{ m}^2$$

$$V_1 = Q/A_1 = 0.14/0.022 = 6.36 \text{ m/s}$$

$$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{6.36}{\sqrt{(9.81 \times 0.018)}}$$

$$= 15.14 > 1$$

Therefore, requred requent depth,

$$y_2 = \frac{y_1}{2}\left(\sqrt{(1 + 8F_{r1}^2)} - 1\right)$$

$$= \frac{0.018}{2}\left(\sqrt{(1 + 8 \times 229.22)} - 1\right)$$

$$= 0.377 \approx 0.38 \text{ m}$$

Therefore, there is a possibility of a jump formation. However, it will be a mildly repelled jump till the oncoming flow attains a value of $y_1 > 0.018$ (due to bed and side friction) for which the sequent depth shall be 0.38 m.

## SAQ 1

Water flows in a horizontal channel with a velocity of 8.0 m/s at a depth of 1.0 m. Find the conjugate depth and the energy loss in the jump.

### SAQ 2

After flowing over a spillway, 4.2 $m^2/s$ water passes over a level concrete apron ($n$ = 0.013). The velocity at the foot of the spillway is 12.5 m/s and the tail water depth is 3.0 m. In order that the jump be contained on the apron, how long should it be built? How much energy is lost from the foot of the spillway to the downstream end of the jump ?

### SAQ 3

A hydraulic jump occurs is a rectangular channel and the depths of flow before and after the jump are 0.5 m and 2.0 m respectively. Calculate the critical depth and the power lost per unit width of the channel.

## 4.5 SUMMARY

In this unit (Unit 4) the concept of specific force (or specific momentum) was explained. Hydraulic jump, as it occurs in a rectangular open channel was defined and its importance emphasised ; moreover various relavant formulae were derived or presented to define the various elements of the jump, as well as, the loss of energy suffered through the jump. The use of these formulae was explained through a solved example.

Formation of a submerged jump (an inefficient energy dissipating device) was also disscussed briely.

## 4.6 KEY WORDS

**Specific Force** : Sum of the momentumof flow per weight of water and force for unit weight of water, as applied at any flow section of an open channel.

**Hydraulic Jump** : When a shooting flow ($F_r > 1$), meets a quiescent flow ($F_r < 1$),there is a sudden (over a comparatively short distance) rise of water surface from upstream to downstream sections in the channel, and a hydraulic jump is said to have been formed .

**Sequent Depth** : Downstream depth required to form a well defined jump when the upstream supercritical flow meets the downstream flow. Sequent depth is also known as conjugate depth. $y_1$ and $y_2$ are the two sequent (or two conjugate) depths with reference to a hydraulic jump.

## 4.7 ANSWERS/ SOLUTIONS TO SAQs

### SAQ 1

$$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{8}{\sqrt{9.8 \times 1.0}} = 2.555$$

Hence the flow is supercritical.

$$\frac{y_2}{y_1} = \frac{1}{2}\left( \sqrt{1+8F_{r1}^2} - 1 \right)$$

$$= \frac{1}{2}\left( \sqrt{1+8 \times 2.555^2} - 1 \right) = 3.148$$

$$E_L = \frac{(y_2 - y_1)^3}{4\, y_1\, y_2} = \frac{(3.147 - 1.0)^3}{4 \times 3.144 \times 1.0} = 0.786 \text{ m}$$

**SAQ 2**

The depth of flow $y_1$ at the foot of the spillway $= \frac{4.2}{12.5} = 0.336$ m

$$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{12.5}{\sqrt{9.8 \times 0.336}} = 6.89$$

Hence the flow is supercritical.

$$V_2 = \frac{4.2}{3.0} = 1.4 \text{ m/s}$$

$$F_{r2} = \frac{V_2}{\sqrt{g y_2}} = \frac{1.4}{\sqrt{9.8 \times 3.0}} = 0.258$$

Hence, the flow at the downstream section is subcritical and a hydraulic jump must take place. The depth, $y_3$ (upstream of $y_2$) conjugate to $y_2$ can be obtained as :

$$\frac{y_3}{y_2} = \frac{1}{2}\left( \sqrt{1 + 8\,F_{r2}^2} - 1 \right)$$

$$= \frac{1}{2}\left( \sqrt{1+8 \times 0.258^2} - 1 \right) = 0.119$$

$$y_3 = 0.119 \times 3 = 0.357 \text{ m} > 0.336 \text{ m}$$

Hence, the jump is repelled through an $H_3$ curve, the depth of 0.336 m changing to 0.357 m at the end of the curve.

We know from the knowledge of gradually varied flow that the length of the $H_3$ profile is given by :

$$L = \frac{y_c}{S_c}\left[ \frac{(y_3/y_c)^{N-M+1} - (y_1/y_c)^{N-M+1}}{N-M+1} - \frac{(y_3/y_c)^{N+1} - (y_1/y_c)^{N+1}}{N+1} \right],$$

but, for a wide rectangular channel $M = 3.00$ and $N = 3.33$

$$\text{Hence,} \quad L = \frac{y_c}{S_c}\left[ \frac{(y_3/y_c)^{1.33} - (y_1/y_c)^{1.33}}{1.33} - \frac{(y_3/y_c)^{4.33} - (y_1/y_c)^{4.33}}{4.33} \right]$$

$$y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{4.2^2}{9.8} \right)^{1/3} = 1.216 \text{ m}$$

$$\therefore \quad \frac{y_3}{y_c} = 0.294, \text{ and } \quad \frac{y_1}{y_c} = 0.276$$

$$q = \frac{1}{n}(y_c)^{5/3}(S_c)^{1/2}$$

$$4.2 = \frac{1}{0.013}(1.216)^{5/3}(S_c)^{1/2}$$

$$= 76.92\,(1.385)\,S_c^{1/2}$$

$$S_c = (0.0394)^2 = 1.554 \times 10^{-3}$$

$$\therefore \; L = \frac{1.216}{1.554\times 10^{-3}}\left(\frac{(0.294)^{1.33}-(0.276)^{1.33}}{1.33} - \frac{(0.294)^{4.33}-(0.276)^{4.33}}{4.33}\right)$$

$$= 782.5\left(\frac{0.196-0.180}{1.33} - \frac{0.00499-0.00379}{4.33}\right)$$

$$= 782.5\,(0.0120-0.0003)$$

$$= 9.16 \text{ m}$$

In case the channel is sloping the length of the surface profile can be calculated using either the step method or the integration method (refer the relevant Block for the purpose).

$$F_{r3} = \frac{q}{y_3\sqrt{g y_3}} = \frac{4.2}{0.357\sqrt{9.8\times 0.357}} = 6.29$$

From experimental results, for $F_{r3} = 6.29$, we have :

$$\frac{L_j}{y_2} = 6.0$$

(Refer Figure 7.8/page 189 of reference (2))

$\therefore$ length of jump = $6\times 3 = 18.0$ m

Hence, the required length of apron = 9.16 + 18.0 = 27.16 m.

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

$$= 3.0 + \frac{1.4^2}{2\times 9.8} = 3.10 \text{ m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 0.336 + \frac{12.5^2}{2\times 9.8} = 8.308 \text{ m}$$

Hence, the total energy loss = 8.308 – 3.100 = 5.20 m

**SAQ 3**

$$y_2 = 2.0 \text{ m}$$

$$y_1 = 0.5 \text{ m}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_{r1}^{\;2}} - 1\right], \text{ and } \frac{y_1}{y_2} = \frac{1}{2}\left[\sqrt{1+8F_{r2}^{\;2}} - 1\right]$$

$$\text{So, } \frac{2}{0.5} = \frac{1}{2}\left[\sqrt{1+8F_{r1}^2} - 1\right], \text{ and } \frac{0.5}{2} = \frac{1}{2}\left[\sqrt{1+8F_{r2}^2} - 1\right]$$

Hence, $F_{r1}$ = Froude Number = 3.16, $F_{r2}$ = Froude Number = 0.40

$$\text{But, } F_{r1} = \frac{V_1}{\sqrt{g y_1}}, \text{ and } F_{r2} = \frac{V_2}{\sqrt{g y_2}}$$

$$3.16 = \frac{V_1}{\sqrt{9.81\times 0.5}}, \text{ and } 0.4 = \frac{V_2}{\sqrt{9.81\times 2}}$$

$$V_1 = 6.99 \text{ m/s, and } V_2 = 1.77 \text{ m/s}$$

$$\therefore \text{ Critical depth } y_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} = \left[\frac{(6.99\times 0.5)^2}{9.81}\right]^{\frac{1}{3}} = 1.076 \text{ m}$$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

$$= 2 + \frac{1.77^2}{2 \times 9.81} = 2.16 \text{ m.}$$

And the power with reference to the flow at section 2 is given by :

$$P_2 = \frac{w\, Q_2\, H_2}{75} = 101.95 \text{ kW}$$

where, $H_2 \equiv E_2$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 0.5 + \frac{6.99^2}{2 \times 9.81} = 2.99 \text{ m.}$$

And the power with reference to the flow at section 1 is given by :

$$P_1 = \frac{W\, Q_1\, H_1}{75} = 139.33 \text{ kW}$$

where, $H_1 \equiv E_1$

$\therefore$ Energy loss = 2.99 – 2.16 = 0.83 m,

and

Power loss = $P_1 - P_2 = 37.38$ kW.

## FURTHER READING


1. Subramanya, K., 1990, *Flow in Open Channels*, Tata McGraw-Hill Publishing Co. Pvt. Ltd.
2. Ranga Raju, K.G., 1993, *Flow Through Open Channel*, Tata McGraw-Hill Publishing Co. Pvt. Ltd.
3. Ven Te Chow, 1973, *Open Channel Hydraulics*, Tata McGraw-Hill Publishing Co. Pvt. Ltd.