UNIT 3 ENERGY PRINCIPLE

Structure

3.1 INTRODUCTION

In this unit, the concept of energy, as applied to open channels, is introduced as 'Specific Energy' of flow. The specific energy curve and its implications are discussed, and the application of specific energy to the solution of practical problems is explained. Lastly, the transition in channel alignment is discussed.

Objectives

After studying this unit, you should be able to

- comprehand the concept of specific energy,
- draw specific energy diagram and draw appropriate inferences, and
- apply the specific energy concept to transition problems.

3.2 CONCEPT OF SPECIFIC ENERGY

In the treatment of an open channel flow, it is very useful to refer the energy of flow at any given section to the bed of the channel instead of a horizontal datum line. This concept helps to solve the problems with least efforts.

3.2.1 Total Energy and Specific Energy

The principle of conservation of energy, in the treatment of hydraulics of open channel flow, occupies a centre-stage position in the process. The Bernoulli's energy equation can be written from the knowledge of elementary fluid mechanics as :

$$H = z + \frac{p}{\gamma} + \frac{V^2}{2g} \qquad \text{...(3.1)}$$

where,

H = total enegy per unit weight

z = elevation of the streamline (or point under consideration) above the datum

p = pressure

γ = specific weight of the fluid

$\frac{p}{\gamma}$ = pressure head

V = average sectional velocity

$\frac{V^2}{2g}$ = velocity head

g = local acceleration of gravity

In practice most uniform flows and gradually varied flows may be regarded as parallel flows (i.e., free surface taken parallel to bed) with hydrostatic pressure distribution (along the depth of flow), as the divergence and curvature of the streamlines are not significant. In a parallel flow, the sum $z + \frac{p}{\gamma}$ is constant (along the flow direction) and equal to the depth of flow y, if the datum is taken as the channel bottom. Then, the specific energy (E) of flow in a channel is defined as the energy at any section per unit weight (say, 1 kg) of water, as measured with respect to the channel bed. It can, then, be expressed as per equation (3.1), in the form :

$$E = y + \alpha \frac{V^2}{2g} \qquad \text{...(3.2)}$$

where, α = kinetic energy correction factor and is used to correct the kinetic head for any nonuniformity of the velocity profile. The assumption inherent, in the present analysis, is that the slope of the channel is small, or $\cos\theta \simeq 1$ and $y \simeq d\cos\theta$, where, d is the depth of flow normal to the bed. In general, if $\theta < 10^\circ$ or $S < 0.018$ this equation is valid.

3.2.2 Specific Energy Curve

An examination of the specific energy equation (3.2) indicates that if $\cos\theta \simeq 1$, $\alpha = 1$ and the channel section and discharge are specified then the specific energy, E, is only a function of the depth of flow y. If y is plotted against E as in Figure 3.1, a curve with two

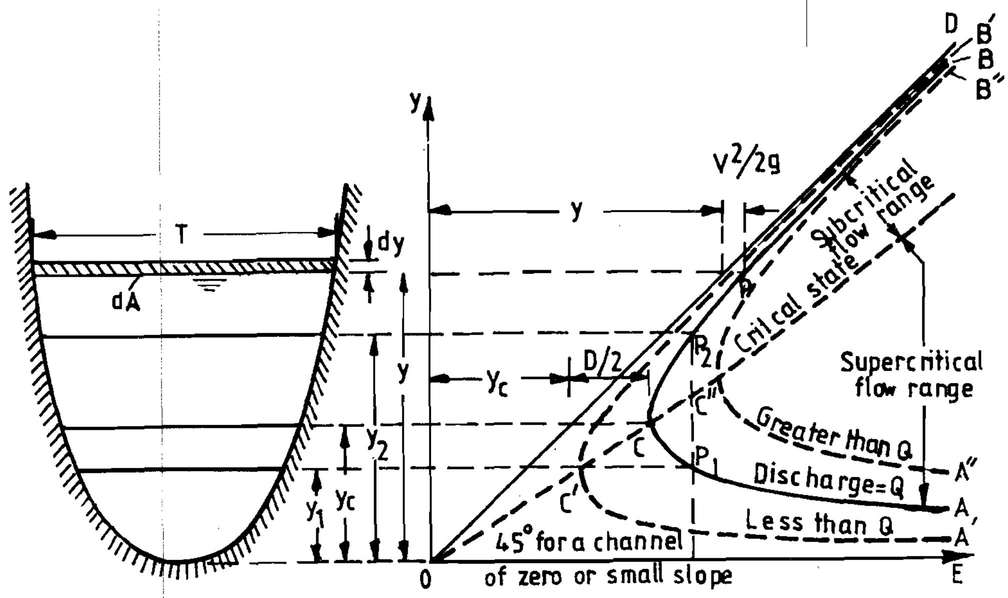

Figure 3.1 : Specific Energy Curve

branches results. The limb AC approaches the E-axis asymptotically as shown.

The other limb, AB, also approaches the line $y = E$ asymptotically. Observe that for all points on the E-axis, beyond point A, there are two possible depths of flow, for a given value of Q, which are known as alternate depths of flow. The low stage depth y_1 is called alternate depth of high stage depth y_2 and vice versa. At critical state of flow (Fr = 1) – represented by point A – these two depths apparently become one, known as the critical depth y_c. Since A represents the minimum specific energy (for a given Q), the coordinates of this point can be found by taking the first derivative of the specific energy equation with respect to y and setting the result to zero :

$$E = y + \alpha \frac{V^2}{2g} = y + \alpha \frac{Q^2}{2gA^2}$$

$$\frac{dE}{dy} = 1 - \alpha \left(\frac{Q^2}{gA^3}\right)\left(\frac{dA}{dy}\right) = 0$$

Substituting $\alpha = 1$ and $D = \frac{A}{T}$, simplifying and rearranging, we have

$$\frac{V^2}{2g} = \frac{D}{2}$$

$$\therefore \quad \frac{V}{\sqrt{gD}} = \text{Fr} = 1 \qquad ...(3.3)$$

This gives the conditon for critical flow in an open channel. Thus, the minimum specific energy occurs at the critical hydraulic depth corresponding to the given Q. Therefore, the limb AC can be interpreted as representing supercritical flows, and limb AB as representing subcrical flows. We can also easily show that, for channels of large slopes (and $\cos\theta$ not equal to 1) the criterion for minimum specific energy is :

$$F = \frac{V}{\sqrt{\left[\frac{gD\cos\theta}{\alpha}\right]}} \qquad ...(3.4)$$

It has to be pointed out that the co-efficient, α, of a channel section actually varies with the depth of flow (y). Hence, our assumption of α being constant, render the equations (3.3) and (3.4) not absolutely exact.

We also observe from the specific energy diagram that the $E - y$ curves for flow rates greater than Q lie to the right of the curve for Q, and the corresponding curves for flow rates less than Q lie to the left of the curve for Q.

3.3 APPLICATION OF SPECIFIC ENERGY PRINCIPLE TO FLOW SITUATIONS

Transitions are an invariable part of a system of open channel necessiated by the ground situations, varying variations in cross-sections are important from the point of view of economy, such as in an aqueduct, flume, and such similar situation.

The Transition Problem

A primary application of the concept of specific energy is in the prediction of changes in the depth of flow in response to channel transitions, i.e., changes in channel width and/or in the elevation of the channel bottom. While examining these problems, the accesssibility of the various points on the $E - y$ curve is to be considered.

Consider a rectangular channel of constant width b, conveying a steady flow, q, per unit width. In an otherwise horizontal channel bed consider a local smooth upward step of height Δz (Figure 3.2). Given this situation, an $E - y$ curve can be constructed for a specified Q (Figure 3.2). Here the flow upstream of the step is represented by point A on the $E - y$ curve.

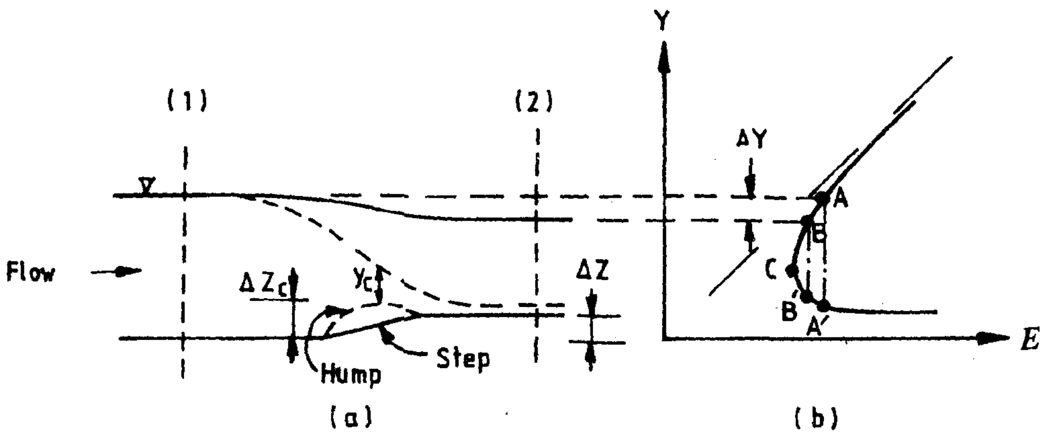

Figure 3.2 : The Accessibility Problem

upstream Froude Number. If point A is chosen to represent the upstream flow, then the flow is subcritical; i.e., Fr < 1.Since q is a constant, the point representing the flow downstream of the step must also lie on the same $E - y$ curve.The location of the downstream point on the $E - y$ axis can be determined by applying the Bernoulli's equation between the upstream and downstream sections :

$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + \Delta z,$$

holding the upstream bed as the datum.

$$\therefore \quad E_2 = E_1 - \Delta z \qquad \text{...(3.5)}$$

It has been tacitly assumed that the energy dissipation between these points is negligible. After determining the value of E_2 from equation (3.2) can be solved to determine the corresponding value of y_2. There would be three values of y satisfying the equation, one value being negative, which has no physical significance. However, the remaining two values, yielding two points, namely, B and B′ are equally valid soutions to the problem. The selection of a value out of these two values, in order to reflect the appropriate physical phenomenon is the essence of the accesssibility problem.

From the analysis of the varied flow equation $\left[\frac{dy}{dx} = \frac{S_o - S_f}{1 - \text{Fr}^2} \right.$, where, $S_o = -\frac{dz}{dx}$

(z being the elevation of the channel bed), and S_f is the slope of total energy line $\left. \right]$

it may be concluded that since, in the present case, $dz / dx > 0$ and if Fr < 1, the depth of flow must decrease from section 1 to 2; however, since both B and B′ represent depths of flow which are less than y_1, this conclusion is not useful while choosing the appropriate value of f. The $E - y$ curve itself provides the solution to the problem. As the width of the channel does not change, and q being constant, the "flow point" can move only along the $E - y$ curve defined by this value of q, i.e. the flow point cannot jump across the space which separates the points B and B′. Thus the flow point must pass through point C if it is to move to point B′ (characterised by a supercritical flow condition); however, attaining to point C is possible only if the increase in the elevation of the channel is greater than the specified change Δz. This situation is represented by the dashed line in Figure 3.2. Thus we can conclude that for the specified situation, only point B is accessible from point A as shown.

The above discussion assumes that a solution to the specified problem exists. In fact, it is quite easy to specify a problem for which there is no solution. For example, in Figure 3.2, if the step height exceeds Δz_c , there is no solution. In essence the three prescribed values q, E and Δz cannot exist simultaneously. A physical interpretation of this situation is that the flow area has been sufficiently obstructed, and the flow is choked. The flow will back up behind this obstruction, q will decrease because the depth of flow increases and a new steady flow will be established on an $E - y$ curve to the left of the one shown in Figure 3.2. An additional observation is that near the critical condition, point C in Figure 3.2, large changes in the water surface can be effected by small changes in the bed level. Thus flows which occur at near critical depth are inherently unstable and should be avoided.

3.4 ILLUSTRATIVE PROBLEMS

Example 3.1

A rectangular channel expands smoothly from a width of 1.5 m to 3.0 m. Upstream of the expansion the depth of flow is 1.5 m and the velocity of flow is 2.0 m. Estimate the depth of flow after the expansion.

Algebraic Solution

Since there is no change in the elevation of the channel bed,the upstream specific energy E_1 is equal to the downstream specific energy E_2,

or $\quad E_1 = E_2$

where, $\quad E_1 = y_1 + \frac{V_1^2}{\cdots} = 1.5 + \frac{4}{\cdots} = 1.7$ m

The velocity at the downstream station is given by :

$$V_2 = \frac{Q}{A_2} = \frac{2 \times 1.5 \times 1.5}{3y_2} = \frac{1.5}{y_2}$$

and therefore,

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{0.11}{y_2^2}$$

or,

$$y_2^3 - 1.7\, y_2^2 + 0.11 = 0$$

Solving this equation yields

$$y_2 = 1.6 \text{ m and } 0.28 \text{ m}$$

Concepts of accessibility indicates that only the subcritical depth of flow is physically possible and hence the appropriate answer is :

$$y_2 = 1.6 \text{ m}$$

Graphical Solution :

The appropriate governing $E - y$ curve for the downstream station can be constructed(Try!). *Plot* E_2 on x-axis (ranging from 0 and 5 m); and depth y on the y – axis (ranging from 0 and 2 m), using $E_2 = y_2 + \frac{0.11}{y_2^2}$.

From the plot ascertain the values of y_2 as 1.6 m and 0.28 m correspoding to E_2 equal to 1.7 m (Figure 3.3).

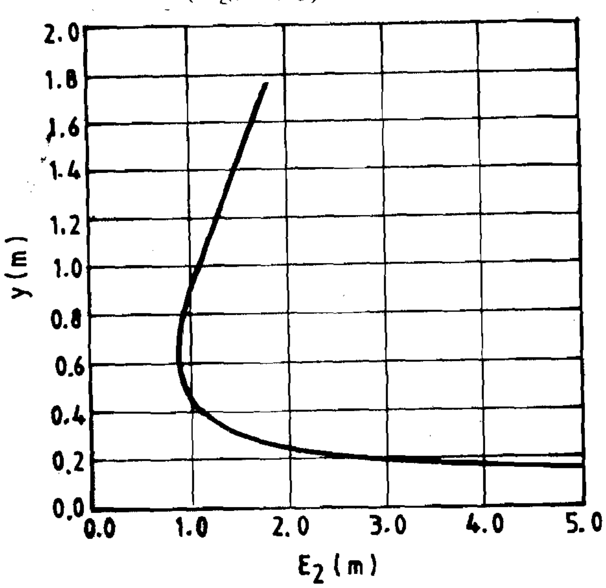

Figure 3.3 : Graphical Solution of Example 3.1

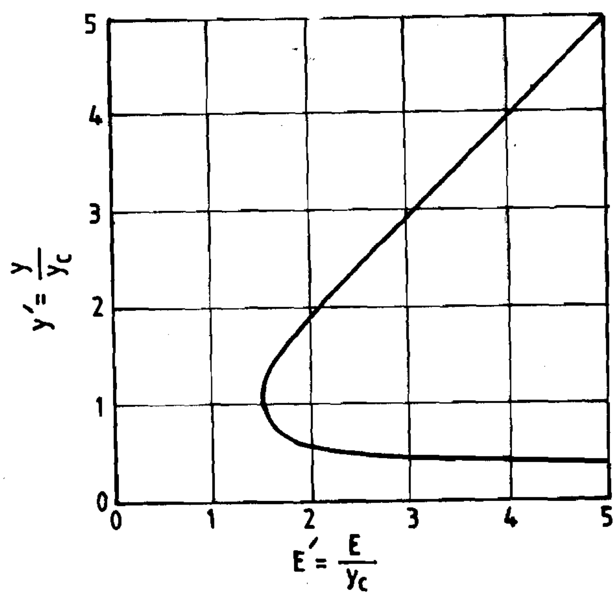

Figure 3.4 : Dimensionless E - y curve

Again the correct value is 1.6 m as already discussed.

Although the above methods provide satisfactory answers they require the solution of a cubic equation or the construction of an E –y curve for each problem. For rectangular channels these computational complexities can be overcome by constructing a dimensionless E –y curve.Dividing both sides of the specific energy equation by the critical depth, y_c :

$$\frac{E}{y_c} = \frac{y}{y_c} + \frac{q^2}{2g\, y^2\, y_c}$$

Defining $E' = \frac{E}{y_c}$ and $y' = \frac{y}{y_c}$, and, noting that $y_c = \left[\frac{q^2}{g}\right]^{1/3}$, we have :

$$E' = y' + \frac{1}{[2\,(y')^2]} \tag{3.6}$$

which is a dimensionless form of specific energy equation. A plot of this equation is shown in Figure 3.4. It is noted that the critical point occurs at (1.5, 1.0). In practice this graph cannot be read with sufficient precision; for this reason Babcock reduced the dimensionless $E - y$ graph to a tabular form as given below (Table 3.1).

Table 3.1 : $E' = \dfrac{E}{y_c}$ as a function of $y' = \dfrac{y}{y_c}$ (Babcock,1959)

E' as a function of y'

y'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		5000	1250	555.6	312.5	200.1	138.9	102.1	78.21	61.82
0.1	50.10	41.13	34.84	29.71	25.65	22.37	19.69	17.47	15.61	14.04
0.2	12.70	11.55	10.55	9.682	8.920	8.250	7.656	7.129	6.658	6.235
0.3	5.856	5.513	5.203	4.921	4.665	4.432	4.218	4.022	3.843	3.677
0.4	3.525	3.384	3.254	3.134	3.023	2.919	2.823	2.734	2.650	2.572
0.5	2.500	2.432	2.369	2.310	2.255	2.203	2.154	2.109	2.066	2.026
0.6	1.989	1.954	1.921	1.890	1.861	1.833	1.808	1.784	1.761	1.740
0.7	1.720	1.702	1.684	1.668	1.653	1.639	1.626	1.613	1.602	1.591
0.8	1.581	1.572	1.564	1.556	1.549	1.542	1.536	1.531	1.526	1.521
0.9	1.517	1.514	1.511	1.508	1.506	1.504	1.502	1.501	1.501	1.500
1.0	1.500	1.500	1.501	1.501	1.502	1.504	1.505	1.507	1.509	1.511
1.1	1.513	1.516	1.519	1.522	1.525	1.528	1.532	1.535	1.539	1.543
1.2	1.547	1.552	1.556	1.560	1.565	1.570	1.575	1.580	1.585	1.590
1.3	1.596	1.601	1.607	1.613	1.618	1.624	1.630	1.636	1.642	1.649
1.4	1.655	1.662	1.668	1.674	1.681	1.688	1.695	1.701	1.708	1.715
1.5	1.722	1.729	1.736	1.744	1.751	1.758	1.766	1.773	1.780	1.788
1.6	1.795	1.803	1.810	1.818	1.826	1.834	1.841	1.849	1.857	1.865
1.7	1.873	1.881	1.889	1.897	1.905	1.913	1.921	1.930	1.938	1.946
1.8	1.954	1.963	1.971	1.979	1.988	1.996	2.004	2.013	2.022	2.030
1.9	2.038	2.047	2.056	2.064	2.073	2.082	2.090	2.099	2.108	2.116
2.0	2.125	2.134	2.142	2.151	2.160	2.169	2.178	2.187	2.196	2.204
2.1	2.213	2.222	2.231	2.241	2.249	2.258	2.267	2.276	2.285	2.294
2.2	2.303	2.312	2.322	2.330	2.340	2.349	2.358	2.367	2.376	2.385
2.3	2.394	2.404	2.413	2.422	2.431	2.440	2.450	2.459	2.468	2.478
2.4	2.487	2.496	2.505	2.515	2.524	2.533	2.543	2.552	2.561	2.571
2.5	2.580	2.589	2.599	2.608	2.618	2.627	2.636	2.646	2.655	2.665
2.6	2.674	2.683	2.693	2.702	2.712	2.721	2.731	2.740	2.750	2.759
2.7	2.769	2.778	2.788	2.797	2.807	2.816	2.826	2.835	2.845	2.854
2.8	2.864	2.873	2.883	2.892	2.902	2.912	2.921	2.931	2.940	2.950

2.9	2.960	2.969	2.979	2.988	2.998	3.008	3.017	3.027	3.036	3.046
3.0	3.056	3.064	3.075	3.084	3.094	3.104	3.113	3.123	3.133	3.142
3.1	3.152	3.162	3.171	3.181	3.191	3.200	3.210	3.220	3.229	3.239
3.2	3.249	3.258	3.268	3.278	3.288	3.297	3.307	3.317	3.326	3.336
3.3	3.346	3.356	3.365	3.375	3.385	3.395	3.404	3.414	3.424	3.434
3.4	3.443	3.453	3.463	3.472	3.482	3.492	3.502	3.512	3.521	3.531
3.5	3.541	3.551	3.560	3.570	3.580	3.590	3.600	3.609	3.619	3.629
3.6	3.639	3.648	3.658	3.668	3.678	3.688	3.697	3.707	3.717	3.727
3.7	3.737	3.746	3.756	3.766	3.776	3.786	3.795	3.805	3.815	3.825
3.8	3.835	3.844	3.854	3.864	3.874	3.884	3.894	3.903	3.913	3.923
3.9	3.933	3.943	3.953	3.962	3.972	3.982	3.992	4.002	4.012	4.021
4.0	4.031	4.041	4.051	4.061	4.071	4.080	4.090	4.100	4.110	4.120
4.1	4.130	4.140	4.150	4.159	4.169	4.179	4.189	4.199	4.209	4.218
4.2	4.228	4.238	4.248	4.258	4.268	4.278	4.288	4.297	4.307	4.317
4.3	4.327	4.337	4.347	4.357	4.366	4.376	4.386	4.396	4.406	4.416
4.4	4.426	4.436	4.446	4.456	4.465	4.475	4.485	4.495	4.505	4.515
4.5	4.525	4.535	4.544	4.554	4.564	4.574	4.584	4.594	4.604	4.614
4.6	4.624	4.634	4.643	4.653	4.663	4.673	4.683	4.693	4.703	4.713
4.7	4.723	4.732	4.742	4.752	4.762	4.772	4.782	4.792	4.802	4.812
4.8	4.822	4.832	4.842	4.851	4.861	4.871	4.881	4.891	4.901	4.911
4.9	4.921	4.931	4.941	4.951	4.960	4.970	4.980	4.990	5.000	5.010

Example 3.2

Water flows in a rectangular channel 3.0 m wide at a velocity of 3.0 m/s and at a depth of 3.0 m. There is an upward step of 0.61 m. What expansion in width must take place simultaneously for this flow to be possible as specified ?

Solution

First, let us examine the upstream flow, with the channel having a step without considering the expansion, and compute the following quantities :

$$q_1 = \frac{Q}{b_1} = \frac{3 \times 3 \times 3}{3} = 9 \ \frac{(\mathrm{m}^3/\mathrm{s})}{\mathrm{m}}$$

$$y_c = \left[\frac{q_c^3}{g}\right]^{1/3} = \left[\frac{9^2}{9.81}\right]^{1/3} = 2.1 \ \mathrm{m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3 + \frac{3^2}{2 \times 9.81} = 3.5 \ \mathrm{m}$$

and $$E_1' = \frac{E_1}{y_c} = \frac{3.5}{2.1} = 1.67$$

The downstream dimensionless specific energy, considering only the step is given by

$$E_2' = E_1' - \frac{\Delta z}{y_c} = 1.67 - \frac{0.61}{2.1} = 1.38$$

From the dimensionless $E-y$ curve (Figure 3.4) and Table 3.1 (Babcock,1959) it can be seen that without an expansion the flow is not possible as specified since the downstream dimensionless specific energy does not lie on the $E'-y'$ curve. If the downstream flow occurs at the critical depth, then the expansion required is a minimum. The downstream condition is thus stated as :

$$E_2' = 1.5 = \frac{E_2}{y_{c_2}}$$

and , $$y_{c_2} = \frac{E_2}{1.5}$$

where, $E_2 = E_1 - \Delta z = 3.5 - 0.61 = 2.89$, and

y_{c_2} = downstream critical depth = $\frac{2.89}{1.5}$ = 1.9 m, and

$$q = \sqrt{g\, y_{c_2}^3} = 8.2\ \frac{(m^3/s)}{m}$$

Therefore, the downstream width is given by :

$$b = \frac{Q}{q} = \frac{27}{8.2} = 3.3 \text{ m}$$

or, the minimum expansion required is :

$$3.3 - 3 = 0.3 \text{ m}$$

SAQ 1

A rectangular channel expands smoothly from a width of [illegible] m to [illegible] m. Upstream of the expansion the depth of flow is 1.8 m, and the velocity of flow is 2.3 m/s. Estimate the depth of flow after the expansion.

SAQ 2

Water flows in a rectangular channel 2.5 m wide at a velocity of [illegible] m/s and at a depth of 2.2 m. There is an upward step of [illegible] m in the channel bed. What expansion in width must take place simultaneously for this flow to be possible as specified ?

3.5 SUMMARY

In this unit the concept of specific energy, and use of specific energy diagram have been discussed. The problem regarding channel transitions are solved by the application of specific energy concept.

3.6 KEY WORDS

Specific energy : Sum of poential energy and kinetic energy referred to

Transition : That portion of a channel which connects two different widths of the same channel.

3.7 ANSWERS/SOLUTIONS TO SAQs

SAQ 1

Algebraic Solution

Since there is no change in the elevation of the channel bed, the upstream specific energy E_1 is equal to the downstream specific energy E_2.

i.e., $$E_1 = E_2$$

where, $$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{2.3^2}{2 \times 9.81}$$

$$= 2.3 \text{ m}$$

The velocity at the downstream station is

$$Q = A_1 V_1$$

$$= (2 \times 1.8) \times 2.3$$

$$A_2 = y_2 B_2$$

$$\therefore \quad V_2 = \frac{Q}{A_2} = \frac{2 \times 1.8 \times 2.3}{\frac{1}{2} \times 4}$$

$$= \frac{2.07}{y_2}$$

and therefore,

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{0.22}{y_2^2}$$

$$y_2^3 - 2.3\, y_2^2 + 0.22 = 0$$

solving this equation by trial and error yields :

$$y_2 = 0.335\,, \ 2.26 \text{ and } -0.290 \text{ m}$$

Rejecting the negative value,

$$\therefore \quad y_2 = 0.335 \text{ m}, \ 2.26 \text{ m}$$

It is obvious that $y_2 < y_1$, due to the channel expansion,

$$\therefore \quad y_2 = 0.225 \text{ m}$$

SAQ 2

$$q_1 = \frac{Q}{b_1} = \frac{(2.5 \times 2.2) \times 2.8}{2.5}$$

$$= 6.2 \ \frac{(\text{m}^3/\text{s})}{\text{m}}$$

$$y_c = \left[\frac{q^2}{g}\right]^{1/3} = \left[\frac{6.2^2}{9.81}\right]^{1/3}$$

$$= 1.58 \text{ m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.2 + \frac{2.8^2}{2 \times 9.81}$$

$$= 2.6 \text{ m}$$

and

$$E_1' = \frac{E_1}{y_c} = \frac{2.6}{1.58} = 1.81$$

The downstream dimensionless specific energy considering only the step is computed as follows :

$$E_2' = E_1' - \frac{\Delta z}{Y_c}$$

$$= 1.81 - \frac{0.72}{1.58}$$

$$= 1.35 \text{ m}$$

$$E_2' = \frac{E_2}{y_c} = 1.35$$

$$y_c = \frac{E_2}{1.35}$$

where,

$$E_2 = E_1 - \Delta z = 2.6 - 0.72$$

$$= 1.88 \text{ m}$$

Hence, referring to the expanded channel section, we have :

$$y_c = \frac{1.88}{1.35}$$

$$= 1.39 \text{ m}$$

Therefore, the downstream width is given by :

$$b_2 = \frac{Q}{q}$$

$$= \frac{15.4}{5.13}$$

$$= 3.00 \text{ m}$$

Hence, the minimum expansion required is

$$3.00 - 2.5 = 0.5 \text{ m}$$