UNIT 2 CHANNEL GEOMETRY AND UNIFORM FLOW

Structure

2.1 INTRODUCTION

Development of open channel flow; the study of velocity and pressure distributions; and the estimation of velocity and pressure at different points or locations, as well as, the various formulae used therein form the areas covered in Unit 2. For this type of flow, also different characteristics of uniform flow are explained for more complete understanding of the flow phenomenon.

Objectives

After studying this unit, you should be able to

- recognise different geometrical sections used in open channels,
- use of uniform flow formulae that are in vogue, and
- compute normal depth and slope as applied to a uniform flow.

2.2 CHANNEL TYPES AND SECTION ELEMENTS

There are various types of open channels, such as :

(1) Canals

(2) Rivers

(3) Minor feeder channels in an irrigation system

It is obvious that an open channel can be either natural or artificial (man-made).

2.2.1 Natural and Artificial Channels

All water courses existing in nature on the surface of earth, varying in size from tiny hillside rivulets through brooks, streams, small and large rivers to tidal estuaries, are all natural channels. Underground streams with a free surface are also natural channels.

An artificial channel is constructed or developed by man. Navigation channels, power canals, irrigation canals and flumes, drainage ditches, trough spillways, floodways, log chutes, roadside gutters, and laboratory channels (for conducting experiments) are all artificial channels.

2.2.2 Prismatic Channels

A prismatic channel is a channel of both, constant cross section and constant bed slope along its length. Channels of varying cross section and bed slope are termed non-prismatic channels. A trough spillway of varying width, for example, is a non-prismatic channel.

2.2.3 Rectangular, Trapezoidal and Curved Sections

Artificial channels are usually designed with sections of regular geometric shapes. Five geometric shapes that are in common use are presented in Table 2.1. Trapezoid is a convenient shape for channels with unlined earth banks, ensuring stability of side slopes. A rectangle and a triangle are special cases of the trapezoidal shape. The vertical side of a rectangle is suitable for channels built of stable materials such as lined masonry, rocks, metals or timber. The triangular section is adopted for small ditches, roadside gutters and sometimes for laboratory flumes. A popular section for sewers and culverts of small and moderate sizes is a circle, while a parabola is used as an approximation of sections of small and medium-sized natural channels.

2.2.4 Depth of Flow and Stage

The depth of flow, y, is the vertical distance from the lowest point of a channel section to the free water surface. In most cases this terminology is used interchangeably with the expression : 'depth of flow of the section, d,' where d is the depth of flow measured perpendicular to the channel bottom.It is obvious that $y = \dfrac{d}{\cos\theta}$, where θ is the angle that the bed makes with the horizontal. If θ is small then $y \approx d$. Only in channels of steep slope the difference between y and d is significant.

The stage of a flow is the elevation of the water surface relative to a datum. If the lowest point of a channel section is taken as the reference then the stage and depth of flow are numerically equal.

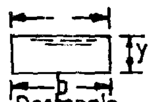
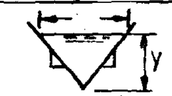
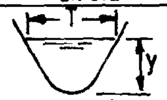
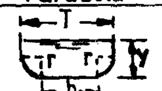
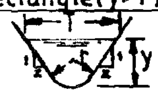
2.2.5 Geometric Elements : Hydraulic Radius, Hydraulic Depth etc.

The top width, T, of a channel is the width of the channel section at the water surface. The flow area, A, as explained earlier, is the cros-sectional area of flow normal to the direction of flow. The wetted perimeter, P, as referred to earlier, is the length of the line of interface between the fluid and the channel boundary. It is to be noted that the top width, T, exposed to atmosphere, is not taken into account in the computation of wetted perimeter, as it offers no significant resistance to flow.

The hydraulic radius, R, as also mentioned earlier, is given by $R = \dfrac{A}{P}$. The hydraulic depth D is the ratio of flow area A to the top width T, i.e., $D = \dfrac{A}{T}$. The section factor, Z, as used in critical flow computation is defined as the product of the flow area, A, and the square root of the hydraulic depth D ; i.e., $Z = A\sqrt{D} = A\sqrt{(A/T)}$ – Table 2.1; whereas, the section factor for uniform flow computation is defined as $Z = AR^{(2/3)}$, i.e. it is the product of the flow area and two-thirds power of hydraulic radius.

2.2.6 Wide Open Channel

It has been observed that in the central region of the section of a wide open channel the velocity distribution is the same as it would be in a rectangular channel of infinite width. Thus, in a very wide channel, the sides (i.e. walls) of the channel have practically no influence on the velocity distribution in the central region. Therefore, the flow in the central region can be regarded as a two dimensional flow for hydraulic analysis. On the basis of these observations a wide open channel can be safely defined as a rectangular channel of width greater than 5 to 10 times the depth of flow because of the existence of this central region, depending on the prevailing roughness condition.

Section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
 Rectangle	by	$b+2y$	$\frac{by}{b+2y}$	b	y	$by^{1.5}$
 Trapezoid	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b+2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$
 Triangle	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$1/2y$	$\frac{\sqrt{2}}{2}zy^{1.5}$
 Circle	$1/8(\theta-\sin\theta)d_0^2$	$1/2\,\theta d_0$	$1/4\left(1-\frac{\sin\theta}{\theta}\right)d_0$	$(\sin 1/2\theta)d_0$ or $2\sqrt{y(d_0-y)}$	$1/2\left(\frac{\theta-\sin\theta}{\sin 1/2\theta}\right)d_0$	$\frac{\sqrt{2}}{32}\frac{(\theta-\sin\theta)^{1.5}}{(\sin 1/2\theta)^{0.5}}$
 Parabola	$2/3\,Ty$	$T+\frac{8y^2}{3T}$ *	$\frac{2T^2y}{3T^2+8y^2}$ *	$\frac{3}{2}\frac{A}{y}$	$(2/3)y$	$(2/9)\sqrt{6}\,Ty^{1.5}$
 Round-corner rectangle $(y>r)$	$(\frac{\pi}{2}-2)r^2+(b+2r)y$	$(\pi-2)r+b+2y$	$\frac{(\pi/2-2)r^2+(b+2r)y}{(\pi-2)r+b+2y}$	$b+2r$	$\frac{(\pi/2-2)r^2}{b+2r}+y$	$\frac{[(\pi/2-2)r^2+(b+2r)y]^{1.5}}{\sqrt{b+2r}}$
 Round-bottomed triangle	$\frac{T^2}{4z}-\frac{r^2}{z}(1-z\cot^{-1}z)$	$\frac{T}{z}\sqrt{1+z^2}-\frac{2r}{z}(1-z\cot^{-1}z)$	$\frac{A}{P}$	$2[z(y-r)+r\sqrt{1+z^2}]$	$\frac{A}{T}$	$A\sqrt{\frac{A}{T}}$

* Satisfactory approximation for the interval $0<x\leq 1$, where $x=4y/T$. When $x>1$ use the exact expression $P=(T/2)[\sqrt{1+x^2}+1/x\ln(z+\sqrt{1+x^2})]$

Table 2.1 : Geometric Elements of an Open Channel

2.3 VELOCITY AND PRESSURE DISTRIBUTION

Any flow field can be characterised by mapping velocity and pressure distribution over any given cross-section of the channel carrying the flow.

2.3.1 Velocity Distribution in a Section

Velocity is not uniformly distributed over a channel section due to the presence of free surface and frictional forces along the channel bottom and sides (i.e., walls). Observations reveal that the maximum velocity in a flow section usually occurs at a distance of 0.05 to 0.25 times the depth from the free surface along any given vertical line. As we approach the banks these maximum velocity points tend to occur comparatively at deeper positions. Other factors like the natural shape of the section, nature of roughness of the channel, and the presence of bends in the channel alignment also influence the velocity distribution profile in the section. Typical curves of equal velocity (isovels) in various channel sections are shown in Figures 2.1, 2.2, 2.3 and 2.4.

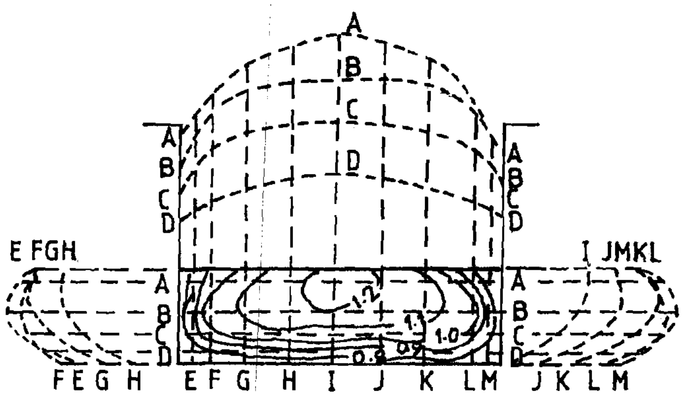


Figure 2.1 : Velocity Distribution in a Rectangular Channel (Chow 1959) in an Open Channel

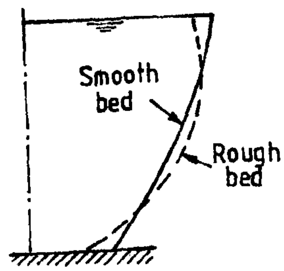


Figure 2.2 : Effects of Roughness on Velocity Distribution in an Open Channel

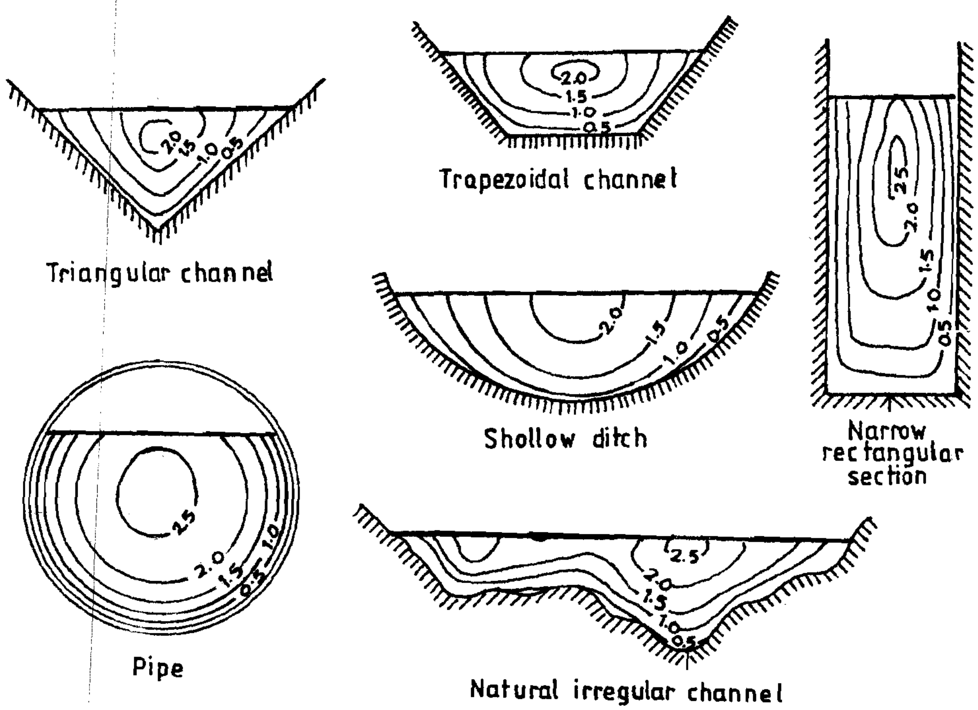


Figure 2.3 : Typical Curves of Equal Velocity in Various Channel Sections

2.3.2 Pressure Distribution in a Section

The height of water column in an open ended glass tube (a piezometer, say) is a measure of the pressure at a point of interest on the cross section of flow, in a channel of small slope. Ignoring minor disturbances due to turbulence etc. it is apparent that this water column

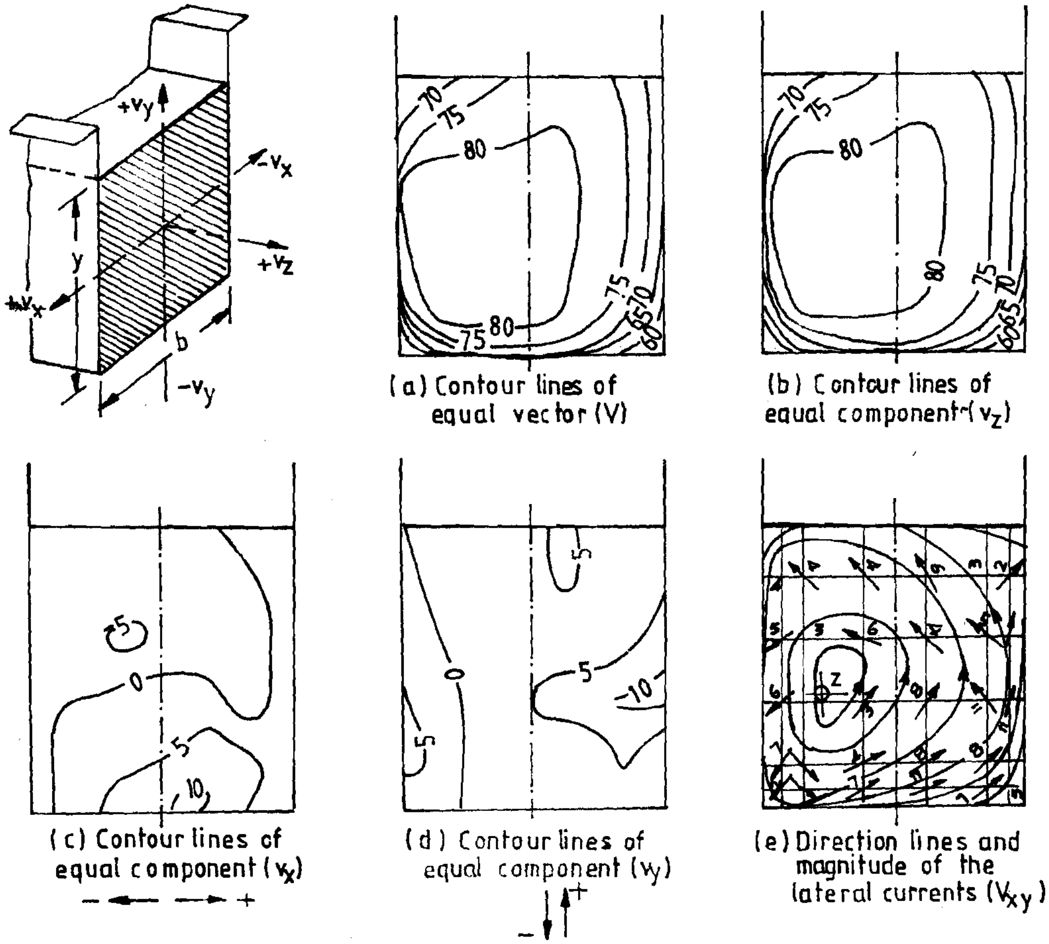


Figure 2.4 : Distribution of the Velocity Components Facing Downstream at the Mid-section of a Straight Flume. Velocities are in cm/s (= 0.0328 fps); $\frac{y}{b}$ = 1.0, Re = 73,500; and Q = 70 litres/s (= 2.47 cfs) (Chow 1959)

surface. Thus, it follows that the pressure at any point on the section is directly proportional to the depth of the point below the free surface. This pressure, in the absence of any extraneous influence, is equal to the hydrostatic pressure corresponding to this depth of flow. This pressure distribution, obviously, is linear and can be represented by a straight line. This hydrostatic law of pressure distribution is valid only if the flow filaments have no acceleration or deceleration, or radial force components, otherwise deviations are bound to occur (Figure 2.5). For practical purposes this law is applicable to uniform as well as gradually varied flow.

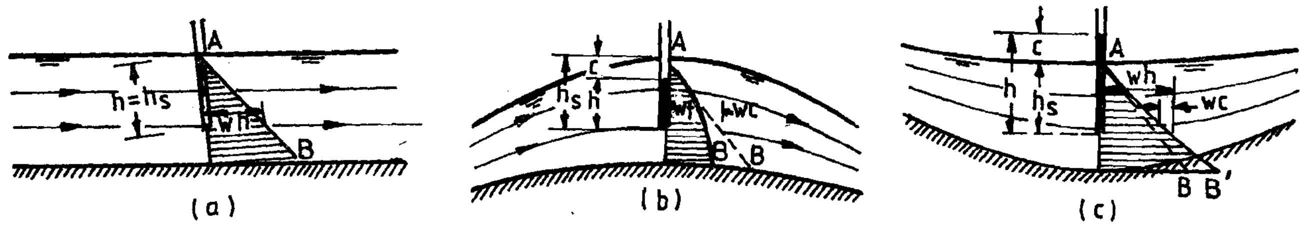


Figure 2.5 : Effect of Bed Curvature on Hydrostatic Pressure Distribution in an Open Channel

2.4 BASIC EQUATIONS

For the solution of flow problems, we need to have a few basic equations, such as incorporating mass balance, resistence to flow offered by the channel boundary, etc., in order to be able to understand the flow characteristics in their completeness.

2.4.1 Continuity Equation

The continuity equation signifies the conservation of fluid mass in space and time. It can be expressed in differential or mass form or in vector notation. Particular expressions with reference to steady flows or incompressible flows can be easily derived from the more general compressible, unsteady flow cases. It is basic to any fluid flow analysis that the flow must satisfy the continuity equation, signifying coservation of mass in space and time irrespective of whether the flow is laminar or turbulent, or taking place in a smooth or rough channel.

Continuity equation for unstedy flow in open channels is written as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad ...(2.1)$$

2.4.2 Basic Resistance Equation

The basic resistance formula, as applied to a uniform flow, is expressed in the following general form:

$$V = C R^a S^b \qquad ...(2.2)$$

where, V is the mean velocity in m/s; R is the hydraulic radius in m; S is the slope of the energy line; a and b are exponents; and, C is a parameter characterising flow resistance, that depends on many factors. It is important to remember that in a uniform flow the free water surface, energy line and the channel bottom are lines that are all parallel to each other; and hence, these lines have the same slope.

2.4.3 Chezy's Equation

In 1769, a French engineer, Antoine Chezy developed probably the first channel flow formula directly applicable to uniform flows :

$$V = C\sqrt{(RS)} \qquad ...(2.3)$$

where, V is the mean velocity in m/s; R is the hydraulic radius in m; S is the slope of the energy line and C is the resistance factor, called Chezy's C or Chezy's coefficient. Many attempts have been made to determine the value of Chezy's C.

2.4.4 Ganguillet–Kutter's Equation

Two Swiss engineers Ganguillet and Kutter in 1869 published an empirical equation to compute Chezy's C :

$$C = \left[\frac{23 + \left(\frac{0.00155}{S}\right) + \left(\frac{1}{n}\right)}{1 + \left(\frac{n}{\sqrt{R}}\right)\left(23 + \frac{0.00155}{S}\right)}\right] \qquad ...(2.4)$$

The coefficient n in this equation is known as Kutter's n. This equation was developed from observations of gaugings of many European rivers and on Mississippi river.

Investigators have been trying to determine the value of C for different boundary conditions, i.e., nature of roughness, obtaining in open channels. As $V \propto C$, therefore, one can say that C does not directly behave as a resistance factor (as does n in equation (2.5)), but, on the contrary, seems to increase the velocity with its increasing values.

2.4.5 Manning's Equation

Irish engineer Robert Manning, in 1889, presented an empirical equation later modified to its present form :

$$V = (1/n)\,R^{(2/3)}\ S^{(1/2)} \qquad ...(2.5)$$

where, V is the mean velocity in m/s; R is the hydraulic radius in m; S is the slope of the energy line and 'n', herein, is known as Manning's rugosity coeffcient. Its simple form and long use by engineers has made the Manning's equation the most widely accepted equation for open channel flow computations. Within the normally encountered range of slope and hydraulic radius, the values of Manning's n and Kutter n are very close to each other. For practical purposes the two values are the same when the slope is greater than 0.0001, and the hydraulic radius lying between 0.3 to 9 m. Chezy's and Manning's co-efficients are related to each other by the following expression (i.e., equating equations (2.3) and (2.5)) :

$$C = (1/n)\left[R^{(1/6)}\right] \qquad ...(2.6)$$

2.5 UNIFORM FLOW : DEVELOPMENT AND COMPUTATIONS

A uniform flow is an idealised flow situation in an open channel, where the energy slope, free water surface, and the bed slope are parallel to each other.

2.5.1 Establishment of Uniform Flow

Whenever a liquid flows in an open channel, it always encounters resistance to its movement due to its interaction with the boundary conditions and shape of the cross section. This resistance gets balanced by the component of gravitational force, acting on the body of water in the direction of motion; A complete balance between the gravitational force and the resisting force (which is achieved after the body of water travels some distance, mobilising more & more resistance) results the establishment of in a uniform flow. In fact, the magnitude of resistance to flow depends, mainly, on the velocity of flow when other factors of the channel are held constant. Therefore, a transition zone (in the beginning of the upstream reach) is necessarily required for the establishment of uniform flow – in this zone, the flow is accelerating under gravity and the water surface profile is varied in its nature. If the channel length is shorter than the transition length required under the given conditions, it is clear that, for the given discharge, a uniform cannot be established. Again, even if the channel is quite long, the resitance in the downstream reach may exceed gravity forces and the flow will become varied again. The required length of the transition zone depends on discharge and physical conditions of the channel such as entrance condition, shape, slope and roughness.

2.5.2 Computation of Normal Depth and Slope

Multiplying both sides of manning's equation by A, and rearranging the terms, we get as follows :

$$Q = (AR^{2/3})\left[\frac{1}{n}S^{1/2}\right] \qquad ...(2.7)$$

$$\therefore\ AR^{2/3} = \frac{Q}{\frac{1}{n}S^{1/2}} \qquad ...(2.8)$$

We note here that the Manning's equation is split into two parts for convenience. The first part, namely, $AR^{2/3}$ (called section factor for uniform flow computation) represents the geometry of the water area in terms of the wetted area (A) and the hydraulic radius (R). The second part (i.e., the R.H.S.) represents the flow characteristics in terms of discharge Q, roughness n and the energy slope S. Thus, it is easy to appreciate that for a given set of flow conditions (Q, n and S) there can be only one possible depth for sustaining uniform flow provided of course $AR^{2/3}$ always increases with increase in depth, which is true in most maintaining a uniform flow. This discharge is known as the normal discharge (Q_n). Dimensionless curves are available for uniform flow computation relating depth and section factor for rectangular, trapezoidal and circular channel sections (Figure 2.6).

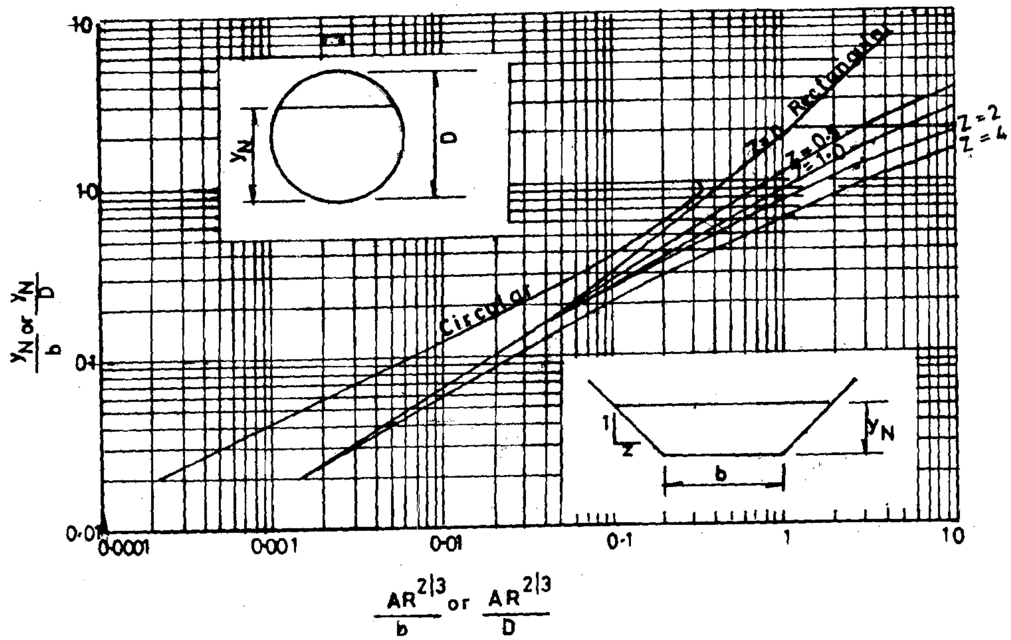


Figure 2.6 : Dimensionless Curves for Determining the Normal Depth of Flow When the Section Factor is Known (Chow, 1959)

When discharge and roughness are given, the slope of a prismatic channel at a specified normal depth (y_n) can be determined in order to yield uniform flow conditions; this value of the slope is called the normal slope S_n .

2.6 ILLUSTRATIVE PROBLEMS

Example 2.1

Given a trapezoidal channel with a bottom width of 3 m, side slopes of 1.5 : 1, a longitudinal slope of 0.0016 and $n = 0.013$, determine the normal discharge if the normal depth of flow is 2.6 m.

Solution

From Table 2.1

$$A = (b + zy)\, y = [\,3 + 1.5 \times 2.6\,] \times 2.6 = 18 \ \text{m}^2$$

$$P = b + 2y\sqrt{(1 + z^2)} = 3 + 2 \times 2.6 \times \sqrt{3.25} = 12 \ \text{m}$$

$$R = \frac{A}{P} = \frac{18}{12} = 1.5 \text{ m}$$

$$Q = AR^{2/3}\frac{S^{1/2}}{n} = 18 \times 1.5^{2/3} \times \frac{\sqrt{0.0016}}{0.013}$$

$$= 73 \ \text{m}^3/\text{s}$$

Example 2.2

Given a trapezoidal channel with a bottom width of 3 m,side slope of 1.5 : 1, a longitudinal slope of 0.0016 and an estimated value of $n = 0.13$, find the normal depth of flow for a discharge of 7.1 m^3/s.

Solution

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.13\ (7.1)}{\sqrt{0.0016}}$$

$$= 23 \ \text{m}^{8/3}$$

$$A = (b + zy)\, y = (3 + 1.5y)\, y$$

$$P = b + 2y\sqrt{1 + z^2} = 3 + 2y\sqrt{3.25} = 3 + 3.6\, y$$

$$R = \frac{A}{P} = \frac{(3 + 1.5\, y)\, y}{(3 + 3.6y)}$$

We construct the following table by assuming different values of y and then computing the corresponding values of $AR^{2/3}$. When the computed value of $AR^{2/3}$ matches with the value computed from the problem statement, the correct value of y_n is said to have been determined.

Trial y	A	P	R	$AR^{2/3}$
m	m^2	m	m	m$^{8/3}$
2.4				
2.7				
2.6	18	12.4	1.5	23.58

Complete the necessary information in this table for arriving, step by step, at the correct value of normal depth of 2.6 m. A second method of solving this problem is by the construction of a graph of the section factor as a function of the depth of flow.

Example 2.3

Given a circular culvert, 0.91 m in diameter, with $S = 0.0016$ and $n = 0.015$, find the normal depth of flow for a discharge of 0.42 m^3/s.

Solution

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.015 \times 0.42}{0.0016}$$

Construct a graph of $AR^{2/3}$ (x-axis – 0 to 0.3) versus y (y-axis – 0 to 0.7). Verify from this graph the correct value of normal depth y_n as 0.51 m. Note also that the section factor first increases with depth and then as the full depth is approached it decreases with increasing depth. Therefore it is possible to have two values of y_n for same values of $AR^{2/3}$ in a circular section.

Example 2.4

A rectangular channel has a bottom width of 6.0 m and an n value of 0.020.

For $y_n = 1.0$ m, and $Q = 11$ m^3/s, find the normal slope.

Solution

Using the given data, we have :

$$A = b\, y = 6 \times 1.0 = 6.0 \text{ m}^2$$

$$P = b + 2y = 6 + 2 \times 1.0 = 8.0 \text{ m}$$

$$R = \frac{A}{P} = \frac{6}{8} = 0.75 \text{ m}$$

$$S_n = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left\{\frac{0.02 \times 11}{6 \times 0.75^{2/3}}\right\}^2$$

$$= 0.002$$

Thus, $S = 0.002$ will maintain a uniform flow of 11 m^3/s in this channel at a depth of 1.0 m.

SAQ 1

Given a trapezoidal channel with a bottom width of 3.3 m, side slopes of 1.5 : 1, a longitudinal slope of 0.0016 and $n = 0.013$, determine the normal discharge if the normal depth of flow is 2.4 m.

SAQ 2

Given a trapezoidal channel with a bottom width of 3.3 m, side slope of 1.5 : 1, a longitudinal slope of 0.0016 and an estimated value of $n = 0.13$, find the normal depths of flow at a discharge of 8 m^3/s.

SAQ 3

Given a circular culvert, 1.0 m in diameter, with $S = 0.0016$ and $n = 0.015$, find the normal depth of flow for a discharge of 0.6 m^3/sec.

SAQ 4

A rectangular channel has a bottom width of 8.0 m and $n = 0.015$.

a) For $y_n = 1.0$ m and $Q = 15$ m^3/s, find the normal slope.

b) Find the normal depths of flow for $Q = 15$ m^3/s.

2.7 SUMMARY

This unit has dealt with the channel geometry and uniform flow conditions therein. Commonly used geometrical sections and section parameters are tabulated for ready reference.

Commonly used resistance formulae were introduced; and uniform-flow computations were illustrated.

2.8 KEY WORDS

Uniform flow	:	A flow where the energy slope and bed slope are parallel.
Normal depth (y_n)	:	Depth of flow at which a uniform flow is sustained in a channel.
Manning's '*n*'	:	The roughness co-efficient, defining the resistance factor as used in Manning's equation.
Chezy's '*C*'	:	The roughness co-efficient as used in Chezy's equation.

2.9 ANSWERS/SOLUTIONS TO SAQs

SAQ 1

From Table 2.1, the area,

$$A = (b + z\,y)\,y = [\,3.3 + 1.5 \times 2.4\,] \times 2.4$$

$$= \mathbf{16.56\ m^2}$$

$$P = b + 2y\,\sqrt{(1 + z^2)} = 3.3 + 2 \times 2.4 \times \sqrt{3.25}$$

$$= 11.95\ \text{m}$$

$$R = \frac{A}{P} = \frac{16.56}{11.95} = 1.38\ \text{m}$$

$$Q = \frac{A\,R^{2/3}\,S^{1/2}}{n}$$

$$= \frac{16.56 \times 1.38^{2/3} \times \sqrt{0.0016}}{0.013}$$

$$= 63.15\ \text{m}^3/\text{s}$$

SAQ 2

$$A\,R^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.13 \times 8.0}{\sqrt{0.0016}}$$

$$= 26.00$$

with

$$A = (b + z\,y)\,y = (3.3 + 1.5\,y)\,y$$

$$P = b + 2y\,\sqrt{(1 + z^2)}$$

$$= 3.3 + 2y\,\sqrt{3.25} = 3.3 + 3.6\,y$$

$$R = \frac{A}{P} = \frac{(3.3 + 1.5\,y)\,y}{3.3 + 3.6\,y}.$$

We construct the following table by assuming different values of y and computing corresponding values of $AR^{2/3}$. When the computed value of $AR^{2/3}$ matches with the value computed from the problem statement, the correct value of y_n stands determined.

Trial, y	**A**	**P**	**R**	**$AR^{8/3}$**
m	m^2	m	m	$\text{m}^{8/3}$
2.40	16.56	11.95	1.38	20.52
2.70	19.84	13.03	1.52	26.22
3.00	23.40	14.11	1.65	32.67

Complete the necessary information in this table for arriving at the required value of normal depth of 2.7 m. A second method of solving this problem, as pointed out earlier, is by the construction of a graph of the section factor versus the depth of flow.

SAQ 3

$$A\,R^{2/3} = \frac{n\,Q}{\sqrt{S}} = \frac{0.015 \times 0.6}{\sqrt{0.0016}}$$

$$= 0.225$$

Construct a graph of $A\,R^{2/3}$ (x-axis – 0 to 0.3) versus y (y - axis– 0 to 0.7). Verify that the section factor first increases with depth, and then as the full depth is approached, it decreases with increasing depth. Therefore, it is possible to have two values of y_n (for some value of $A\,R^{2/3}$), the section being circular in shape.

SAQ 4

From given data

$$A = b\,y = 8 \times 1.0 = 8.0\ \text{m}^2$$

$$P = b + 2y = 8 + 2 \times 1.0 = 10.0\ \text{m}$$

$$R = \frac{A}{P} = \frac{8}{10} = 0.80\ \text{m}$$

$$S = \left[\frac{n\,Q}{A\,R^{2/3}}\right]^2$$

$$= \left[\frac{0.015 \times 15}{8 \times (0.8)^{2/3}}\right]^2 = 0.001065$$

Thus, a bed slope, $S = 0.001065$ will maintain a uniform flow of a discharge of 15 m^3/s, in this channel at a depth of 1.0 m.