UNIT 16 HYDRAULIC MODELS

Structure

16.1 INTRODUCTION

There are many problems related to hydraulic engineering and fluid mechanics which defy a complete analytical solution. Model studies are usually conducted to find solutions to these problems. Even if analytical methods are available, experiments are still necessary to verify these methods, because invariably such methods are based on certain assumptions or have some empirical constants. Therefore use of models have steadily increased. To give few examples, an aeronautical engineer obtains data from model tests in wind tunnels; the naval architect tests ship models in towing basins; the mechanical engineer tests models of turbines and pumps; the civil engineer works with models of hydraulic structures and rivers to obtain more reliable solutions to his design problems. Therefore one would like to know the various similarity criteria for the design of these models in a manner that they show complete similarity with prototypes. It is rarely feasible to attain this complete similarity; in fact especially for large river models, the horizontal and vertical scale ratios are kept different, resulting in distorted models. In some cases forces that are negligible in prototype, become quite significant in models, this effect is known as **scale effect**. Some basic principles of dimensional analysis, significance of dimensionless parameters governing the fluid flow phenomena alongwith the types of various similarities required to be attained by the model,have already been studied in Unit 5.

In this unit some aspects regarding laws of similitude in respect of few specific fluid flow problems are proposed to be discussed. Need for distorted models and correcting model results for scale effects will then be discussed.

Objectives

After studying this Unit, you should be able to

* appreciate need for hydraulic models,
* write the similarity criteria for designing the models,
* interpret the model results, and
* appreciate the scale effects in these models.

16.2 LAWS OF SIMILITUDE

In all model studies, similarity in behaviour between the model and the prototype must be ensured. There are three types of similarity to be attained :

16.2.1 Geometric Similarity

This is similarity of shape. It implies that model and prototype would be geometrically similar if, linear dimension ratios or scale factors are same. In the model and prototype shown in Figure 16.1, it means

$$\frac{d_p}{d_m} = \frac{l_p}{l_m} = l_r \qquad (16.1)$$

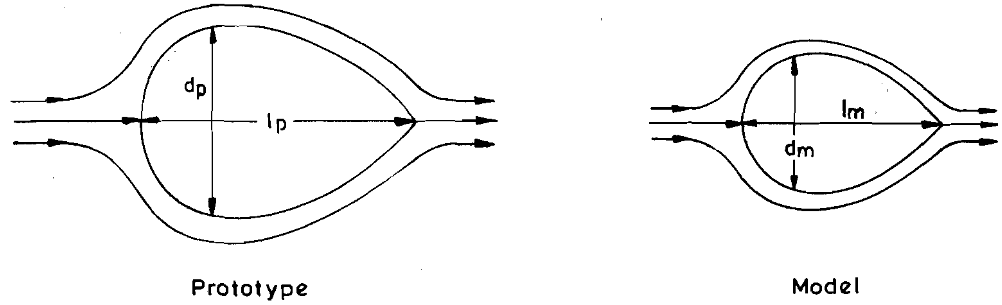

Figure 16.1 : Geometric Similarity in Model and Prototype

In equation (16.1) suffix *p* is for prototype, *m* for model and *r* for length scale. Quite often perfect geometric similarity is not easy to attain. This problem comes especially in building river models requiring study of sediment motion. Normally these river models are quite large and their size is limited by the available floor space. But if the vertical and horizontal scale ratios are kept same, the result may be a stream, so shallow that surface tension has a considerable effect and may be flow in the model is laminar instead of turbulent. Therefore, in river models, horizontal and vertical scale ratios are invariably kept different and this results in distorted models. The characteristics of distorted models would be discussed at a later stage of this unit.

16.2.2 Kinematic Similarity

Kinematic similarity is similarity of motion. This is attained if velocity ratios at corresponding points in the model and prototype are same.

16.2.3 Dynamic Similarity

Dynamic similarity is similarity of forces. In problems related flow of fluids, forces may be due to many causes : viscosity, gravity, difference in pressure, surface tension etc. For a perfect dynamic similarity, the ratio of all these forces should be same. However, in practice it is not possible to satisfy all these forces in the model. Fortunately, in many instances, some of the forces do not come into picture, or have negligible effect, and so it becomes possible to concentrate on the similarity of the most significant forces governing that particular problem. Therefore in order to identify these significant forces, a detailed knowledge of various forces governing the particular problem is required.

Let us now consider some of the modelling techniques and the relevant force ratios that need to be considered in the design of specific models.

16.3 MODELLING TECHNIQUES

In the design of models, the various force ratios are expressed in terms of dimensionless parameters. For example, Reynolds number which signifies the ratio of inertial force to viscous force, Froude number which is the ratio of inertial force to gravitational force, likewise there are many such parameters. Once in a particular problem, the important governing dimensionless parameters are known, the model can be built holding those parameters constant in the model and prototype. The required model scales can then be obtained on the basis of this equality and these scales will then be used to convert the model results to prototype results.

To illustrate the above modelling technique, let us study the testing of an aircraft model in a wind tunnel in order to find the drag force F_D on the prototype.

16.3.1 Drag of an Aircraft Model

In this problem, froces due to gravity and surface tension do not affect the flow field. If compressibility effects are also ignored, the only forces to be considered are viscous and inertia forces, thus for the dynamic similarity, the relevant dimensionless parameter to be considered is the Reynolds number. Thus dynamic similarity is obtained between the model and the prototype

when

$$\left(\frac{U\,l}{\nu}\right)_p = \left(\frac{U\,l}{\nu}\right)_m \qquad (16.2)$$

provided model and prototype are geometrically similar and their orientation with respect to oncoming flow is identical. Here l is some characteristic length of the aircraft may be span of the wing of the aircraft.

Equation (16.2) places no restriction on the fluids of the model and prototype, the prototype could move through air while the model could be tested in water as long as the Reynolds number are kept same in model and prototype. If for practical reasons the same fluid is used in model and prototype, equation (16.2) suggests that the product (Ul) must be the same in both. When a model aircraft is being tested, the size of the model is naturally less than that of prototype; this means to keep (Ul) same, the velocities around the model will be larger than the corresponding ones around the prototype. To give the idea of velocity required for this model study, let us assume that the prototype is intended to fly at 300 km/hr speed. Obviously, the velocity in the wind tunnel must be greater than 300 km/hr since such velocities can not be attained, one way of obtaining a sufficiently high Reynolds number without using inconveniently high velocities is to test the model in air of higher density. Such wind tunnels which use compressed air are known as **variable density wind tunnels.** In such tunnels $\nu_m < \nu_p$, the equality of Reynolds number could be achieved easily. Once equality of Reynolds number is obtained, the drag force could be estimated as follows :

$$F_D = C_D\,\rho\,\frac{U^2 l^2}{2}$$

or

$$C_D = \left(\frac{F_D}{\frac{\rho U^2 l^2}{2}}\right)$$

since drag coefficient C_D will be same both in model and prototype it gives

$$\left(\frac{F_D}{\frac{\rho U^2 l^2}{2}}\right)_p = \left(\frac{F_D}{\frac{\rho U^2 l^2}{2}}\right)_m \qquad (16.3)$$

Equation (16.3) will then give the drag coefficient for the prototype directly using the corresponding model values and no correction for scale effect is needed as Re values are same in model and prototype.

In case variable density tunnel is not available, complete similarity has to be sacrificed and a compromise solution sought. We have already studied that C_D varies with Re. Therefore if the Reynolds numbers of model and prototype are different, the values of C_D will also differ, as such one can not directly transfer the model results to prototype. In this case some extrapolation of model results will be required. For this extrapolation, C_D versus Re relationship would be required to be developed from experiments in which models have been compared with their prototypes.

In problems where more than one force ratio is relevant, it is difficult to achieve complete similarity. Similarity is still achieved but with some departure. It is, nevertheless, essential that these departures are justified. An example to illustrate this procedure is regarding model testing of a ship model.

16.3.2 Drag of a Ship Model

The resistance experienced by a ship moving through water is generally due to three causes

(i) Viscosity

(ii) Eddies formed in the wake

(iii) Surface waves

Due to viscosity of the fluid, viscous forces are set up between the fluid layers close to the surface of the ship and those farther away and these forces cause a frictional resistance to the motion of the ship. This part of the total drag is usually termed as **skin friction.** In addition, as the ship moves forward, part of the flow towards the rear of the ship breaks away from its surface to form a 'wake' of eddying motion. The eddies give rise to a distribution of pressure round the body different from that to be expected in an ideal fluid, and so provide another contribution to the total drag force. This part of the drag is known as **pressure** or **form drag**.

These two types of resistances - the skin friction and form drag - are experienced by any solid body moving through the fluid. A ship, however, is only partly submerged in a liquid, and its motion through liquid gives rise to waves on the surface. The formation of these waves requires energy, and since this energy must be derived from the motion of ship, the ship experiences an increased resistance to its motion and this is called **wave resistance**. Thus for a ship moving through water the total drag is the sum of friction drag (skin friction), form drag and drag due to surface waves. Since the ships are generally streamlined, the form drag is relatively small and within the usual range of Reynolds number, it can be assumed to remain constant. Moreover, it is usual practice to consider this portion of the total drag along with the drag due to surface waves which are associated with gravitational action. As such the resistance to the motion of a ship is affected by viscosity as well as gravity. Making use of dimensional analysis, the functional relationship for the total resistance F_D can be written as

$$F_D = \rho U^2 l^2 \phi \left(\frac{U l}{\nu}, \frac{U^2}{g l} \right) \qquad (16.4)$$

or

$$F_D = \rho U^2 l^2 \phi \left(\text{Re}, F_r^2 \right) \qquad (16.4a)$$

The total resistance thus depends both on the Reynolds number and on the Froude number. Therefore, for complete similarity these two numbers must be the same in model and prototype. That is,

$$\frac{\rho_p U_p l_p}{\mu_p} = \frac{\rho_m U_m l_m}{\mu_m}$$

and

$$\frac{U_p}{\sqrt{g_p l_p}} = \frac{U_m}{\sqrt{g_m l_m}}$$

Since in practice $g_m = g_p$, the combination of above equations give

$$\left(\frac{l_p}{l_m} \right)^{3/2} = \left(\frac{\nu_p}{\nu_m} \right)$$

As both model and prototype must operate in water giving $\frac{\nu_p}{\nu_m} = 1$, the above condition demands $l_p = l_m$, i.e.model be as large as prototype. Obviously such a condition can not be satisfied. Thus equality of Re and Fr can not be met simultaneously. Therefore, following approach as suggested by Froude may be adopted to determine the total resistance of a ship.

The total resistance F_D experienced by a ship may be assumed to be consisting of two portions viz.

(a) the wave resistance F_w due to the action of waves;

(b) the frictional resistance F_f due to the frictional effects on the wetted surface of the ship.

That is

$$F_D = F_w + F_f \qquad (16.5)$$

A similar subdivision can also be made for the total resistance F_{Dm} of the model. Thus if F_{wm} and F_{fm} represent the wave and frictional resistances respectively for the model, then

$$F_{Dm} = F_{wm} + F_{fm} \qquad (16.6)$$

The frictional resistances may be estimated by assuming that they have the same values as that for a thin flat plate of given length and wetted surface area moving through water at the same velocity. In earlier unit, we have already studied the methods for estimating the drag on a flat plate, hence there is no difficulty in estimating the frictional resistances.

A geometrically similar model of the ship is then tested by towing it in water taking model velocity according to Froude law i.e. $u_m = u_p \left(\frac{l_m}{l_p}\right)^{\frac{1}{2}}$ and total drag F_{Dm} is measured. The wave drag of the model is then found as

$$F_{wm} = F_{Dm} - F_{fm}$$

since F_{fm} for the model has already been estimated as indicated above. Now for dynamic similarity between the model and the prototype considering only wave resistance, we have

$$\frac{F_w}{\rho_p\, l_p^2\, u_p^2} = \frac{F_{wm}}{\rho_m\, u_m^2\, l_m^2} \qquad (16.7)$$

The wave resistance F_w for the prototype can be obtained from equation (16.7) as

$$F_w = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{l_p}{l_m}\right)^2\left(\frac{u_p}{u_m}\right)^2 F_{wm}$$

But from equality of Froude number we have

$$\frac{u_p}{u_m} = \left(\frac{l_p}{l_m}\right)^{\frac{1}{2}}$$

This gives

$$F_w = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{l_p}{l_m}\right)^3 F_{wm} \qquad (16.8)$$

The total drag F_D for the ship may then be obtained by adding the two i.e. F_w and F_f.

SAQ 2

(i) What is the physical significance of Reynolds number and Froude number ?

(ii) Is it possible to maintain both Reynolds number and Froude number same in the model & prototype?

(iii) Define wave resistance.

16.4 RIVER MODELS

Rivers carry water and sediment, the bed and banks of rivers are generally erodible. Quite often hydraulic engineers are required to build structures across these rivers. For example a dam, a bridge and a barrage. As a result of this construction many changes take place in the river both upstream and downstream of these structures, as well as, around the structures. Model studies are, therefore, required to study these changes. Since these flows are characterised by free surface, gravitational forces dominant over other forces, therefore, invariably these river models are built on the basis of equality of Froude number. Thus assuming horizontal and vertical scales to be same, equality of Froude number and continuity equation for discharge gives

$$\frac{u_p}{\sqrt{g\, l_p}} = \frac{u_m}{\sqrt{g\, l_m}}$$

or

$$\frac{u_p}{u_m} = \left(\frac{l_p}{l_m}\right)^{\frac{1}{2}}$$

or

$$u_r = l_r^{1/2}$$

Continuity equation gives $Q_r = A_r\ u_r$ where A is the area of flow cross- section.

Since $A_r = l_r^2$ we have

$$Q_r = l_r^2\ u_r$$

$$Q_r = l_r^{5/2} \tag{16.9}$$

Equation (16.9) gives the relation between discharge scale and length scale. Obviously two considerations will govern the model dimensions, the available discharge and the available space to build the model. If discharge is going to govern the model scale, the length scale could be found from equation (16.9). Thus if this scale ratio is used to reduce the vertical dimensions of the prototype and size of the prototype roughness, the resulting depth of flow and the height of roughness element at times become unrealistic. The depth of flow in the model could be so small that forces due to surface tension start affecting model results. Also reduced depth of flow could make flow in the model laminar instead of turbulent flow which prototype invariable has.

In view of above, the horizontal and vertical scales are invariably kept different in river models, this introduces what we call as geometric distortion. Another distortion in such models is force distortion because similarity of only Froude number is maintained whereas Reynolds number is allowed to be kept different in the model and in the prototype. Yet another distortion, that is given to river models is material distortion, this is needed to have the motion of sediment on the bed. This requires use of light weight material which could be easily moved in the model under reduced depths of flow. Thus river models are invariably distorted models and it requires lot of experience and judgement to interpret model results and their transfer to predict prototype behaviour.

The other details of these river models are beyond the scope of this unit.

SAQ 3

(i) Why are river models distorted?

(ii) List the various types of distortions that are given to these models.

(ii) River models are built on the basis of equality of Froude number. Comment.

16.5 ILLUSTRATIVE EXAMPLES

Example 16.1 :

A geometrically similar model of a spillway discharges 0.10 m^3/s discharge per metre length of the spillway at a head of 0.14 m. If the scale ratio is 10.0, determine the prototype head and discharge per metre length of the spillway.

Solution :

According to Froude's law

$$Q_r = l_r^{5/2}$$

Since $$Q_r = \frac{Q_p}{Q_m} = \frac{q_p\, l_p}{q_m\, l_m} = q_r\, l_r$$

$$\therefore \quad q_r\, l_r = l_r^{5/2} \quad \text{or} \quad q_r = l_r^{3/2}$$

As $$l_r = 10$$

$$q_r = 10^{3/2} = 31.623$$

$$\therefore \quad q_p = q_m \times 31.623$$

$$= 0.10 \times 31.623 = 3.162 \ m^3/s$$

Also $$\frac{h_p}{h_m} = l_r \quad \text{or} \quad h_p = h_m\, l_r$$

$$= 0.14 \times 10 = 1.40 \ m$$

Example 16.2 :

A ship model 1.0 m long with negligible frictional resistance is tested in a towing tank at a velocity of 0.60 m/s. To what ship velocity does this correspond to if the ship is 60.0 m long. A force of 5.0 N is required to tow this model. Calculate the force required in the prototype.

Solution :

Since frictional force is negligible, for dynamic similarity, Froude number must be same in model and prototype

$$U_r = l_r^{1/2}$$

As $$l_r = 60.0 \qquad U_r = 60^{1/2}$$

or $$U_p = U_m \times 60^{1/2}$$

$$= 0.6 \times 60^{1/2} = 4.65 \ m/s$$

Force ratio

$$F_r = l_r^3 \quad \text{(From Newton's second law of motion)}$$

$$F_p = F_m\, l_r^3 = 5.0 \times 60^3 = 1080 \ kN$$

16.6 SUMMARY

In this Unit, you have learnt the basic need for model studies in understanding the behaviour of flow in prototype structures. For building these models, the laws governing the phenomenon must be known, in other words, the dimensionless parameters likely to affect the flow should be known. Models are built on the basis of similarity criteria which at times becomes difficult to satisfy; this leads to what is known as **scale effect** in the model. The various distortions that are given to a river model were also discussed. Thus on the basis of the principles discussed in this Unit, it should be possible to finalise the various scale ratios of a given model for a given set of conditions.

16.7 KEY WORDS

Model	:	Structure - built on the bas of similarity criteria - which represents prototype on a smaller size. Occasionally model could be larger than prototype.
Scale Ratio	:	Defined as the value of a particular quantity in prototype to that in the model.
Froude's Model Law	:	Model built on equality of Froude number
Scale Effect	:	Model having some of the dimensionless parameters different from the prototype is supposed to have scale effect.
Distorted Models	:	Models having horizontal and vertical scales different are called distorted models.

16.8 ANSWERS TO SAQs

Check your answers of all SAQs with respective preceeding text of each SAQs.