UNIT 14 LIFT

Structure

14.1 INTRODUCTION

In the previous unit on drag forces around bodies immersed in fluid, we had observed that the total force exerted by the fluid on the body can be resolved into two components - one in the direction of fluid motion known as the **drag** and the other perpendicular to the direction of fluid motion known as the **lift**. This unit is devoted to the elementary concepts about lift force.

Objectives

After studying this unit, you should be able to

* understand the cause of lift in irrotational flow,
* compute the lift force exerted on a body, and
* appreciate the lift on an aerofoil.

14.2 CIRCULATION IN IRROTATIONAL FLOW

Circulation refers to such flow which follows a circuitous course back to the starting point. Generally, the circulatory motion is superposed upon the basic translatory motion of the fluid and, hence, it becomes essential to express circulation in terms of an arbitrary curve passing through a system of streamlines as shown in Figure 14.1.

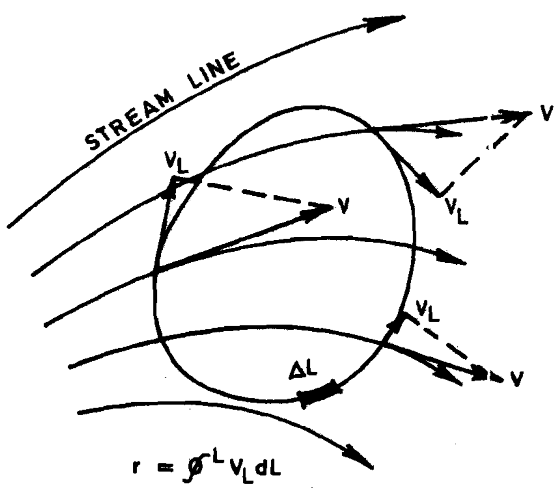

Figure 14.1 : Definition of Circulation

If v_L represents the tangential component of the velocity at any point on the curved line then the circulation Γ around a closed curve is defined as the integral of the quantity $v_L \, dL$ around the closed curve. Thus,

$$\Gamma = \oint v_L \, dL$$

If we have to consider the case of flow in concentric circles (refer Figure 14.2), according to the condition $v = \omega r$, then the circulation around any stream line (which is a circle of radius r) is obviously equal to $\omega r\ (2\pi r)$ i.e. $2\pi r^2\omega$. One may similarly find circulation around any other curve such as that composed of the two circular arcs and two segments of radii (along which the tangential velocity is zero) as indicated in the figure. In this case,

$$\Gamma = \omega r_2 . \theta r_2 - \omega r_1 . \theta r_1 = \omega \theta \left(r_2^2 - r_1^2 \right)$$

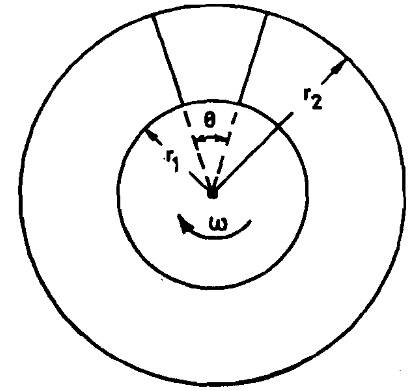

Figure 14.2 : Circulation around Two Concentric Circles

Now consider a cylinder placed in an irrotational flow. The streamline pattern is as shown in Figure 14.3 (a). Let us superimpose the streamline pattern of constant circulation around a cylinder (Figure 14.3 (b)) so that the resulting flow pattern is as shown in Figure 14.3 (c).

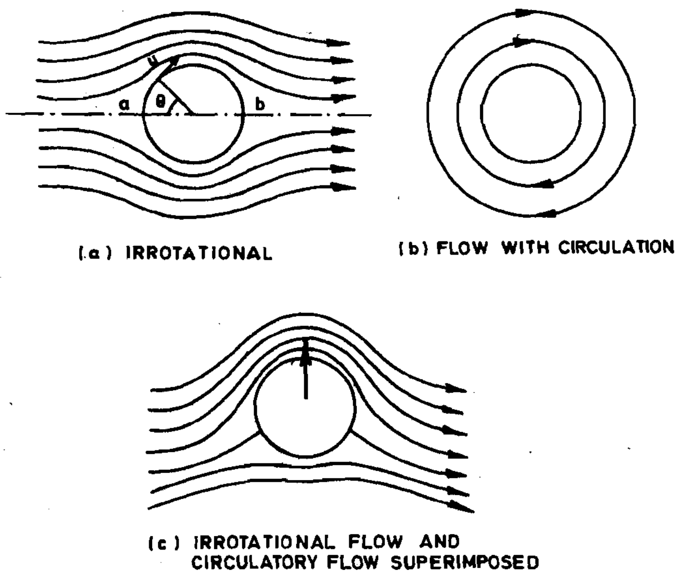

Figure 14.3 : Generation of Lift around a Cylinder

Obviously, this has caused increased velocity (and hence decreased pressure) on one side and the decreased velocity on the other. The streamline pattern is, however, still symmetrical about the vertical axis. Therefore, the circulation has not changed the longitudinal force upon the cylinder from its initial magnitude of zero. But, the asymmetry of the flow pattern about the horizontal axis has resulted in decreased pressure intensity in the upper part and increased pressure intensity in the lower part. Thus it is obvious that the circulation in irrotational flow causes a resultant force in the lateral direction.

14.3 MAGNUS EFFECT

In a real fluid flow, local circulation can be produced through surface drag by rotating the cylinder itself. The circulation decreases instead of remaining constant as in the irrotational case. Nevertheless, this local circulation does produce considerable lateral thrust - commonly known as lift force. This phenomenon was first observed by a German scientist Magnus and hence it is named as Magnus effect. The sudden deviation of a ball, which has been chopped (as in volley ball or table tennis) or sliced (as in lawn tennis) by player, from its normal trajectory is simple illustration of the Magnus effect.

14.4 COMPUTATION OF LIFT FORCE

If U_0 is the free stream velocity, the velocity v at any point on the surface of a cylinder of radius r can be obtained from the principles of hydrodynamics. Thus,

$$v = 2\, U_0 \sin\theta$$

Similarly, the velocity u_c on the surface of the cylinder due to circulation Γ is given by

$$u_C = \frac{\Gamma}{2\pi r}$$

Therefore, the resultant velocity u of the superimposed flow

$$u = v + u_C = 2\, U_0 \sin\theta + u_C$$

or

$$\frac{u}{U_0} = 2 \sin\theta + \frac{u_C}{U_0}$$

For a specified value of $\frac{u_C}{U_0}$, one can obtain the value of θ for $u = 0$. That is

$$\theta = \sin^{-1}\left(-\frac{1}{2}\frac{u_C}{U_0}\right)$$

This gives us the location of stagnation points. If p_0 is the pressure in the ambient stream and one applies Bernoulli's equation between any point in the unaffected ambient flow and any point on the surface of the cylinder, one gets,

$$p - p_0 = \Delta p = \frac{\rho}{2}\left(U_0^2 - u^2\right)$$

The lift force F_L acting on the surface of the cylinder is given as

$$F_L = \int_0^{2\pi} \Delta p \,\sin\theta \,.\, L\, r\, d\theta$$

$$= \frac{L\rho r}{2}\int_0^{2\pi}\left[U_0^2 - \left(2\, U_0 \sin\theta + \frac{\Gamma}{2\pi r}\right)^2\right]\sin\theta\, d\theta$$

which reduces to a very useful relation

$$F_L = L\, \rho\, U_0\, \Gamma$$

The derivation of this expression does not take into account the viscous effects. The actual magnitude of the lift force (or so called Magnus effect) F_L must obviously depend upon the Reynolds number of the flow alongwith other parameters such as the density and velocity of the fluid and the size and peripheral speed of the cylinder. However, at higher Reynolds number, viscous effects upon the lift should approach an asymptotic value, just as in the case of drag. Under such circumstances the lift force F_L may be expressed as,

$$F_L = C_L\, L\, D\, \frac{\rho\, U_0^2}{2}$$

which is similar to the expression for the drag force. Here, C_L is the lift coefficient and D is the diameter of the cylinder. Thus, $L\,D$ represent the projected area of the cylinder in a plane normal to the direction of the lift force.

For irrotational flow, therefore,

$$C_L = \frac{F_L}{L\,D\left(\frac{1}{2}\rho\, U_0^2\right)} = \frac{L\,\rho\, U_0\,\Gamma}{L\,D\,\frac{1}{2}\rho\, U_0^2} = \frac{2\,\Gamma}{D\, U_0}$$

$$= \frac{2\pi\, D\, u_C}{D\, U_0}$$

$$C_L = 2\pi \frac{u_C}{\ldots}$$

This value of the lift coefficient for irrotational flow may be termed as theoretical value which has been compared with the experimental values of C_L and the corresponding coefficient of drag C_D in Figure 14.4. It is obvious that the local circulation produced through surface drag and viscous shear is about half as effective as the constant circulation required for truly irrotational flow.

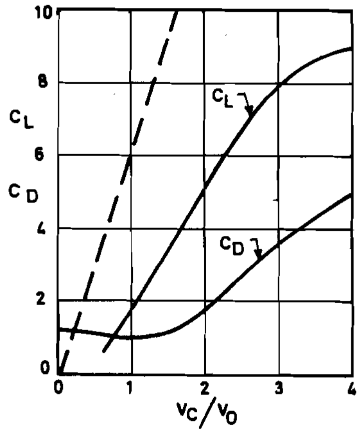

Figure 14.4 : Experimental Variation of C_L and C_D as a function of $\frac{u_c}{U_o}$

14.5 LIFT ON AEROFOIL

In aeroplanes, wings (which are also known as lifting vanes or aerofoils) are used to produce lift force as they move through the fluid which is air. These vanes (or aerofoils) can be symmetrical or unsymmetrical and are characterised by the geometric chord C and the angle of attack θ_0 between the geometric chord and the direction of flow (see Figure 14.5).

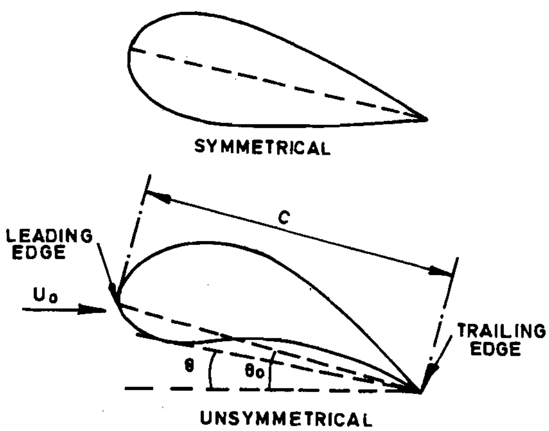

Figure 14.5 : Nomenclature for an Aerofoil

When a circulatory flow around an aerofoil is superimposed over an irrotational flow around the same aerofoil, the resulting flow pattern appears similar to the one shown in Figure 14.6.

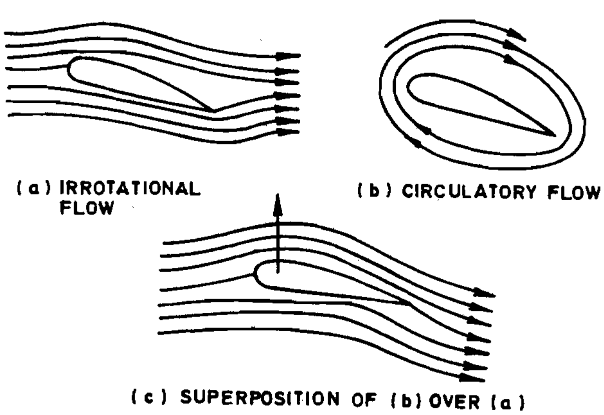

Figure 14.6 : Schematic superposition of Circulation on an Irrotational Flow Field

It has been shown that to make the streamline at the trailing edge tangential to the trailing edge the circulation required is given as

$$\Gamma = \pi\ C\,U_0 \sin\theta_0$$

$$\therefore \quad F_L = \pi\ C\,L\,\rho\ U_0^2\ \sin\theta_0$$

$$\therefore \quad C_L = \frac{\dfrac{F_L}{CL}}{\dfrac{1}{2}\ \rho\ U_0^2}$$

$$= \frac{\dfrac{\pi\ CL\,\rho\ U_0^2\ \sin\theta_0}{CL}}{\dfrac{\rho\ U_0^2}{2}}$$

$$= 2\,\pi\ \sin\theta_o$$

The theoretical values of C_L obtained from this expression are quite close to the actual values in real fluid flow.

SAQ 1

Why do aeroplanes take off and land against the wind?

14.6 ILLUSTRATIVE EXAMPLES

Example 14.1 :

An aeroplane weighing 22,000 Newtons has a wing area of 22 m^2 and span of 12.0 m. What is the lift coefficient if it travels at a speed of 360 km/hr in the horizontal direction. Also compute the theoretical value of circulation and angle of attack.

Solution :

$$U_0 = 360 \times \frac{1000}{3600} = 100 \text{ m/s}$$

$$\therefore \quad 22000 = 22 \times C_L \times 1.208\,\frac{100^2}{2}$$

$$\therefore \quad C_L = 0.166$$

$$= 2\,\pi \sin\theta_0$$

$$\therefore \quad \theta_0 = 1.51^\circ$$

$$\Gamma = \pi\,C\,U_0\ \sin\theta_0 \quad \text{and} \quad C = \frac{22}{L} = \frac{22}{12} = 1.833 \text{ m}$$

$$= 3.14 \times 1.833 \times 100 \times 0.026 = 14.97 \text{ m}^2/\text{s}$$

Example 14.2 :

A circular cylinder 2.0 m in diameter and 12 m long is rotated at 360 rpm with its axis perpendicular to the air stream having a velocity of 37.7 m/s. Assuming ρ to be 1.236 kg/m^3. Determine (i) circulation (ii) theoretical lift (iii) position of stagnation points and (iv) actual lift. Assume $C_L = 1.5$.

Solution :

$$u_c = \frac{\pi DN}{60} = \frac{3.14 \times 2.0 \times 360}{60} = 37.70 \text{ m/s}$$

$$\therefore \quad \frac{u_c}{U_o} = \frac{37.70}{37.70} = 1.0$$

Circulation,

$$\Gamma = \pi D u_c$$

$$= 3.14 \times 2.0 \times 37.7$$

$$= 236.76 \text{ m}^2/\text{s}$$

Theoretical lift,

$$F_L = \rho L U_o \Gamma$$

$$= 1.236 \times 12 \times 37.7 \times 236.76$$

$$= 132.4 \text{ kN}$$

Points of Stagnation

$$\frac{u}{U_o} = 2 \sin \theta + \frac{u_c}{U_o} \qquad \text{and } u = 0$$

$$\therefore \quad \sin \theta = -\frac{1}{2}$$

$$\therefore \quad \theta = -30^\circ \quad \text{and} \quad 210^\circ$$

Actual lift,

$$F_L = C_L \times D \times L \times \frac{1}{2} \rho U_o^2$$

$$= 1.5 \times 2.0 \times 12 \times \frac{1}{2} \times 1.236 \times (37.7)^2$$

$$= 31.6 \text{ kN}$$

14.7 SUMMARY

In this Unit we discussed the generation of lift force on a body immersed in either irrotational flow or viscous flow. Magnus effect was also explained. Lift force on an aerofoil was also briefly described.

14.8 KEY WORDS

Lift Force : Component of the total force exerted by the fluid on the body in the direction perpendicular to the flow is known as Lift Force.

Airfoil : Body shape which yields high lift and low drag values.

Magnus effect : Lift force can be caused by superposition of rectilinear and circulatory flows around a cylinder was first discovered by Magnus in 1852. It is known as Magnus effect.

14.9 ANSWERS TO SAQs

SAQ 1

An aeroplane moving against the wind will have greater speed relative to the air and hence greater lift than, when moving in the direction of the wind.

The opposing wind velocity helps in slowing down the velocity of aeroplane when it lands.