UNIT 12 PIPE FLOW PROBLEMS

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12.1 INTRODUCTION

In the previous unit on pipe flow we studied basic concepts of fluid mechanics associated with the frictional loss of energy in pipe flow. Now we shall use those concepts to obtain solutions to different kinds of pipe flow problems.

Pipelines are used to convey fluids from one point to another. While conveying the fluids, the pipelines also convey the energy (or power) of the flowing fluid. It is obvious that part of the fluid power would be utilized in overcoming the frictional and form resistances in pipelines. The form resistance is on account of the changes in the shape and or size of the pipeline, the changes in the direction of flow and different types of fittings in the pipeline. The loss(es) of head in overcoming the form resistances is termed the minor loss(es).

Pipe flow problems can be grouped into two categories depending upon whether the solution is obtainable by direct computations or it requires a trial-and-error method.

Objectives

By the end of this unit, you should be able to

* compute frictional and minor losses in a pipeline of given size, characteristics and for known discharge rate,
* compute the flow rate for known pipeline characteristics and the head loss,
* compute the diameter of the pipeline for known flow rate and the head loss,
* obtain the distribution of flow in a pipe network,
* solve three-reservoir problems, and
* solve problems related to transmission of fluid power.

Only incompressible flow has been considered for the subject matter of this unit.

12.2 VARIOUS LOSSES IN PIPELINES

In addition to the loss of head caused by friction in a pipeline, there are losses due to change in the cross-section and presence of bends, valves and different kinds of fittings. In a long pipeline these additional losses (usually termed **minor** or **secondary** losses) may be a small fraction of the **ordinary** friction loss and hence, are considered negligible in long pipelines. The minor losses may, however, exceed the frictional losses in a shorter pipeline and should, therefore, be accounted for in such situations. These minor losses are generally caused due to sudden changes in the magnitude and/or direction of the velocity of flow. These changes, in turn, generate large-scale turbulence in which the energy is dissipated as heat. The turbulence so geneated affects the flow for a considerable distance downstream of the section where the change in velocity occurred. For the purpose of analysing this complicated flow, it is assumed that the effects of friction and the additional large-scale turbulence can be separated and the additional loss (i.e. the minor loss) is assumed to occur at the device causing it. The total head loss in a pipeline is, obviously, the sum of the friction loss for the pipeline and the minor loss due to various fittings in the pipeline.

The minor losses, in general, cannot be determined theoretically (one exception is sudden enlargement which can be analysed theoretically). Since the losses are proportional to the square of the average velocity of flow (U) in turbulent flow, the minor losses (H_L) are generally expressed in the following form :

$$H_L = K \frac{U^2}{2g}$$

Here, K is an empirical coefficient which would depend on the type of fitting causing minor loss and the Reynolds number representing the flow condition. Obviously, at high Reynolds number, the value of K remains constant for a given type of fitting.

12.2.1 Head Loss Due to Sudden Enlargement

Consider the sudden enlargement of cross-section of pipeline as shown in Figure 12.1. Due to sudden change in the boundary, the fluid emerging from the smaller pipe is unable to follow the boundary. As a result, the flow separates and pockets of eddies are formed in the corners. These eddies result in dissipation of energy as heat.

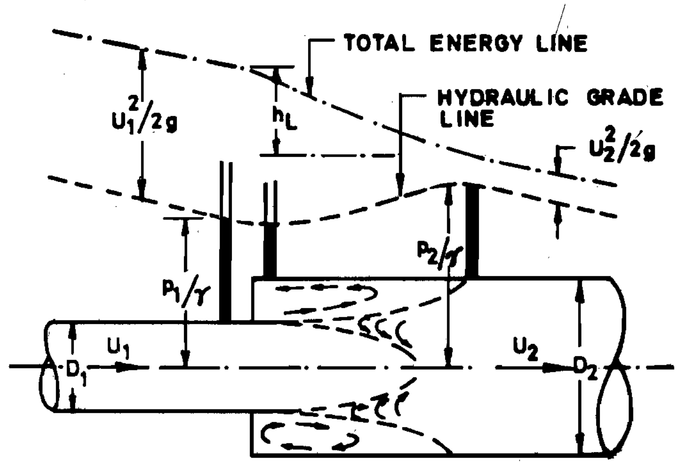

Figure 12.1

Assuming that (i) the velocity distribution is uniform at sections 1 and 2, (ii) the mean pressure of the eddying fluid is the same as the pressure at section 1 and (iii) the pipe axis is horizontal, one can write the momentum equation for the control volume between sections 1 and 2 as follows :

$$(p_1 - p_2) A_2 = \rho Q (U_2 - U_1)$$

Here, p represents the mean pressure, A the cross- sectional area, ρ the mass density, Q the discharge rate and U is the mean velocity of flow. The subscripts indicate the section at which the values are being considered.

$$\therefore \quad \frac{p_1 - p_2}{\rho g} = \frac{Q}{gA_2}(U_2 - U_1) = \frac{U_2}{g}(U_2 - U_1)$$

From the energy equation, one obtains

$$\frac{p_1}{\rho g} + \frac{U_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{U_2^2}{2g} + z + H_L$$

$$\therefore \quad H_L = \frac{p_1 - p_2}{\rho g} + \frac{U_1^2 - U_2^2}{2g}$$

or

$$H_L = \frac{U_2}{g}\left(U_2 - U_1 \right) + \frac{U_1^2 - U_2^2}{2g}$$

$$\therefore \quad H_L = \frac{\left(U_1 - U_2 \right)^2}{2g}$$

$$\therefore \quad H_L = \frac{U_1^2}{2g}\left[1 - \frac{A_1}{A_2} \right]^2 = K\,\frac{U_1^2}{2g} \qquad (12.2)$$

This equation is known as **Borda - Carnot** equation.

The factor K, known as loss coefficient, depends only on the sizes of the two pipes. When a pipeline discharges into a large reservoir, Figure 12.2, A_2 is very large compared to A_1 and, therefore, K is unity. Thus, exit loss is given as

$$H_L = \frac{U_1^2}{2g}$$

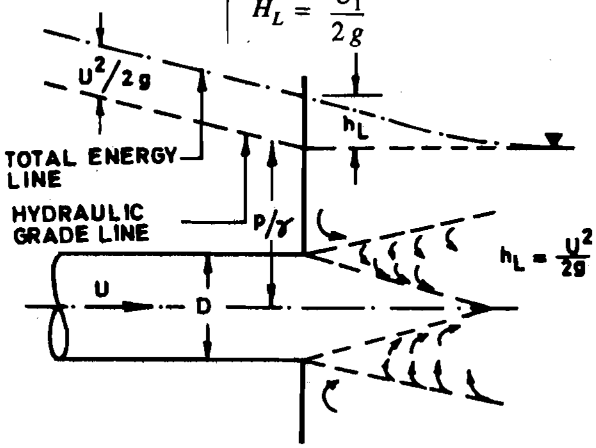

Figure 12.2

12.2.2 Head Loss Due to Sudden Contraction

In the case of sudden contraction, Figure 12.3, there forms vena contracta immediately downstream of the contraction and thereafter the jet expands. Upstream of the vena contracta, the flow is accelerating and the energy loss is, therefore, relatively small compared to the loss that occurs downstream of the vena contacta in which region the flow is decelerating.

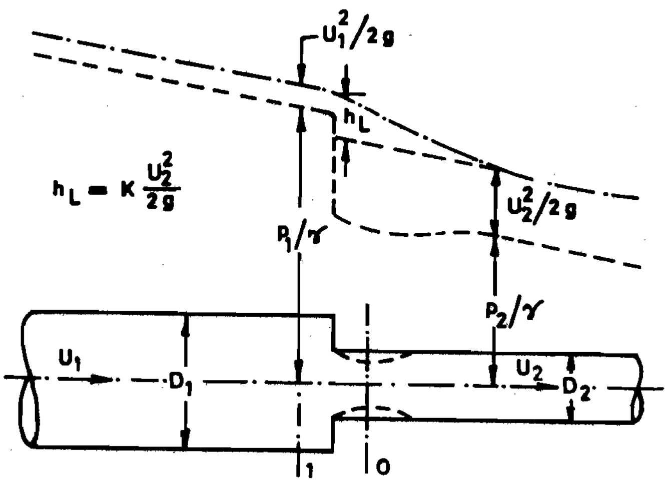

Figure 12.3

The total loss of head is, obviously, the sum of these two losses, i.e.

$$H_L = K_1 \frac{U_0^2}{2g} + \frac{(U_0 - U_2)^2}{2g}$$

Since $A_0 = C_c A_2$

$$\therefore \quad U_0 = \frac{U_2}{C_c}$$

Hence,

$$H_L = \left[\frac{K_1}{C_c^2} + \left(\frac{1}{C_c} - 1 \right)^2 \right] \frac{U_2^2}{2g} = K \frac{U_2^2}{2g} \tag{12.3}$$

The loss coefficnet K for sudden contraction depends on the coefficient of contraction C_c which, in turn, depends on the ratio of the areas A_2/A_1. The observed values of C_c and K are given in Table 12.1.

Table 12.1 : The Coefficient of Contraction and Loss Coefficient for Sharp-edged Sudden Contraction

A_2/A_1	C_c	K
0	0.617	0.50
0.1	0.624	0.46
0.2	0.632	0.41
0.3	0.643	0.36
0.4	0.659	0.30
0.5	0.681	0.24
0.6	0.712	0.18
0.7	0.755	0.12
0.8	0.813	0.06
0.9	0.892	0.02
1.0	1.000	0.00

When a pipeline is connected to a reservoir for receiving water from the reservoir, the ratio A_2/A_1 is almost zero and the loss coefficient is 0.5. Thus, for the sharp-edged entrance, the entrance loss is taken as $0.5 \frac{U_2^2}{2g}$. For a rounded or bell-mouthed entry, the loss coefficient can be as low as 0.02.

12.2.3 Head Loss in Gradual Transitions

Two pipes of different cross-sections are usually connected through a fitting which accomplishes the change gradually. This fiiting is called transition which may be either convergent or divergent. Convergent transition results in accelerating flow which converts the potential energy into kinetic energy and is, therefore, inherently stable and free from separation. The energy loss in gradual contraction is, therefore, very small and is given as

$$H_L = 0.04 \frac{U_2^2}{2g} \tag{12.4}$$

in which U_2 is the average velocity of flow in the contracted section.

In the diverging transitions, however, the flow is retarding and the kinetic energy is converted into potential energy. Further, if the angle of expansion is larger than about 10°, the flow separates from the boundary and thus results in the formation of eddies. Hence, the energy loss in the expanding transitions is relatively more. The head loss due to gradual expansion can be expressed as

$$H_L = K \frac{(U_1 - U_2)^2}{2g} \tag{12.5}$$

in which the loss coefficient K depends upon the flow condition and the geometry of the expansion. For relatively large Reynolds number, the loss coefficient depends only on the geometry of the expansion. The variation of the loss coefficient K with the angle of expansion for a conical diffuser is shown in Figure 12.4.

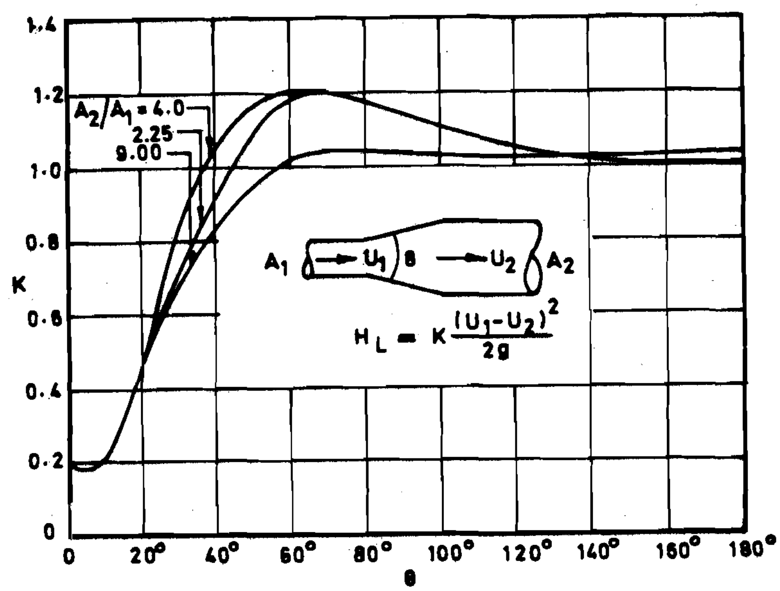

Figure 12.4

It may be noted that when $\theta = 180°$ (i.e. sudden expansion), $K = 1.0$.

SAQ 1

Why head loss in diverging transition is more than that in converging transition?

12.2.4 Head Loss in Pipe Fittings

In any pipeline, it is usual to use different kinds of fittings such as Tee joints, valves, bends etc.. The head loss on account of these fittings can be expressed as

$$H_L = K \frac{U^2}{2g} \tag{12.6}$$

The value of K has to be determined experimentally for any type of fitting as it depends on the Reynolds number as well as the geometry of the fitting. Some typical values of K for some pipe fittings are given in Table 12.2.

Table 12.2 : Typical Values of Loss Coefficient K for Pipe Fitings

Pipe Fitting	K
Standard Tee joint	1.80
Standard 90° elbow	1.90
Standard 45° elbow	0.42
Return bend	2.20
Gate Valve fully open	0.19
Gate Valve 3/4 fully open	1.15
Gate Valve 1/2 fully open	5.60
Gate Valve 1/4 fully open	24.0

12.3 EQUIVALENT PIPE SYSTEMS

Two pipe systems are said to be equivalent when the same amount of discharge flowing through the two pipe systems causes the same head loss. This concept can be advantageously used to express the form as well as friction effects in terms of an equivalent length of straight pipe of uniform diameter which would result in the same head loss at the same rate of discharge. The total head loss h_L in a pipeline can always be expressed as

$$h_L = K \frac{U^2}{2g}$$

in which K is a loss coefficient which includes the effects of all non-uniformities as well as friction. Another pipeline of straight length l_e and uniform diameter D would be equivalent to the given pipeline having total head loss h_L only if the following relation is satisfied.

$$K \frac{U^2}{2g} = f \cdot \frac{l_e}{D} \cdot \frac{U^2}{2g}$$

from which one obtains

$$\frac{l_e}{D} = \frac{K}{f} \qquad (12.7)$$

Here, l_e is termed the equivalent length of pipe and f is the friction factor of the selected equivalent pipeline.

Likewise two straight pipes of diameters D_1 and D_2, length l_1 and l_2 and friction factors f_1 and f_2 would be equivalent if

$$\frac{8 f_1 l_1 Q^2}{\pi^2 g D_1^5} = \frac{8 f_2 l_2 Q^2}{\pi^2 g D_2^5}$$

i.e.

$$\frac{f_1 l_1}{D_1^5} = \frac{f_2 l_2}{D_2^5} \qquad (12.8)$$

Similarly, pipes in series and pipes in parallel can be simplified to **equivalent single pipe** of specified diameter having an equivalent pipe length l_e.

The concept of equivalent pipe length enables simplification of a compound pipeline into a single pipeline of suitable length i.e. the equivalent pipe length.

SAQ 2

What is meant by "equivalent pipe length"?

SAQ 3

When are the minor losses really minor ?

When is the terms "minor loss" a misnomer?

12.4 FLOW THROUGH PIPES IN SERIES

When pipes of different cross-sections are connected end to end to form a pipeline, so that the fluid flows through each in turn, the pipes are said to be in series. The total loss of head for the entire pipeline would, obviously, be the sum of the friction and minor losses for each pipe together with the losses that might occur at their junctions. The discharge would be the same in all these pipes which have been connected in series.

12.5 FLOW THROUGH PIPES IN PARALLEL

When two or more pipes are connected so as to first divide the flow and subsequently bring it together as shown in Figure 12.5, the pipes are said to be in parallel.

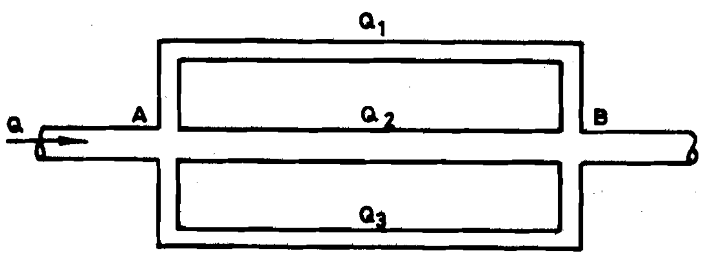

Figure 12.5

Referring to the Figure, it is obvious that,

$$Q = Q_1 + Q_2 + Q_3 \qquad (12.9)$$

and

$$H_L = H_{L_1} = H_{L_2} = H_{L_3} \qquad (12.10)$$

Here, H_L is the headloss between points A and B and H_{L_1}, H_{L_2} and H_{L_3} are head losses in the three pipes.

For known pipe and fluid characteristics, one can easily determine the total discharge Q for known head loss H_L by determining Q_1, Q_2 and Q_3 as H_{L_1}, H_{L_2} and H_{L_3} are all equal to H_L.

Other type of problems related to pipes in parallel concerns the distribution of the given amount of total discharge Q and the determination of the head loss H_L. Such problems can be solved by the following procedure.

(i) Assume a suitable discharge Q_1' for pipe 1 and compute the head loss H_{L_1}' with the assumed discharge Q_1'.

(ii) For known $H_{L_1}' \left(= H_{L_2}' = H_{L_3}' \right)$ obtain Q_2' and Q_3' and determine

$$\Sigma Q' = Q_1' + Q_2' + Q_3'.$$

(iii) Distribute the total given discharge Q in the following manner :

$$Q_1 = \frac{Q_1'}{\Sigma Q'} \cdot Q \qquad (12.11a)$$

$$Q_2 = \frac{Q_2'}{\Sigma Q'} \cdot Q \qquad (12.11b)$$

$$Q_3 = \frac{Q_3'}{\Sigma Q'} \cdot Q \qquad (12.11c)$$

(iv) Check the correctness of the discharges Q_1, Q_2 and Q_3 by computing the loss in each pipe.

In such problems, minor losses may be either completely neglected or could be considered by adding their equivalent pipe length to the actual length of the relevant pipe.

12.6 PIPE NETWORK

For distribution of municipal water, a network of pipelines is used. In addition, the friction equation must also be satisfied for each pipe. It should be noted that Darcy's formula for friction loss takes no account of the direction of flow. When the direction of flow is in doubt, one has to assume it and see if the assumption yields physically possible solution. These pipe networks often are difficult problems to analyse. The fundamental principles of continuity and uniqueness of the head at any junction form the basis of solution of pipe networks. According to the principle of continuity, the total incoming discharge at any junction is equal to the total outgoing discharge at that junction. The uniqueness of the head at a given junction requires that the net head loss (or the algebraic sum of the head loss) round any closed loop in the network must be zero. For the purpose of obtaining the net headloss, the headloss in a pipe of a given loop may be considered as positive if the flow in the pipe is clockwise. One has to obtain the distribution of discharge (magnitude as well as direction and flow) in different pipes of the network for known pipe characteristics. Since the flow direction in many of the pipes of a pipe network may be unknown, it is often very time-consuming if one solves the resulting equations by ordinary trial and error method. Hardy Cross has developed a systematic method of computation which gives results of acceptable accuracy with relatively small number of trials. The method consists of the following steps :

1. By careful inspection, assume the most reasonable distribution of flows which satisfies the continuity principle at every junction.
2. Obtain head loss h_L (usually due to friction i.e. h_f) in each pipe by writing

$$h_f = r\,Q^n$$

where r is a constant for each pipe. n is usually taken as 2.0. While minor losses within any circuit may be included, the minor losses at the junction points are neglected.

3. Compute the algebraic sum of the head losses around each elementary circuit i.e.

$$\Sigma\, h_f = \Sigma\, r\, Q^n$$

considering losses from clockwise flows as positive and those from counter clockwise flows as negative.

4. Adjust the flow in each circuit by a correction ΔQ to balance the head loss in the circuit so that

$$\Sigma\, r\, Q^n = 0$$

ΔQ is determined in the following manner :

For any pipe, one may write

$$Q = Q_0 + \Delta Q$$

where Q is the correct discharge and Q_0 is the assumed discharge. Then, for each pipe,

$$h_f = rQ^n = r\left(Q_0 + \Delta Q\right)^n = r\left(Q_0^n + n\,Q_0^{n-1}\Delta Q + \ldots\right)$$

If ΔQ is small compared with Q_0, one may neglect the terms of the series after the second term and

$$h_f = rQ_0^n + rn\,Q_0^{n-1}\Delta Q$$

Now for a circuit, with ΔQ the same for all pipes,

$$\sum h_f = \sum rQ_0^n + \Delta Q \sum rn\,Q_0^{n-1} = 0$$

$$\therefore \quad \Delta Q = -\frac{\sum rQ_0^n}{\sum rn\,Q_0^{n-1}} = -\frac{\sum h_f}{n\sum |h_f/Q_0|} \qquad (12.12)$$

It should be noted that the numerator of this equation is to be summed algebraically with due regard to sign and the denominator is summed arithmatically. The direction of ΔQ is, obviously, clockwise when it is positive and counter clockwise when negative.

5. After each circuit is given a first correction, the losses will still not balance because of the interaction of one circuit upon another due to which pipes, common to two circuits, receive two independent and different corrections - one for each circuit. The procedure is, therefore, repeated until the corrections become negligible small.

12.7 THREE RESERVOIR PROBLEMS

Pipe system, involving flow among three reservoirs, consists of three pipes meeting at a junction (Figure 12.6). The flow in such a pipe system can also be analysed by satisfying the continuity principle at any junction, uniqueness of head at any point and the Darcy's equation for each pipe. Because of uncertainty in the flow direction in one or more pipes of the pipe system, the solution of three reservoir problem requires trial.

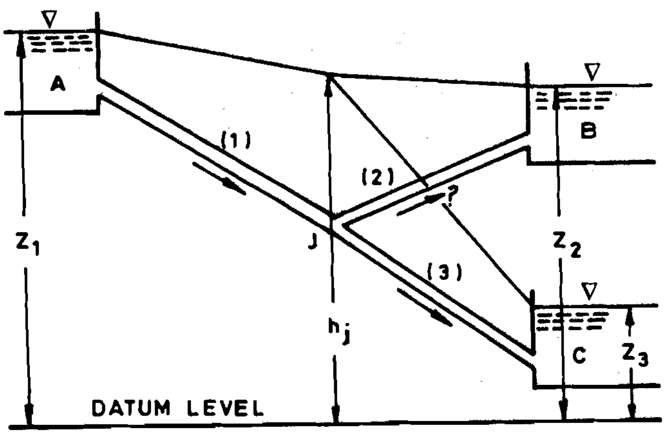

Figure 12.6

In Figure 12.6, three reservoirs A, B and C are connected to common junction J by pipes 1, 2 and 3 in which the head losses are h_{f1}, h_{f2} and h_{f3} respectively. The characteristics of pipes 1, 2 and 3 as well as surface levels in reservoirs A, B and C are known. The head at junction J is unknown.

For such pipe system, following method is used for the solution :

Since the head at A is the highest and that at C the lowest the direction of flow in pipes 1 and 3 is as indicated by the arrows. The direction of flow in pipe 2, however, is not immediately evident. If h_j, the head at J, is intermediate between the heads at A and B then flow occurs from J to B and for steady conditions the following equations apply:

$$z_1 - h_j = h_{f1} \qquad (12.13a)$$

$$h_j - z_2 = h_{f2} \qquad (12.13b)$$

$$h_j - z_3 = h_{f3} \qquad (12.13c)$$

$$Q_1 = Q_2 + Q_3 \qquad (12.13d)$$

Since h_f is a function of Q, these four equations involve the four unknowns h_j, Q_1, Q_2 and Q_3. Even when f is assumed constant and minor losses are neglected so that

$$h_f = \frac{fL}{d} \frac{Q^2}{\left(\pi d^2/4\right)^2 2g}$$

albegraic solution is tedious (and for more than four pipes impossible). Trial values of h_f substituted in the first three equations, however, yield values of Q_1, Q_2 and Q_3 to be checked in the fourth equation. If the calculated value of Q_1 exceeds $Q_2 + Q_3$, for example, the flow rate towards J is too great and a larger trial value of h_j is required. Values of $Q_1 - (Q_2 + Q_3)$ may be plotted against h_j as in Figure 12.7 and the value of h_j for which $Q_1 - (Q_2 + Q_3) = 0$ readily found. If, however, the direction of flow in pipe 2 was incorrectly assumed no solution is obtainable.

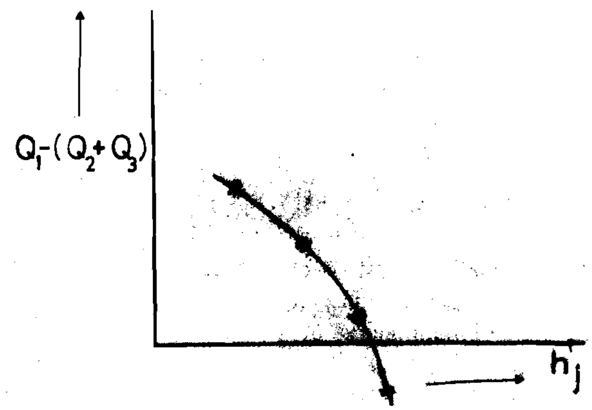

Figure 12.7

For the, opposite direction of flow in pipe 2 the equations are :

$$z_1 - h_j = h_{f1} \tag{12.14a}$$

$$z_2 - h_j = h_{f2} \tag{12.14b}$$

$$h_j - z_3 = h_{f3} \tag{12.14c}$$

$$Q_1 + Q_2 = Q_3 \tag{12.14d}$$

It will be noticed that the two sets of equations (12.13) and (12.14), become identical when $z_2 = h_j$ and $Q_2 = 0$. A preliminary trial with $h_j = z_2$ may, therefore, be used to determine the direction of flow in pipe 2. If the trial value of Q_1 is greater than that of Q_3, that is, if the flow rate towards J exceeds that leaving J, then a greater value of h_j is required to restore the balance. On the other hand, if $Q_1 < Q_3$ when h_j is set equal to z_2, then h_j is actually less than z_2.

12.8 TURBULENT FLOW IN NON-CIRCULAR CONDUITS

Normally the conduits for carrying a fluid are circular in shape and one can use Darcy-Weisbach equation alongwith Moody's diagram for solving flow problems. Experiments have indicated that the relations developed for circular pipes yield reasonable results if the diameter is replaced by another term known as **equivalent diameter** such that the hydraulic mean readius R (i.e. the ratio of area of cross- section A and the wetted perimeter P) remains the same. For a circular section

$$R = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

i.e.

$$D = 4R$$

This means that the **equivalent diameter** of a non-circular pipe is four times its hydraulic mean radius. Thus, Darcy-Weisbach equation, for non-circular conduits, is written as

$$h_f = \frac{f l U^2}{2g(4R)}$$

and the Reynolds number would be calculated as

$$\text{Re} = \frac{U(4R)}{\nu}$$

It should, however, be noted that this kind of simplification yields only approximate results depending upon the deviation of non-circular shape from the circular shape. This is on account of the assumption involved in the concept of **equivalent diameter** that the mean shear stress at the boundary is the same as for a circular section. Further, this concept is not applicable to laminar flows.

12.9 POWER REQUIREMENTS OF A PIPELINE

Problems of flow through pipes involve the estimation of power required to maintain a certain flow through the pipe. Since power P is the rate of doing work, it is equivalent to the product of force (i.e. $-\frac{\partial p}{\partial x}.Al$) and the mean velocity U, i.e.

$$P = -\frac{\partial p}{\partial x} \cdot A l U$$

$$= -\frac{\partial p}{\partial x} l \cdot Q$$

or $$P = Q(p_1 - p_2)$$

For inclined pipes, $(p_1 - p_2)$ is replaced by $\rho g (h_1 - h_2)$

$$\therefore \quad P = \rho g Q (h_1 - h_2)$$

Here, power P is in watts and Q in m^3/s, ρ in kg/m^3 and h is in metres.

12.10 POWER DELIVERED BY A PIPELINE

Pipelines are often used to transmit water from higher elevation to lower elevation. In the process the water loses its potential energy, part of which is lost in overcoming friction in the pipe and the remaining in the turbine. Thus, if z is the elevation difference and h_f is the head loss due to friction in a pipe line carrying discharge Q, then the power P delivered to the turbine is

$$P = \rho g Q (z - h_f)$$

$$= \rho g Q z - \rho g Q h_f$$

$$= \rho g Q z - \rho g r Q^3 \qquad \text{as } h_f = rQ^2$$

For maximum power delivery, $\frac{dP}{dQ} = 0$.

$$\therefore \quad \frac{dP}{dQ} = \rho g z - 3\rho g r Q^2 = \rho g(z - 3rQ^2) = \rho g(z - 3h_f) = 0$$

$$\therefore \quad z - 3h_f = 0$$

or $$h_f = \frac{1}{3} z$$

This means that the power delivered by a given pipe is maximum when the flow is such that one-third of the static head is consumed in friction. However, such a wastage would not be desirable and the pipelines are of such a size that these could deliver water with a loss of only a few percent.

12.11 NOZZLES

A nozzle is a converging tube which is usually fitted at the end of a pipeline to obtain a high velocity jet at the end of the pipeline. The pressure at the exit end of the nozzle is atmospheric. A practical example of nozzle is the flow from a reservoir to an impulse turbine through a penstock that ends in a nozzle.

12.12 ILLUSTRATIVE PROBLEMS

Example 12.1 :

A 300 mm diameter pipe with friction factor of 0.02 has a pipe fitting with loss coefficient of 1.9 and 200 mm diameter pipe of 50 m length with friction factor of 0.022. Determine their equivalent lengths in terms of 300 mm diameter pipe.

Solution :

Since

$$\frac{KU^2}{2g} = f \cdot \frac{l_e}{D} \cdot \frac{U^2}{2g}$$

$$\therefore \quad l_e = \frac{KD}{f}$$

Equivalent length for the pipe fiiting $= \dfrac{1.9 \times 0.3}{0.02} = 28.5$ m

Also, since

$$\frac{8 f_1 l_1 Q^2}{\pi^2 g D_1^5} = \frac{8 f_2 l_2 Q^2}{\pi^2 g D_2^5}$$

$\therefore$ Equivalent length of 50 m pipe in terms of 300 mm diameter pipe

$$= \frac{l_1 f_1}{f_2} \left(\frac{D_2}{D_1} \right)^5$$

$$= \frac{50 \times 0.022}{0.02} \left[\frac{0.3}{0.2} \right]^5$$

$$= 417.66 \text{ m}$$

$\therefore$Equivalent length for the fitting and 200 mm diameter pipe in terms of 300 mm diameter pipe = 28.5 + 417.66

= 446.16 m .

Example 12.2 :

Determine the size of galvanised steel pipe needed to carry water for a distance of 180 m at 85 lit./s with a head loss of 9.0. Take k_s = 0.15 mm.

Solution :

$$h_f = f\frac{L}{D}\frac{U^2}{2g} = f\frac{L}{D}\frac{Q^2 16}{\pi^2 D^4 2g}$$

$$= \frac{8 f . L Q^2}{\pi^2 g D^5}$$

Substituting the values,

$$D^5 = \frac{8 f \times 180 \times 0.085^2}{9.0 \times \pi^2 \times 9.81} = 0.01194 f$$

Taking $\quad \nu = 1.0 \times 10^{-6}$ m²/s

$$\text{Re} = \frac{UD}{\nu} = \frac{4Q\,D}{\pi\,D^2\nu} = \frac{4Q}{\pi\,D\,\nu}$$

$$= \frac{4\times\ 0.085}{\pi\times\ D\times\ 10^{-6}} = \frac{1.08\times\ 10^5}{D}$$

Assume

$$f = 0.018$$

$$D = 0.1846 \text{ m}$$

$$\text{Re} = \frac{1.08\times\ 10^5}{0.1846} = 5.84\times\ 10^5$$

$$\frac{k_s}{D} = \frac{0.15\times\ 10^{-3}}{0.1846} = 0.000812$$

From Moody's diagram for Re = 5.48×10^5 and $\frac{k_s}{D}$ = 0.000812, the value of f is 0.019.

Assumption is O.K.

Hence D = 0.1846 m

Example 12.3:

Determine the head loss due to the flow of 100 lit./s of water through 100 metre length of 15 cm diameter pipe having relative roughness of 0.01.

Solution:

$$\text{Re} = \frac{\frac{Q}{(\pi D^2/4)}D}{\nu} = \frac{4\,Q}{\pi\,D\,\nu}$$

$$= \frac{4\times\ 100\times\ 10^{-3}}{\pi\times\ 0.15\times\ 10^{-6}}$$

$$= 8.4\times\ 10^5$$

Using Moody's diagram for Re = $8.49\times\ 10^5$ and k/D = 0.01, one obtains f = 0.038

$$h_f = \frac{8fL\,Q^2}{\pi^2 gD^5}$$

$$= \frac{8\times\ 0.038\times\ 100\times\ (100\times\ 10^{-3})^2}{\pi^2\times\ 9.81\times\ (0.15)^5}$$

$$= 41.35 \text{ m of water}$$

Example 12.4:

Determine the distribution of flow in the pipe network as shown in Figure 12.8 : Assume n = 2

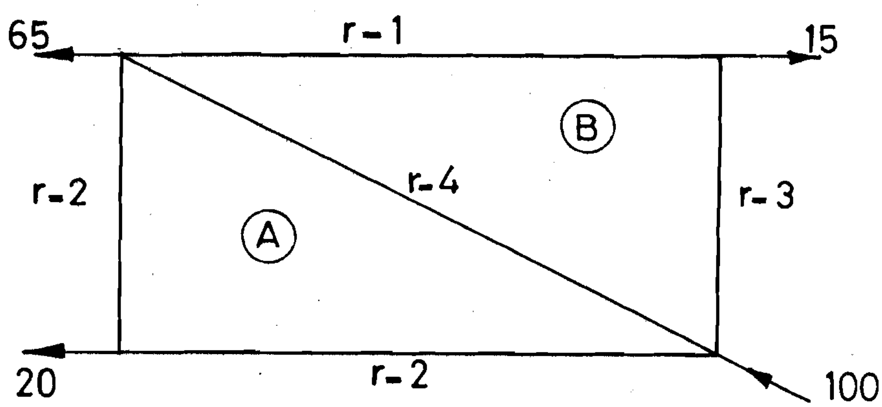

Figure 12.8

Solution :

Let the distribution be as follows :

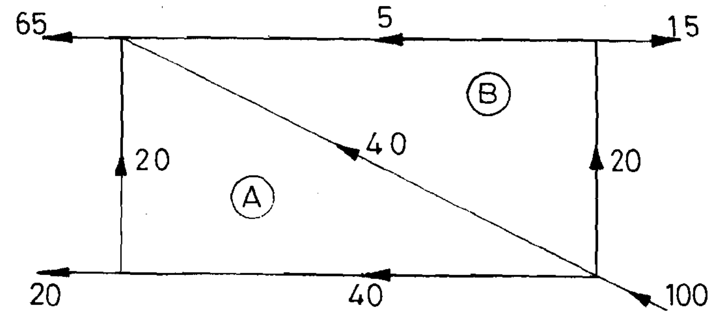

Figure 12.9 (a)

For Circuit A		**For Circuit B**	
rQ_0^2	$2rQ_0$	rQ_0^2	$2rQ_0$
$2\times 40^2 = 3200$	$2\times 2\times 40 = 160$	$4\times 40^2 = 6400$	$2\times 4\times 40 = 320$
$2\times 20^2 = 800$	$2\times 2\times 20 = 80$	$-3\times 20^2 = -1200$	$2\times 3\times 20 = 120$
$-4\times 40^2 = -6400$	$2\times 4\times 40 = 320$	$-1\times 5^2 = -25$	$2\times 1\times 5 = 10$
$\sum rQ_0^2 = -2400$	$\sum 2rQ_0 = 560$	$\sum rQ_0^2 = 5175$	$\sum 2rQ_0 = 450$

$\Delta Q = -\left[\frac{-2400}{560}\right] = 4.3$ $\qquad$ $\Delta Q = -\left[\frac{5175}{450}\right] = -11.5$

Applying these corrections to the assumed distribution, the new distribution is as follows :

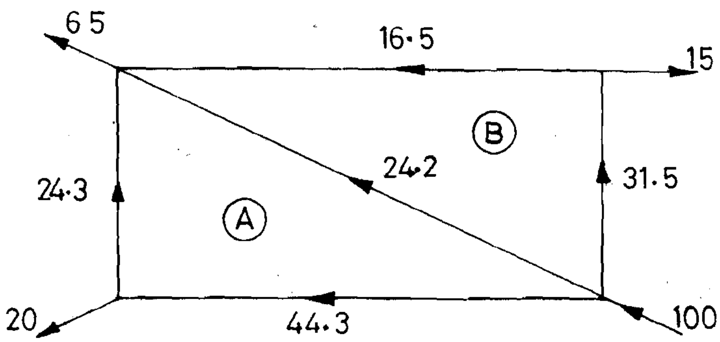

Figure 12.9 (b)

For Circuit A		**For Circuit B**	
rQ_0^2	$2rQ_0$	rQ_0^2	$2rQ_0$
$2\times 44.3^2 = 3925$	$2\times 2\times 44.3 = 177.2$	$-3\times 31.5^2 = -2976.75$	$2\times 3\times 31.5 = 189$
$2\times 24.3^2 = 1181$	$2\times 2\times 24.3 = 97.2$	$-1\times 16.5^2 = -272.25$	$2\times 1\times 16.5 = 33$
$-4\times 24.2^2 = -2342.56$	$2\times 4\times 24.2 = 193.6$	$+\ 4\times 24.2^2 = 2342.56$	$2\times 4\times 24.2 = 193.6$
$\sum rQ_0^2 = 2763.44$	$\sum 2rQ_0 = 468$	$\sum rQ_0^2 = -906.44$	$\sum 2rQ_0 = 415.6$

$\Delta Q = -\left[\frac{2763.44}{468}\right] = -5.9$ $\qquad$ $\Delta Q = -\left[\frac{-906.44}{415.6}\right] = +2.2$

Applying these corrections to the previous distribution, the revised distribution is as follows:

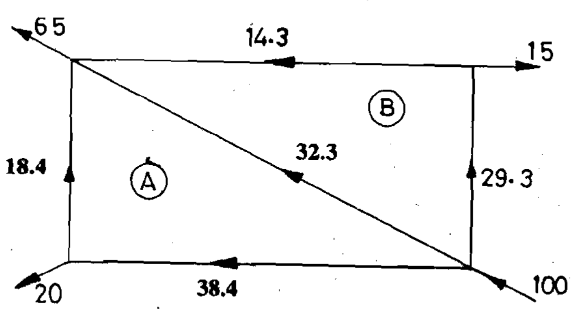

Figure 12.9 (c)

On further calculation in the same manner one would finally obtain the following distribution with acceptable level of error.

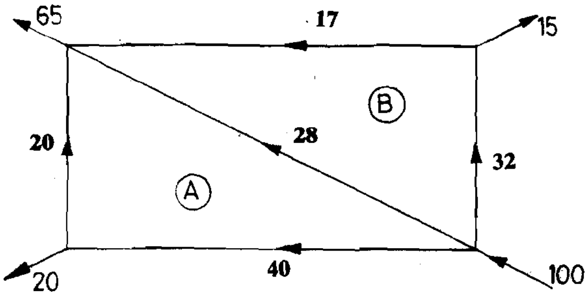

Figure 12.9 (d)

Examples 12.5 :

For the problem shown in Figure 12.10, find elevation of level of the reservoir C and distribution of discharge. Assume $f = 0.03$ for all pipes.

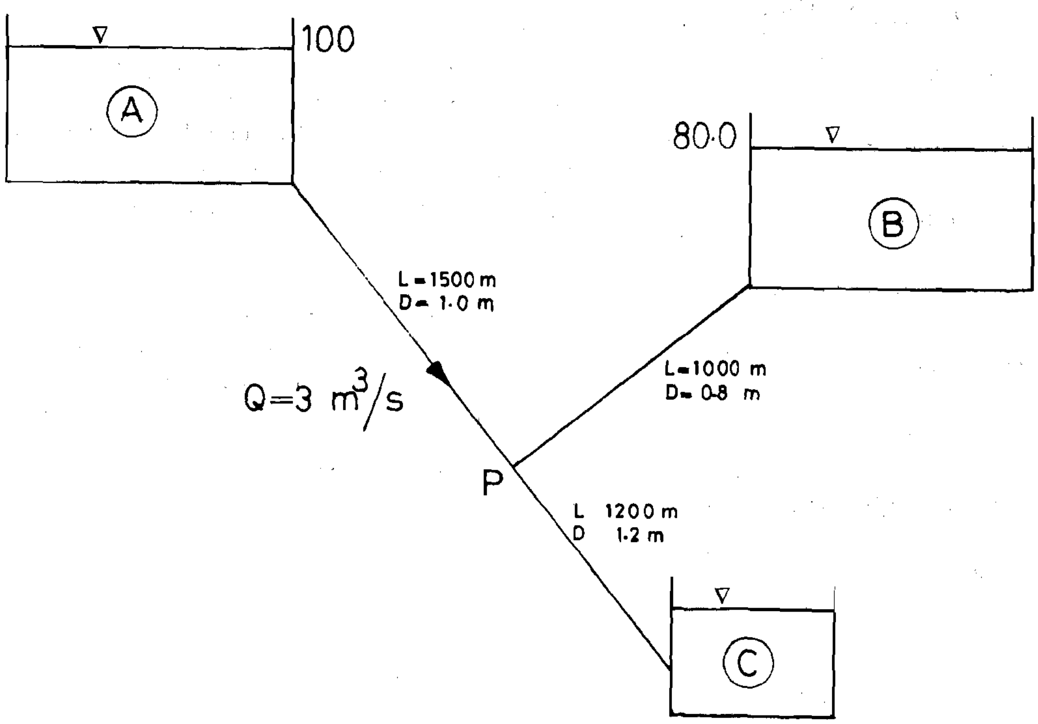

Figure 12.10

Solution :

Velocity in pipe from reservoir A $= \dfrac{3}{\frac{\pi}{4}(1.0)^2} = 3.82$ m/s

$$\therefore \quad h_{f_1} = f\frac{L_1}{D_1}\frac{V_1^2}{2g} = 0.03 \times \frac{1500}{1.0} \times \frac{(3.82)^2}{2 \times 9.81} = 33.47 \text{ m}$$

Therefore, total head at P = 100 - 33.47 = 66.53 m

Hence the flow would be from reservoir B to P with head loss h_{f_2} equal to 80-66.53 = 13.47 m

$$\text{i.e.} \qquad 13.47 = 0.03 \times \frac{1000}{0.8} \times \frac{V_2^2}{2 \times 9.81}$$

$$\therefore \qquad V_2 = 2.655 \text{ m/s}$$

$$\therefore \ Q_2 = \frac{\pi}{4}(0.8)^2 \times 2.655 = 1.335 \text{ m}^3\text{/s}$$

$$\therefore \ Q_3 = 3 + 1.335 = 4.335 \text{ m}^3\text{/s}$$

$$\therefore \quad V_3 = \frac{4.335}{\frac{\pi}{4}(1.2)^2} = 3.833 \text{ m/s}$$

$$\therefore \quad h_{f_3} = 0.03 \times \frac{1200}{1.2} \times \frac{(3.833)^2}{2 \times 9.81}$$

$$= 22.47 \text{ m}$$

$\therefore$ Elevation of reservoir level $\quad C = 66.53 - 22.47$

$$= 44.06 \text{ m.}$$

12.13 SUMMARY

In this Unit we studied the concepts related to hydraulic losses on account of different types of pipe fittings. The methods to solve different kinds of pipe flow problems including pipe network and three reservoir problems were illustrated. There is no alternative to solving many problems on pipe flow to comprehend the methods illustrated in this Unit.

12.14 KEY WORDS

Pipe Fittings : Bends, elbows, joints, valves etc., in a pipe line systems are known as pipe fittings.

Equivalent Pipe : A hypothetical pipe line of uniform diameter resulting in the same head loss as in the actual pipe line.

Pipe Network : Pipes of different length and diameters connected in different ways.

Three Reservoir Problems : Problems involving flow among three reservoirs, consists of three pipes meeting at a junction.

Minor Loss : Hydraulic loss in pipe lines on account of different forms and fittings in pipe line and other than those due to friction.

12.15 ANSWERS TO SAQs

SAQ 1

See text (Sec. 12.2.3)

SAQ 2

See text (Sec. 12.3)

SAQ 3

For long pipelines the combined loss due to all pipe fittings, termed "minor loss" is small compared to the loss due to friction. But, for short pipe lengths, the reverse may be true and the minor loss may be larger than the loss due to friction.