UNIT 10 BOUNDARY LAYER ANALYSIS

Structure

10.1 INTRODUCTION

In this unit we will study the basic concepts of boundary layer as given by Prandtl in 1904. In fact the theory of boundary layer flow constitutes the backbone of the modern fluid dynamics as it has helped in a rapid advances in various branches of engineering especially aeronautical engineering.

It is our common observation that bodies moving through water or air experience lot of frictional resistance during their motion. For example, motion of a ship, aeroplane, automobiles, turbine blades etc. The theory of hydrodynamics which considers the motion of ideal fluids fails to explain this frictional resistance of such moving bodies. Prandtl realised this shortcoming of the hydrodynamics and showed that for the motion of bodies through water or air, the effect of fluid viscosity is limited in a thin layer close to the body. It is this thin layer which is known as boundary layer. Since in several branches of engineering one is concerned with relative motion between solid wall or on immersed body and the real fluid, concepts and analysis of boundary layers assumes prime importance.

Objectives

After going through this unit, you will be able to

- explain development of boundary layer along a flat plate,
- estimate thickness of boundary layer,
- identify the hydrodynamic nature of the boundary surface i.e. smooth or rough,
- describe flow characteristics inside the boundary layer and their effects on velocity distribution, and
- estimate the frictional drag on a flat plate as a result of boundary layer development.

10.2 DESCRIPTION OF THE BOUNDARY LAYER

In order to understand the concept of a boundary layer, let us consider the flow of a real fluid over a thin stationary plate held parallel to the flow in a uniform stream of velocity U_0 as shown in Figure 10.1.

Plate held parallel to the flow as in Figure 10.1 is also called plate held at zero incidence and the uniform velocity just upstream of the leading edge of the plate is known as **ambient velocity, free stream velocity** or **potential velocity**. In the case of real fluids, however small their viscosity may be, the fluid particles adhere to the boundary and hence the condition of **no slip** prevails. The condition of no slip implies that if the wall or the solid boundary is stationary, the fluid velocity at the wall will be zero whereas if the boundary is moving, the fluid adhering to the boundary will have the same velocity as that of the boundary.

Thus, when the fluid approaches the plate with uniform velocity U_0 as shown in Figure 10.1, Further away from the wall, the velocity will change rapidly from zero at the boundary to its uniform velocity U_0 over a short distance as shown in Figure 10.1. As such there is a velocity

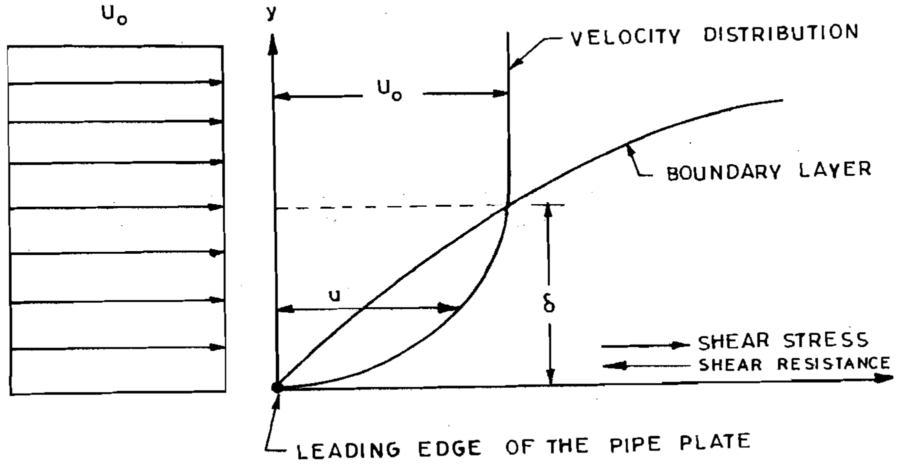

Figure 10.1:Definition Sketch

gradient $\frac{\partial u}{\partial y}$ causing shear or tangential stress on the surface of the plate in the direction of motion. This shear stress on the surface of the plate is given by the following equation :

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (10.1)$$

The force caused by this stress in the direction of motion is known as surface drag or simply drag whereas the corresponding equal and opposite force exerted by the wall on the fluid is known as shear resistance to the flow. As the fluid passes over the plate, this viscous shear retards more and more fluid consequently the thickness of the boundary layer increases in the downstream direction.

It can be seen from the equation (10.1) that even if the fluid has very small viscosity, the presence of high velocity gradient near the boundary is responsible for high shear stresses. Following important observations could be made on the basis of above description of boundary layer :

1. Viscous effects are confined to a very thin layer, called the boundary layer, near the solid boundary.
2. The thickness of the boundary layer will increase along the downstream direction accordingly velocity gradient will reduce in the downstream direction.
3. Reduction in velocity gradient will mean reduction in τ_0 as per equation (10.1) in the downstream direction.
4. The flow outside the boundary layer can be considered as a potential flow of an ideal fluid i.e. the viscous effects could be ignored.

SAQ 1

(i) What is 'no slip' condition?

(ii) Define free stream velocity.

(iii) What is surface drag?

10.2.1 Boundary Layer Thickness and its Characteristics

It was seen above that in case of flow over a flat plate, the thickness of the boundary layer increases in the direction of flow. In many boundary layer problems, the knowledge of this thickness is required. It was also shown that inside the boundary layer, the change in the velocity - zero at the boundary to the free stream velocity U_0, takes place asymptotically. Hence there is some difficulty in defining the exact thickness of the boundary layer in which this change in the velocity takes place. Some of the commonly used definitions of boundary layer thickness are as below :

Nominal Thickness δ

It is defined as that distance from the boundary (measured in the y direction) where the velocity differs by 1 percent from the free stream velocity. Thus we have the following condition for the boundary layer thickness or nominal thickness δ.

$$y = \delta \text{ for } u = 0.99\ U_0$$

Displacement Thickness δ^*

The boundary layer formation and the resulting velocity distribution indicate a reduction in the flow rate as compared to the one that occurs in the absence of the boundary layer. To account for this, a thickness known as displacement thickness δ^* is defined. To clarify this concept, consider Figure 10.2 which shows a graphical representation of this displacement thickness.

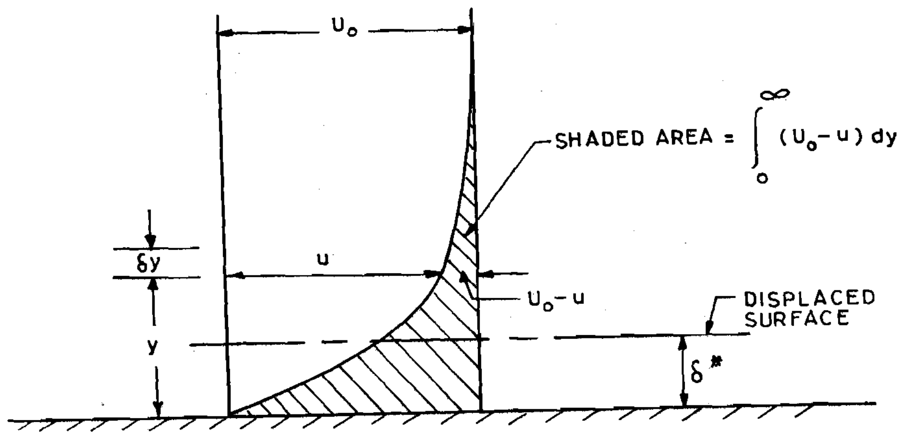

Figure 10.2: Displacement Thickness

Let u be the velocity at a distance y from the boundary, the discharge per unit width through an element of thickness δy is $u.\delta y$. In the absence of any boundary layer, this discharge would have been $U_0\,\delta y$. Thus, the total reduction in discharge caused by the boundary layer is given by

$$\int_0^\infty \left(U_0 - u \right) dy$$

If we now define δ^* such that

$$U_0\,\delta^* = \int_0^\infty \left(U_0 - u \right) dy \tag{10.2}$$

The value of δ^* so defined is known as the **displacement thickness**. In other words, to reduce the total discharge of a frictionless fluid by the same amount as caused by the boundary layer, the solid surface would have to be displaced outwards by a distance δ^*.

Thus, the concept of displacement thickness allows us to consider the main flow as that of a frictionless fluid past a **displaced** surface instead of the actual flow past the actual surface.

Since for y greater than δ, the local velocity is equal to U_0, equation (10.2) can be simplified as

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_0} \right) dy \tag{10.3}$$

Momentum Thickness θ

On lines similar to displacement thickness, we now define momentum thickness denoted as θ. Consider Figure 10.2 again, the fluid passing through an element of the boundary layer carries momentum at a rate $(\rho u dy)u$ per unit width whereas in frictionless flow, the same amount of fluid would have its momentum $(\rho u dy)U_0$. Thus total reduction in momentum is given by

$$\int_0^\infty \rho u \left(U_0 - u \right) dy$$

If we now define θ such that

$$\left(\rho U_0 \theta\right) U_0 = \int_0^{\infty} \rho u \left(U_0 - u\right) dy \qquad (10.4)$$

the value of θ so defined is known as **momentum thickness**. Equation (10.4) can be simplified as,

$$\theta = \int_0^{\delta} \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy \qquad (10.5)$$

From above expressions of boundary layer thicknesses, it can be seen that the displacement thickness is smaller than nominal boundary layer thickness while momentum thickness is smaller than the displacement thickness.

10.2.2 Laminar and Turbulent Boundary Layers and Laminar Sublayer

In the previous unit, we had defined laminar and turbulent flows and pointed out that such flows are identified on the basis of Reynolds number. For example, flow in a pipe is characterised as laminar if the value of the Reynolds number defined as $\frac{UD}{\nu}$ is less than 2100. Here U is the mean velocity of flow in the pipe, D is the diameter of the pipe and ν is the kinematic viscosity of the fluid. Likewise the flow in a boundary layer may be either laminar or turbulent. Figure 10.3 may be referred to visualise the development of laminar and turbulent boundary layers on a flat plate.

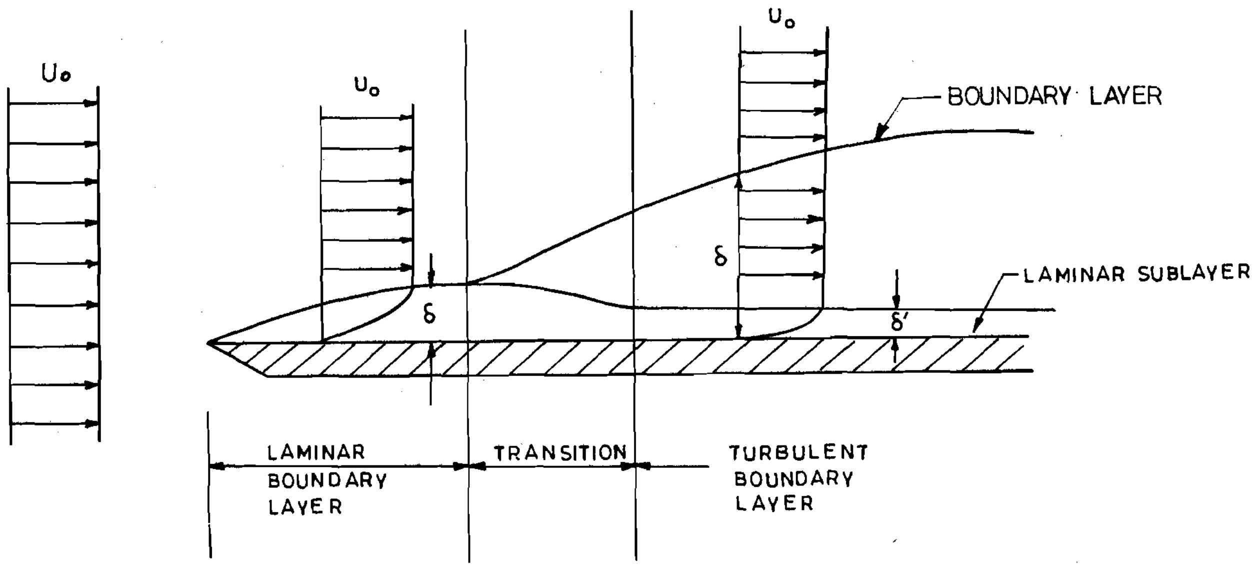

Figure 10.3 : Development of Boundary Layer over a Flat Plate

For boundary layer flow along a flat plate, the characteristic Reynolds number is defined as $R_{ex} = \frac{U_0 x}{\nu}$ where x is the distance along the flat plate measured from the leading edge. It has been found that when the fluid of small viscosity flows over a flat plate, the flow in the boundary layer changes from laminar to turbulent if the Reynolds number R_{ex} exceeds a certain value. From the leading edge upto a certain length, the flow in the boundary layer has all the characteristics of laminar flow, this is irrespective of whether the ambient flow is laminar or turbulent. With increasing thickness of the boundary layer, the laminar boundary layer becomes unstable and the flow within it starts changing into turbulent flow. The region within which this change in flow takes place is known as **transition region**. Downstream of the transition region, the boundary layer is known as turbulent boundary layer and its thickness increases further. It is worthwhile to mention at this stage that for the case of flow over the flat plate as shown in Figure 10.3, the pressure is uniform as such there is no pressure gradient and secondly the thickness of the boundary layer increases continuously provided there is no separation.(The phenomenon of flow separation is discussed later on). Further, the scale in the y direction as shown in Figure 10.3 is greatly enlarged and the thickness of boundary layer δ at any distance x is very small compared to x.

The value of Reynolds number R_{ex} at which a laminar boundary layer becomes unstable depends on number of factors such as roughness of the surface, intensity of turbulence in the main stream, pressure gradient etc. In general, the critical Reynolds number R_{ex} defined above varies between 3×10^5 to 6×10^5. However, for practical purposes, the value of critical R_{ex} is taken as 5×10^5.

Laminar Sublayer

Turbulent flow is characterised by the presence of random components of velocity in all directions. Since fluid particles can not pass through the solid boundary over which the flow is taking place, these random components of velocity must die out very close to the solid boundary. It therefore, follows that the turbulent flow can not exist immediately in contact with the solid boundary.

Thus even when the main flow possesses considerable turbulence, and even when a greater part of the boundary layer is also turbulent, there is still an extremely thin layer, adjacent to the solid surface, in which the flow has negligible fluctuations of velocity. This layer which is very thin and generally denoted by δ' is known as **laminar sublayer** or **viscous sublayer**, see Figure 10.3. This layer is not to be confused with the laminar boundary layer because the velocity at the outer edge of the laminar boundary layer is equal to the velocity of flow U_0 of the main stream whereas the velocity at the outer edge of the laminar sublayer is different than U_0, see Figure 10.3.

10.2.3 Prandtl's Boundary Layer Equations

As discussed above, the effects of viscosity for flow of real fluids are confined within the boundary layer, with some simplification, Navier-Stokes equations could be simplified to yield boundary layer equations.

The flow past a flat plate of infinite extent is essentially a two dimensional flow and is divided into following two regions :

(a) The region within the boundary layer where the velocity gradient, $\frac{\partial u}{\partial y}$, is very large giving large shear stress $\tau = \mu \frac{\partial u}{\partial y}$. It is this region where Navier-Stokes equations apply.

(b) The region outside the boundary layer, here velocities are of the order of free stream velocity U_0. The flow in this region is governed by the potential flow theory; the viscous effects are negligible and the velocity gradients are also very small. Thus the boundary conditions for analysing the flow in the boundary layer can be written as :

(i) no slip condition at the solid boundary,

$$\text{i.e.} \quad u = v = 0 \quad \text{at} \quad y = 0$$

(ii) outside the boundary layer, the velocity u tends to become equal to free stream velocity U_0

$$\text{i.e.} \quad u \to U_0 \quad \text{for} \quad y > \delta$$

(iii) $\frac{\delta}{x} < 1.0$ i.e. the dimensionless boundary layer thickness $\frac{\delta}{x}$ is very small as compared to unity.

Neglecting the body force, the Navier-Stokes equation along the flow direction for flow outside the boundary layer is reduced to

$$\frac{\partial U_0}{\partial t} + U_0 \frac{\partial U_0}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (10.6)$$

For steady flow, the pressure p is a function of x only, the partial derivative $\frac{\partial p}{\partial x}$ can therefore be replaced by the total derivative $\frac{dp}{dx}$ and the equation (10.6) thus reduces to

$$U_0 \frac{dU_0}{dx} = -\frac{1}{\rho}\frac{dp}{dx} \qquad (10.7)$$

or

$$p + \frac{1}{2}\rho\, U_0^2 = \text{constant} \qquad (10.8)$$

The simplified Navier-Stokes equation is thus written as

(i) **in the x direction**

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (10.9)$$

for steady flow and substituting the value of $\frac{dp}{dx}$, equation (10.9) is simplified to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_0\frac{dU_0}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (10.10)$$

(ii) **in the y direction**

Taking order of magnitude of various quantities, the Navier-Stokes equation in the y direction results in

$$\frac{\partial p}{\partial y} = 0$$

and the continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

SAQ 2

(i) Define nominal thickness of boundary layer.

(ii) Write down the definitions of Reynolds number for flow in a pipe as well in the boundary layer. Explain the terms involved in these definitions.

(iii) Define laminar sublayer

10.3 HYDRODYNAMICALLY SMOOTH AND ROUGH BOUNDARIES

Consider the flow of a fluid over a solid boundary. If we examine the boundary closely, we will notice that its surface has some irregularities or projections. The height of these irregularities will depend on the nature of the surface. Let us assume that the average height of these projections is k_s. One would consider this surface as rough for a large value of k_s whereas it will be considered as smooth for smaller values of k_s. However, in problems related to flow of fluids, the absolute size of the roughness elements is not a measure of boundary roughness, it is related to fluid and flow properties.

Consider the case of a boundary of roughness height k_s and for this case also consider that the thickness of laminar sublayer δ' is much greater than k_s, see Figure 10.4 (a). Since all the rough elements are well within the laminar sublayer, these elements do not affect the flow in the turbulent boundary layer. In this case even though the surface is rough, it is defined as hydrodynamically smooth boundary. Experiments have shown that when $\frac{k_s}{\delta'}$ is less than 0.25, the boundary is treated as hydrodynamically smooth boundary.

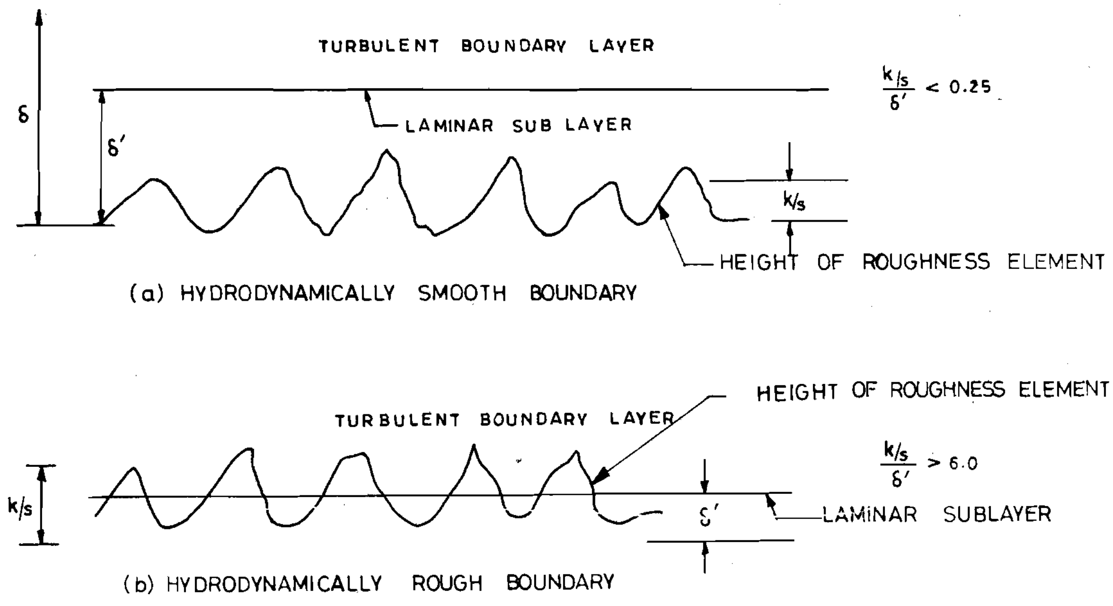

Figure 10.4 : Definition Sketch for Smooth and Rough Boundaries

On the other hand if the thickness of the laminar sublayer δ' is smaller than k_s, see Figure 10.4 (b), the roughness elements project outside the laminar sublayer as a result this layer gets completely destroyed. Wakes are formed behind each roughness element and there is considerable energy loss. The boundary in this case is known as hydrodynamically rough boundary and for this $\frac{k_s}{\delta'}$ is greater than 6.0. For $0.25 < \frac{k_s}{\delta'} < 6.0$, the boundary is classified as boundary in transition.

10.3.1 Velocity Distribution near Smooth and Rough Boundaries

Let us now study the variation in velocity with distance y from the boundary. We have already seen that for laminar flow over a plane boundary, the velocity distribution is obtained by integrating the following equation.

$$\frac{du}{dy} = \frac{\tau}{\mu} \qquad (10.11)$$

Here u is the velocity at a distance y from the boundary, and μ is the dynamic viscosity of the fluid. Likewise for turbulent flow we have

$$\frac{d\bar{u}}{dy} = \frac{\bar{\tau}}{\eta} \qquad (10.12)$$

Here bars denote the temporal mean values of velocity and the shear stress, and η is eddy viscosity. Therefore to obtain velocity distribution for turbulent flow, one needs to integrate equation (10.12). One difficulty arises in this integration because the eddy viscosity η varies with y. Fortunately, experiments indicate that in turbulent flow the intensity of shear and the eddy viscosity vary in such a manner that their ratio is, as a first approximation, directly proportional to $\sqrt{\frac{\tau_0}{\rho}}$ (a parameter known as shear velocity U_*) and inversely proportional to the distance y from the boundary. For simplicity let us omit bars from equation (10.12), this equation can be rewritten as :

$$\frac{du}{dy} = \frac{2.5}{y} U_* \qquad (10.13)$$

in which the factor 2.5 is the constant of proportionality determined from experiments. This factor is also expressed as 1/K where K is Karman's constant having its value equal to 0.40.

Integration of equation (10.13) leads to

$$\frac{u}{U_*} = 2.5 \ln y + C$$

which states that in turbulent flow the velocity will vary directly with the logarithm of the distance from the boundary as shown in Figure 10.5. Here C is a constant of integration.

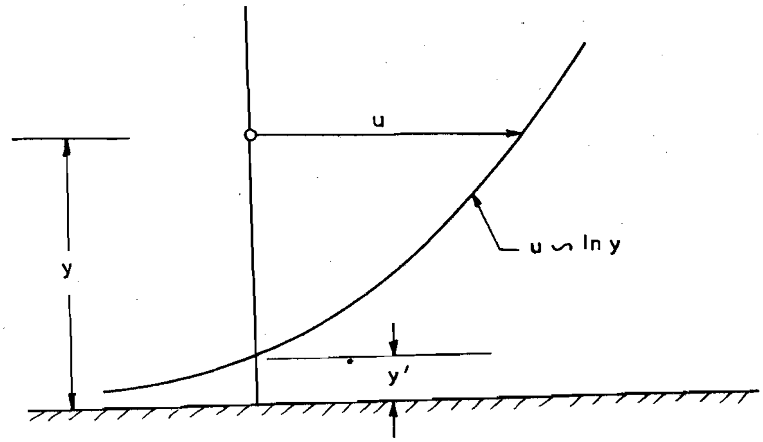

Figure 10.5 : Logarithmic Velocity Distribution in Turbulent Flow

The constant of integration C can be evaluated if we write $u = 0$ at $y = y'$. This condition yields following :

$$\frac{u}{U_*} = 2.5 \ln \frac{y}{y'}$$

or

$$\frac{u}{U_*} = 5.75 \log_{10} \frac{y}{y'} \qquad (10.14)$$

In obtaining equation (10.14) no assumption was made as to whether the boundary is hydrodynamically rough or smooth. Hence equation (10.14) is applicable for both of these boundaries. Further the condition $u = 0$ at $y = y'$ assumed in obtaining this equation at a first glance would appear to be in disagreement with physical fact. In fact it is not so and could be better understood if we recall that close to the boundary there is a laminar sublayer where effects of viscosity are more pronounced. In this layer, the velocity distribution may be assumed to be linear (which implies constant shear stress in this layer) and one can use equation (10.11) to obtain the velocity distribution in this layer i.e.

$$\tau_0 = \tau = \mu \frac{du}{dy} = \mu \frac{u}{y}$$

Dividing this by ρ and simplifying taking $U_* = \sqrt{\frac{\tau_0}{\rho}}$, one get

$$\frac{u}{U_*} = \frac{U_* y}{\nu} \qquad (10.15)$$

Thus, a close examination of equations (10.14) and (10.15) will show that the zone of laminar sublayer must extend well beyond the distance y' if a smooth transition is to exist between the velocity distributions for the laminar and turbulent flows see Figure 10.6.

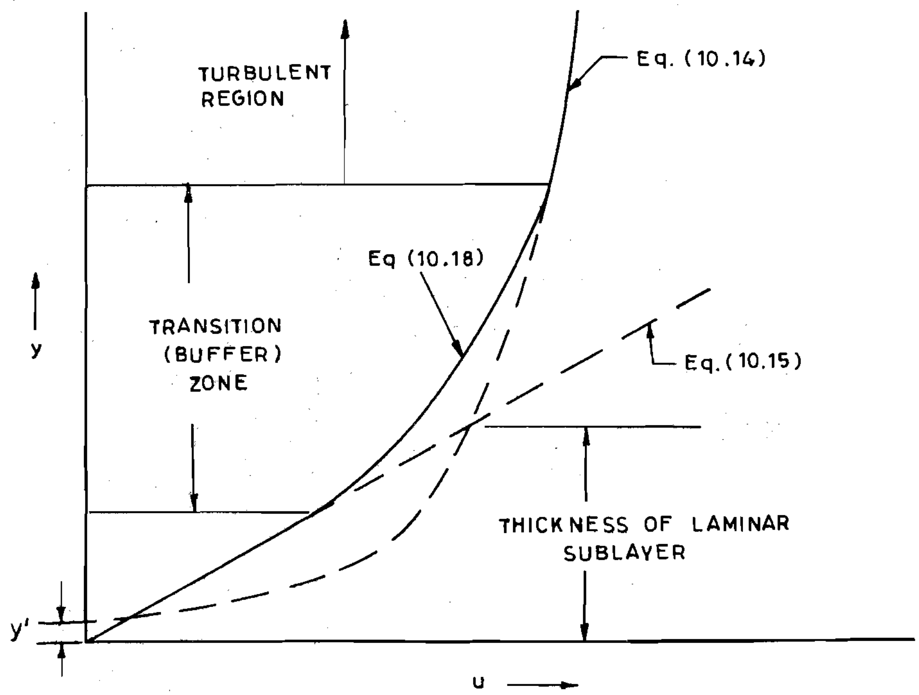

Figure 10.6 : Velocity Distribution in Different Zones

If one arbitrarily selects the intersection of curves given by equations (10.14) and (10.15) as the thickness of laminar sublayer δ' one gets

$$\delta' = \frac{11.6\,\nu}{U_*} \tag{10.16}$$

Nikuradse's experimental data indicate that for hydrodynamically smooth boundaries

$$y' = \frac{\delta'}{107}$$

substituting this value in equation (10.14) one gets

$$\frac{u}{U_*} = 5.75 \log_{10} \frac{U_* y}{\nu} + 5.5 \tag{10.17}$$

This is known as the Karman-Prandtl's equation for velocity distribution for turbulent flow near hydrodynamically smooth boundaries. As can be see from Figure 10.6, equation (10.17) is not valid close to the boundary, data indicate that equation (10.1) is valid for $\frac{U_* y}{\nu} > 30$ while for values of $\frac{U_* y}{\nu} < 5.0$, equation (10.15) is valid. For the transition region i.e. $5 < \frac{U_* y}{\nu} < 30$, following equation has been suggested

$$\frac{u}{U_*} = 11.5 \log_{10} \frac{U_* y}{\nu} - 3.05 \tag{10.18}$$

In the case of hydrodynamically rough boundaries, it is hardly to be presumed that a laminar sublayer will exist at the boundary if the roughness height k_s is greater than δ'. Nikuradse's experimental data indicate that for rough boundaries

$$y' = \frac{k_s}{30}$$

and introducing this value of y' in equation (10.14) one gets

$$\frac{u}{U_*} = 5.5 \log_{10} \frac{y}{k_s} + 8.5 \tag{10.19}$$

Equation (10.19) is known as Karman-Prandtl's equation for the velocity distribution in turbulent flow near rough boundaries.

Inspite of inherent limitations of equations (10.17) and (10.19), these are used for all practical purposes to determine the velocity distribution in a vertical section of a given boundary.

SAQ 3

(i) How are boundaries classified as smooth and rough?

(ii) Write down Karman-Prandtl's equations of velocity distributions in turbulent flow near smooth and rough boundaries.

(iii) Explain why logarithmic law of velocity distribution is not valid at the boundary.

10.4 BOUNDARY RESISTANCE

From the above discussions on boundary layer, it is now clear that the viscous effects are limited in a narrow region close to the boundary. Let us now discuss the magnitude of the resulting forces exerted by the fluid on the boundary. This aspect is proposed to be discussed separately for laminar and turbulent boundary layers.

10.4.1 Boundary Shear for Laminar Boundary Layer

Consider again the flow over a flat plate held longitudinally in a moving fluid. The boundary layer would develop on either side of this plate, giving rise to boundary shear stresses. The magnitude of these stresses depends on R_{ex} and the boundary layer thickness δ. As the Reynolds number is a ratio of inertial force to viscous force, a smaller value of it will signify larger viscous forces whereas a large value of Reynolds number would correspond to small viscous forces. Obviously, δ/x should be a function of R_{ex}. Indeed it has been shown both analytically and experimentally that for laminar boundary layer,

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{R_{ex}}} \tag{10.20}$$

or

$$\delta = 5\sqrt{\frac{x\,\nu}{U_0}} \tag{10.21}$$

Equations (10.20) or (10.21) is known as Blasius equation for the boundary layer thickness.It can be seen from this equation that the boundary layer thickness increases with x and ν and decreases with U_0.

With increase in the boundary layer thickness, the velocity gradient at the plate will decrease in magnitude with a consequent reduction in boundary shear stress τ since $\tau = \mu \frac{du}{dy}$.

Expression for the intensity of shear τ_0 at the surface of the plate can be obtained as :

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} \approx \mu \frac{U_0}{\delta} \tag{10.22}$$

Substituting the value of δ from equation (10.21) in equation (10.22), we get,

$$\tau_0 = \text{Const.} \sqrt{\frac{U_0^3 \rho \mu}{x}}$$

or

$$\frac{\tau_0}{\frac{1}{2}\rho U_0^2} = c_f = \frac{0.664}{\sqrt{R_{ex}}} \tag{10.23}$$

after introducing the experimental value of the proportionality constant obtained by Blasius. In equation (10.23), c_f is known as local drag coefficient.

The total drag force exerted by the fluid on one side of a plate of width B and length L can now be evaluated as

$$F_D = B \int_0^L \tau_0 \, dx$$

Evaluation of this integral will show that

$$F_D = C_f \, B \, L \, \frac{\rho U_0^2}{2} \tag{10.24}$$

In which C_f is the mean drag coefficient and its value is given by

$$C_f = \frac{1.328}{\sqrt{R_{eL}}} \tag{10.25}$$

where $R_{eL} = \frac{U_0 L}{\nu}$. Equation (10.25) is valid for R_{eL} less than 5×10^5.

10.4.2 Boundary Shear for Turbulent Boundary Layer

As we have studied in earlier units, the turbulent flow is characterised by intense mixing of different fluid layers, as a result velocity distribution in a turbulent boundary layer is more uniform as compared to that in laminar boundary layer. This at the same time produces a very rapid change in velocity near the wall. The corresponding expressions for δ, c_f, C_f for the turbulent boundary layer are as follows :

$$\frac{\delta}{x} = \frac{0.377}{R_{ex}^{1/5}} \qquad (10.26)$$

$$c_f = \frac{0.059}{R_{ex}^{1/5}} \qquad (10.27)$$

and

$$C_f = \frac{0.074}{R_{eL}^{1/5}} \qquad (10.28)$$

It can be seen from equations (10.21) and (10.26) that the boundary layer thickness increases as $x^{0.50}$ for laminar boundary layer while it increases as $x^{0.80}$ for turbulent boundary layer. Two important characteristics of the turbulent boundary layers may be noted. Firstly, the thickness of the turbulent boundary layer increases at a faster rate than the laminar boundary layer thickness. Secondly, the velocity gradients are much larger for turbulent boundary layer than for laminar boundary layer giving large shear stresses.

Equation (10.28) is applicable within the range 5×10^5 to 2×10^7 of R_{eL} provided the boundary layer is turbulent from the leading edge, see Figure 10.7. This means that some special measures must be taken to make the boundary layer turbulent right from the leading edge, or one assumes the length of the laminar boundary layer small as compared to the total length of the plate. Beyond $R_{eL} = 2 \times 10^7$, following equation may be used for C_f.

$$\frac{1}{\sqrt{C_f}} = 4.13 \log_{10}\left(R_{eL}\, C_f \right) \qquad (10.29)$$

In case where the plate is covered with laminar and turbulent boundary layers, following equation as given by Prandtl may be used for C_f

$$C_f = \frac{0.074}{R_{eL}^{1/5}} - \frac{1700}{R_{eL}} \qquad (10.30)$$

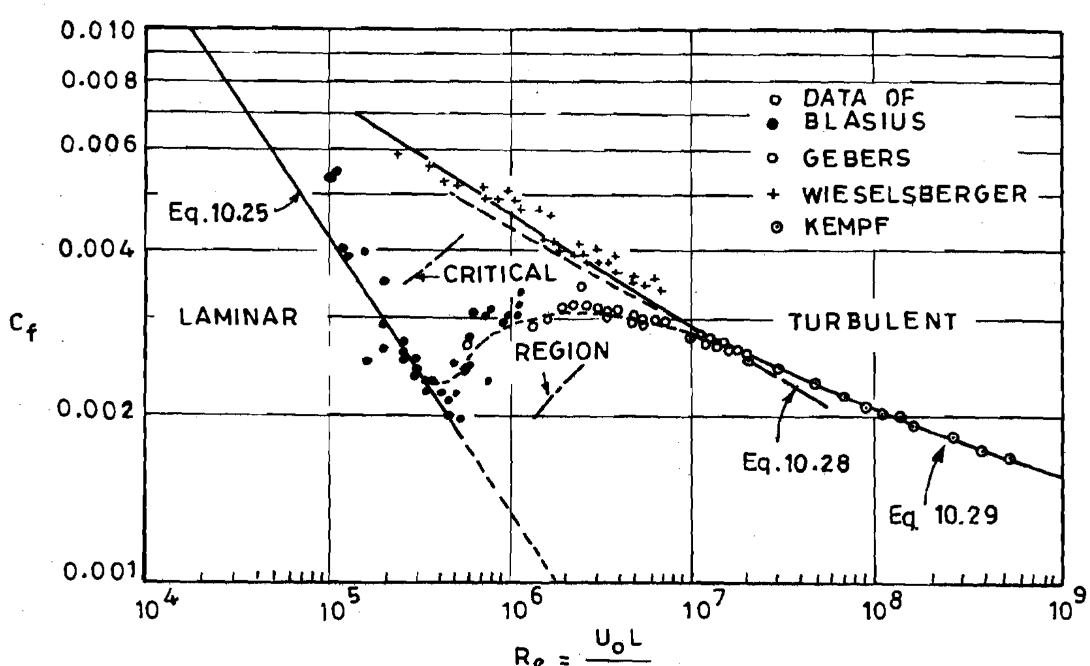

Figure 10.7 : The Mean Drag Coefficient for a Plane Boundary as a Function of the Reynolds Number

The value of constant 1700 in equation (10.30) depends on the value of critical Reynolds number R_{ex} at which the laminar boundary layer becomes turbulent and is given in Table 10.1.

TABLE 10.1 : Variation of Constant in equation (10.30) with R_{ex}

R_{ex}	3×10^5	5×10^5	10^6	3×10^6
Constant	1050	1700	3300	8700

Equation (10.30) is valid for R_{eL} values between 5×10^5 and 10^7 while equation (10.31) is to be used for $R_{eL} > 10^7$.

$$C_f = \frac{0.455}{\left(\log_{10} R_{eL}\right)^{2.58}} - \frac{A}{R_{eL}} \qquad (10.31)$$

Equation (10.31) is applicable for R_{eL} values 5×10^5 to 10^9, and the value of A is same as the value of constant given in Table 10.1.

SAQ 4

(i) Assuming boundary layer to be laminar, write down the equation that gives the boundary layer thickness at a given section.

(ii) Define local drag coefficient.

(iii) Is the statement that turbulent boundary layer thickness varies as $x^{4/5}$ correct?

10.5 ILLUSTRATIVE PROBLEMS

Example 10.1 :

Air at 20° C (ρ_{air} = 1.208 kg/m^3, $\mu = 1.85 \times 10^{-5}$ kg/ms) flows over a 2.0 m wide plate at 10.0 m/s velocity. Determine

(i) τ_0 and δ at a place where the boundary layer ceases to be laminar.

(ii) Drag force on one side of the plate in the laminar region.

Solution :

Let us first calculate the value of x upto which the boundary layer remains laminar

$$R_{ex} = \frac{U_0 x}{\nu} = 5 \times 10^5$$

or

$$x = \frac{5\times 10^5 \nu}{U_0} = \frac{5\times 10^5 \times 1.85\times 10^{-5}}{1.208\times 10.0} = 0.765 \text{ m}$$

Thickness of the laminar boundary layer at x = 0.765 m

$$\delta = 5\sqrt{\frac{x\nu}{U_0}} = 5\sqrt{\frac{0.765\times 1.85\times 10^{-5}}{1.208\times 10.0}} = 5.41 \times 10^{-3} \text{ m}$$

or δ = 5.41 mm

Local drag coefficient

$$C_f = \frac{0.66}{\sqrt{R_{ex}}} = \frac{0.66}{\sqrt{5\times 10^5}} = 9.33 \times 10^{-4}$$

$$\tau_0 = c_f \, \rho_f \frac{U_0^2}{2} = 9.33 \times 10^{-4} \times 1.208 \times \frac{10^2}{2} = 0.056 \text{ N/m}^2$$

Drag force on one side of the plate in the laminar region is given by taking $x = L$

$$F_D = C_f \, B \, L \, \rho_f \frac{U_0^2}{2}$$

where

$$C_f = \frac{1.328}{\sqrt{R_{eL}}} = \frac{1.328}{\sqrt{5 \times 10^5}} = 1.878 \times 10^{-3}$$

$$F_D = 1.878 \times 10^{-3} \times 2 \times 0.765 \times 1.208 \times \frac{10^2}{2}$$

$$= 0.1735 \text{ N}$$

Example 10.2 :

A rectangular plate of sides a and b is towed through water once in a direction of a at the velocity U_a and then in the direction of b at the velocity U_b. If in both the cases boundary layer is laminar over the entire length of the plate, determine the ratios of U_a and U_b which will give equal force on the plate.

Solution :

Case 1   Case 2

Case 1 :

$$R_{eL_1} = \frac{U_a a}{\nu}$$

$$F_{D_1} = \frac{1.328}{\sqrt{R_{eL_1}}} \cdot a \, b \, \rho \frac{U_a^2}{2}$$

Case 2 :

$$R_{eL_1} = \frac{U_b b}{\nu}$$

$$F_{D_2} = \frac{1.328}{\sqrt{R_{eL_2}}} \cdot a \, b \, \rho \frac{U_b^2}{2}$$

Equating F_{D_1} and F_{D_2} and simplifying the expression one gets

$$\frac{U_a}{U_b} = \left(\frac{a}{b} \right)^{1/3} \qquad \text{Answer}$$

10.6 SUMMARY

In this unit we have studied some of the basic concepts of boundary layer alongwith the various characteristics of laminar and turbulent boundary layers on a flat plate. The importance of laminar sublayer in classifying the nature of boundary roughness was also studied. The nature of velocity distributions over smooth and rough boundaries was then examined. Finally, the various equations for evaluating the boundary layer thicknesses and the drag force on the boundary surface were discussed.

10.7 KEY WORDS

Boundary Layer : The layer adjacent to the boundary where fluid is retarted is known as boundary layer.

Boundary Layer Thickness : This is also known as nominal thickness of boundary layer. It is defined as the distance from the boundary where the velocity differs by one percent from the ambient velocity U_0

Viscous Layer : Layer in which viscous forces are more pronounced.

Laminar Sublayer : Thickness of turbulent boundary layer in which the flow is laminar is known as laminar sublayer.

Drag : Force exerted by the fluid on the body in the direction of flow.

Velocity Distribution : Variation in velocity with distance from the boundary of a given section.

Rough & Smooth Boundaries : Boundary is rough if $\frac{k_s}{\delta'} > 6.0$. Boundary is smooth if $\frac{k_s}{\delta'} < 0.25$

Drag Coefficient : Designated as C_D and is interpreted as drag force per unit area divided by the dynamic head.

Reynolds Number : It is the raito of inertial force per unit volume to viscous force per unit volume. It is defined as $\frac{UL}{\nu}$ where L is some characteristic length dimension.

10.8 ANSWERS TO SAQs

Check your answers of all SAQs with respective preceeding text of each SAQs.