UNIT 9 TURBULENT FLOW

Structure

9.1 INTRODUCTION

In this unit we will learn about turbulent flow, which is an irregular kind of flow as against the well-ordered laminar flow. Under certain conditions, laminar flow is not possible and we get a transition from laminar to turbulent flow. Most of the flows we encounter in nature are turbulent and hence the importance of this kind of flow. Further, turbulent flow also results in transfer of momentum, mass and heat across the flow and, therefore, assumes importance in many natural phenomena as well as in engineering applications.

Objectives

After studying this unit you should be able to

* identify transition from laminar to turbulent flow,
* describe turbulent flows,
* understand preliminary concepts related to analysis of turbulent flows, and
* analyse effects of turbulence.

9.2 TURBULENT FLOW

We now know that laminar flow is a well-ordered flow in which the fluid flows in parallel layers or laminas and transfer of momentum takes place at the molecular level. Turbulent flow occurs at relatively higher velocities. In turbulent flow chunks of fluid particles migrate from one layer to another. This migration results in transfer of mass as well as momentum across different layers at a macroscopic level. Most of the flows occurring in nature as well as in engineering applications are turbulent. Examples of turbulent flows include flows in natural streams, irrigation canals, pipes and sewers, flow of wind over earth's surface, flow of natural gas and oil in pipe lines etc..

Turbulent flow is an irregular condition of fluid flow in which various flow parameters show a random variation with respect to space and time coordinates such that statistically distinct average values of the flow parameters can be obtained.

Such random variations are observed in velocity components, pressure, concentration of suspended matter, fluid forces on structures such as suspension bridges, chimneys etc.. Qualitative variation of x-component of velocity u with respect to time and space is shown in Figures 9.1(a) and (b).

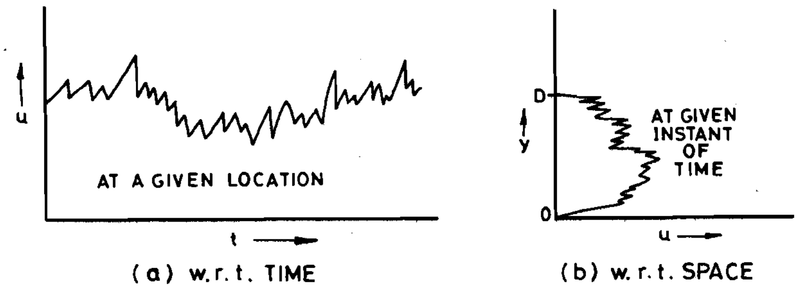

Figure 9.1 : Variation of *u*

Turbulence can be generated in two ways: (1) by friction forces at solid walls (or surfaces) and (2) by the flow of fluid layers with different velocities past the another. These two types of turbulence are, respectively, known as **wall turbulence** and **free turbulence**.

Turbulent flow, like all other real viscous fluid flow, is dissipative in nature and requires continuous external source of energy for the continuous generation of the turbulent motion. The source of energy is in the form of mean shear for both types of turbulence and hence turbulent flows are also known as **turbulent shear flows**. Viscosity of the fluid also makes the turbulence more homogeneous and less dependent on direction. If the structure of the turbulence is the same everywhere in the flow field, the turbulence is said to be homogeneous. Otherwise, it is non-homogeneous. The turbulence is isotropic if it is independent of the direction. Otherwise, it is non-isotropic (or anisotropic).

SAQ 1

(i) Differentiate between laminar and turbulent flows.

(ii) List the characteristics of turbulent flows.

(iii) Define wall turbulence and free turbulence.

9.3 TRANSITION FROM LAMINAR TO TURBULENT FLOW

9.3.1 Reynolds' Criterion

Prior to 1880, the difference between laminar and turbulent flows was not easily understood. Reynolds' experiment in 1880 clearly demonstrated the nature of these two flows. The experimental set-up, Figure 9.2, consisted of a straight length of circular glass tube with a smoothly rounded and flared inlet placed in a large glass-walled tank full of water. Other end of the tube, having a suitable valve to control the flow of water through the

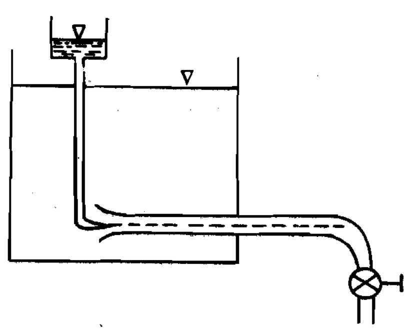

Figure 9.2 : Reynold's Experimental Set-up

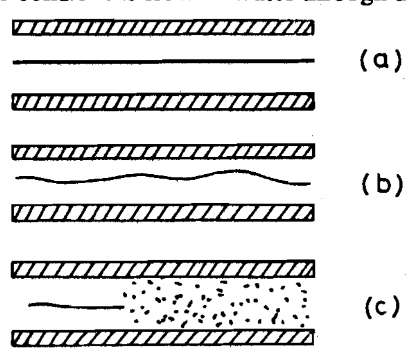

Figure 9.3 : Behaviour of Coloured Filament

tube, was outside the tank. An arrangement was made so that a small reservoir of a liquid dye discharged a coloured filament into the inlet of the glass tube. By observing the behaviour of the coloured filament, Figure 9.3, at different discharges (hence, velocity) of water through the glass tube enabled Reynolds to study the way in which water was flowing in the tube. The flow pattern of Figure 9.3(a) shows the condition corresponding to low discharge and Figure 9.3(c) corresponds to much higher discharge. The pattern of the coloured filament indicates that at low discharge, Figure 9.3(a), the water flows in parallel laminas and the filament remains a distinct streak.

At higher discharge, Figure 9.3(c), there occurs intermixing of fluid layers and hence the coloured filament gets mixed with water across the entire section of the tube. This flow condition, obviously, corresponds to the turbulent motion. Figure 9.3(b) shows the intermediate stage when the turbulent motion starts.

Through these experiments, Reynolds observed that the transition of flow from laminar to turbulent depends on the velocity of flow in the tube, the diameter of the tube and the kinematic viscosity of the liquid. Using the basic concepts and the experimental observations, he concluded that flow in a circular pipe remains laminar if the dimensionlesss number UD/ν, known as the Reynolds number, Re is less than 2100. Under normal conditions, the flow becomes turbulent if the Reynolds number is greater than 3000. When the Reynolds number is between 2100 and 3000, the flow is said to be in transition and is sometimes laminar and at other times turbulent. Under extremely controlled laboratory conditions the flow has been maintained laminar at Reynolds number as high as 50000.

We have seen that the flow patterns in laminar and turbulent flows are considerably different. In addition, the relationship between the energy gradient and the velocity of flow is different for the two types of flow, Figure 9.4. While the energy loss is proportional to the first power of the velocity of flow in case of laminar flow, energy loss is proportional to U^n in which n generally varies from 1.7 to 2.

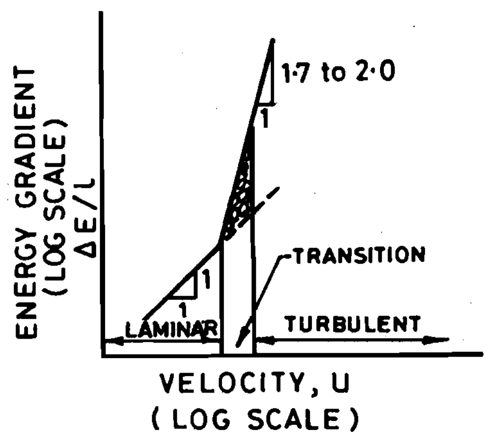

Figure 9.4 : Energy Loss in Laminar and Turbulent Flow

The Reynolds number, Re $(= \frac{UD}{\nu}$ or $\frac{UD\rho}{\mu})$ indicates the importance of the viscous force relative to the inertial force. Larger Reynolds number signifies lesser influence of viscosity upon the flow pattern. An infinite value of Re corresponds to a flow in which the viscous resistance to deformation plays no role in comparison with the intertial resistance to acceleration. On the other hand, a small value of Re indicates more important role of viscosity in the flow.

9.3.2 Rouse's Criterion

Turbulence can also be seen as breakdown of laminar flow in some regions of flow due to local disturbances. Under certain circumstances, local fluctuations (i.e.disturbances) in velocity will gradually be dampened by the viscous stresses and the flow eventually becoming laminar. Under other conditions, the viscous stresses may not be sufficient enough to dampen the local disturbances before these have spread throughout the flow region. This flow is termed turbulent flow. It is obvious that some minor disturbances would always be present in a moving fluid. These distrubances would dempen or spread depending upon whether the flow is stable or unstable. The stability of the flow at a given location would be influenced by the velocity gradient du/dy, mass density ρ, viscosity μ and the wall distance y at the location. Rouse combined these quantities to form a dimensionless parameter, known as **stability parameter** χ, having the following form:

$$\chi = \frac{y^2 \rho \frac{du}{dy}}{\mu}$$

Obviously, smaller the viscosity μ or greater the numerator or this parameter, greater will be the local instability. On the other hand, a larger value of viscosity (i.e. smaller χ) would result in greater stabilizing effect. Also the value of χ would be zero at the wall where y is zero and far away from the wall where boundary effects are negligible and hence $du/dy = 0$. At some intermediate value of y, the stability parameter shall attain a maximum value as

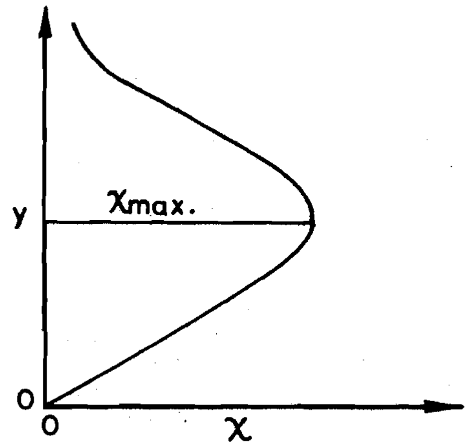

Figure 9.5 : Variation of Stability Parameter

shown in Figure 9.5. Experiments have indicated that the critical value of χ is 500. This means that if χ is less than 500 at all points in a given flow field the flow is inherently stable. If the value of the stability parameter in a flow exceeds 500, the flow becomes unstable and the flow has a tendencey to become turbulent.

9.4 MEAN AND FLUCTUATING COMPONENTS OF VELOCITY

In a turbulent flow, quantities such as velocities, pressures, forces etc. fluctuate about their mean values. If, for example, one plots the velocity u with time t at a given section, this

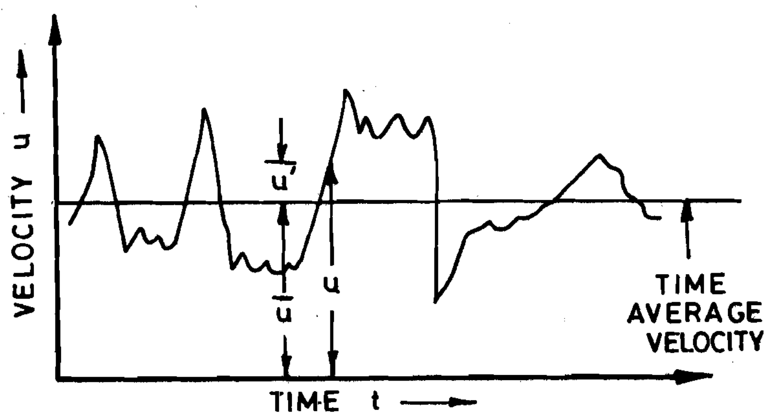

Figure 9.6 : Velocity Variation with Time at a given Section

variation of u with time will be as shown in Figure 9.6. Thus, the variation in velocity u could be considered in terms of instantaneous velocity, fluctuating velocity and the time average velocity, i.e. at any instant, the instantaneous velocity u is equal to the sum of the time average velocity $\bar{u}$ and the turbulent fluctuating velocity u'. The component u' can be positive, negative or zero. These fluctuations in velocity result in an extensive intermixing between different fluid layers on a scale much larger than the molecular diffusion in laminar flows.

Extending the concept of velocity variation to the conventional three co-ordinate directions x, y, z, the corresponding instantaneous velocities u, v and w can be expressed as

$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

$$w = \bar{w} + w'$$

The time average velocity $\bar{u}$ can be found by integrating the instantaneous values of velocity over a time interval T, thus $\bar{u} = \frac{1}{T}\int_0^T u\, dt$

Here the averaging is considered over a sufficiently long period of time T. Further, it can be easily shown that the time average of fluctuations u' is zero, i.e.

$$\bar{u}' = \frac{1}{T}\int_0^T u'\, dt = 0$$

Therefore, from the above result it can be concluded that the mean value of any fluctuating component in a turbulent flow is zero. This means $\bar{u}' = \bar{v}' = \bar{w}' = 0$

Statistical analysis of these fluctuations shows that the fluctuations follow a normal or Gaussian distribution. And from the property of the normal distribution, it can be shown that the maximum value of any instantaneous quantity could be as high as three times its time averaged value. Thus if velocity is considered one such quantity, it implies that the maximum value of instantaneous velocity could be as high as three times, the time-averaged mean velocity of flow.

SAQ 2

i) List the factors which affect the transition from laminar to turbulent flow.

ii) Why is energy loss in turbulent flow much larger than that in laminar flow?

iii) What factors influence the stability of flow ? Define stability parameter.

iv) Show that $\bar{u}' = 0$

9.5 QUANTITATIVE DESCRIPTION OF TURBULENCE

As pointed out above, turbulent flow results in a complex motion which varies continuously with time. In such a motion, lumps of fluid particles in the form of eddies or vortices of various sizes travel haphazardly due to fluctuations in velocity. Thus one may visualize turbulent flow as a haphazard and ever changing system of eddies superimposed on the mean motion of the fluid. Some of the examples of this disorderly flows are smoke coming out of chimneys, dust patterns behind cars, flow in rivers etc. The diffusion of a column of smoke and its subsequent disappearance into the atmosphere is on account of turbulent mixing.

In order to get an idea about the vigourness of turbulent motion, one uses the root mean square value of u' name $\sqrt{\overline{u'^2}}$. This r.m.s. value of u' is a quantitative measure of the intensity of turbulence.

9.6 EDDY VISCOSITY AND MIXING LENGTH CONCEPTS

9.6.1 Eddy Viscosity

It has been already discussed in sections 9.2 to 9.4 that there are rapid fluctuations in velocity components in all the three directions which cause turbulent mixing of layers. This mixing of one layer to the other results in transfer of momentum. Thus, in turbulent flow, there is presence of additional shear which is over and above the viscous shear (discussed in Unit 8 on laminar flow). Therefore, an expression, similar to that of viscous shear for laminar flow, can be expressed as:

$$\bar{\tau} \propto \frac{d\bar{u}}{dy} \qquad ...(9.1)$$

Where $\bar{\tau}$ and $\bar{u}$ represents the time average shear and velocity at any point. The above equation is written in exactly the same way as that proposed by Newton for laminar flow which states that shear at any point is proportional to the velocity gradient at that point. Equation (9.1) can be written in the following simplified form by introducing constant of proportionality,

$$\bar{\tau} = \eta \frac{d\bar{u}}{dy}$$

This equation was proposed by Boussinesq for the first time and the constant of proportionality ,η is called "eddy viscosity". This is analogous to dynamic viscosity μ. The difference between them is quite significant. While dynamic viscosity μ, is only the fluid property and depends upon temperature of the fluid alone , eddy viscosity η, is dependent upon fluid properties as well as the flow properties. Hence η varies from place to place in any given flow field whereas μ remains constant (if the temperature remains constant).

Similar to kinematic viscosity, ν eddy kinematic viscosity ε, can be expressed as: $\varepsilon = \frac{\eta}{\rho}$

9.6.2 Mixing Length

Consider two fluid layers A and B, some distance apart. As shown in Figure 9.7, let the time

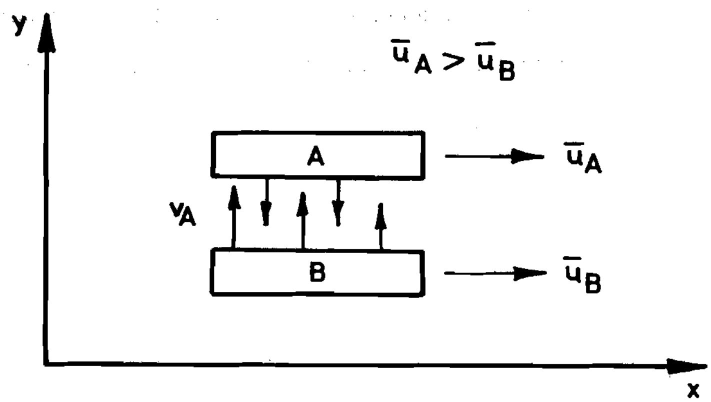

Figure 9.7 : Turbulent Mixing

average velocity of these layers be $\bar{u}_A$ and $\bar{u}_B$ respectively. Let us assume that $\bar{u}_A > \bar{u}_B$. Hence, it can be easily said that layer A moves with respect to layer B with velocity $u_A(= \bar{u}_A - \bar{u}_B)$. Let small mass of fluid move from layer B to layer A with an average velocity of v_A. This mass will now merge with layer A. If this cross motion is assumed to be uniform over a small area, say a, the mass $a\rho v_A$ has moved upwards and started moving with velocity u_A and therefore a transfer of momentum equal to $\rho a u_A v_A$ has taken place. Hence a shear stress now will act on layer A towards left and it will be equal to

$$\tau = -\frac{\rho\, a u_A\, v_A}{a} = -\rho\, u_A\, v_A$$

The negative sign indicates that the shear stress acts in a direction opposite to that of flow direction. Thus a slow moving layer exerts a retarding force on a fast moving layer and similarly the fast moving layer will also exert an equal and opposite force on the slow moving layer.

In this manner one can explain that the additional shear stress in turbulent flow is due to the rate of change of momentum of fluid mass due to their motion across imaginary plane parallel to the main flow. The turbı lent fluctuations u' and v' will naturally be responsible for such a shear. Hence, one can express the time average turbulent shear stress as:

$$\bar{\tau} = -\rho\,\overline{u'v'}$$

Now, if one has to compute this turbulent shear stress it is necessary to know the magnitudes of $\overline{u'v'}$. Since it is very difficult to measure these quantities, Prandtl proposed a mixing length hypothesis which can be used to express $\overline{u'v'}$ in terms of easily measurable quantities.

According to him, the mixing length l, can be considered as a transverse distance between two layers such that the lumps of fluid of one layer could reach to the other one and get embedded in it, while retaining their momentum in x direction during their travel through l.

Prandtl has further assumed that

$$u' = l\,\frac{d\bar{u}}{dy}$$

Further, he assumed that v' is proportional to u' and hence one can write

$$u'v' = l^2\left(\frac{d\bar{u}}{dy}\right)^2$$

wherein constant of proportionality is included in l^2. From this, Prandtl evaluated the turbulent shear stress $\bar{\tau}$ as:

$$\bar{\tau} = -\rho\overline{u'v'} = -\rho l^2\left(\frac{d\bar{u}}{dy}\right)^2$$

Hence, the total shear stress at any point for turbulent flow can be written as.

$$\tau = \mu\,\frac{d\bar{u}}{dy} + \rho l^2\left(\frac{d\bar{u}}{dy}\right)^2$$

Wherein the first term on the right side of the equation is viscous shear and the second one is turbulent shear.

SAQ 3

i) What is the role of eddies in a turbulent flow ?

ii) List the similarities and dissimilarities between μ and η.

iii) Define mixing length and state the relationship that exists between the turbulent shear stress and the mixing length.

9.7 ILLUSTRATIVE PROBLEMS

Example 9.1 :

Below are given the instantaneous values of velocity in m/s measured at a given point in flow at an interval of 1.0 sec.

7.93, 3.69, 4.21, 4.30, 5.37, 2.62, 4.85, 4.97, 8.54, 7.50, 7.62, 2.87, 8.75, 4.15, 6.43, 5.18, 3.96, 4.65, 7.56, 2.99

Determine $\bar{u}$, $\overline{u'^2}$, $\frac{\sqrt{\overline{u'^2}}}{\bar{u}}$

Solution :

Total number of values of u = 20

$$\Sigma u = 108.14$$

$$\bar{u} = \frac{\Sigma u}{20} = \frac{108.14}{20} = 5.407 \text{ m/s Ans.}$$

As $u' = u - \bar{u}$, values of u' corresponding to u values are:

2.523, -1.717, -1.197, -1.107, -0.037, -2.787, -0.557, -0.437, 3.133, 2.093, 2.213, -2.537, 3.343, -1.257, 1.023, -0.227, -1.447, -0.757, 2.153, -2.417

Hence $$\overline{u'^2} = \frac{\Sigma u'^2}{20} = \frac{72.77}{20} = 3.639 \text{ Ans}$$

and $$\frac{\sqrt{\overline{u'^2}}}{\bar{u}} = 0.352 \text{ Ans.}$$

Example 9.2 :

Two open wagons A and B are moving on parallel tracks in the same direction at velocities of 20.0 m/s and 30.0 m/s respectively. If gravel is transported from B to A at the rate of 1000 kg per second, determine the tangential force acting on A.

Solution :

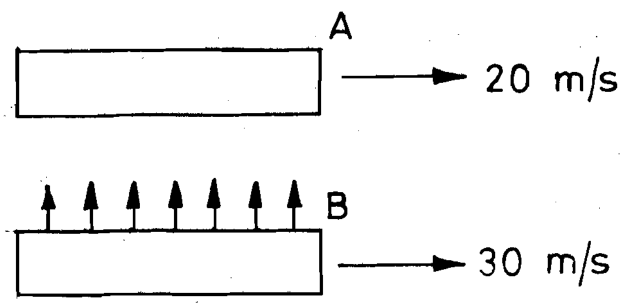

Figure 9.8

As shown in the Figure 9.8, the mass of gravel transferred from B to A per unit time is 1000 kg/s.

Relative velocity of B.w.r.t. A = 10 m/s.

Force acting on A = 1000 (10-0) = 10.000 N. Ans.

9.8 SUMMARY

In this unit we have studied the basic concepts of turbulent flow and its characteristics as they differ from laminar flow. Conditions at which laminar flow changes to a turbulent flow were also examined. The role of velocity fluctuations in intense mixing that results in such flows was emphasised. Finally the concept of mixing length was discussed evaluating the turbulent shear stresses in these flows.

9.9 KEY WORDS

Turbulent Flow : Flow characterised by intense intermixing of fluid particles.

Instantaneous Velocity : Velocity at a given time and is equal to the time average velocity plus the turbulent fluctuation velocity.

Critical Reynolds Number : Value of Reynolds number at which transition from laminar to turbulent flow is expected to occur.

Isotropic Turbulence : Turbulence in which the mean-square velocity fluctuations in the three co-ordinate directions are equal to each other:

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$$

Wall Turbulence : Turbulence, generated due to the presence of solid wall is known as wall turbulence.

Free Turbulence : Turbulence, generated by the flow of fluid layers at different velocities, in the absence of a solid wall is known as free turbulence.

Mixing Length : Average distance perpendicular to the flow which a small fluid mass travels before its momentum is changed by the new environment.

9.10 ANSWERS TO SAQs

Check your answers of all SAQs with respective proceeding text of each SAQ.