UNIT 7 FLOW MEASUREMENT

Structure

7.1 INTRODUCTION

Flow measurement is one of the most frequently encountered problems in fluid mechanics by field engineers. Efficient and accurate measurements are absolutely essential to arrive at correct estimates with minimum errors. For this purpose, the engineer should be well equipped with the basic knowledge of various methods to measure fluid properties and phenomena. The purpose of this unit is to give the principle of fluid measurements by various devices.

Objectives

After studying this unit, you should be able to measure

* static pressure,
* dynamic pressure,
* velocity, and
* discharge.

7.2 MEASUREMENT OF PRESSURE

The measurement of fluid pressure is essential in determining the velocity of the fluid or the rate of flow using energy equation.

7.2.1 Static Pressure

The static pressure of a fluid in motion indicates the pressure measured without disturbing its velocity. When the flow is uniform and streamlines are parallel, the pressure variation across the flow will be hydrostatic. Hence, in such cases if the pressure intensity at the boundary is measured, the pressure at any point may be computed. Figure 7.1 shows a manometer connected to a small hole drilled in the boundary of a pipe line. The manometer reading indicates the static pressure at the boundary. For accurate results, the tube must be normal and flush with the boundary and the diameter should be very small. In the case of rough surface at the boundary, the static pressure can be measured at the mid-section using a static tube as shown in Figure 7.2.

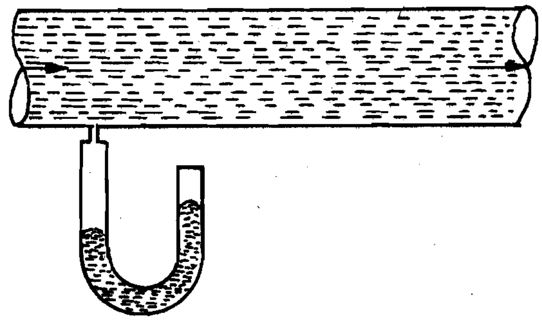

Figure 7.1 : Static Pressure Measurement by Manometer

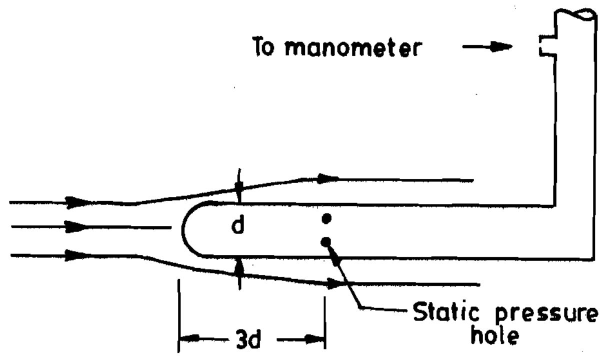

Figure 7.2 : Static Tube

The tube consists of a smooth cylinder with round nose having radial holes at a minimum distance of $3d$ from the upstream end of static tube. After the disturbance at the upstream end, the streamlines become parallel again hereafter and hence the flow is undisturbed. Smaller diameter for the tube increases the accuracy of the measurement. Another way of measuring static pressure is based on the flow net analysis using a cylindrical tube.

7.2.2 Dynamic Pressure

This pressure is due to the velocity of the fluid and can be expressed as

$$h = \frac{V^2}{2g} \qquad (7.1)$$

The sum of static pressure and dynamic pressure constitutes the total pressure or stagnation pressure. By measuring static pressure and total pressure and connecting to opposite ends of a differential manometer yields the dynamic pressure head. The above procedure to measure the dynamic pressure is essential, since practically it is difficult to read the dynamic pressure head from a free surface using a simple pitot tube.

7.2.3 Total Pressure

The total pressure is the pressure which includes the pressure due to hydrostatic force and velocity. To measure the total pressure, a tube having a short right angled limb with a rounded and having a hole is inserted facing the flow. This causes stagnation at the entrance of the tube. At this point the velocity is reduced to zero and the velocity head is converted into pressure head indicating the total pressure. This tube is known as **pitot tube** or **total head tube**.

7.2.4 Piezometric Head

Energy of a fluid in motion consists of pressure energy, kinetic energy and potential energy. The sum of pressure head and potential head is known as **piezometric head**. Piezometers are the simplest manometers which are used to measure the pressure of a liquid at rest or in motion. The piezometer is a tube connected vertically to a small hole in the wall of the conduit with a minimum diameter of 0.625 cm to reduce capillary effects (Figure 7.3).

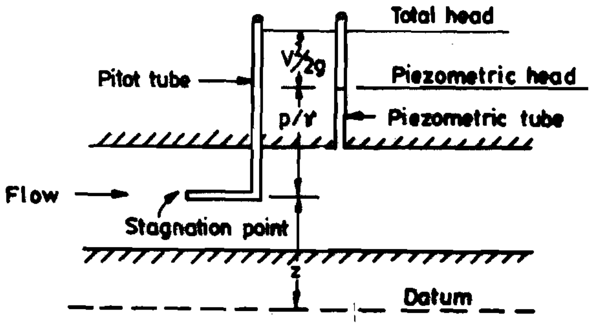

Figure 7.3 : Measurement of total Head

Example 7.1 :

Water flows through a pipe of 25 cm diameter at a discharge of 69.5 litres/sec. At a section which is 6 m above the datum, the pipe pressure is 160 kN/m^2. Determine the total pressure (head) of the flow.

Solution :

Velocity, $V = \frac{Q}{A}$

$$= \frac{0.0695}{0.785 \times 0.25^2} = 1.417 \text{ m/sec.}$$

Total pressure per unit weight of fluid

$$= \frac{V^2}{2g} + \frac{p}{\gamma} + z$$

$$= \frac{1.417^2}{2 \times 9.81} + \frac{160}{9.797} + 6$$

$$= 22.43 \text{ m}$$

SAQ 1

(i) A piezometer opening is used to measure

a) the pressure of a static fluid

b) the velocity in a flowing fluid

c) the total pressure

d) the dynamic pressure

e) the undisturbed fluid pressure.

(ii) A static tube is used to measure

(a) pressure in a static fluid

(b) the velocity of a flowing fluid

(c) the total pressure

(d) the dynamic pressure

(e) the undisturbed fluid pressure.

Select the correct answer from the above.

7.3 MEASUREMENT OF VELOCITY

7.3.1 Pitot Tube

Velocity measurement is one of the most important measurements in fluid mechanics. It is useful in determining the discharge, velocity profile and shear stress across a section.

Pitot tube which is named after Henry Pitot (1730) is used to measure the velocity of the fluid. (Figure 7.3). The difference in the readings of pitot tube and piezometer (h) indicates the velocity head.

i.e.
$$\frac{V^2}{2g} = h$$
$$V = \sqrt{2gh} \qquad (7.2)$$

Example 7.2 :

A pitot tube inserted at the centre of 0.22 m diameter pipe is connected to a static tube through inverted U-tube. When the pipe carries water, a difference of 8 cm was indicated in the U-tube. Determine the velocity of flow (Figure 7.4).

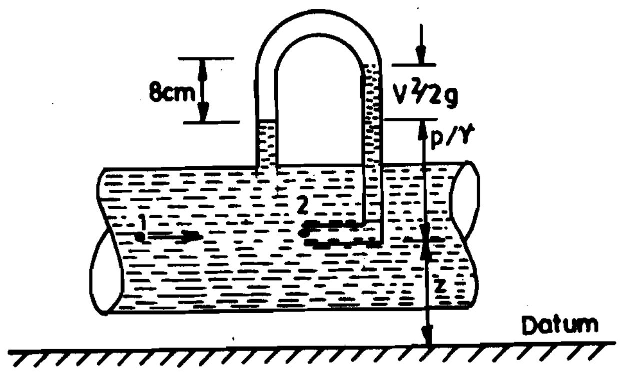

Figure 7.4 : Pitot Tube

Solution :

Applying Bernoulli's theorem between points 1 and 2 (stagnation point)

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z = \frac{p_2}{\gamma} + z$$

i.e.,
$$\frac{V_1^2}{2g} = \frac{p_2 - p_1}{\gamma} = 0.08 \text{ m}$$
$$V_1 = \sqrt{2 \times 9.81 \times 0.08} = 1.2528 \text{ m/sec}$$

7.3.2 Prandtl Tube

The combination of static pressure tube and stagnation pressure (total) tube is known as **Prandtl pitot tube**. When the boundary surface is rough and the static pressure measurements at the boundary are not possible, this device can be used. The front portion of the tube is rounded (Figure 7.5) to avoid separation of the flow and on its shaft, holes are provided at certain distance where stream lines become parallel. The two tubes are enclosed in the same tube and are connected to a differential manometer. The manometer

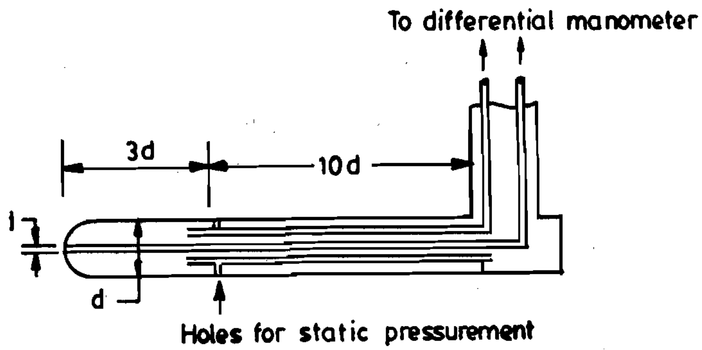

Figure 7.5 : Prandtl Pitot Tube

reading indicates the velocity head from which velocity can be determined. A coefficient C_d (0.95 to 0.99) is evaluated to eliminate the error in the velocity caused due to slight imperfection in the fabrication of the device. Hence the actual velocity V_{actual} is given as

$$V_{actual} = C_d \sqrt{2gh} \qquad (7.3)$$

SAQ 2

Tick (√) the correct answer or most appropriate from among the alternatives given

(i) The simple pitot tube measures the
   - (a) static pressure
   - (b) dynamic pressure
   - (c) total pressure
   - (d) velocity at the stagnation point
   - (e) difference in total and dynamic pressure.

(ii) The pitot static tube measures
   - (a) static pressure
   - (b) dynamic pressure
   - (c) total pressure
   - (d) difference in static and dynamic pressure
   - (e) difference in total and dynamic pressure.

7.4 MEASUREMENT OF DISCHARGE

7.4.1 Orifices

An orifice is an opening in the wall of a container. It can be of any shape. A standard orifice has a sharp edge and the flow through such an orifice is shown in Figure 7.6.

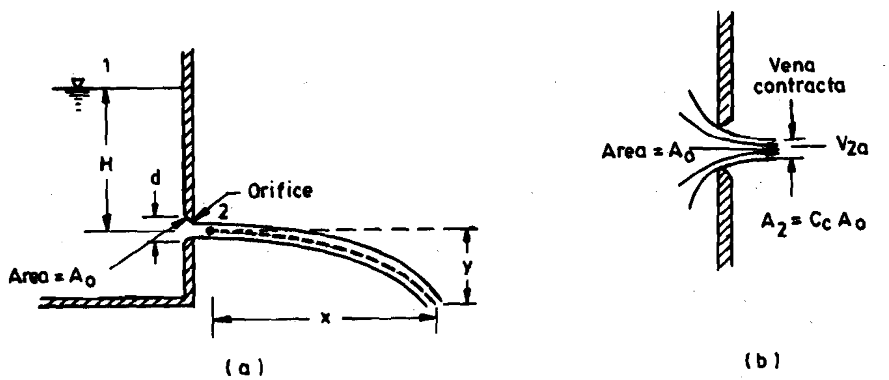

Figure 7.6 (a) Flow through an Orifice (b) An Orifice

As the fluid flows, the streamlines converge to form a jet having a cross section smaller than the orifice, known as **vena contracta.** Since the fluid particles can not change their direction abruptly, the position of vena contracta is slightly away from the wall. Streamlines are thus curved between the orifice and vena contracta. However, at the vena contracta the streamlines are parallel and the pressure is atmospheric.

Let a constant head H be maintained above the centre of the orifice. Bernoulli's equation applied between point 1 on the free surface and point 2 at the centre of vena contracta gives

$$0 + 0 + H = \frac{V_2^2}{2g} + 0 + 0$$

$$V_2 = \sqrt{2g\,H} \qquad (7.4)$$

The above expression indicates theoretical velocity of the jet which doesn't include the frictional losses on the periphery of the orifice. Hence a coefficient known as **velocity coefficient,** C_v, is introduced in calculating the actual velocity which is the ratio of the actual velocity V_a to the theoretical velocity V_t.

Hence, $$V_a = C_v \sqrt{2g\,H} \qquad (7.5)$$

The actual discharge Q_a through the orifice is the product of actual velocity at the vena contracta and the area of the jet. The area of the jet at vena contracta A_2 is smaller than the area of the orifice A_0 and their ratio is known as **coefficient of contraction** (C_c). The actual discharge can thus be given as

$$Q_a = C_v C_c A_0 \sqrt{2g H} \tag{7.6}$$

The product of the above two coefficients is known as discharge coefficient C_d

i.e., $$C_d = C_c C_v$$

Hence, $$Q_a = C_d A_0 \sqrt{2g H} \tag{7.7}$$

If the downstream side of the orifice is submerged, the discharge equation gets modified as

$$Q_a = C_d' \sqrt{2g H'} \tag{7.8}$$

in which H' is the difference between upstream and downstream levels, C_d' is the new discharge coefficient, the value of which is slightly lesser than C_d.

The value of coefficient of velocity C_v for circular orifices varies from 0.95 to 0.98. For an orifice of given dimensions, C_v slightly increases with increase in H. For circular orifices, with the diameter less than H, C_c is equal to 0.61. The values of C_v and C_c however are to be determined experimentally. The following are some of the methods used in determining the hydraulic coefficients, C_v, C_c and C_d.

(1) **Trajectory method :** By measuring the co-ordinates of a point 3 on the trajectory with respect to the centre of vena contracta (Figure 7.6 (a)), the actual velocity, V_a can be determined.

Let 't' be the time required for the fluid particle to reach the point 3 from vena contracta.

Then $$x_3 = V_a t$$

The vertical distance travelled by the particle at the same time, when there is no initial velocity in that direction is

$$y_3 = \frac{1}{2} g t^2$$

Eliminating t from the above two equations.

$$V_a = \frac{x_3}{\sqrt{2y_3/g}} \tag{7.9}$$

The coefficient of velocity $$C_v = \frac{V_a}{V_t} = \frac{V_a}{\sqrt{2g H}}$$

$$= \frac{x_3}{\sqrt{4 y_3 H}} \tag{7.10}$$

(2) **Pitot tube method :** The actual velocity V_a at the vena contracta is measured using a pitot tube. However, care must be taken while inserting the pitot tube so that the jet velocity is not altered. The coefficient C_v can thus be determined knowing the value of V_c.

(3) **Callipers method :** The diameter of the jet at vena contracta may be determined using a callipers. Knowing the diameter of the orifice, the coefficient of contraction C_c may be determined.

(4) **Momentum method :** A small tank suspended on knife edges is shown in Figure 7.7. The tank is balanced initially by putting weight, when the orifice is closed. When the orifice is functioning, a force imparts momentum to the jet by creating an equal and opposite force F which acts against the tank. The tank is now levelled by putting additional weight W. Using the momentum equation,

$$F = \frac{\gamma Q_a V_a}{g} \tag{7.11}$$

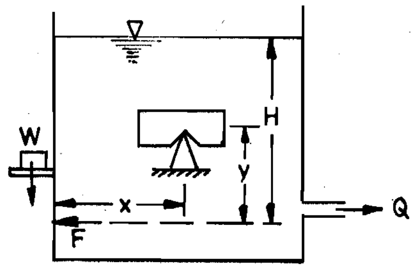

Figure 7.7 : Momentum method

Also, by taking moments about the knife edge

$$F y = W x \qquad \therefore \quad F = W \frac{x}{y} \qquad (7.12)$$

From equations (7.11) and (7.12)

$$\frac{\gamma Q_a V_a}{g} = W \frac{x}{y} \qquad (7.13)$$

Since the actual discharge Q_a is measured, V_a is the only unknown in the equation.

(5) **Through measurements :** The coefficient of discharge, C_d can be obtained by measuring the area A_o, the head H and the discharge Q_a by gravimetric or volumetric means.

Example 7.3 :

A large tank resting on the floor maintains water upto a height of 8 m. A circular orifice of 12 mm diameter is located 2 m above the floor level. For $C_v = 0.97$ and $C_c = 0.61$, determine the discharge and horizontal distance from the tank where the jet will strike the ground. Also determine C_d.

Solution :

The acting head, $H = 8 - 2 = 6$ m

Theoretical velocity, $V = \sqrt{2gH}$

$= \sqrt{2 \times 9.81 \times 6} = 10.85$ m / sec.

Actual velocity, $V_a = C_v\ V$

$= 0.97 \times 10.85 = 10.52$ m / sec.

Area of the jet at vena contracta $= C_c\ a$

$= 0.61 \times 1.13 \times 10^{-4}$

$= 6.899 \times 10^{-5}$ m^2

Actual discharge, $Q_a = V_a \times$ Area at Vena contracta

$= 7.258 \times 10^{-4}$ m^3/sec

The coefficient of discharge, $C_d = C_v\ C_c$

$= 0.97 \times 0.61$

$= 0.5917$

Vertical distance travelled by the jet, $y = 2$ m

If time required is t, then $y = \frac{1}{2} g t^2$, so

$$2 = \frac{1}{2} \times 9.81\ t^2$$

$$\therefore \quad t = \sqrt{\frac{4}{9.81}} = 0.639 \text{ sec.}$$

Horizontal distance travelled during this period, $x = V_a\, t = 6.72$ m

Losses in Orifice Flow :

The loss of energy in flow through orifice can be determined by applying the Bernoulli's equation between point 1 and vena contracta (Figure 7.6).

$$H = \frac{V_a^2}{2g} + H_L$$

Where H_L is the head loss from point 1 to vena contracta.

$$H_L = H - \frac{V_a^2}{2g}$$

$$\text{i.e.,} \quad H\,(1 - C_v^2) \quad \text{or} \quad \frac{V_a^2}{2g}\left(\frac{1}{C_v^2} - 1\right) \qquad (7.14)$$

The above equation indicates the losses in terms of H and C_v or V_a and C_v.

The orifice discussed above (Figure 7.6) is known as standard orifice, where the sharp edge of the orifice provides only a line contact with the fluid. The characteristic of this orifice is that the thickness of the wall or plate is small compared to the size of the opening. When the head of fluid above the orifice is less than 1-5 times the diameter or height of the orifice, it is classified as large orifice. The discharge through a large rectangular orifice is given as

$$Q_a = \frac{2}{3}\, C_d\, B\, \sqrt{2g}\, (H_2^{3/2} - H_1^{3/2}) \qquad (7.15)$$

in which B is the width of the orifice, H_1, and H_2 are the heads above the top and bottom edges of the orifice.

SAQ 3

(i) Explain briefly how the coefficient of velocity of a jet issuing through an orifice can be determined experimentally.

Find an expression for head loss in an orifice flow in terms of coefficient of velocity and jet velocity.

The head loss in flow through a 8 cm diameter orifice under a certain head is 20 cm of water and the velocity of water in the jet is 7m / sec. If the coefficient of discharge is 0.61, determine

(a) head on the orifice

(b) diameter of the jet, and

(c) C_v.

(ii) A 75 mm diameter orifice discharges 907.6 kg of water in 32.6 sec under a head of 4.90 m. The x and y co-ordinates of a point on the jet are 4.80 m and 1.3 m respectively. Determine C_v, C_c and C_d and the head loss.

7.4.2 Mouth Pieces

A mouth piece is a short tube connected to the orifice whose length is usually not more than two or three times its diameter. The mouth pieces may be cylindrical, converging or diverging. The mouth pieces are also used as measuring devices and the different types are shown in Figure 7.8 .

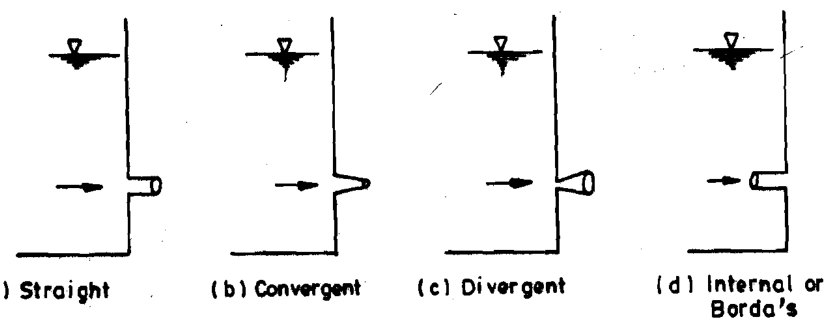

Figure 7.8 : Different types of Mouth Pieces

A standard mouth piece is a cylindrical tube having a length of 2 to 3 times diameter with a sharp cornered entrance (Figure 7.8 a). In such tubes the jet contracts first and then expands filling the tube. Since the tube is full at the outlet, C_c is usually considered to be equal to unity and mean value of C_v is about 0.82 .

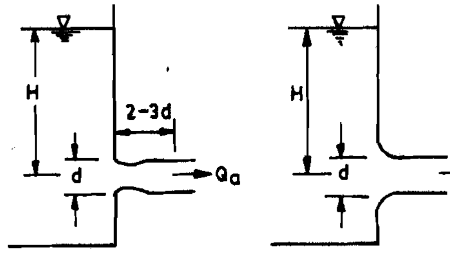

Figure 7.9 : Flow through Cylindrical Mouth Pieces

In the case of standard mouth pieces, if the entrance is rounded, it reduces the losses due to re-expansion to fill the mouth piece as shown in Figure 7.9. In such cases the coefficient of velocity C_v is relatively higher i.e. 0.98. In the case of diverging mouth piece, its length should be equal to 9 times its diameter at the inlet and the angle of divergence equal to 5° for better results. Borda's mouth piece is a short cylindrical re-entrant tube which projects inside. Its length is equal to its diameter and has sharp edges to ensure contraction and in this case the jet will not touch the sides of the tube. The average hydraulic coefficient in this case are $C_v = 0.98$, $C_c = 0.52$, $C_d = 0.51$.
A relationship can be established between the hydraulic coefficients using the principle of momentum as

$$2\, C_d C_v = 2\, C_v^2 C_c = 1 \qquad (7.16)$$

Example 7.4 :

A borda mouth piece 8 cm in diameter has a discharge coefficient of 0.51, what is the diameter of the issuing jet?

Solution :

We have $\quad 2\, C_d C_v = 1$

$$\therefore \quad C_v = \frac{1}{2 C_d}$$

$$= \frac{1}{2 \times 0.51} = 0.98$$

$$C_c = \frac{C_d}{C_v}$$

$$= \frac{0.51}{0.98} = 0.52$$

Area of the jet $= C_c\, A$

$$= 0.52 \times \pi \times \frac{8^2}{4}$$

$$= 26.14 \text{ cm}^2$$

$$\therefore \quad \text{Diameter of the jet} = \sqrt{\frac{4A}{\pi}}$$

$$= 5.77 \text{ cm}$$

SAQ 4

An external mouth piece converges from inlet upto the vena contracta to the shape of the jet and then diverges gradually as shown in Figure 7.10. The diameter at the vena contracta is 3 cm and the head over the centre of the mouth piece is 1.6 m. The head loss in the contraction may be taken as 1% and that in the divergent portion 5% of the total energy head before the jet. What is the maximum discharge that can be drawn through the outlet and what should be the corresponding diameter at the

Assume that the pressure in the system may be permitted to fall upto 8 m below atmosphere, the liquid conveyed being water.

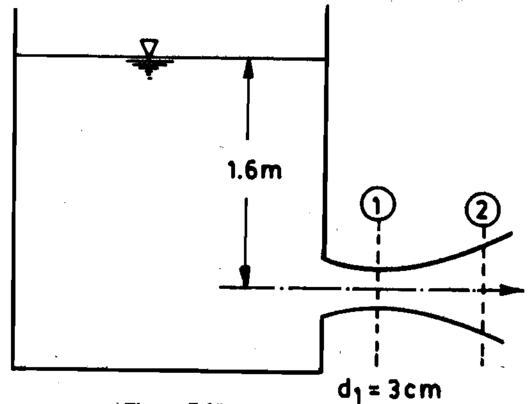

Figure 7.10

7.4.3 Notches and Weirs

The flow in open channel may be measured by a weir or notch which is an obstruction in the channel that causes the liquid to rise behind it and flow over it or through it. By measuring the height of upstream liquid surface, the rate of flow is determined. The notches or weirs may be constructed from a sheet of metal, masonry or from other material. Basically, there is no difference between a notch and a weir except that a notch is of smaller size while a weir will be usually of bigger size. Also, a notch is made of metal plate whereas weir is usually made of masonry or concrete. The common shapes of the notches are rectangular, triangular, trapezoidal, composite, parabolic and proportional. Figure 7.11 shows a rectangular notch. The bottom edge over which the

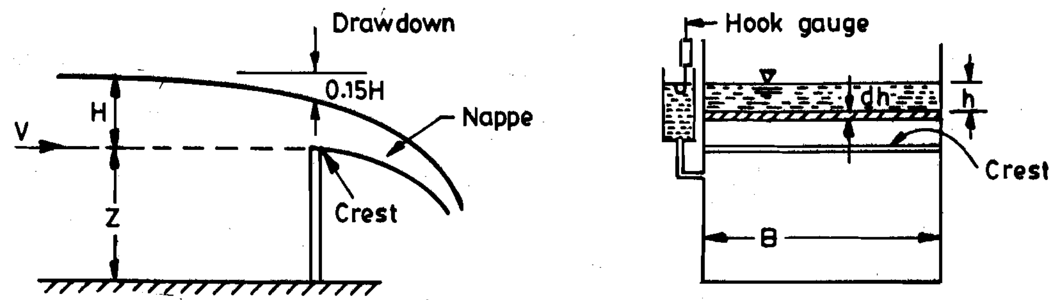

Figure 7.11 : Sharp Crested Rectangular Notch

liquid flows is called the **sill** or **crest** of the notch. The sheet of liquid which springs free from the crest is known as **nappe**. If the jet of liquid springs free as it leaves the crest, the weir is known as **sharp crested weir**. In the case of broad crested weirs, there is a support for the falling nappe over the crest in the longitudinal direction. As the liquid approaches the crest, there is a drop in the liquid surface in the form of convex curve known as **draw down**. The draw down at the weir crest is about 0.15 H where H is the head of liquid surface above the crest. Considering an elementary strip of area $B\,dh$ at a depth h below the free surface, the theoretical flow through it can be given as

$$dQ = B\,dh\,\sqrt{2gh}$$

The theoretical flow passing over the notch can be obtained by integrating the above equation from 0 to H

$$Q = \int_0^H B\,\sqrt{2gh}\,dh$$

$$= \frac{2}{3}\,B\,\sqrt{2g}\,H^{3/2} \qquad (7.17)$$

However, the losses such as fluid friction, draw down at the crest, a non-zero velocity at $h = 0$ and curved streamlines in the plane of the crest reduce the actual discharge to certain extent. Hence a coefficient C_d is introduced to determine the actual discharge

$$Q_a = \frac{2}{3}\,C_d\,B\,\sqrt{2g}\,H^{3/2} \qquad (7.18)$$

In the above section the length of crest of the notch is the same as the width of the channel. Such notches are called **suppressed notch** or **weir**. If the crest length is less than the width of the channel, such weirs are known as **contracted weirs**. In such cases, when the sheet of liquid passes over the crest, there will be lateral contraction of the sheet at both the ends known as **end contractions**. The value of end correction per end is $0.1\ H$. Hence the effective length of sheet of liquid is equal to $B - 0.2\ H$.

Example 7.5 :

If there is an error of 6% in observing the head over a rectangular weir, determine the error in the discharge resulting from it.

Solution :

The head-discharge relationship for rectangular weir can be written as

$$Q = KH^{3/2}$$

$$\frac{dQ}{dH} = \frac{3}{2} KH^{1/2}$$

i.e., $$dQ = 1.5\,K\,\sqrt{H}\,dH$$

We have $$\frac{dQ}{Q} = 1.5\,\frac{dH}{H}$$

The error in head measurement, $$\frac{dH}{H} = 6\%$$

$\therefore$ Error in the discharge $= \dfrac{dQ}{Q} = 1.5 \times 6 = 9\%$

Triangular Notch

If smaller discharges are to be measured accurately to avoid surface tension effects and to obtain appreciable measurable heads, triangular notch is the best. Figure 7.12 shows a triangular notch with a vertex angle 2θ.

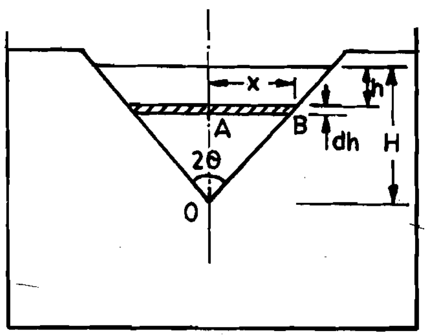

Figure 7.12 : Triangular Notch

The discharge through an elementary strip of area $dA = 2\,x\,dh$ is $dQ = 2x\,dh\,\sqrt{2gh}$ where h is the depth of the strip below the liquid surface. From triangle OAB, we have

$$\frac{x}{H-h} = \tan\theta$$

or $$x = (H-h)\tan\theta$$

The total discharge Q can be obtained by integrating the above expression over the limits $h = 0$ to $h = H$

$$Q = \int_0^H 2\,x\,dh\,\sqrt{2gh}$$

$$= \int_0^H 2\,(H-h)\tan\theta\,\sqrt{2gh}\,dh$$

$$= \frac{8}{15}\sqrt{2g}\,\tan\theta\,H^{5/2}$$

The actual discharge however is less due to losses as mentioned earlier and coefficient C_d is introduced in determining it.

$$Q_a = \frac{8}{15}\,C_d\,\sqrt{2g}\,\tan\theta\,H^{5/2} \qquad (7.19)$$

For a given notch, the discharge varies as $H^{5/2}$ and the discharge-head relationship may be expressed as

$$Q_a = KH^{5/2} \tag{7.20}$$

in which K is constant for a given notch

Example 7.6 :

When water flows through a right angled V-notch, show that the discharge is given by $KH^{5/2}$ in which K is a constant and H is the height of water above the bottom of the notch. If H is measured in cm and Q in litres/sec, and the coefficient of discharge is 0.61, what is the value of K ?

Solution :

The discharge through V-notch is given as

$$Q_a = \frac{8}{15} C_d \sqrt{2g} \tan\theta \ H^{5/2}$$

$$= KH^{5/2}$$

For right angled V-notch, $\theta = 90^o$

$$\therefore \quad Q = \frac{8}{15} C_d \sqrt{2g} \ H^{5/2}$$

If Q is expressed in litres/sec, it has to be converted to cm^3/sec and the discharge equation becomes

$$Q \times 1000 = \frac{8}{15} \times 0.61 \times \sqrt{2 \times 981} \ H^{5/2}$$

i.e.,
$$Q = \frac{8}{15} \times \frac{0.61 \times \sqrt{1962}}{1000} H^{5/2}$$

$$= 14.41 \times 10^{-3} \ H^{5/2}$$

$$\therefore \quad K = 14.41 \times 10^{-3}$$

Trapezoidal Notch

A trapezoidal notch is a combination of rectangular and triangular notch (Figure 7.13). The advantage in this type is that the end contractions are either completely eliminated or considerably reduced increasing the discharge.

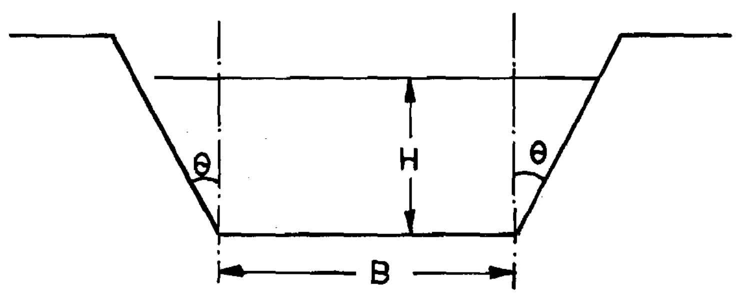

Figure 7.13 : Trapezoidal Notch

The discharge through such a notch is the sum of the discharges through rectangular and triangular portions.

$$Q_a = \frac{2}{3} C_d \sqrt{2g} \ BH^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan\theta \ H^{5/2} \tag{7.21}$$

Where the coefficients of discharge C_d for the two portions may be slightly different.

If the coefficients are the same, equation (7.21) reduces to

$$Q_a = \frac{2}{3} C_d \sqrt{2g} \ H^{3/2} \left[B + \frac{4}{5} H \tan\theta \right] \tag{7.21(a)}$$

Example 7.7 :

A trapezoidal weir has the dimensions as shown in Figure 7.14. Develop the equation for discharge and find the discharge when the head of water acting is 40 cm. Assume that the coefficient of discharge is 0.6.

Solution :

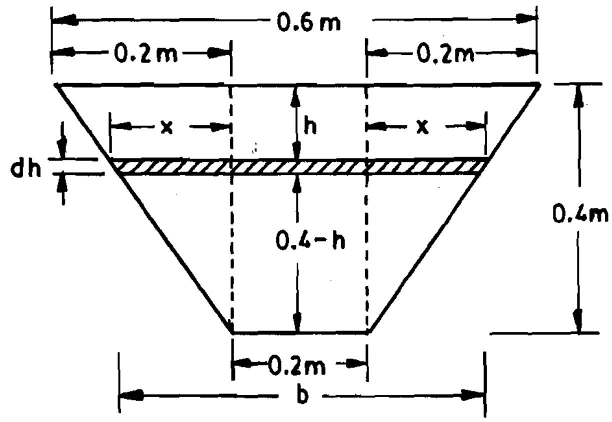

Figure 7.14 : Trapezoidal Weir

Consider a strip parallel to weir crest at a depth h and of width b and depth dh. From the geometry,

$$\frac{0.2}{0.4} = \frac{x}{0.4-h}$$

$$\therefore \quad x = \frac{1}{2}(0.4-h)$$

and $$b = 0.2 + 2x = 2(0.1 + x)$$

The discharge through the elementary strip of area $b\,dh$ is

$$dQ = b\,dh\,\sqrt{2gh}$$

If the head over the crest is H, the total discharge through the weir

$$Q_a = C_d \int_0^H 2\,(0.1 + x)\,dh\,\sqrt{2gh}$$

$$= C_d \int_0^H (0.2 + 0.4 - h)\,\sqrt{2gh}\,dh$$

$$= C_d\,\sqrt{2g} \int_0^H (0.6\sqrt{h} - h^{3/2})\,dh$$

$$= C_d\,\sqrt{2g}\left(0.4\,H^{3/2} - \frac{2}{5}\,H^{5/2}\right)$$

For $H = 40$ cm, the discharge, $Q_a = 0.6\,\sqrt{2\times 9.81}\,(0.060716)$

$$= 0.1614 \text{ m}^3/\text{sec.}$$

Cipolletti Notch

This is a trapezoidal notch with the sides in the ratio of 1 horizontal to 4 vertical. The advantage in this type of notch is that the decrease in the value of discharge due to end contractions in the rectangular notch is eliminated. The loss is made up by the triangular portion and the same formula for discharge as that of rectangular notch holds good here also. The reduction in the discharge due to end contractions can be given as

$$Q_L = \frac{2}{3}\,C_d\,\sqrt{2g}\,(0.2\,H)\,H^{3/2}$$

The discharge through the triangular portion is

$$Q = \frac{8}{15}\,C_d\,\sqrt{2g}\,\tan\theta\,H^{5/2}$$

Since these two discharges must balance

$$Q_L = Q$$

i.e., $$\frac{2}{15} C_d \sqrt{2g}\, H^{5/2} = \frac{8}{15} C_d \sqrt{2g} \tan\theta\, H^{5/2} \qquad (7.22)$$

Solving for θ, $\tan\theta = \frac{1}{4}$

The above fact was the conclusion drawn based on the experiments by Cipolletti and hence named after him Cipolletti gave an equation for discharge through such a notch as

$$Q = 1.86\, B H^{3/2} \qquad (7.23)$$

in which B and H are in metres and Q in m³/sec.

Proportional Weir or Sutro Weir

In this weir, the discharge is linearly proportional to the head H i.e. $Q = KH$ where K is a constant.

In many cases, such as in constant velocity sedimentation tanks, it is desirable to employ this type of weir. The shape of the sides of the weir diverge downward in the form of a hyperbola as shown in Figure 7.15.

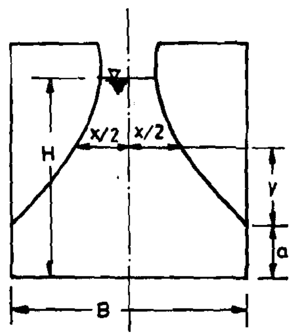

Figure 7.15 : Proportional Weir

The weir consists of a rectangular portion joined to a curved portion with the profile equation

$$\frac{x}{B} = 1 - \frac{2}{\pi} \tan^{-1} \sqrt{\frac{y}{a}}$$

The actual discharge in this type of weir is given by

$$Q_a = C_d\, a^{1/2}\, B \sqrt{2g} \left(H - \frac{a}{3} \right) \qquad (7.24)$$

C_d for this weir is mainly dependent on a and B and ranges from 0.60 to 0.65. Even though the weir is considered to be ideal, its construction may be costly and hence not popular.

Parabolic Weir

As the name indicates, the weir has a parabolic shape as shown in Figure 7.16 and here the discharge is proportional to the head by the following relationship

$$Q = K H^2 \qquad (7.25)$$

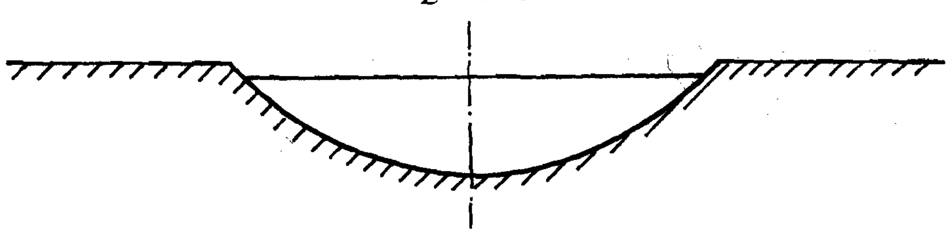

Figure 7.16 : Parabolic Weir

The discharge through parabolic weir is given by the following equation

$$Q_a = 1.512\ a^{0.479}\ H^{1.99} \qquad (7.26)$$

Where $a = \dfrac{x^2}{2y}$ between limits of 0.03 m to 0.06 m ; x and y are the co-ordinates of the curved boundary.

Broad Crested Weir

Weirs in which the sheet of liquid (nappe) touches the surface of the crest are called **broad-crested weirs**. The length of such a weir should be $> \dfrac{2}{3} H$ as shown in Figure 7.17.

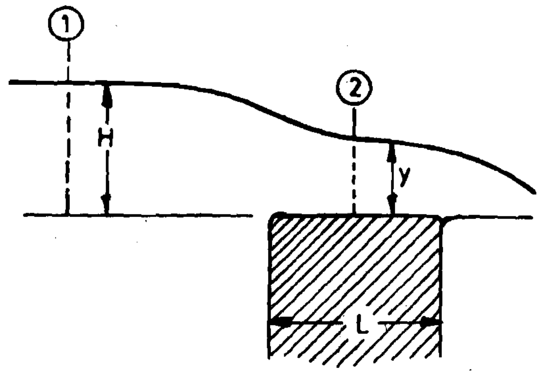

Figure 7.17 : Broad Crested Weir

Applying Bernoulli's equation between points 1 and 2

$$H + 0 + 0 = \frac{V_2^2}{2g} + 0 + y$$

i.e.

$$V_2 = \sqrt{2g\ (H - y)}$$

The theoretical discharge can thus be given by

$$Q = B\,y\,\sqrt{2g\ (H - y)} \qquad (7.27)$$

in which B is the length of the weir.

From the above equation, for a given value of H, theoretical discharge is found to be maximum when $y = \dfrac{2}{3} H$ which leads to

$$Q = 1.705\ B\,H^{3/2} \qquad (7.28)$$

Experiments for a well rounded upstream edged weir show the following relationship for the discharge

$$Q_a = 1.67\,B\,H^{3/2} \qquad (7.29)$$

Which is 2 percent within the theoretical value. The flow, therefore, adjusts itself to the maximum discharge.

SAQ 5

(i) Show that the time required to reduce the water level from H_1 to H_2 by rectangular weir is given by $t = \dfrac{3A}{C_d\,L\,\sqrt{2g}} \left(\dfrac{1}{\sqrt{H_2}} - \dfrac{1}{\sqrt{H_1}} \right)$ in which A is the area of the reservoir, C_d is the discharge coefficient and L is the length of the weir.

(ii) If a discharge of 120 litres/sec is to be measured over a triangular notch with a vertex angle of 60°, what will be the head over the notch, given $C_d = 0.6$. If at this level, there is an error of 1 mm in measuring the head, what is the corresponding error in the discharge.

(iii) A trapezoidal notch has a base width of 0.8 m, and a side slope of 1 horizontal to 2 vertical. Find the discharge for a head of 0.6 m. Assume $C_d = 0.62$

7.4.4 Venturimeter

The venturimeter is used to measure the rate of flow in a pipe. It is based on the Bernoulli's principle, that is, when the velocity head increases in an accelerated flow, there is a corresponding reduction in the piezometric head. The device consists of the following parts as shown in Figure 7.18.

(a) a converging entrance cone of angle of about 20°

(b) a cylindrical throat, and

(c) a divergent part known as **diffuser** with cone angle of 5° to 7^0.

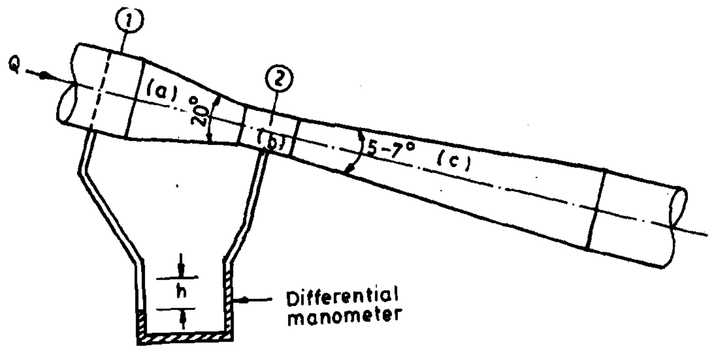

Figure 7.18 : Venturimeter

The size of the venturimeter is specified by the pipe and throat diameter, e.g. 10 by 6 cm venturimeter.

The accelerated flow is achieved by the converging cone which increases the velocity in the flow direction. The function of the converging cone is therefore, to convert the pressure energy into kinetic energy. A differential manometer is attached at sections 1 and 2 to measure the difference in the piezometric heads. The discharge through a venturimeter can be given as

$$Q_a = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} \qquad (7.30)$$

in which A_1 and A_2 are known areas at 1 and 2; h is the manometer reading indicating the difference in piezometric heads. The discharge coefficient varies with Reynold's number, the variation is small and if an average value is considered, the discharge equation reduces to

$$Q = K \sqrt{h} \qquad (7.31)$$

in which K is known as **venturimeter constant**. The value of K can be determined by calibrating the venturimeter.

Example 7.8 :

Water flows through a 30 cm × 15 cm venturimeter as shown in Figure 7.19, at the rate of 42.5 litres/sec and the differential gauge reads 115 cm. If the relative density of the liquid in the differential gauge is 1.25, what is the coefficient of the meter?

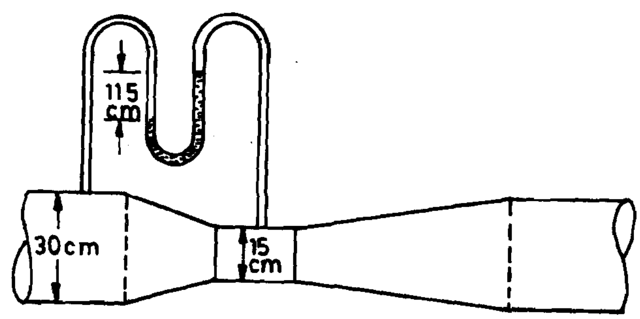

Figure 7.19 : Venturimeter

Solution :

The discharge equation is

$$Q = \frac{C_d\, A_1\, A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$42.5 \times 10^{-3} = \frac{C_d \times 0.070686 \times 0.017671}{\sqrt{4.9965 \times 10^{-3} - 3.1228 \times 10^{-4}}} \sqrt{2 \times 9.81 \times 1.15\,(1.25 - 1)}$$

∴ Coefficient of discharge, $C_d = 0.98$

∴ Coefficient of the meter $= \dfrac{C_d A_1 A_2 \sqrt{2g}}{\sqrt{A_1^2 - A_2^2}} = 0.08$

SAQ 6

(i) Estimate the discharge of kerosene (Relative density = 0.8) through the venturimeter as shown in Figure 7.20.

(ii) For the venturimeter of 150 mm × 75 mm dimensions, determine Δh in the mercury manometer if the pipe carries a discharge of 50 litres/sec. of oil of relative density 0.8. Take $C_d = 0.97$.

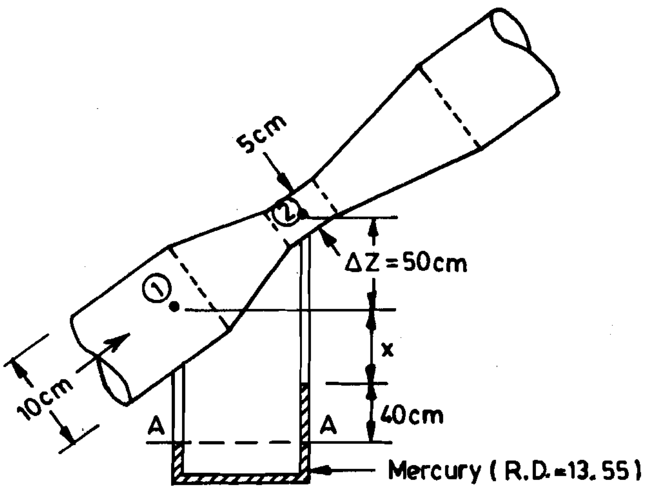

Figure 7.20

7.4.5 Orificemeter

Orificemeter is extensively used for flow measurements in pipes. It is also one of the oldest devices for measuring or regulating the flow of fluids. The device consists of a circular hole in a thin flat plate which is clamped between the flanges at a joint in a pipe line so that its plane is perpendicular to the axis of the pipe and the hole is concentric with the pipe as shown in Figure 7.21.

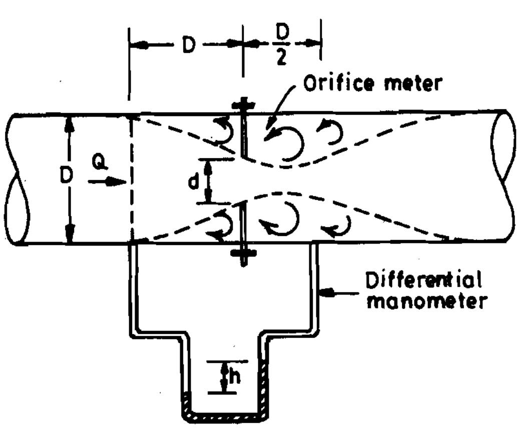

Figure 7.21 : Flow through Orifice Meter

The difference of pressure or differential head between two sections on either side of the orifice at a distance of D and $D/2$ is measured through differential manometer.

The fluid flow approaching the orifice gets accelerated and the boundary streamlines assume the shape as shown in figure. The radially inward component of fluid acceleration causes reduction in the flow area of the jet forming vena contracta within a distance of one pipe diameter i.e. at $D/2$. The discharge through the orifice is given by

$$Q_a = C_d \frac{\pi}{4} d^2 \sqrt{2gh} \quad (7.32)$$

in which h is the differential head across the orifice; C_d is the coefficient of discharge given by

$$C_d = \frac{C_c}{\sqrt{1 - C_c^2 (d/D)^4}} \quad (7.33)$$

The discharge coefficient C_d, in general is dependent on the Reynold's number and the diameter ratio d/D.

7.4.6 Flow Nozzle

The flow nozzle is a truncated form of the venturimeter without the diverging cone. It is simply a constriction with well rounded entrance placed in the pipe line as shown in Figure 7.22. The flow nozzle is simpler than the venturimeter and can be installed between the flanges of a pipeline.

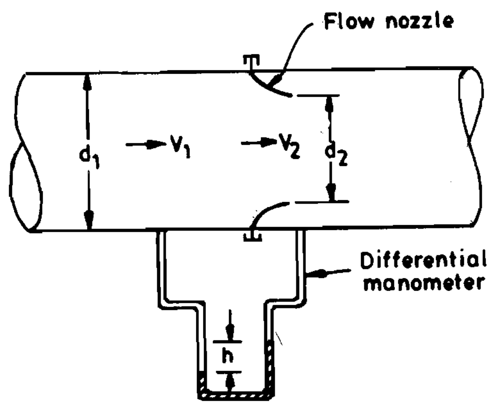

Figure 7.22 : Flow Nozzle

It serves the same purpose, though at an increased energy loss due to the sudden expansion of flow downstream of the nozzle. The discharge equation for the flow nozzle is the same as for the venturimeter. The coefficient of contraction $C_c = 1$ in this case leading to a higher value of coefficient of velocity than for the venturimeter. For accurate results, the flow nozzle must be calibrated in place. The discharge through the nozzle is given by

$$Q_a = C A_2 \sqrt{2gh} \quad (7.34)$$

in which C is a coefficient $= \dfrac{C_v}{\sqrt{1 - \left(\dfrac{A_2}{A_1}\right)^2}}$;

A_1 and A_2 are the cross sectional areas of the pipe and flow nozzle respectively, C_v is the coefficient of velocity.

The head loss through the nozzle is given by

$$H_L = \frac{V_2^2}{2g} \left(\frac{1}{C_v^2} - 1\right) \quad (7.35)$$

in which V_2 is the velocity of the fluid through the nozzle.

7.4.7 Venturi Flume

A venturi flume is an artificial constriction in a channel to measure the flow rate. It is usually constructed in concrete, masonry, steel or timber. The flume consists of a bell-mouthed entry, parallel throat and downstream diverging portion as shown in

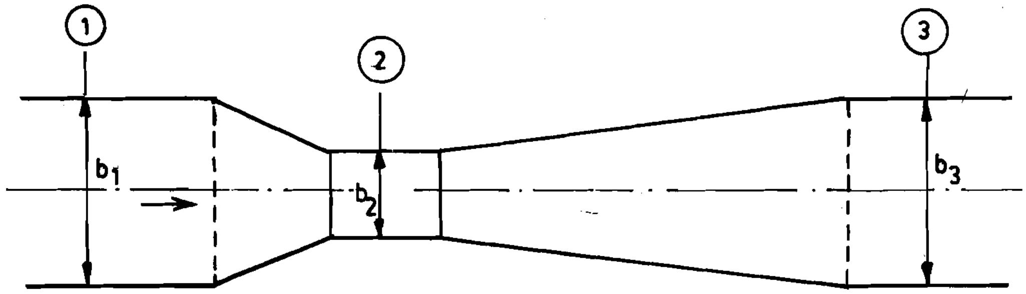

Figure 7.23 : Venturi Flume

The discharge through venturi flume is given by

$$Q_a = C_d \frac{(b_1 y_1) \ (b_2 y_2)}{\sqrt{(b_1 \ y_1)^2 - (b_2 \ y_2)^2}} \ \sqrt{2g \ (y_1 - y_2)} \qquad (7.36)$$

in which b_1, b_2 are the channel widths and y_1 and y_2 are the depths of water at sections 1 and 2 respectively and C_d is the coefficient of discharge.

In case the flume is designed to produce critical flow at the throat, the discharge can be determined by the following formula

$$Q_a = C_d \ b_2 \ y_2 \ \sqrt{g y_2} \qquad (7.37)$$

However, since the exact position in the throat , where the critical condition occurs is difficult to determine, the discharge may be computed using the depth of water at section 1 as

$$Q_a = 1.705 \ C_d \ b_2 \ y_1^{3/2} \qquad (7.38)$$

The value of C_d for the venturi flume varies between 0.94 to 0.99. The advantages of the flume over the broad crested weir is that the loss of energy in the flume is minimum and there is no silt deposition problem in the flume. The venturi flume is well suited to measure the flow in streams, small rivers and purification plants. The discharge capacity ranges from a few litres/second to about 25 m^3/sec.

Example 7.9 :

A venturi flume is 2 m wide at entrance and 1.2 m at the throat. Neglecting the hydraulic losses in the flume, calculate the discharge, if the depth at the entrance and throat is 0.9 m and 0.8 m respectively.

Solution :

From continuity equation

$$Q = b_1 y_1 V_1 = b_2 y_2 V_2$$

$$= 2 \times 0.9 \times V_1 = 1.2 \times 0.8 \ V_2$$

$$\therefore \quad V_2 = 1.875 \ V_1$$

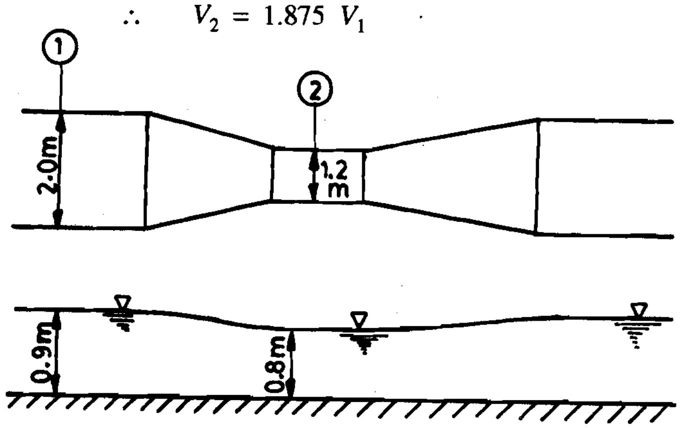

Figure 7.24

Applying energy equation at 1 and 2

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$0.9 + \frac{V_1^2}{2g} = 0.8 + \frac{(1.875\ V_1)^2}{2g}$$

$$\therefore \quad V_1 = \sqrt{0.1 \times 2 \times \frac{9.81}{2.5156}} = 0.883 \text{ m / sec.}$$

The discharge

$$Q = b_1\, y_1\, V_1$$
$$= 2 \times 0.9 \times 0.883$$
$$= 1.589 \text{ m}^3\text{/sec.}$$

7.5 SUMMARY

This unit introduces you to various methods and devices used to measure the fluid flow. Bernoulli's principle is used to evaluate the flow parameters such as pressure, piezometric head, velocity and discharge. Application of Bernoulli's equation to closed conduit and free surface flows are illustrated. The structure and working principles of various flow measuring devices are discussed. The numerical examples solved illustrate the usefulness of the devices in practical problems.

7.6 ANSWERS TO SAQs

SAQ 1

(i) (e)

(ii) (e)

SAQ 2

(i) (c)

(ii) (b)

SAQ 3

(i) Applying Bernoulli's equation between the reservoir surface and vena contracta (See Figure 7.6 (a))

$$H = \frac{V^2}{2g} + h_L$$
$$= \frac{7^2}{2 \times 9.81} + 0.2$$
$$= 2.698 \text{ m}$$

Coefficient of velocity, $C_v = \frac{V}{\sqrt{2gH}}$

$$= \frac{7}{\sqrt{2 \times 9.81 \times 2.698}}$$
$$= 0.96$$

Coefficient of contraction, $C_c = \frac{C_d}{C_v} = \frac{0.61}{0.96} = 0.635$

$$C_c = \frac{A_{jet}}{A} = \frac{d_{jet}^2}{d^2}$$

$\therefore$ diameter of the jet $= d_{jet} = d\ \sqrt{C_c} = 8\ \sqrt{0.635}$

$$= 6.375$$

(ii) Theoretical velocity, $V = \sqrt{2gH}$

$$= \sqrt{2 \times 9.81 \times 4.9} = 9.805 \text{ m/sec.}$$

The vertical distance travelled, $y = \frac{gt^2}{2}$

$\therefore$ Time required to travel this distance, $t = \sqrt{\frac{2y}{g}}$

$$= \sqrt{\frac{2 \times 1.3}{9.81}} = 0.515 \text{ sec.}$$

The horizontal distance travelled, $x = V_a t$

$\therefore$ Actual velocity, $V_a = \frac{x}{t} = \frac{4.8}{0.515} = 9.32 \text{ m/sec.}$

Coefficient of velocity, $C_v = \frac{V_a}{V}$

$$= 0.95$$

The actual discharge, $Q_a = \frac{907.6}{32.6 \times 1000}$

$$= 0.0278 \text{ m}^3/\text{sec}$$

Coefficient of discharge, $C_d = \frac{Q_a}{A \cdot \sqrt{2gH}}$

$$= \frac{0.0278}{\pi (0.0375)^2 \sqrt{2 \times 9.81 \times 4.9}}$$

$$= 0.64$$

Coefficient of contraction, $C_c = \frac{C_d}{C_v}$

$$= 0.67$$

Head loss $= H(1 - C_v^2) = 0.478 \text{ m.}$

SAQ 4

Applying Bernoulli's equation between free surface and section 1 (Figure 7.10), we have

$$1.6 = -8 + \frac{V_1^2}{2g} + \frac{1}{100} \times 1.6$$

$$\therefore \quad V_1 = 13.71 \text{ m/sec.}$$

Maximum discharge, $Q = \frac{\pi}{4} \times \frac{3^2}{10^4} \times 13.71$

$$= 9.69 \times 10^{-3} \text{ m}^3/\text{sec.}$$

Applying Bernoulli's equation between the free surface and section 2, we have

$$1.6 = \frac{V_2^2}{2g} + \frac{6}{100} \times 1.6$$

$$\therefore \quad V_2 = 5.432 \text{ m/sec.}$$

Using continuity equation

$$Q = \frac{\pi D^2}{4} V_2$$

$$D = \sqrt{\frac{4Q}{\pi V_2}}$$

$$= 4.77 \text{ cm}$$

SAQ 5

(i) The discharge over the rectangular weir of length L and head H is

$$Q_a = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

The rate at which the reservoir level is falling is given by

$$-\frac{dH}{dt} = \frac{Q}{A}$$

The time required to lower the reservoir level from H_1 to H_2 is given by

$$t = -A \int_{H_1}^{H_2} \frac{dH}{Q}$$

$$= -A \int_{H_1}^{H_2} \frac{dH}{\frac{2}{3} C_d L \sqrt{2g} H^{3/2}}$$

$$= \frac{3A}{2 C_d L \sqrt{2g}} \int_{H_2}^{H_1} \frac{dH}{H^{3/2}}$$

$$= \frac{3A}{C_d L \sqrt{2g}} \left(\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right)$$

(ii) Using equation (7.19)

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\theta \, H^{5/2}$$

$$0.12 = \left[\frac{8}{15} \times 0.6 \sqrt{19.62} \times \tan 30^\circ \right] H^{5/2}$$

$$0.12 = 0.818 \, H^{5/2}$$

$$\therefore \quad H = 0.46 \text{ m}$$

The discharge equation may be written as

$$Q = KH^{5/2} \qquad (1)$$

$$dQ = \frac{5}{2} KH^{3/2} dH \qquad (2)$$

Dividing 2nd equation by the 1st

$$\therefore \quad \frac{dQ}{Q} = \frac{5}{2} \frac{dH}{H}$$

In this case, $dH = 0.001$ m, $H = 0.46$ m

$$\therefore \quad \frac{dQ}{Q} = \frac{5}{2} \times \frac{0.001}{0.46} = 5.43 \times 10^{-3}$$

$$= 0.543 \ \%$$

$$Q = 120 \text{ litres/sec} \therefore \quad dQ = 0.65 \text{ litres/sec}$$

(iii) Using equation (7.21 (a))

$$Q = \frac{2}{3} C_d \sqrt{2g} H^{3/2} \left[B + \frac{4}{5} H \tan\theta \right]$$

In this case, $\tan\theta = \frac{1}{2} = 0.5$

$$C_d = 0.62$$

$$H = 0.6 \text{ m}$$

$$B = 0.8 \text{ m}$$

$$Q = \frac{2}{3} \times 0.62 \times \sqrt{19.62} \times (0.6)^{3/2} \left[0.8 + \left(\frac{4}{5} \times 0.6 \times 0.5 \right) \right]$$

$$= 0.885 \text{ m}^3/\text{s}$$

SAQ 6

(i) Applying Bernoulli's equation between sections 1 and 2

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \Delta z$$

$$\frac{p_1}{\gamma} - \left(\frac{p_2}{\gamma} + \Delta z\right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

From continuity equation

$$V_2 = 4V_1$$

$$\therefore \quad \frac{p_1}{\gamma} - \left(\frac{p_2}{\gamma} + \Delta z\right) = \frac{15\, V_1^2}{2g}$$

By equating pressures at A-A

$$\frac{p_1}{\gamma} + x + 0.4 = \frac{p_2}{\gamma} + 0.5 + x + \left(0.4 \times \frac{13.55}{0.8}\right)$$

$$\text{i.e.,} \quad \frac{p_1}{\gamma} - \left(\frac{p_2}{\gamma} + 0.5\right) = 6.775 - 0.4$$

$$= 6.375 \text{ m}$$

$$\therefore \quad \frac{15\, V_1^2}{2g} = 6.375$$

$$\text{i.e.} \quad V_1 = 2.888 \text{ m/sec.}$$

Hence $Q = A_1 V_1$

$= 0.0227 \text{ m}^3\text{/sec.}$

(ii)

$$Q_a = \frac{C_d\, A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$\therefore \quad \sqrt{h} = \frac{Q_a \sqrt{A_1^2 - A_2^2}}{C_d A_1 A_2 \sqrt{2g}}$$

$$= \frac{50 \times 10^{-3} \times \sqrt{3.1228 \times 10^{-4} - 1.95175 \times 10^{-5}}}{0.97 \times 0.017671 \times 4.41786 \times 10^{-3} \times 4.4294}$$

$$\therefore \quad h = 6.505 \text{ m of oil}$$

$$= \frac{6.505}{(13.55 - 0.8)} \text{ of mercury}$$

$$= 0.51 \text{ m}$$