UNIT 2 BUOYANCY

Structure

2.1 INTRODUCTION

This unit forms the concluding part of Fluid statics. For most part of this unit, we will still be dealing with fluids at rest, as we did in Unit 1. Determination of buoyant force, its point of action, metacentric height are some of the important exercises you will be carrying out in this unit. However, towards the end of the unit we shall deal with some special cases of fluids under motion where the methods of fluid statics still apply.

Objectives

After studying this unit, you should be able to

* determine the magnitude, direction and point of action of the buoyant force,
* conclude whether a floating body is stable against angular displacements, and
* determine the inclination/shape assumed by the fluid free surface when the fluid container is subjected to: a) linear acceleration and b) rotation.

2.2 BUOYANT FORCE

A body immersed completely or partially in a fluid is acted upon by an upward buoyant force equal to the weight of the fluid displaced by the body. This force acts through the centre of gravity of the displaced fluid. Buoyant force F_B may therefore be written as,

$$F_B = \gamma V \tag{2.1}$$

where V is the volume of the fluid displaced (equal to the volume of the submerged portion of the body) and γ is the specific weight of the fluid. For example, consider a rectangular block immersed in water as shown in figure 2.1 The volume of the portion

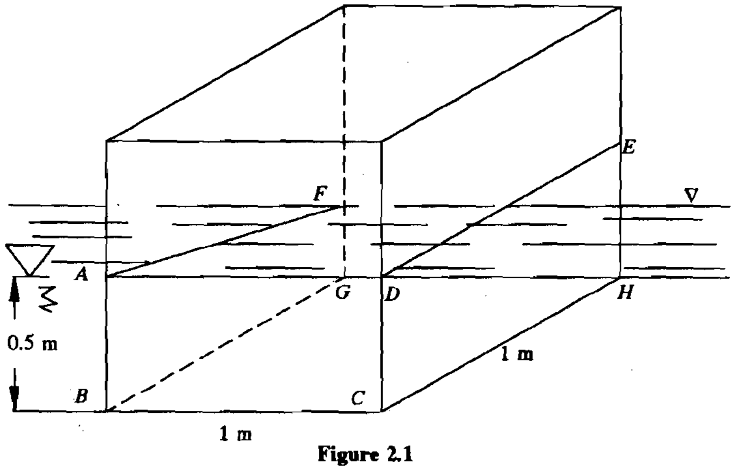


Figure 2.1

of block immersed in the fluid is equal to $1 \times 1 \times 0.5 = 0.5\ m^3$. The specific weight of water being equal to 1000 N/m^3, the weight of water displaced and hence the magnitude of the buoyant force is,

$$F_B = \gamma V = 1000 \times 0.5 = 500\ N$$

This force acts vertically upwards through the centre of gravity of the water in the portion *ABCDEFGH*. You can readily see that for the block to be in equilibrium the buoyant force must be equal to the weight of the block, since there are no other forces acting in the vertical direction. As the weight of the block increases, a greater buoyant force will be necessary to keep it in equilibrium and therefore more and more portion of the block will be submerged in the fluid. The point of action of the buoyant force, which coincides with the centre of gravity of the fluid displaced is called the **Centre of Buoyancy**.

Example 1 :

What fraction of an iceberg would be above the free surface in the ocean, if the density of ice = 920 kg/m^3 and density of sea water = 1030 kg/m^3.

Solution :

In figure 2.2, V_1 is the volume of the iceberg above the free surface and V_2 is the volume below it.

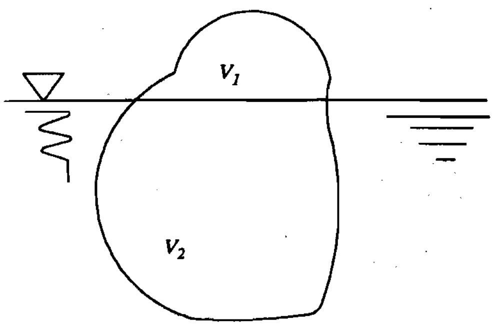


Figure 2.2

Total weight of the ice berg

$$= \rho_{ice}\, g\, (V_1 + V_2)$$

$$= 920 \times 9.81\, (V_1 + V_2) \text{ Newtons.}$$

Buoyant Force

$$F_B = \text{Weight of the fluid displaced}$$

$$= \rho_w \times g \times V_2$$

$$= 1030 \times 9.81 \times V_2 \text{ Newtons}$$

Since the iceberg is in equilibrium, the two forces must be equal.

$$\therefore \quad 920 \times 9.81\, (V_1 + V_2) = 1030 \times 9.81 \times V_2$$

$$\therefore \quad \frac{V_2}{V_1 + V_2} = \frac{920}{1030}$$

$$\text{or} \quad \frac{V_1}{V_1 + V_2} = 1 - \frac{920}{1030} = \frac{1}{9.36}$$

That is, one part of 9.36 parts is above the free surface.

Example 2 :

Find the density of a metallic body which floats at the interface of mercury (specific gravity, 13.6) and water such that 40% of its volume is submerged in mercury and 60% in water.

Solution :

Let the volume of the metallic body be V.

Then volume of the body immersed in mercury

$$= \frac{40}{100} V = 0.4V\ m^3$$

Similarly volume of the body immersed in water 0.6 $V\ m^3$

Now, for equilibrium, weight of the body = Total buoyant force on the body

= buoyant force due to water + buoyant force due to mercury.

= weight of water displaced + weight of mercury displaced

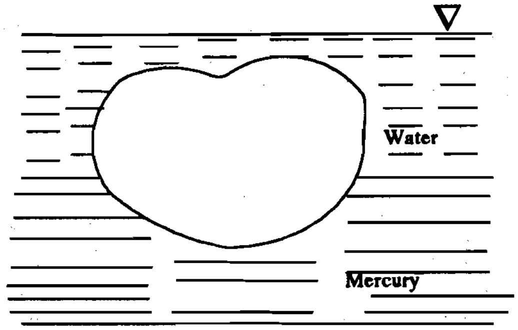


Figure 2.3

$$= 0.6\,V \times 1000 \times g + 0.4\,V \times 13.6 \times 1000\,g$$

Weight of the body $= \rho\, g\, V$

$$\therefore \quad \rho g\, V = 0.6\,V \times 1000\,g + 0.4\,V \times 13.6 \times 1000\,g$$

$$\therefore \quad \rho = 600 + 13.6 \times 0.4 \times 1000$$

$$= 6040\ \text{kg/m}^3$$

SAQ 1

A piece of irregularly shaped metal weighs 300 N in air. When the metal is completely submerged in water it weighs 232.5 N. Find the volume of the metal.

SAQ 2

A hollow cube 1 m on each side weighs 2.4 kN. The cube is tied to a solid concrete block weighing 10 kN. Will these two objects tied together float or sink in water ? Sp. gravity of concrete is 2.4.

2.3 STABILITY OF BODIES IMMERSED IN A FLUID

A wooden bar with a heavy lead ball attached at its lower end is shown floating in a fluid in figure 2.4(a). If you disturb this position slightly by tilting the bar and releasing it, you will observe that the bar quickly gets back to its original position of equilibrium. This original position of equilibrium AB is then called the position of stable equilibrium. In figure 2.4(b), the same bar is shown with the lead ball attached to the upper end. The bar is still in equilibrium. However, if you slightly tilt the bar and release it, the bar will completely turn over and will not regain its original position of equilibrium. Such a position is said to be one of unstable equilibrium.

Bodies floating in a fluid are in stable equilibrium against vertical linear displacement because a vertical displacement causes a change in buoyant force on the body and returns the body to its original position of equilibrium. For example, if the spherical ball shown in figure 2.5(a) is pushed vertically downward (producing a vertical linear displacement), and released, it quickly bounces back to its original position of

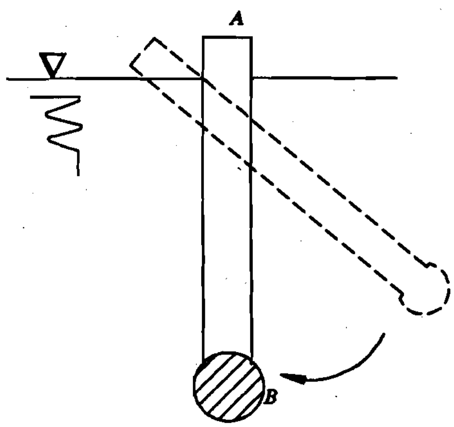


Figure 2.4 (a)

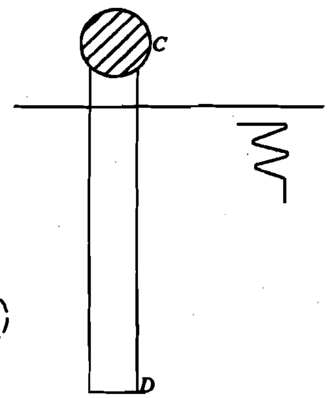
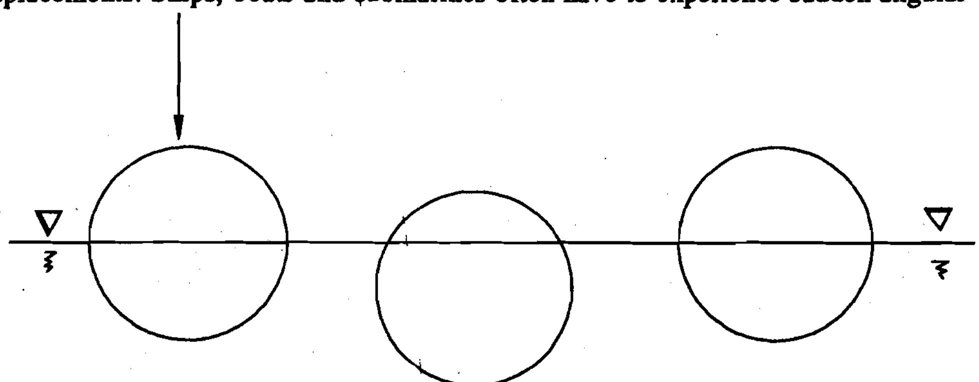


Figure 2.4 (b)

equilibrium. This is because, as more portion of the body is submerged, the buoyant force increases (see figure 2.5(b)). As this increased buoyant force is more than that required to keep the body in equilibrium the body is pushed up, till the buoyant force becomes equal to the weight of the body, which occurs in its original position.

An interesting engineering problem is the stability of floating bodies against angular displacements. Ships, boats and submarines often have to experience sudden angular



(a) (b) (c)

Figure 2.5

displacements. It is important that they are stable against such small angular displacements. Let us now see under what conditions the bodies are stable when they are immersed in a fluid.

2.3.1 Stability of Submerged Bodies

In case of completely submerged bodies, the position of centre of gravity *(CG)* and centre of buoyancy *(CB)* get fixed with respect to the body and hence the stability of the body is completely determined by the relative positions of the *CG* and *CB*. Look at figure 2.6(a). The centre of buoyancy *CB* is above the centre of gravity *CG*. The

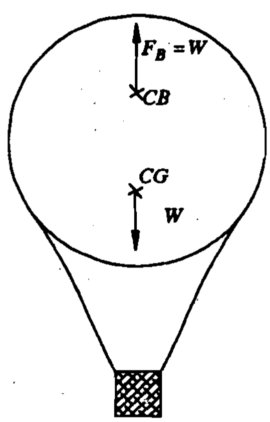


Figure 2.6 (a)

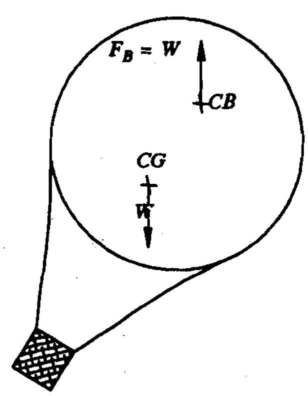


Figure 2.6 (b)

buoyant force F_B acts vertically upwards at *CB* and the weight of the body, *W* acts downwards at *CG*. If you slightly tilt the body as shown in figure 2.6(b), the force F_B and *W*, will form a couple that tends to restore the body to its original position. If, however, the centre of buoyancy *CB* is below the centre of gravity *CG* as shown in figure 2.7(a), then the couple formed by F_B and *W* in the new position (figure 2.7(b)) will only increase the angular displacement and therefore the body will be unstable. Thus, for a completely submerged body to be stable against small angular displacements, the centre of buoyancy must lie above the centre of gravity.

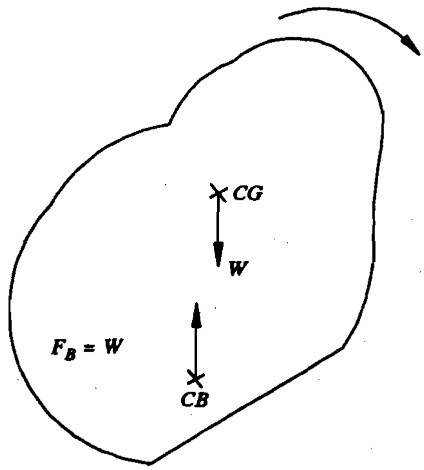


Figure 2.7 (a)

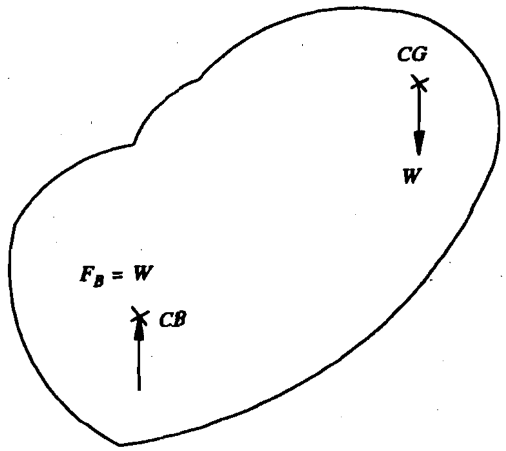


Figure 2.7 (b)

2.3.2 Stability of Floating Bodies

Any floating body with its centre of gravity below the centre of buoyancy is also in stable equilibrium for the same reasons as explained earlier in the case of completely submerged bodies. However, in practical situations, the *CG* of floating bodies like ships and vessels may often lie above the *CB* and still they may be in stable equilibrium. We shall now see under what conditions the floating bodies with their *CG* above the *CB* will be stable against angular displacements.

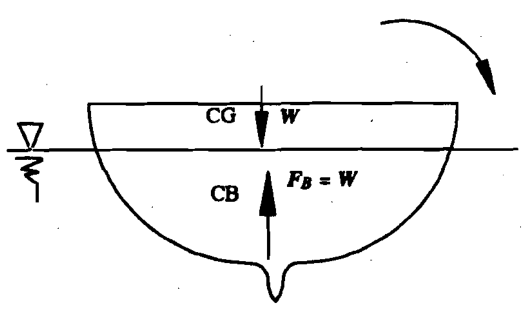


Figure 2.8 (a)

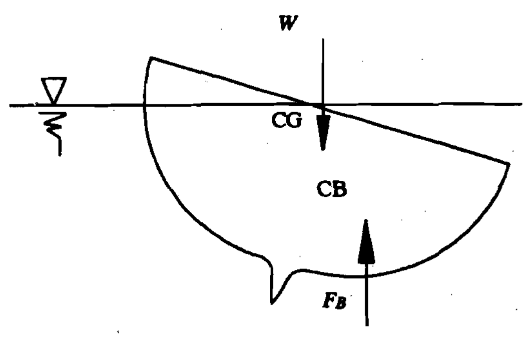


Figure 2.8 (b)

In the case of floating bodies, although the *CG* is fixed with respect to the body, the *CB* keeps varying with rotation, since the submerged volume varies. If the point *CB* shifts to the right of *CG* for a clockwise rotation as shown in figure 2.8(a) then a restoring couple is set up (figure 2.8(b)) and the body will be stable. Remember, however, that this is true only for small angular displacements. If, for a clockwise angular displacement the *CB* shifts to the left of *CG*, figure 2.8(c), then a clockwise moment is set up and hence the body will be unstable. Whether *CB* shifts to the left or right of the *CG* depends on the shape of the body.

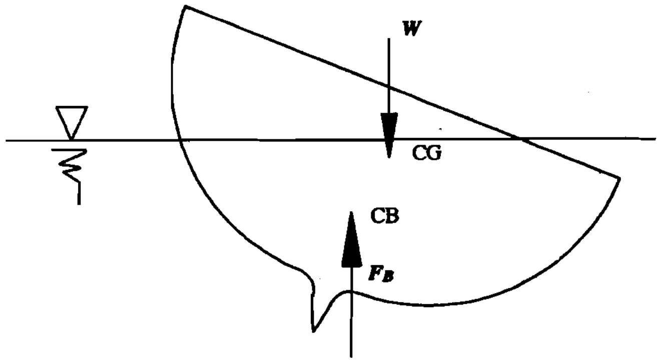
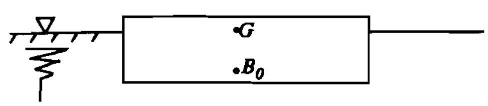


Figure 2.8 (c)

To make this more clear, let us consider the simple rectangular floating body as shown in figure 2.9(a).

For a small angular displacement, B′ is the new position of the centre of buoyancy, figure 2.9(b). The point of intersection of the vertical through B' (which is the line of ation of F_B in the new position) and the line B_0G (extended) is called the **Meta Centre.** The position of meta centre in a floating body determines whether the body will be stable against angular displacement or not. If M is above G, as shown in figure 2.9(b), a restoring moment will be set up and therefore the body will be stable.



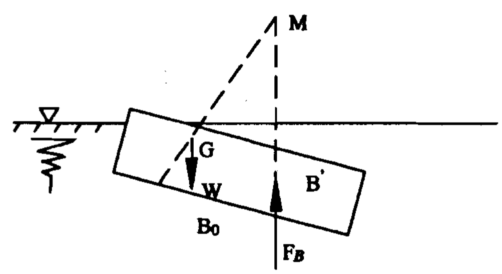


Figure 2.9 (a)

Fgure 2.9 (b)

If, on the other hand, the point M is below G it will be unstable. The position of metacentre M with respect to G, thus determines the stability of a body against angular displacements.

The distance MG is called the **Metacentric height.** It is positive if the point M is above G and negative if it is below G. It can be shown that

$$\overline{MG} = \frac{I}{V} \mp \overline{GB} \qquad (2.2)$$

where $\overline{MG}$ is the metacentric height, I is the moment of inertia of the cross section at the level of water surface about its centroidal axis, V is the volume of water displaced and $\overline{GB}$ is the distance GB. The negative sign is taken in equation (2.2) if the point G is above B and positive sign is taken if it is below B. For stability $\overline{MG}$ must be positive.

Example 3 :

A rectangular pontoon is 5m long, 3m wide and 1.2m high. The depth of immersion is 0.80m in sea water. If the CG is 0.6 m above the bottom of the pontoon, dertermine the metacentric height. Density of sea water = 1025 kg/m^3.

Solution :

Figure 2.10(a) shows how the body is floating in water. In figure 2.10(b) the cross section of the body at the water level is shown; Since G is above B, the negative sign is used for $\overline{BG}$ in equation (2.2).

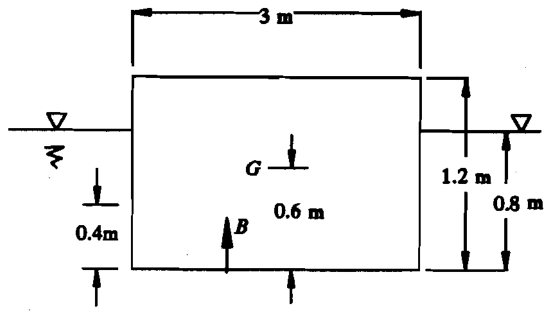


Figure 2.10 (a)

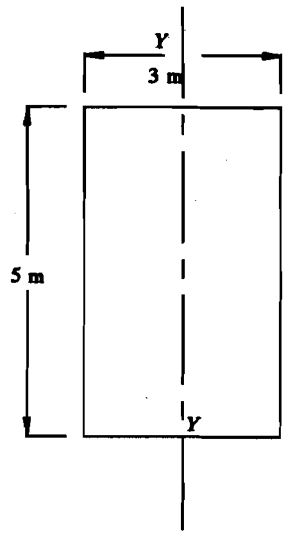


Figure 2.10 (b)

i.e. $\overline{GM} = \frac{I}{V} - \overline{BG}$

$I =$ MI about axis Y–Y in figure 2.10(b).

$= \frac{b\,d^3}{12}$

$= \frac{5 \times (3)^3}{12} = \frac{45}{4}\,\text{m}^4$

$V =$ Volume of the pontoon submerged

$= 3 \times 0.8 \times 5 = 12\ \text{m}^3$

$BG = AG - AB = 0.6 - 0.4 = 0.2\ \text{m}$

$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12} - 0.2$

$= 0.9375 - 0.2 = 0.7375$

Since *GM* is positive, the pontoon is stable against small angular displacements.

Example 4 :

A cube of side *a* and relative density *S* floats in water. Determine the conditions for its stability against angular tilt.

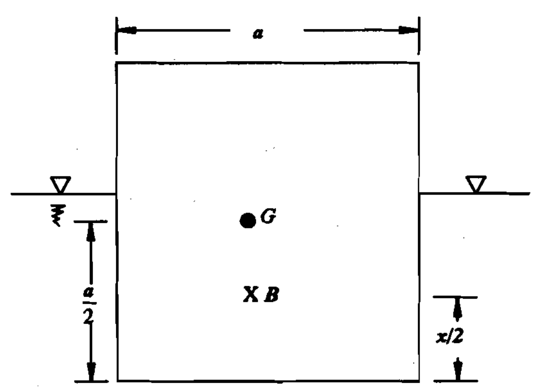


Figure 2.11 (a)

Solution :

For equilibrium
weight of the cube = Buoyant force on the cube.

$\therefore \quad \gamma a^3 S = \gamma a^2 x$, where γ is the specific weight of water

$$\therefore \quad x = a S \qquad \text{(A)}$$

CG of cube is at $\frac{a}{2}$ and *CB* at $\frac{x}{2}$ (from bottom)

$$\therefore \quad BG = \frac{a}{2} - \frac{x}{2} = \frac{1}{2}(a - x)$$

$$MG = \frac{I}{V} - BG$$

$$I = \frac{bh^3}{12} = \frac{a \times a^3}{12} = \frac{a^4}{12}$$

V = Submerged volume

$$= a^2 x$$

$$\therefore \quad MG = \frac{a^4}{12 a^2 x} - \frac{1}{2}(a - x)$$

$$= \frac{a^2}{12x} - \frac{1}{2}(a - x)$$

For stability,

$$MG > 0$$

$$\therefore \quad \frac{a^2}{12x} - \frac{1}{2}(a - x) > 0$$

$$\frac{a^2}{12x} > \frac{1}{2}(a - x)$$

or $\quad 1 > 6\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$

Since $\quad \frac{x}{a} = S$, (from (A))

$$6S - 6S^2 < 1$$

or $\quad 6S^2 - 6S + 1 > 0$

From solution of $6S^2 - 6S + 1 = 0$, we get $S = 0.789$ or 0.211.

$$\therefore \quad 0.211 \geq S \geq 0.789$$

This is the condition to be satisified for stability.

SAQ 3

Would the wooden cylinder (sp. gr. 0.61) be stable if placed vertically in oil (sp. gr. 0.85) as shown in figure 2.11 (b) ?

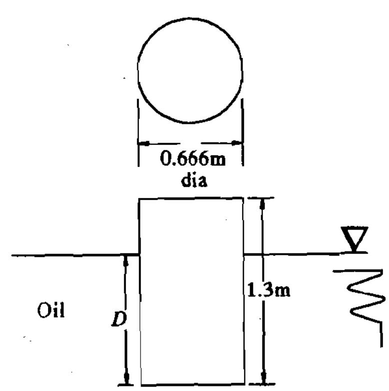


Figure 2.11 (b)

SAQ 4

A cube of side length L and sp. gravity 0.8 floats in water. Is the cube stable ?

2.4 LIQUIDS IN A CONTAINER SUBJECTED TO LINEAR ACCELERATION

Under certain conditions there may be no relative motion between the particles of a fluid mass, yet the fluid mass as a whole may be in motion. For example a liquid being transported in a tanker moves as a single body without any relative motion

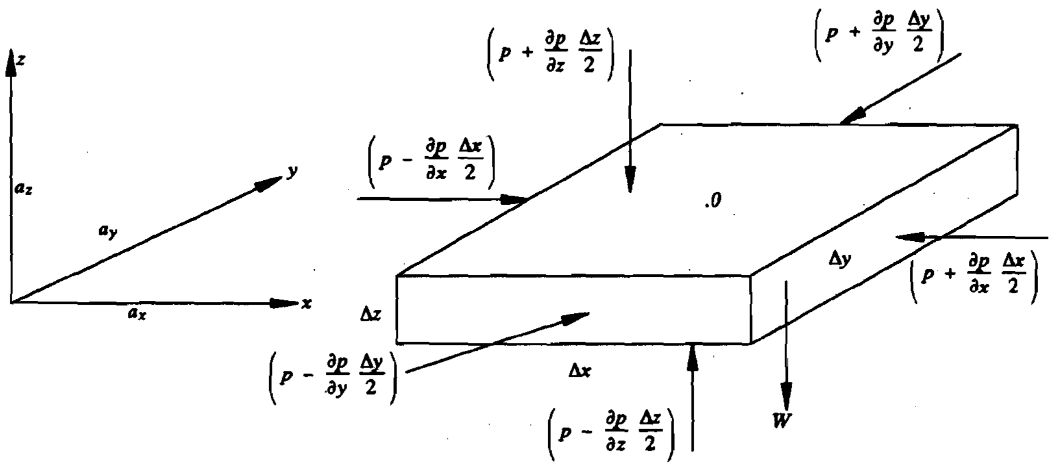


Figure 2.12

between particles. If the liquid is being transported at a uniform velocity, the conditions are those considered in fluid statics. But if the liquid mass is subjected to acceleration, then other forces need to be considered. You know that for a fluid at absolute rest, the hydro-static pressure distribution applies. Let us now see what happens to a fluid which is in an accelerated motion.

Consider a volume element of a fluid whose container is subjected to accelerations a_x, a_y and a_z in the three directions as shown in figure 2.12. With pressure at the centre of the element 0 being p, pressures of the centre of the sides are as indicated in the figure. (For an explanation of the convention used in arriving at these pressures, you may refer to your unit 1 on fluid statics, where the hydrostatic pressure equation was derived with this convention). Since the element as a whole is in motion we may apply the Newton's second law of motion in the three directions to get.

$$ma_x = -\left(\frac{\partial p}{\partial x}\,\Delta x\right)\Delta y\,\Delta z \qquad (2.3)$$

$$ma_y = -\left(\frac{\partial p}{\partial y}\Delta y\right)\Delta x\,\Delta z \qquad (2.4)$$

$$ma_z = -\left(\frac{\partial p}{\partial z}\Delta z\right)\Delta x\,\Delta y - mg \qquad (2.5)$$

Where m is the mass of the element. Notice that the weight of the fluid mg is accounted for in the z-direction. Since the mass m of the element is equal to the density times volume,

$m = \rho\,(\Delta x\,\Delta y\,\Delta z)$, the above set of equations (2.3) to (2.5) result in

$$\frac{\partial p}{\partial x} = -\rho a_x \qquad (2.6)$$

$$\frac{\partial p}{\partial y} = -\rho a_y \qquad (2.7)$$

$$\frac{\partial p}{\partial z} = -\rho(a_z + g) \qquad (2.8)$$

when the acceleration in the three directions is zero, $a_x = a_y = a_z = 0$, the above equations result in,

$$\frac{\partial p}{\partial x} = 0 = \frac{\partial p}{\partial y};\ \frac{\partial p}{\partial z} = -\rho g$$

which is just the hydrostatic pressure distribution you have studied in your first unit. When the container is in an accelerated motion, the liquid level in the container assumes an inclined position instead of the horizontal level it has in purely static conditions. This is because of the pressure distribution given by equations (2.6), (2.7) and (2.8). With a little treatment of these general pressure distribution equations, it may be shown that the inclination θ, of the free surface in a container, (see figure 2.13) in the $x - z$ plane is given by

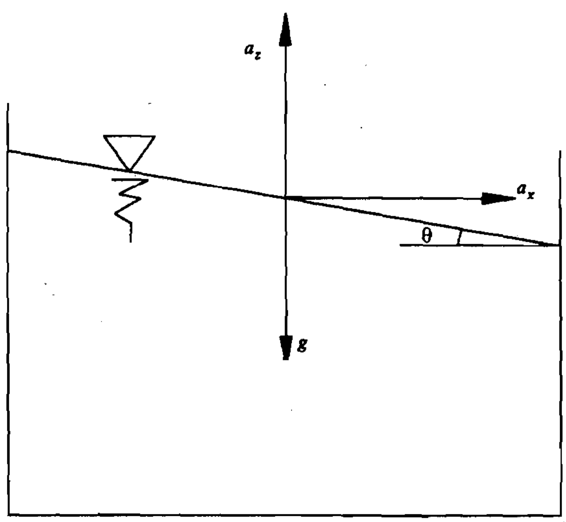


Figure 2.13

$$\frac{dz}{dx} = \tan\theta = -\frac{a_x}{a_z + g} \qquad (2.9)$$

For a purely horizontal acceleration, $a_z = 0$ in equation (2.13), in which case,

$$\tan\theta = -\frac{a_x}{g} \qquad (2.10)$$

Example 5 :

A thin walled, open topped tank in the form of a cube of 500 mm side is initially full of oil of sp. gravity 0.88. It is accelerated uniformly at 5 m/sec^2 up a long straight slope, $\tan^{-1}(1/4)$ to the horizontal.

Calculate a) the volume of oil left in the tank when no more spill occurs
b) the pressure at the lowest corner of the tank

Solution :

See figure 2.14 (a) for the definition of various terms.

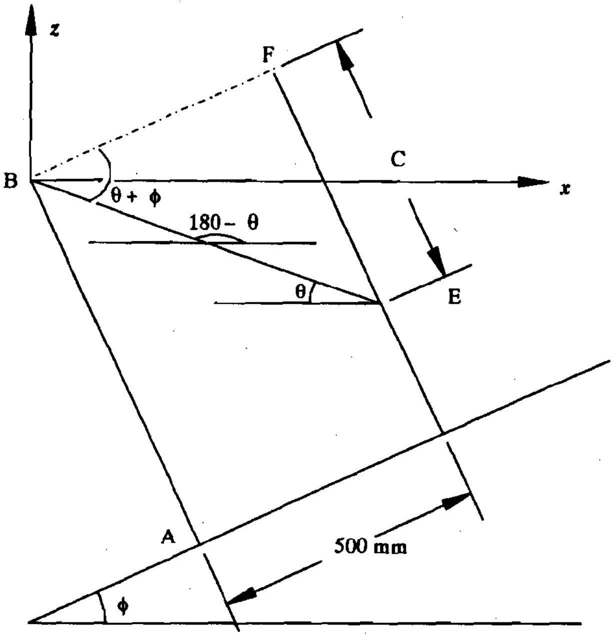


Figure 2.14 (a)

We have, $$\tan\theta = \frac{a_x}{a_z + g}$$

At the point when no more spill occurs, the liquid surface will be as shown in figure 2.14. To get the volume of water left in the tank, we need to determine the distance C.

We shall proceed as follows:

$$a = 5\ \text{m/sec}^2 \quad \therefore\ a_x = a\cos\phi = 5 \times \frac{4}{\sqrt{17}}$$

Similarly, $$a_z = a\sin\phi = 5 \times \frac{1}{\sqrt{17}}$$

Now,

$$\tan\theta = \frac{\frac{20}{\sqrt{17}}}{\left(\frac{5}{\sqrt{17}}\right) + 9.81} = 0.440$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$= \frac{0.25 + 0.44}{1 - 0.25 \times 0.44} = 0.775$$

From figure, $$\tan(\theta + \phi) = \frac{C}{0.5}$$

$$\therefore\quad C = 0.775 \times 0.5 = 0.3875\ \text{m}$$

$\therefore$ Volume left = Volume of the tank − Volume of portion BEF

$$= 0.5\left(0.5^2 - \frac{1}{2} \times 0.5 \times 0.3875\right) = 0.0765\ \text{m}^3$$

= 76.5 litres.

Further,

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial z}dz$$

$$\therefore \quad p = -\rho a_x x - \rho\,(g + a_z)\,z + \text{constant}$$

(We have substituted for $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial z}$ and then integrated)

With the origin at B, $p = 0$ at $x = 0$ and $z = 0$

$$\therefore \text{constant} = 0$$

$$\therefore p = -\rho a_x x - \rho(a_z + g)\,z$$

We want the pressure at A.

Co-ordinates of $A = (0.5 \sin\phi\,, -\ 0.5 \cos\phi)$

$$\therefore \quad p_A = -\ \rho(a\cos\phi)\cdot(0.5\sin\phi) - \rho(a\sin\phi + g)\,(-\,0.5\cos\phi)$$

$$= \rho g\,(0.5\cos\phi)$$

$$= 0.88 \times 1000 \times 9.81 \times 0.5 \times \frac{4}{\sqrt{17}}$$

$$= 4190\ \text{Pa}$$

SAQ 5

A tank shown in figure 2.14(b) contains oil of sp. gravity 0.80. If it is given an acceleration of 5.0 m/sec^2 along a 30° inclined plane in the upward direction, determine the slope of free surface and pressure at B.

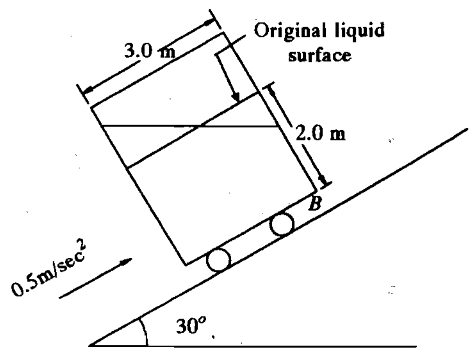


Figure 2.14 (b)

2.5 LIQUIDS IN A CONTAINER SUBJECTED TO ROTATION

When a vessel containing a liquid is rotated about an axis, the fluid elements experience both centrifugal and gravitational body forces. Consider a cylindrical container with liquid rotated about a vertical axis at a constant angular speed ω. Under the action of centrifugal and gravitational forces the liquid line assumes a curved free surface, as shown in figure 2.15.

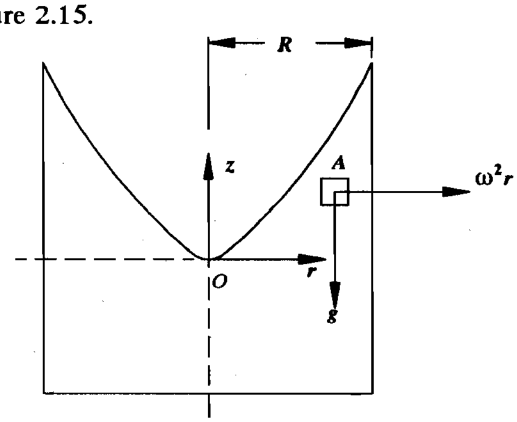


Figure 2.15

For the fluid element A in figure 2.15, the acceleration a_r in the r-direction is given by $a_r = \omega^2 r$

The fluid element will be in equilibrium, if

$$-\frac{\partial p}{\partial r} + \rho\omega^2 r = 0, \text{ and}$$

$$-\frac{\partial p}{\partial z} - \rho g = 0$$

The slope of the free surface, $\frac{dz}{dr}$ is therefore obtained as,

$$\frac{dz}{dr} = \frac{-\frac{\partial p}{\partial r}}{\frac{\partial p}{\partial z}}$$

$$= \frac{-\rho\omega^2 r}{-\rho g}$$

i.e

$$\frac{dz}{dr} = \frac{\omega^2 r}{g} \qquad (2.11)$$

Integration of equation (2.11) yields,

$$z = \frac{\omega^2 r^2}{2g} + c$$

With $z = 0$ at $r = 0$, $c = 0$

$$\therefore \quad z = \frac{\omega^2 r^2}{2g} \qquad (2.12)$$

The maximum height to which the liquid level can rise is thus, $\frac{\omega^2 R^2}{2g}$ where R is the radius of the container.

Example 6 :

A cylindrical vertical container 50 cm internal dia. is rotated about its axis. The container has a height of 1 m and was initially filled to 60 cm. Calculate the speed of rotation at which the water shall begin to spill over the container, and the pressure at a point 20 cm radial position and 5 cm above the base. (See figure 2.16)

Solution :

At the condition of spilling over

$$\frac{1}{2}\frac{\omega^2 R^2}{2g} = 40 \text{ cm (half the maximum height)}$$

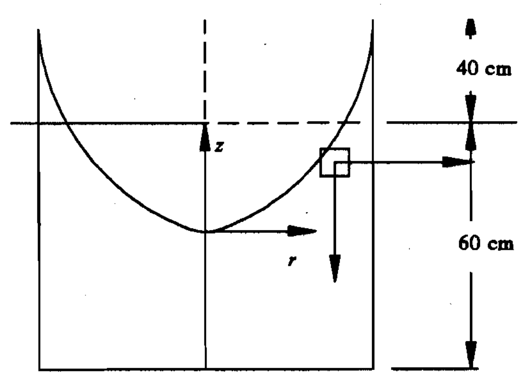


Figure 2.16

$$\therefore \quad \omega^2 = \frac{0.4 \times 4 \times 9.81}{(0.25)^2} = 251.136$$

$$\omega = 15.84 \text{ rad/sec}$$

The point whose vertical position is 5 cm above the base corresponds to $z = -15$ cm

$$p(r, z) = \frac{\rho\, \omega^2 R^2}{2} - \rho g z$$

$$= \frac{1000 \times 251.136 \times (0.2)^2}{2} + 1000 \times 9.81 \times 0.15$$

$$= 6.494 \times 10^3 \text{ N/m}^2$$

SAQ 6

If the cylinder in the above example was closed with a lid and was rotated so that the point at the centre of the base was just clear of water, what would have been the angular speed ?

2.6 SUMMARY

Now let us summarise what you have learnt in this unit.

a) The buoyancy force on a wholly or partially submerged surface is equal to the weight of the fluid displaced and it acts vertically upwards through the centre of buoyancy.

b) The point of intersection of the buoyancy force from its new position after a slight angular tilt and the line joining the centre of buoyancy in the undisturbed position & the centre of gravity of the body is called the Meta Centre. For stability the metacentre should be above the centre of gravity.

c) The free surface of a liquid in a container under acceleration inclines upward in a direction against that of the acceleration. Under the condition of rotation of the container at a constant speed the free surface assumes a paraboloid shape.

2.7 ANSWERS TO SAQs

SAQ 1

$F_B = W \qquad \therefore \; (300 - 232.5) = 9.8 \times 1000 \times V$

$\therefore \; V = 0.00689 \text{ m}^3$

SAQ 2

W = Weight of hollow cube plus solid concrete block.

F_{b_1} = buoyant force on hollow cube

F_{b_2} = buoyant force on solid concrete block

$$\therefore \; W = 2.4 + 10 = 12.4 \text{ kN}$$

$$F_{b_1} = \frac{9.8 \times 1 \times 1 \times 1 \times 10^3}{10^3} = 9.8 \text{ kN}$$

$$V_{\text{block}} = \frac{10}{2.4 \times 9.8 \times 10^3} = 0.4256 \times 10^{-3} \text{ m}^3$$

$$F_{b_2} = 9.8 \times 0.4256 \times 10^3 \times 10^{-3} = 4.17\,\text{kN}$$

$$F_{b_1} + F_{b_2} = 9.8 + 4.17 = 13.96\,\text{kN}$$

Since $[W = 12.4] < \left[F_{b_1} + F_{b_2} = 13.96\right]$ the two objects will float in water

SAQ 3

The first step is to determine the submerged depth of the cylinder when placed in oil.

$F_B = W$

$$\therefore \quad 0.85 \times 9.8 \times \frac{\pi \times (0.666)^2}{4} \times D = 0.61 \times 9.8 \times 1.3 \times \frac{\pi \times (0.666)^2}{4}$$

$$\therefore \quad D = 0.9333\,\text{m}$$

The centre of buoyancy is located $\frac{0.9333}{2} = 0.466\,\text{m}$ from the bottom of the cylinder.

$$\overline{MB} = \frac{I}{V} = \frac{\pi \times (0.666)^4 \times 4}{64 \times 0.933 \times \pi \times (0.666)^2} = 0.030\,\text{m}$$

That is, the metacentre is located 0.030 m above the centre of buoyancy. This places the metacentre $\frac{1.30}{2} - 0.466 - 0.030$ or 0.154 m below the centre of gravity. Since the metacentre is below the centre of gravity, the cylinder is not stable.

SAQ 4

The cube's centre of gravity is at 0.5 L above its bottom. Since the cube's sp.gr. is 0.8, it will float at a submerged depth of 0.8 L, and its centre of buoyancy will be at 0.4 L above its bottom.

$$\text{MB} = \frac{I}{V} = \frac{\left[\frac{L \times L^3}{12}\right]}{L \times L \times 0.8L} = 0.1042\,L$$

That is, the metacentre is located 0.1042 L above the centre of buoyancy and 0.1042 L + 0.4 L - 0.5 L or 0.0042 L and therefore the cube is stable.

SAQ 5

The acceleration a_s is resolved as

$$a_x = 5 \cos 30° = 4.33\ \text{m/sec}^2$$

$$a_y = 5 \sin 30° = 2.5\ \text{m/sec}^2$$

Slope of water surface is given by

$$\tan\theta = \frac{a_x}{a_y + g} = \frac{4.33}{12.31} = 0.3519$$

or $\theta = 19.38°$

Depth at $B = 2 - 1.5 \tan\theta = 1.472\,\text{m}$

$$\frac{P_b}{\gamma} = \left(1 + \frac{a_y}{g}\right) \times h_b$$ (See the solved example in text for hint).

$$\frac{P_b}{\gamma} = \left(1 + \frac{2.5}{9.81}\right) \times 1.472$$

$$= 1.847\ \text{m}$$

$\therefore \quad P_b = 1.847 \times 9.81 \times 1000 \times 0.87$

$= 15763.59 \text{ N/m}^2$

$= 15.763 \text{ kN/m}^2$

SAQ 6

Since the forces continue to be the same two body forces even when the container is closed, the equipressure surface will still be paraboloid. However, as the water is not allowed to spill out and it in turn begins to readjust the volume there is no true free surface.

With this hint, you should be able to work out the problem. You will get an angular speed of 19.81 red/sec as the answer.