UNIT 5  EARTH PRESSURE AND RETAINING STRUCTURES

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5.1 INTRODUCTION

This unit seeks to introduce you to the evaluation of lateral earthforces acting on retaining structures and to determine the stability of such structures. You know that structures which are constructed to hold back a soil mass are called retaining structures. Retaining walls, sheet pile bulkheads, braced excavations and basement walls are examples of retaining structures (Figure 5.1). A retaining wall helps in maintaining the surface of the ground at different elevations on either side of the structure. If the retaining wall was not there, the soil at higher elevation would tend to move down till it acquires its natural, stable configuration (Figure 5.2). Consequently the soil that is now retained at a slope steeper than it can sustain by virtue of its shear strength exerts a force on the retaining wall. The force is called earth pressure. The gravity retaining wall is the simplest type of retaining wall and the other common type is the cantilever retaining wall.
It should be clear to you that a retaining wall is constructed whenever space requirements do not allow the natural slope to be formed for an excavation. Such conditions may arise when a roadway or storage area is needed immediately adjacent to an excavation and the retaining wall forms a permanent wall of the excavation. Let us see how a wall is constructed. A temporary slope is formed at the edge of excavation and the wall is built (Figure 5.2). Then the backfill is dumped into the space between the wall and the temporary slope. For gravity walls, masonry or concrete is used. Reinforced concrete is used for cantilever and counterfort retaining walls. Reinforced earth walls are now widely used in developed countries.

Figure 5.3 shows in a general way the forces that act on a gravity retaining wall. The bearing force resists the weight of the walls plus the vertical components of the other forces. The active thrust is developed due to the placement of the backfill and any surcharge on its surface and tends to push the wall outward. The outward motion is resisted by sliding resistance along the base of the wall and by the passive resistance of the soil lying above the toe of the wall. The overturning is resisted by the weight of the wall, weight of the soil and the vertical component of the active thrust. The weight of the wall and the soil resists overturning and causes sliding resistance at the base of the wall. In gravity retaining wall the weight of the wall is the major component of resistance while in cantilever wall, the weight of soil is the major component.

A retaining wall together with the backfill the wall retains and the soil that supports the wall is a highly indeterminate system. The magnitudes of the forces that act upon a wall are difficult to determine. Hence the design is based on the analysis of forces that would exist if the wall started to fail (i.e.) to overturn or to slide outward.
The first step in the analysis is to envision the pattern of deformation that would accompany such a failure. In the case of failure of gravity retaining wall it has been observed that the soil moved toward the wall and downward within the soil. These motions indicated that the shear failure occurred throughout the active zone and the full frictional resistance was mobilized throughout the zone. A second zone of shear failure (the passive zone) developed at the toe of the wall when the wall was pushing against the soil.

The approach to the design of retaining walls can be started as follows:

i) Select a trial dimension of the wall

ii) Determine the active thrust against the wall

iii) Determine the sliding resistance at the base of the wall due to the weight of the wall and the soil

iv) Determine the passive resistance at the toe of the wall

v) Check whether the resistance exceeds the active thrust whether the resisting moment exceeds the overturning moment, and whether tension is created in any horizontal section of the wall. Adopt suitable factors of safety.

**Objectives**

After studying this unit you should be able to:

- identify the field situations where Active, Passive and at rest earth pressures development,
- calculate the lateral earth thrusts by Rankine and Coulomb methods for cohesionless and cohesive soils, and
- to check the stability of gravity and cantilever retaining walls against sliding bearing capacity and overturning failures.

Now let us consider the methods for determining the active thrust and passive resistance.

### 5.2 EFFECT OF WALL MOVEMENT ON EARTH PRESSURE

About fifty years ago Terzaghi conducted a series of tests on large scale model retaining walls to ascertain the variation of magnitude of earth pressures with the movement of the walls. When the wall is rigid and unyielding, there are no deformations and the soil mass is in a state called earth pressure at rest. This is represented by the point A in (Figures 5.4(a)). When the wall rotates about its toe, moving away from the soil (Figure 5.4(b)), the soil mass expands, resulting in a decrease of earth pressure. Any element of soil will then behave just like a specimen in a triaxial test in which the
confining pressure is decreased while the axial stress remains constant. When the horizontal stress is decreased to a certain magnitude, the full shear strength of the soil will be mobilised. No further decrease in the horizontal stress is possible even with further movement of the wall. This is called active earth force. This is represented by point B in the (Figure 5.4(a)).

If the wall is pushed towards the wall (Figure 5.4(c)) the soil is compressed and the soil offers resistance to this movement by shearing resistance. Any element of the soil can now be considered to be in the condition of a triaxial specimen being failed by increasing the confining pressure while holding the vertical stress constant. The horizontal stress cannot be increased beyond a magnitude called passive stress. At this stress failure occurs and the passive earth force is denoted by the point C in (Figure 5.4(a)).

Active and Passive earth pressures develop corresponding to two limiting states of equilibrium. The soil mass is said to be in a state of plastic equilibrium in these two stages. A small increase in stress at this stage will cause the plastic flow condition—a continuous increase in the corresponding strain.

It is to be noted that for sands very little horizontal strain, less than 0.5% is required to reach the active state while little horizontal compression, about 0.5% is required to reach one-half the maximum passive resistance. However about 2% of horizontal compression is needed to reach the full maximum passive resistance in dense sands while in loose sands horizontal compression needed to reach full passive resistance may be as large as 15%.

### 5.3 EARTH PRESSURE AT REST

You have seen that the active pressure is the minimum lateral pressure while the passive pressure is the maximum lateral pressure. Active pressure is associated with the expansion of the soil and the passive pressure with the compression of soil. In both the conditions the soil mass is in a state of incipient failure. The soil in its natural state at a depth \( z \) below the ground level is not subjected to any strain. The element is in a condition known as the “at rest” condition. The corresponding lateral pressure, called the earth pressure at rest is expressed in the form

\[
P_0 = K_0 \sigma_z = K_0 \gamma_z
\]  \hspace{1cm} \text{(5.1)}

Where, \( K_0 \) is called the coefficient of earth pressure at rest. \( \sigma_z \) is the effective vertical stress at depth \( z \). At rest condition it also termed as \( K_0 \) condition.

If the soil mass is considered to be semi-infinite, homogeneous, elastic and isotropic material, the magnitude of can be shown to be

\[
K = \frac{\mu}{1 - \mu}
\]  \hspace{1cm} \text{(5.2)}
where, \( \mu \) is the Poisson’s ratio of the material.

For sands and normally consolidated clays, the magnitude of can be related to the \( \theta \) value as follow

\[
K_0 = (1 - \sin \theta)
\]

Typical values of \( k_0 \) are shown in Table 5.1. The earth pressure at rest condition exists in the case of basements and abutments where the wall earth system is rigid.

### Table 5.1: Typical Values of \( K_0 \)

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Soil</th>
<th>( K_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Dense sand</td>
<td>0.40 - 0.45</td>
</tr>
<tr>
<td>2)</td>
<td>Loose sand</td>
<td>0.45 - 0.50</td>
</tr>
<tr>
<td>3)</td>
<td>Compacted sand</td>
<td>0.80 - 1.50</td>
</tr>
<tr>
<td>4)</td>
<td>Normally consolidated clay</td>
<td>0.50 - 0.60</td>
</tr>
<tr>
<td>5)</td>
<td>Over consolidated clay</td>
<td>1.0 - 4.0</td>
</tr>
</tbody>
</table>

**SAQ 1**

i) Sketch of wall movement you expect in Cantilever sheet pile walls and anchored sheet pile walls.

ii) Derive the relation \( K = \frac{\mu}{1 - \mu} \).

iii) Estimate the value of \( K_0 \) for water.

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### 5.4 RANKINE ACTIVE AND PASSIVE STATES

Rankine theory considers the condition when shear failure is imminent at every point within the soil mass and it is in a state of plastic equilibrium. Let us now analyse a semi-infinite mass of soil bounded by a horizontal surface and a vertical boundary formed by the vertical smooth wall surface (Figure 5.5). The soil mass is assumed to be homogeneous dry and cohesionless. Please note that the theory has been extended to include cohesive and submerged soils also.

In Figure 5.5, a soil element at depth \( z \) is subjected to a vertical stress \( \sigma_x \) and a horizontal stress \( \sigma_y \). Since a semi-infinite mass of soil is considered, there are no shear stresses on the vertical and horizontal planes. So \( \sigma_y \) and \( \sigma_x \) are the principal stresses. The Mohr’s circle representing the at rest condition is shown in Figure 5.6. When the wall moves away from the backfill, the soil element expands and the value of \( \sigma_n \) decreases. The value of \( \sigma_n \) decreases to a minimum when the expansion is large enough developing a state of plastic equilibrium. In Figure 5.6, this condition is obtained when the Mohr circle representing the stressed state touches the failure envelope for the soil. In this case the horizontal stress \( \sigma_y \).
is the minor principal stress and the vertical stress \( \sigma_v \) is the major principal stress. At this stage the sand is said to be in active Rankine state.

The vertical stress \( \sigma_v \) (or \( \sigma_y \)) on the soil element is the weight of sand above the plane, \( \gamma' \). The minimum value of \( \sigma_v \) (or \( \sigma_3 \)) when the state of plastic equilibrium is reached can be determined as follows:

\[
\sin \phi = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}
\]

\[
1 + \sin \phi = \frac{\sigma_1}{\sigma_3}
\]

\[
\sigma_3 = \frac{\sigma_1 (1 - \sin \phi)}{(1 + \sin \phi)}
\]

The minimum value of \( \sigma_3 \) is defined as the active earth pressure, \( P_a \). Hence \( P_a \) can be written as

\[
P_a = \gamma' \frac{1 - \sin \phi}{1 + \sin \phi}
\]

\[
= \gamma' \tan^2 \left(45 - \frac{\phi}{2}\right)
\]

\[
P_a = K_a \gamma' z
\]

Where, \( K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45 - \frac{\phi}{2}\right) \)

\( K_a \) is termed as coefficient of active Rankine pressure.

When the soil mass is in the active Rankine state two sets of failure planes develop each inclined at an angle \( 45 + \phi/2 \) to the horizontal which is the direction of major principal plane as seen in Figure 5.5.

Let us now consider the case when the wall moves towards the backfill. There will be uniform compression in the horizontal direction. This leads to an increase in the value of \( \sigma_h \) while the value of \( \sigma_v \) remains constant. As the deformation increases, \( \sigma_v \) goes on increasing. When \( \sigma_v = \sigma_v \) the Mohr's circle is a point. Then the horizontal stress becomes greater than vertical stress. The maximum value of \( \sigma_v \) is reached when the diameter of the Mohr’s circle touches the failure envelope (Figure 5.6). At this point a state of plastic equilibrium is reached and the soil is then said to be in the passive Rankine state. The corresponding horizontal stress is defined as the passive earth pressure, \( P_p \). Then we can write

\[
P_p = \gamma' \frac{1 + \sin \phi}{1 - \sin \phi} = \gamma' \tan^2 \left(45 + \frac{\phi}{2}\right)
\]
In the passive state two sets of failure planes make an angle of 45° - $\phi/2$ with the horizontal which is the direction of minor principal plane as shown in Figure 5.5.

SAQ 2

1) Show that $\frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left(45 + \frac{\phi}{2}\right)$

2) Estimate the value $K_o$, $K_p$ and $K_o$ for a cohesionless soil whose angle of shear resistance is 30°.

3) Estimate the values of $K_o$ and $K_p$ for water and soft clays.

5.5 ACTIVE PRESSURE ON RETAINING WALLS

5.5.1 Cohesionless Backfill

i) Dry Backfill - No Surcharge

Figure 5.7 shows a retaining wall with a smooth vertical back. The backfill is dry and cohesionless. There is no surcharge. At any depth $z$, the lateral pressure $P_{az}$ can be written as

$$P_{az} = K_z \gamma_z$$  \hspace{1cm} (5.13)

where, $\gamma$ is the dry unit weight of the soil. At the base of the wall, where $z = H$,

$$P_a = K_z \gamma H$$  \hspace{1cm} (5.14)

The total active thrust $P_a$ is given by the area of the active pressure distribution diagram. It acts through the centre of gravity of the area at a height $H/3$ above the base of the wall.

$$P_a = \frac{1}{2} K_o \gamma H \cdot H = \frac{1}{2} K_o \gamma H^2$$  \hspace{1cm} (5.15)

**Example 5.1**

A retaining wall with a smooth vertical back retains dry sand backfill for a depth of 3 m. The backfill has a level surface and has the following properties.
Calculate the magnitude of the total active earth thrust against the wall assuming the wall is free to move and its point of application.

\[ F_a = 0; \phi = 30^\circ; \gamma = 16 \text{ kN/m}^3 \]

Solution

Since the wall is free to move, assume active condition.

Coefficient of active earth pressure \( K_a = \frac{1 - \sin 30}{1 + \sin 30} = \frac{0.5}{1.5} = 0.33 \)

\[ P_a = K_a \gamma H = 0.33 \times 16 \times 3 = 16 \text{ kN/m}^2 \]

Total thrust per metre length of wall (Figure 5.8)

\[ P_a = \frac{1}{2} K_a \gamma H^2 = 24 \text{ kN/m}^3 \]

This acts at a height of 1 m above base

Example 5.2

In example 5.1 assume the wall is restrained against yielding and determine the active lateral thrust.

Solution

Since the wall is not free to move, assume earth pressure at rest condition.

Coefficient of earth pressure at rest \( K_0 = 1 - \sin \phi \)

\[ = 1 - \sin 30 = 0.5 \]

\[ P_0 = 0.5 \times 16 \times 3 = 24 \text{ kN/m}^2 \quad \text{(Figure 5.9)} \]

Total lateral thrust per metre length = \( \frac{1}{2} K_0 \gamma H^2 \)
2) **Dry Back Fill - Uniform Surcharge**

If a uniformly distributed surcharge load of intensity \( q \) per unit area is acting over the surface of the backfill, the effective vertical pressure at any depth is increased by \( q \). The increase in active pressure is uniform throughout the depth and is equal to \( K_a q \) (Figure 5.10). The total active thrust can be written as

\[
P_a = \frac{1}{2} K_a \gamma H^2 + K_a q \cdot H
\]

... (5.16)

**Example 5.3**

In Example 5.1, a surcharge load of 30 kN/m\(^2\) is acting on the backfill. Determine the total lateral force and its line of action.

**Solution**

The active earth force = 24 kN/m\(^2\)
and it acts at a height of 1.0 m above the base
The lateral force due to surcharge = \( K_a q \cdot H \).

\[
= 0.33 \times 30 \times 3 = 30 \text{ kN/m}
\]

Total lateral force = 54 N/m

The line of action of the force can be determined by taking moments about the base.

\[
54 \times y = 24 \times 1 + 30 \times 1.5
\]

\[
y = \frac{24 + 45}{54} = \frac{69}{54} = 1.28 \text{ m}
\]

Note that the thrust due to surcharge acts at a height \( H/2 \) above the base (Figure 5.11).
3) Sloping Backfill

In the case of an inclined backfill at an angle $\beta$ to the horizontal, the lateral earth pressure is assumed to act parallel to the backfill surface (Figure 5.12). It is also assumed that the vertical stress and the lateral pressure acting on the soil element are conjugate stresses, (i.e.) the direction of one is parallel to the plane on which the other acts. In this case the vertical stress and the lateral pressure are not principal stresses. The relation between them can be obtained by means of a Mohr diagram.

\[ \sigma_z = \gamma z \cos \beta \]

The active earth pressure at a depth $z$ acting parallel to the slope is given by

\[ P_a = K_a \gamma z \quad ... (5.17) \]

Where, \[ K_a = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \cos \beta \quad ... (5.18) \]

The total active thrust $P_a$ is given by

\[ P_a = \frac{1}{2} K_a \gamma H^2 \]

Example 5.4

In Example 5.1 assume that the backfill is inclined at an angle of 20° to the horizontal. Estimate the total lateral earth thrust.

Solution

The coefficient of active pressure for inclined backfill, $K_a$

\[ = \frac{0.9397 - \sqrt{0.8830 - 0.75}}{0.9397 + \sqrt{0.8830 - 0.75}} \times 0.9397 = 0.4144 \]

Active earth thrust $= \frac{1}{2} \times 16 \times 0.4144 \times 9$

$= 29.84 \text{ N}$

4) Inclined Back of Wall

A modified procedure which includes the weight of backfill on the retaining wall can be used to determine the active earth thrust for the walls with a batter. Figure 5.13 shows a
horizontal backfill with an inclined back. The suggested procedure for calculating the active earth pressure is

i) Draw a vertical line through the base of the wall to intersect the backfill surface at B.

ii) Calculate the total active thrust, $P_a$ on the vertical plane AB.

iii) Determine the weight of soil, $W$ included in the triangular wedge ABC.

iv) The lateral thrust on the wall $P$ is the vector sum of $P_a$ and $W$.

$$P = \sqrt{P_a^2 + W^2} \quad \ldots (5.20)$$

Similar procedure can be adopted for determining the total active thrust in the case of inclined backfill for a retaining wall with a batter (Figure 5.14).

5) **Completely Submerged Backfill**

Figure 5.15 shows the case of completely submerged backfill. For the submerged portion of the backfill by the presence of natural water table, submerged unit weight of the soil $\gamma$ is to be used for computing the active thrust. The total lateral thrust will be equal to the sum of active thrust and the hydrostatic pressure. Then lateral thrust $P$

$$P = K_a \gamma_{sub} H^2 + \gamma_w H^2 \quad \ldots (5.21)$$

Example 5.5

In example 5.1, assume that water table is at the top of the wall and there is no drainage. The saturated unit weight of sand is 20 kN/m$^3$. Determine the total lateral force acting on the wall and its line of action.

Assume active lateral earth pressure.

Solution

Since the sand is submerged, submerged unit weight have to be used in calculating earth pressure. However there will be no reduction in angle of shearing resistance due to submergence.
Active lateral earth pressure = $K_a \gamma_{sub} H$

\[ = 0.33 \times (19 - 9.8) \times 3 = 10.2 \text{kpa} \]

Active lateral earth force = \(\frac{1}{2} \times 10.2 \times 3\)

\[ = 15.3 \text{ N/m} \]

Lateral force due to water = \(\frac{1}{2} \gamma_w H^2\)

\[ = \frac{1}{2} \times 9.8 \times 9 = 44.1 \text{N/m} \]

Total lateral force = 59.4 N/m

Both the forces act at a height of 1.0 m above base. Hence the resultant force also acts at 0 height of 1.0 m above base (Figure 5.16).

6) **Partly Submerged Backfill**

In the case of backfill submerged partially, the moist or dry unit weight of soil is to be taken for the portion above the water table and the submerged unit weight below the water table. The total lateral thrust will be equal to the sum of the active thrust due to soil and the hydrostatic pressure. The lateral pressure on the base of the wall $P$ can be written as (Figure 5.17).

\[ P = K_a \gamma H_1 + K_a \gamma_{sub} (H - H_1) + \gamma_w (H - H_1) \]  \( \cdots (5.22) \)

7) **Stratified Backfill**

Rankine’s theory can also be used to calculate the lateral thrust due to stratified backfills. For the top layer Rankine’s earth thrust is calculated as indicated in earlier sections. For determining the thrust due to bottom layers, the weight of the layers above them is treated as surcharge and earth thrust calculated. You can get a clear idea about the procedure in example 5.6.

**Example 5.6**

A retaining wall with a smooth vertical back retain a two layer dry cohesionless backfill with the following properties.
0 - 4 m depth  \( c = 0, \phi = 30^\circ \) :  \( P = 17 \text{kN/m}^2 \)

4 - 8 m depth  \( c = 0, \phi = 34^\circ \) :  \( P = 20 \text{kN/m}^2 \)

Determine the total lateral earth force acting on the wall and its line of action.

**Solution**

When there are two layers, lateral earth pressure for the bottom layer \( P_a \), is calculated assuming that the top layer is a surcharge force (Figure 5.18).

\[
K_a = \frac{1 - \sin 30}{1 + \sin 30} = 0.33
\]

\[
K_b = \frac{1 - \sin 34}{1 + \sin 34} = 0.283
\]

For the top layer

\[
(P_a)_{4\text{m}} = 0.33 \times 4 \times 17 = 22.67 \text{kN/m}^2
\]

For the bottom layer

\[
(P_a)_{4\text{m}} = 0.283 \times 4 \times 17 = 19.24 \text{kN/m}^2
\]

\[
(P_a)_{8\text{m}} = 0.283 \times 4 \times 17 + 4 \times 20 = 41.88 \text{kN/m}^2
\]

The active pressure diagram is shown in Figure 5.18.

Note that there is a break in the pressure distribution diagram at the interface.

The total lateral earth force

\[
= \frac{1}{2} \times 22.67 \times 4 + 19.24 \times 4 + \frac{1}{2} (41.88 - 22.64) \times 4
= 45.34 + 76.96 + 45.28 = 167.58 \text{kN/m}^2
\]

The line of action of the resultant can be calculated by taking moments about the base.

\[
167.58 \times y = 45.34 \times 4 + 76.96 \times 2 + 45.28 \times 4
\]

\[
y = \frac{241.66 + 153.92 + 60.22}{167.58} = 2.78 \text{ m}
\]

The resultant acts at a height of 2.78 m above base.

### 5.5.2 Cohesive Backfill

The Rankine theory has been extended to the case of soils having both friction and cohesion. For a soil having both friction and cohesion, the relation between major principal stress \( \sigma_1 \) and minor principal stress \( \sigma_3 \) at failure can be expressed in the form (Figure 5.19).
\[ \sigma_1 = \sigma_3 \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2C \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \]

\[ \sigma_3 = \sigma_1 \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) + 2C \sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}} \]

In the case of a retaining wall of height \( H \) with a smooth vertical back retaining cohesive backfill, at any depth \( z \) (Figure 5.20).

\[ \sigma_v = \gamma z \]
\[ \sigma_h = \gamma z K_a - 2c \sqrt{K_a} \]

From Equation 5.27 at \( z = 0 \), \( P_a = -2C \sqrt{k_a} \)

The distribution of active earth pressure is shown in Figure 5.21. The negative pressure exists up to a depth \( z_0 \) where the active pressure becomes zero. The soil is in a state of tension in the zone between the ground and depth \( z_0 \). In practice this tension cannot be taken to act on the wall. Tension cracks develop in the soil within the tension zone and the soil may not remain adhering to the wall. Hence in calculating the total active thrust on the wall, the tension zone is usually ignored and only the area of the pressure diagram between depth \( z_0 \) and \( H \) is considered. The active thrust on the wall is,

\[ P_a = \frac{1}{2} K_a \gamma H^2 - 2C \sqrt{K_a} \]
You can observe that the net active thrust is zero for a depth equal to $2z_0$. It is implied that in a cohesive soil, a vertical cut can be made up to a depth $2z_0$ without any lateral support. The critical depth of a vertical cut $H_c$ in a cohesive soil is given by

$$H_c = 2z_0 = \frac{4C}{\gamma \sqrt{K_a}} \quad \text{... (5.31)}$$

However, the failure conditions in a cut differ from those in a retaining wall and the actual unsupported depth of a cut is likely to be smaller than what is given by the equation.

**Example 5.7**

A retaining wall 6 m high retains a clay backfill with $c = 20$ kN/m$^2$, $\phi = 15^\circ$ and $\gamma = 18$ N/m$^3$. Assume that the wall is smooth and the back vertical. It is expected that tension cracks may develop to the full theoretical depth. Calculate the total active earth force $a$ acting on the wall.

**Solution**

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \frac{1 - \sin 15}{1 + \sin 15} = 0.589$$

$$P_a = K_a \gamma z = 2c \sqrt{K_a}$$

at $z = 0, \quad P_a = -30.7$ kN/m$^2$

at $z = 6$ m; $P_a = 63.6 - 30.7 = 32.9$ kN/m$^2$

The depth of tension crack $= \frac{2c}{\gamma \sqrt{K_a}} = \frac{2 \times 20}{18 \times 0.589} = 2.90$ m

The lateral pressure diagram is shown in Figure 5.22. It is recommended that the tensile stress up to the depth of 2.9 m is to be ignored. Hence the total active thrust is

$$P_a = 1/2 \times 32.92 \times 3.10 = 51.03$$ kN/m
Example 5.8

An excavation is to be made in a clay having a cohesive strength of 24 kN/m². The unit weight of clay is 20 kN/m³. What is the depth of vertical cut that can be made without any support?

Solution

You know that the depth of tension crack is \( \frac{2c}{\gamma \sqrt{K_s}} \) and the depth of vertical cut is \( \frac{4c}{\gamma \sqrt{K_s}} \).

Let us assume \( \phi_u = 0 \)

Then \( K_u = 1 \)

So critical depth \( = \frac{4 \times 24}{20} = 4.8 \text{ m} \)

5.6 PASSIVE PRESSURE OF RETAINING WALLS

The passive earth pressure is mobilized when the soil is compressed. This can occur in the field in front of the toe of a retaining wall which is being pushed by a backfill. In some cases passive pressure will be mobilized to a partial extent only.

5.6.1 Cohesionless Soil

In the case of dry cohesionless level backfill the passive pressure at a depth \( z \) is given by

\[ P_p = K_p \gamma z \]

where, \( K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \tan^2 \left( \frac{45 + \phi}{2} \right) \)

The total passive resistance \( P_p \) for the full height of the retaining wall is

\[ P_p = \frac{1}{2} K_p \gamma H^2 \]

If a uniform surcharge load of \( q \) is applied over the surface, the passive earth pressure is increased by \( k_p q \) at every depth. The total, passive resistance can then be written as

\[ P_p = \frac{1}{2} K_p \gamma H^2 + K_p q . H \]

If the backfill is inclined at an angle \( \beta \) to the horizontal, the passive pressure \( P_p \) at a depth \( z \) is given by

\[ P_p = K_p \gamma^2 \cos \beta \left( \frac{\cos^2 \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos^2 \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}} \right) \]

The total passive resistance \( P_p \) of the wall of height \( H \) can be written as

\[ P_p = \frac{1}{2} \gamma k_p . H^2 \]

The force acts parallel to the slope of the fill.

Example 5.9

A retaining wall 5 m high is pushed against a cohesionless backfill. The surface is horizontal. The angle of shearing resistance of the soil is 30° and its unit weight is 17 kN/m³. Calculate the total Rankine passive resistance?

Solution

Coefficient of passive earth pressure \( = \frac{1 + \sin \phi}{1 - \sin \phi} = 3.0 \)
Passive earth pressure = $k_p \gamma H = 3 \times 17 \times 5 = 255 \text{kN/m}$ (Figure 5.23)

Passive thrust = $1/2 \times 255 \times 5 = 637.5 \text{kN/m}$

5.6.2 Cohesive Soil

You have already seen that the relation between the major principal stress $\sigma_1$, and the minor principal stress $\sigma_3$ for a cohesive soil can be written as

$$\sigma_1 = \sigma_3 K_p + 2c \sqrt{K_p} \quad \ldots \ (5.39)$$

For the case of passive earth pressure,

$$\sigma_3 = \sigma_v = \gamma z , \quad \sigma_1 = \sigma_n = P_p \quad \ldots \ (5.40)$$

$$P_p = \gamma z K_p + 2c \sqrt{K_p} \quad \ldots \ (5.41)$$

The total pressure earth resistance $P_p$, for the full height $H$ of the retaining wall is given by (Figure 5.24).

$$P_p = \frac{1}{2} \gamma H^2 K_p + 2c \sqrt{K_p} H \quad \ldots \ (5.42)$$

The two components of act $P_p$ at heights of $H/3$ and $H/2$ respectively from the base of the wall.

Example 5.10

A retaining wall 5 m high is pushed against a cohesive back fill. The uniform surcharge on the level backfill is 40 kN/m². The cohesive strength of soil 30 kN/m² and angle of shearing resistance is 20°. The unit weight of soil is 20 kN/m³. Determine the total Rankine passive thrust and its point of application.
Solution

\[ K_p = \frac{1 + \sin 20}{1 - \sin 20} = \frac{1 - 0.3420}{1 + 0.3420} = \frac{1.342}{0.658} = 2.04 \]

At any depth \( z \), passive earth pressure

\[ P_p = K_p \gamma z + K_p q + 2 c \sqrt{K_p} \]

at \( z = 0 \),

\[ = 2.04 \times 40 + 2 \times 20 \times \sqrt{2.04} = 81.60 + 57.13 = 138.73 \text{kN/m}^2 \]

at \( z = 5 \text{ m} \),

\[ = 2.04 \times 20 \times 5 + 81.60 + 57.13 = 342.73 \text{kN/m}^2 \]

The passive earth pressure diagram is shown in Figure 5.25.

The total passive thrust is

\[ P_p = \frac{1}{2} \times 204 \times 5 + 81.6 \times 5 + 57.13 \times 5 = 510 + 408 + 285.7 = 1203.65 \text{kN/m}^2 \]

The point of application can be determined by taking moments about the base

\[ 1203.65 \times y = 510 \times 5/3 + 408 \times 5/2 + 285.7 \times 5/2 \]

\[ = 850 + 1620 + 714.25 = 2584.25 \]

\[ y = \frac{2584.25}{1203.65} = 2.15 \text{ m} \]

The resultant acts horizontally at a height of 2.15 m from the base Figure 5.25.

5.7 LIMITATIONS OF RANKINE’S THEORY

Generally the movement of the retaining wall by the rotation of the wall about its base will not be able to produce the state of plastic equilibrium in the entire backfill. The active

minimum strain \( \delta l/l \) is constant at every depth

(a) Active Case  (b) Passive Case

Figure 5.26: Acting Pressure Cohesive Soil
plastic equilibrium will be developed in the wedge of soil between the wall and a failure plane passing through the heel of the wall at an angle of $45^\circ + \phi/2$ to the horizontal (Figure 5.26). Within this wedge the strain is constant and is of magnitude sufficient to produce the active Rankine state. The rest of the backfill will be in elastic equilibrium. The active pressure distribution along the back of the wall however, is the same for the local state of plastic equilibrium as for the general Rankine state. The wedge of the soil in the passive state will have its failure plane at $45^\circ - \phi/2$ to the horizontal. Here also the rotation of the wall about the base should create the minimum deformation about the base.

Rankine's theory assumes the back of wall to be smooth. No frictional forces are assumed to exist between the soil and the wall. Hence the lateral pressure is assumed to act parallel to the surface of the backfill. But in practice, considerable friction will be developed between the soil and the wall due to the movement of the wall. As a consequence earth pressure will be inclined at a certain angle to the normal to the wall. The assumption of a smooth wall surface results in an overestimation of active earth pressure and an underestimation of passive earth pressure. The error however is on the safe side.

In cases of irregular backfill surface and complex surcharge loads, Rankine's theory is difficult to use.

**SAQ 3**

i) A 5 m high gravity retaining wall is designed on the basis of dry backfill active earth pressure with a factor of safety of 1.5 against sliding. Check whether the wall will be safe in the following conditions.

   a) the ground water level rises to the top of the backfill and there is no drainage.
   b) the retaining wall remains rigid and does not deform.

ii) How do you prevent the conditions stated in problem 1 occurring during the life of the wall?

### 5.8 COULOMB'S THEORY OF EARTH PRESSURE

Instead of considering the equilibrium of an element in a stressed mass, Coulomb's theory consider a sliding wedge which tends to break away from the rest of the backfill when the wall moves. A cohesionless soil will slump down to its angle of shearing resistance $\phi$ on the plane BD, if the wall is suddenly removed (Figure 5.27). If the wall is moved slightly, rupture plane BC would develop somewhere between AB and BD. The wedge of soil ABC would then move down the back of the wall AB and along the rupture plane BC. If the wall is pushed toward the soil, the sliding wedge would move inwards and upwards. Coulomb's theory takes into account the friction between the wall and the soil in its analysis. The assumptions in the Coulomb's analysis are as follows:

i) The backfill soil is dry, cohesionless, homogeneous and isotropic
ii) The failure surface is a plane surface which passes through the heel of the wall. Note that the actual failure surface will be curved since wall friction is considered. The error introduced by this assumption in the evaluation of active pressure is of small magnitude. However, in the case of passive pressure, error will be significant when the angle of wall friction $\phi$ exceeds $\phi/3$.

iii) The sliding wedge is considered to be a rigid body.

5.8.1 Cohesionless Backfill

i) Active Earth Pressure

The three forces keeping the wedge ABC in equilibrium are the weight of the wedge ABC, the soil reaction $R$ on the failure plane, and the reaction to the active earth force $P_a$ between the wall and the soil. The forces acting on the wall are shown in Figure 5.28(a). The earth pressure reaction acts at an angle $\phi$ below the normal to the back of the wall. The direction of soil reaction $R$ at failure is at an angle $\phi$ measured below the normal to the failure plane to oppose the downward movement of the wedge. The triangle of forces is shown in Figure 5.28(b). Since the magnitude of $W$ is known, the other forces can be determined. The magnitude of $P_a$ is thus known. The critical failure plane is the one for which the soil reaction is the maximum. The wall must resist the maximum lateral force before it moves out. The value of $P_a$ (max) is the lateral active earth pressure.

The coefficient of active earth pressure for the case shown in Figure 5.28 is

$$K_a = \frac{\sec \theta \cos (\phi - \theta)}{\sqrt{\cos (\theta + \delta) + 1 \frac{\sin (\theta - \delta) \sin (\phi - \delta)}{\cos (\beta - \theta)}}}$$

where, $\theta =$ angle of back of wall to the vertical

$\sigma =$ angle of wall friction

$\beta =$ angle of inclination of surface of retained soil to the horizontal

$\phi =$ angle of shearing resistance of the soil

when $\sigma = 0$ and $\sigma = \beta$, $K_a$ reduces to the Rankine's equation

$$K_a = \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

when $\theta = 0$ and $\delta = \beta = 0$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

The value of angle of wall friction varies from $2/3 \phi$ to $3/4 \phi$.

Table 5.2 gives typical values of $K_a$. It is seen that an increase in $\delta$ causes a decrease in $K_a$ but the increase is of the order of 5 to 10 percent only. But the more
significant factor in inclusion of friction is in changing the line of action of earth pressure. This will reduce the overturning moment and the lateral sliding force.

**SAQ 4**

Show that in the case of a retaining wall with a smooth vertical back retaining a level cohesionless soil with a failure wedge making an angle of \(45 + \frac{\phi}{2}\) to the horizontal, Coulomb’s theory gives the same magnitude of lateral active earth thrust as the Rankine’s theory.

**ii) Culmann’s Graphical Procedure**

When the surface of the retained soil is irregular, it is simpler to use the graphical method proposed by Culmann in 1866. The procedure is versatile and can deal with irregular soil surfaces and with irregular combinations of uniform and line loads.

The procedure is to select a series of trial wedges and find one that exerts the greatest thrust on the wall. A wedge is acted upon by the three forces:

- \(W\) – the weight of the wedge
- \(P_a\) – the reaction from the wall
- \(R\) – the reaction on the plane of failure.

At failure the reaction on the failure plane will be inclined at maximum obliquity, \(\phi\) to the normal to the plane. If the angle of wall friction is \(\delta\) then the reaction from the wall will be inclined at \(\delta\) to the normal to the wall. Please note that \(\delta\) cannot be greater than \(\phi\). As active pressures are being developed the wedge is tending to move downwards and consequently \(R\) and \(P_a\) will be on the downward side of the normals (Figure 5.29). \(W\) is of known magnitude (area \(ABD \times \text{unit weight}\)) and acts vertically downwards. The line of action of \(R\) and \(P_a\) are known. Hence the magnitudes of \(R\) and \(P_a\) can be determined by constructing the triangle of forces.

**Figure 5.29: Culmann Construction – Active Case Cohesionless Backfill**

In Figure 5.29, the total thrust on the wall due to earth pressure is to be evaluated. Four trial wedges have been selected with failure surfaces BC, BD, BE and BF. At some point along each failure surface, a line normal to it is drawn. Then a second line is constructed at \(\phi\) to the normal. The resulting four lines give the lines of action of the reactions on each of
the trial planes of failure. The direction of wall reaction is similarly obtained by drawing a line normal to the wall and then another line at an angle \( \delta \) to it.

The weight of each trial slice is next obtained. Starting at a point \( X \) these weights are set off vertically upwards as points \( d_1, d_2, d_3 \) etc. \( X_d \) represents the slice \( ABC, X_d \) the weight of slice \( ABD \) etc.

A separate triangle of forces is now completed for each of the four wedges. The directions of the corresponding reaction on the failure plane and of \( P_a \) are obtained from space diagram. The point of intersection of \( R \) and \( P_a \) are given the symbol \( e \) with suffix that tallies with the wedge analysed (i.e.) the point \( e \), represents the intersection of \( P_{a1} \) and \( R_1 \).

The maximum thrust on the wall is represented by the maximum value of length \( ed \). To obtain this length a smooth curve (the Culmann line) is drawn through points \( e_1, e_2, e_3, \) and \( e_4 \). A tangent to the Culmann line parallel to \( x d \) will cut the line at point \( e \); hence the line \( ed \) can be drawn on the force diagram and the length \( ed \) represents the thrust on the back of the wall due to the soil.

The position of the actual failure plane can be plotted on the space diagram. The angle \( e_1, e_2, e_3, \) equals angle \( EBD \) on the space diagram. Similarly angle \( e_1, e_2, e_3, e_4 \) equals the angle \( GBD \) when \( BG \) is the failure plane.

iii) **Point of Application of Active Thrust**

The total active thrust \( P_a \) on the wall for both Rankine and Coulomb analytical methods is given by the expression

\[
P_a = \frac{1}{2} \gamma H^2 K_a
\]

where, \( K_a = \) coefficient of active earth pressure

\( \gamma = \) unit weight of retained soil

![Figure 5.30: Point of Application of Active Thrust - Rankine Theory](image)

The point of application of \( P_a \) on the back of the wall is largely indeterminate. Locations suitable for design purposes when Rankine’s theory is used are given in Figure 5.30.

![Figure 5.31: Point of Application of Active Thrust Coulomb Theory](image)
These are based on the assumption of triangular distribution of pressure obtained by Rankine's theory. For most practical purposes these locations of $P_a$ can also be used in conjunction with values obtained from a Coulomb analytical solution.

When Coulomb line construction is used, the magnitude of can be obtained directly from the force diagram. Its point of application may be assumed to be where a line drawn through the centroid of the failure wedge and parallel to the failure plane intersects the back of the wall (Figure 5.31).

Example 5.11

The retaining wall shown in Figure 5.32 retains cohesionless soil having an angle of shearing resistance of $34^\circ$ and unit weight of 18 kN/m$^3$. The angle of wall friction can be taken as $20^\circ$. The backfill is sloping at an angle of $15^\circ$ to the horizontal. Determine the total active lateral thrust due by Culmann's graphical procedure and its point of action.

Solution

Refer to Figure 5.32
Weight of wedges

$\text{ABC} = W_1 = 138.6 \text{ kN} (\times d_1)$

$\text{ABD} = W_2 = 277.2 \text{ kN} (\times d_2)$

$\text{ABF} = W_4 = 554.4 \text{ kN} (\times d_4)$

$\text{ABG} = W_3 = 693.0 \text{ kN} (\times d_3)$

The space and Force diagrams are shown in Figure 5.32.

![Force and space diagrams](image)

**Figure 5.32**

Maximum $P_a = e_1 d_3 = ed = 346.5$ kN/m. A parallel line is drawn through $c$ and $q$ the failure wedge $ABE$ to intersect the back of the wall to determine the line of action. The active earth force $P_a$ acts at an inclination of $0^\circ$ to the normal to the face of the wall as shown.

iv) Surcharges

The extra loading carried by a retaining wall is known as a surcharge and can be a uniform load (roadways stacked goods), a line load (track running parallel to a wall) or isolated load (column footing).

The uniform load can be added to the weight of each slice. The weight of each wedge plus its surcharge is plotted as $x d_{11}', x d_{21}$ etc. and the procedure is as described before. Even when a retaining wall is not intended to support a uniform surcharge, it is to be noted that...
it may be subjected to surface loading due to plant movement during its construction. It is therefore preferable to design the wall for a nominal uniform surcharge of 5 to 10 kN/m².

The weight \( W_f \) due to the line load is simply added to the trial wedges affected by it (Figure 5.29). The Culmann line is first constructed as before ignoring the line load. On this basis the failure plane would be BC and \( P_a \) would have a value \( ed \) to some force scale.

Slip occurring on BC, and all places further from the wall will be due to the wedge weight plus \( W_L \). For plane BC, set off \( (W_f + W_L) \) from \( X \) to \( d_1 \) and continue the construction of Culmann line as before – for every trial wedge to the right of plane BC add \( W_L \) to its weight. The Culmann line jumps from \( e_1 \) to \( e_1^* \) and then continues to follow a similar curve.

The wall thrust is again determined from maximum \( ed \) value by drawing a tangent, the maximum value of \( ed \) being in this case \( e_1 \ d_1 \). If is located far enough back from the wall, \( ed \) may be greater than \( e_1 \ d_1 \); in this case is taken as having no effect on the wall.

Neither Rankine theory nor the Coulomb theory can be adapted for isolated loads. The lateral pressures due to isolated loads have to be solved by elastic theory.

**Example 5.12**

In the Example 5.11, assume that a vertical line load of 50 kN/m is acting at a horizontal distance of 3 m from the crest of the wall. Determine the magnitude of the total active lateral earth thrust.

**Solution**

Refer Figure 5.33

weight of wedges

\[ \begin{align*}
ABC &= 138.2 \text{ kN} (x d_1) \\
ABD &= 277.2 + 50 = 327.2 \text{ kN} (x d_1) \\
ABE &= 415.8 + 50 = 465.8 \text{ kN} (x d_1) \\
ABF &= 554.4 + 50 = 604.4 \text{ kN} (x d_1) \\
ABG &= 693.0 + 50 = 743.0 \text{ kN} (x d_1)
\end{align*} \]

The culmann line with live load are shown in dotted line.

Maximum \( P_a = e_1 \ d_1 = 415.8 \text{ kN/m} \)

**5.8.2 Cohesive Backfill**

Coulomb theory assumes that at the top of the wall there is a zone of soil within which there are no friction or cohesion effects along both the back of the wall and the plane of rupture (Figure 5.33). The depth of zone is taken as \( z_o \).
\[ R \] - reaction of plane of failure
\[ W \] - weight of whale wedge ABED
\[ P \] - resultant thrust on the wall
\[ C_u \] - adhesive force along length BF of wall
\[ C \] - cohesive force along rupture plane BE.

Note that unit wall adhesion \( C_a \) cannot be greater than \( C_a \). For soils with a cohesion value greater than 50 kN/m², \( C_u \) should be taken as 50 kN/m². \( C_u \) can be taken as equal to \( C_a \) for \( C_u \) values less than 50 kN/m².

The magnitudes of \( W \), \( C_u \) and \( C \) can be determined as follows:

\[
C = C_u \cdot BE \\
C_w = C_a \cdot BF
\]  

(5.46)

(5.47)

There are only two unknowns \( R \) and \( P_a \) and they can be determined by constructing a polygon of forces. As the force \( C_a \) is common to all wedges it is set off first and the \( C_u \) force is then plotted. The direction of \( P_a \) is drawn from point \( d \) and the direction of \( R \) is drawn from the end of force \( C_a \). The two points intersect at the point \( e \) on the Culmann line.

Analytical solutions with the Coulomb theory are possible but extremely complicated. Hence graphical procedures are generally preferred for stratified Backfill.

### 5.8.3 Stratified Backfill

Culmann’s graphical construction can be used to solve the problems of stratified backfills. Figure 5.34 shows a two layer backfill.

Assume BC as the back of wall retaining a single backfill \( (\gamma_1, \phi_1) \). The earth pressure is determined by the Culmann method. The point of application of \( P_{a1} \) is obtained by drawing a line through centre of gravity of the sliding wedge parallel to the critical plane to intersect the back of the wall at BC. \( P_{a1} \) is drawn at an angle \( \delta \) to the normal to BC.

Next earth pressure \( P_{a2} \) is determined by the Culmann method for a retaining wall of height CA by assuming the lower stratum \( (\gamma_2, \phi_2) \) as a backfill acted upon by a surcharge equal in magnitude to the weight of the upper stratum (Figure 5.34).

![Figure 5.34: Stratified Backfill](image)

The same procedure is continued when the backfill consists of more than the two strata.

### 5.9 CHOICE OF METHOD FOR PREDICTION OF ACTIVE PRESSURE

It should be obvious to you that the assumptions made in Rankine’s theory are not realistic. The wall will never be perfectly smooth and will have some degree of roughness. Hence there will invariably be friction/adhesion developed between the wall and the soil. Hence the assumption that no shear forces develop on the back of the wall is not true. So there is now a general tendency to use the Coulomb theory whenever possible.
You should realise however that it is not easy to obtain measured values of wall friction $\delta$ and of wall adhesion $C_u$. They are usually estimated. The value of $\delta$ is taken to be $1/2 \phi$ to $2/3 \phi$ while the magnitude of $C_u$ is estimated to be $1/2 \sigma$ to $C$. The actual operating value of $\delta$ depends on the amount of relative movement between the soil and the wall. A significant downward movement of the soil relative to the wall will result in the development of maximum $\delta$ value. In the case of gravity and sheet pile walls, there may not be significant downward movement of the soil and a value of $\delta$ less than the maximum can be used. In the case of reinforced concrete cantilever or counterfort wall where the retained soil is supported by a foundation slab, there will be virtually no movement of the soil relative to the wall. Hence Rankine’s theory can be justifiably used.

Rankine method also enables a relatively faster method for determining a conservative value of active pressure which can be useful in preliminary design work.

However the general consensus amongst soil engineers is that for the solution of most earth pressure problems, the Coulomb theory should be used.

### 5.10 DESIGN PARAMETERS FOR DIFFERENT SOIL TYPES

Though the retaining wall performs in a state of plane strain, the parameters obtained from the triaxial shear tests give realistic estimates of active earth pressure. Please note that the strength parameters vary with soil type and drainage conditions.

The appropriate strength parameter for sands and gravels is the angle of shearing resistance, $\phi$. The cohesion is taken as zero.

In soft and normally consolidated clays, the strength of a soil immediately after construction is minimum and the value of active earth pressure exerted on the back of the wall is maximum. After construction and after sufficient time is elapsed the soil will achieve a drained condition. The soil will then be at its greatest strength. The active earth pressure acting on the wall will then be minimum. It is generally advisable to use the undrained strength parameters in earth pressure calculations. $\phi$ is assumed zero and the undrained strength of clay is $C_u$.

In overconsolidated clay negative pore pressures are generated during shear. The stiff clay is strongest immediately after construction and exerts the minimum lateral pressure. The maximum value of active earth pressure will occur when the clay attains a fully drained condition. The effective stress parameters $C$ and $\phi$ are to be used for determining the pressure. It may be safer to assume $C_u = 0$ and estimate the earth pressure based on $\phi$ since $C_u$ values may decrease with time.

Silts can be assumed purely granular with the characteristics of fine sand or purely cohesive with the characteristics of soft clay. When such a classification is not possible, total stress parameters $C$ and $\phi$ should be used for evaluation of earth pressure.

### 5.11 CHOICE OF BACKFILL MATERIAL

The ideal backfill material is granular such as suitably graded stone, gravel, clean sand with small percentage of fines. Such material will be durable, strong and free draining. However it can be expensive even if obtained locally.

Economy can sometimes be achieved by using granular material in retaining wall construction in the form of a wedge as shown in Figure 5.35. The wedge separates the finer material making up the bulk of the backfill material from the back of the wall. The lateral pressures exerted on the wall can then be calculated with the assumption that the backfill is entirely made up of granular material.

Slag, clinker, burnt colliery shale and other manufactured materials similar to granular materials will generally prove satisfactory. However they should not contain any harmful chemicals. Inorganic silts and clays may require special drainage arrangements; shrinkage and swell problems may also be encountered in such materials. Peat, organic soil, chalk, and pulverized fuel ash should be avoided as backfill material.
5.12 BACKFILL DRAINAGE

Drainage of backfill is of great importance in retaining walls. A retaining wall is designed generally to withstand only the lateral pressures exerted by the soil it is supporting. Appropriate drainage system should be designed so that hydrostatic pressure does not develop behind the wall.

Provision of weepholes that go through the wall and are spaced at about 3 m centres both horizontally and vertically is sufficient for granular backfills. The diameter of the holes can vary from 75 mm to 150 mm. They should be protected against clogging by the provision of gravel pockets placed in the backfill immediately behind each weephole (Figure 5.36 (a)).

An alternative arrangement for granular backfill is shown in Figure 5.36 (b). A continuous longitudinal backdrain is placed at the foot of the wall and consists of open jointed pipes packed around with gravel.

If the granular backfill consists more than 5% fine sand or fine grained soil, then it is only semi-Pervious. The provision of weepholes alone may not provide sufficient drainage. Additional drainage may have to be provided in the form of vertical strips of filter material (about 0.33 x 0.33 m) placed between the weepholes and down to a continuous longitudinal strips of the same filter material of the same cross section (Figure 5.36 (c)).

Blanket drains of suitable material are necessary for clayey materials. These blankets should be 0.33 m thick and typical arrangements are as shown in Figure 5.36 (d) and (e). If the surface of the soil can be protected with some form of imperious covering, drainage shown in 5.36 (d) will be sufficient. If such protection cannot be given then there is a
chance of greater seepage pressure created during heavy rain. In such cases the provision of inclined filter as shown in Figure 5.36 (e) will substantially reduce seepage pressures.

SAQ 5
i) Explain why cohesionless soils are preferred as backfills.
ii) Rankine’s theory can be used to estimate lateral earth thrust in cantilever walls. Do you agree? Why?
iii) Why are counterfort walls considered more economical than buttressed walls?

5.13 TYPES OF EARTH RETAINING STRUCTURES

You have already seen that the major types of retaining structures are:

a) Gravity walls
b) Cantilever walls
c) Counterfort walls
d) Buttress walls
e) Crib walls
f) Sheet pile walls
g) Diaphragm walls
h) Reinforced earth walls
i) Anchored earth walls

A brief description of these walls are given in the following section. Design procedures are discussed for gravity and cantilever walls along with examples.

5.13.1 Gravity Walls

This wall depends on its self weight for its stability. It is designed so that the overturning effect of the lateral earth pressure does not induce tensile stresses within the section. This is used for walls of low height and is not economical for large heights. Gravity walls have been built of stone, bricks, mass concrete and precast concrete blocks.

The cross section of the wall is trapezoidal with a base width between 0.3 and 0.5 \( H \), where \( H \) is the height of the wall. The top width varies from 0.2 to 0.3 m. For concrete, a top width of 0.3 m is recommended for proper placement of concrete.

5.13.2 Cantilever Walls

Reinforced concrete cantilever retaining walls are suitable for heights upto 7 m. It has a vertical stem monolithic with the base. The slender sections are possible as the tensile
stresses within the stem and the base are resisted by steel reinforcement. If the face of the wall is to be exposed a small backward batter of about 1 in 50 is provided in order to compensate for any forward tilting of the wall (Figure 5.37).

1) Relieving Platforms

Both shear and bending stresses act in a retaining wall due to lateral pressures induced by the supporting soil. Since the thickness of gravity wall is large, it can resist those stresses. But in a cantilever retaining wall enough steel reinforcement must be provided in the stem to resist the bending moment and it should have sufficient thickness to withstand the shear forces.

It is this situation that imposes a practical height limitation of about 7m on the wall stem of a conventional retaining wall. As the dimensions of the wall are increased, it becomes less flexible. Consequently the lateral pressures exerted on the wall by the soil tend to be higher than the active pressures assumed in the design. A peculiar situation is thus created - if a wall is strengthened to withstand increased lateral pressure, then its rigidity is increased and the lateral pressures are increased.

One of the solutions to the above problem is the provision of one or more horizontal concrete slabs or platforms. The platforms are placed within the backfill and are rigidly connected to the wall stem (Figure 5.38). A platform carries the weight of the material above it (up as far as the next platform if there are more than one). The vertical force exerts a cantilever moment on the back of the wall in the opposite direction to the bending moment caused by the lateral soil pressure. The resulting bending moment diagram becomes a series of steps and the wall is subjected to a maximum bending moment that is considerably less than the value when there are no platforms.

Since the bending moments are reduced to manageable level the stem of the wall can then be kept slim so that assumption of active pressure is realistic. This will also result in more economical construction.

5.13.3 Counterfort Wall

These walls are used for heights greater than about 6.0m. Its wall stem acts as a slab spanning between the counterfort supports. The spacing between supports is about 2/3 H but should not be less than 2.5 m. Details of the walls are given in (Figure 5.39).

A form of counterfort wall is the buttressed wall where the counterforts are built on the face of the wall and not within the backfill. These walls are not very popular because of the exposed buttressed which consume space and spoil the appearance.

5.13.4 Crib Wall

Figure 5.40 show the crib wall. It consists of a series of boxes made from timber, precast concrete or steel members which are filled with granular soils. It acts as a gravity wall with the advantage of quick erection. It can also withstand relatively large displacements due to its flexible nature. It is usually fitted so that its face has a batter of 1 in 6. The width
of the wall varies from 0.5 to 1.0 $H$ and is suitable for walls up to a height of about 7.0 m. Note that the crib wall should not be subjected to surcharge loadings.

5.13.5 Gabion Wall

A gabion wall is built of rectangular metal cages or baskets. They are made from a square grid of steel fabric, generally 5 mm in diameter and spaced 75 mm apart. These baskets are usually 2 m long and 1 m in cross section. A central diaphragm fitted in each metal basket divides it into two equal $1 \times 1$ m sections and adds stability. During construction, the stone filled baskets are secured together with steel wire of 2.5 mm diameter. The base of the gabion wall is about 0.5$H$. A typical wall is illustrated in Figure 5.41. A front face batter can be provided by slightly stepping back each succeeding layer.
5.13.6 Sheet Pile Walls

These walls are made up from a series of interlocking piles individually driven into the foundation soil. Most modern sheet pile walls are made of steel. Sometimes timber or precast concrete sections are also used.

Cantilever sheet pile walls are held in the ground by the active and passive pressures that act on its lower part. (Figure 5.1.b).

Anchored sheet piles walls are fixed at the base and are supported by a row or two rows of ties or struts placed near its top (Figure 5.1.b).

5.13.7 Diaphragm Walls

A diaphragm wall can be classified either as a reinforced concrete wall or sheet pile wall. It consists of a vertical concrete reinforced concrete slab fixed in position. It is held in position by the passive and active pressures acting on its lower portion.

A diaphragm wall is constructed by a machine digging a trench in panels of limited length filled with the bentonite slurry as the digging proceeds to the required depth. In clays there is no penetration of bentonite slurry into the soil. But in sands and silts, bentonite slurry initially penetrates into the soil and creates a virtually, imperious skin of bentonite particles, only a few mm thick, on the sides of the trench. The lateral pressure created by slurry acts on the sides of the short trench panel and prevents its collapse. The required steel reinforcement is lowered into position when excavation is complete. The trench is then filled with concrete by means of a tremie pipe, the displaced slurry being collected for cleaning and further use.

A wall is constructed in alternating short panel lengths. When the concrete has developed sufficient strength, the remaining intermediate panels are excavated and constructed to complete the walls. The various construction stages are shown in Figure 5.42.

![](image)

Figure 5.42: Construction of Diaphragm Wall

5.13.8 Reinforced Earth Walls

The use of reinforcement to strengthen the soil has been known for centuries. Straw has been used to strengthen unburnt bricks and fascine mattresses have been used to strengthen soft soil deposits prior to road construction. The principle of reinforced earth is that a mass of soil can be given tensile strength in a specific direction if lengths of a material capable of carrying tension are embedded within it in the required direction. A rational approach to the design of reinforced earth was presented by Vidal in 1966.

Reinforced earth has been used in many geotechnical applications. In this section we are only concerned with retaining structures.

A reinforced earth wall is a gravity structure. A simple form of such a wall is illustrated in Figure 5.43. The components listed are described below:

The soil fill should be granular and free draining. The reinforcing elements can be either metal strips or geosynthetics. This metallic strips 50-100 mm wide and 3 to 5 mm thick are generally used. Metal grids have also been employed in some cases. Galvanised steel strips are the most common reinforcement. Aluminium alloy, copper and stainless steel are the other metals used. All these materials have a high modulus of elasticity and negligible strains are created within the soil mass.

There have been increasing use of geosynthetics as reinforcement in reinforced earth from 1975. Woven geotextiles and geogrids have the advantage of greater durability than metals.
in corrosive soil. Their tensile strength can approach that of steel. Geogrids can achieve high frictional properties between itself and the surrounding soil. However all the geosynthetics undergo creep deformation under sustained loading which can lead to large strains within the soil mass.

At the boundary of reinforced earth structure it is necessary to provide a facing so that fill is contained. The facing does not contribute to the structural strength of the wall. The facing is usually built up from prefabricated units small and light enough to be handled by manual labour. The most common facing material is precast concrete though steel, aluminium and plastic units have been used. A concrete foundation is required to form a platform from which facing units can be built up.

Reinforced earth can provide a satisfactory method for retaining soil when existing conditions do not allow construction by conventional methods. A compressible soil may be capable of supporting a reinforced earth structure while pile foundation may be required in the case of gravity or cantilever walls. The technique can also be used when there is insufficient land space to construct the sloping side of an earthen embankment.

Please note that in developed countries reinforced earth is often the first choice for design engineers when considering an earth retaining structure.

5.14 CAUSES OF FAILURE IN EARTH RETAINING STRUCTURE

The more common causes of failure of retaining wall are as follows:

i) By a rotational slip failure of the surrounding soil (Figure 5.44 (a)). This can occur in cohesive soils and can be analysed as a slope stability problem.

ii) By bearing capacity failure of the soil beneath the structure (Figure 5.44 (b)). The overturning moment created by the lateral earth thrust causes high bearing pressures at the toe of the wall. These values must be within safe limits.

iii) By overturning about the toe of the wall. The resultant thrust must fall within the base for the wall to be safe against overturning. Generally most walls are so designed that the resultant thrust is within the middle third of the base.

iv) By sliding forward of the wall along the base. This is caused by insufficient base friction or lack of passive resistance in front of the wall (Figure 5.44 (c)).
5.14.1 Bearing Pressure on Soil

The resultant of the forces due to lateral earth thrust and the weight of the wall subject the foundation to both direct and bending stresses.

Let \( R \) be the resultant force on the wall per unit length and let \( R \) be its vertical component (Figure 5.45) considering unit length of the wall.

Section modulus of wall \( Z_w = \frac{1}{6} B^2 \) ...

\[ (5.48) \]

maximum pressure on base = Direct pressure + pressure due to bending

\[ P_{\text{max}} = \frac{R_v}{B X_1} + \frac{6 R_v e}{B^2} \]

\[ (5.49) \]

\[ = \frac{R_v}{B} \left( 1 + \frac{6 e}{B} \right) \]

\[ (5.50) \]

\[ P_{\text{min}} = \frac{R_v}{B} \left( 1 - \frac{6 e}{B} \right) \]

\[ (5.51) \]

When \( R_v \) is on the middle third

\[ P_{\text{max}} = \frac{2 R_v}{B} ; \quad P_{\text{min}} = 0 \]

\[ (5.52) \]

If the resultant \( R \) lies outside the middle third, then

\[ P_{\text{max}} = \frac{2 R_v}{3 x} ; \quad P_{\text{min}} = 0 \]

5.14.2 Factor of Safety for Passive Resistance

The precise determination of passive resistance is difficult. Hence the factor of safety should not be less than 2.5 for clays and should be increased to 3.5 for sands and silts. A conservative approach is to calculate the passive resistance by using Rankine's theory on the assumption that wall friction and adhesion are both equal to zero. In this case a factor of safety of 2.0 is recommended for the calculated passive resistance.

5.14.3 Sliding Resistance of Base

The coefficient of friction \( \mu \) between the base of gravity or reinforced concrete wall and the granular soils or drained clays may be assumed to be

\( \mu = \tan \phi \) for cast-in-situ concrete

\( \mu = 2/3 \tan \phi \) for precast concrete or stone

The resistance to sliding, \( R_s \) is

\[ R_s = \mu R_v \]
where, $R_v = \text{is the vertical reaction on wall base.}$

For undrained clays the adhesion between the supporting soil and the base of gravity or reinforced concrete wall can be obtained from $C_v$ as explained in earlier section. Then the resistance to sliding $R_v$ is

$$R_v = C_v \times \text{area of a base of wall}$$

The factor of safety against sliding should not be less than 2.0.

**Example 5.14**

A gravity wall is to be constructed as shown in Figure 5.46. The wall is to be constructed of masonry whose unit weight is 24 kN/m$^3$. The properties of the backfill material are: Unit weight $\gamma = 17$ kN/m$^3$; angle of shearing resistance $\phi = 33^\circ$; Cohesion $c = 0$. The safe bearing capacity of the soil is 250 kN/m$^2$. The coefficient of friction at the base is 0.60. Check the safety of the wall against sliding and bearing capacity.

**Solution**

Let us first use Rankine’s theory.

Let BC be the vertical surface from heel of the wall to the top of the soil

$BC = 6.0 + 0.6 \tan 20 = 6.218$ m

$$K_a = \cos 20 \frac{\cos 20 - \sqrt{\cos^2 20 - \cos^2 33}}{\cos 20 + \sqrt{\cos^2 20 - \cos^2 33}} = 0.356$$

![Figure 5.46](image)

Thrust on BC = $1/2 K_a \gamma H = 1/2 \times 0.356 \times 17 \times 6.22^2 = 117.0$ N/m

The thrust acts parallel to the backfill.

Horizontal component $P_h = P_v \cos 20 = 110.0$ kN/m

Vertical Component $P_v = P_v \sin 20 = 40.0$ kN/m

<table>
<thead>
<tr>
<th>Part</th>
<th>Vertical Force (kN)</th>
<th>Horizontal Force (kN)</th>
<th>Lever arm from A (m)</th>
<th>Moment about A (kNm)</th>
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<td>0.4</td>
<td>17.3</td>
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<td>2</td>
<td>259.2</td>
<td>-</td>
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<td>3</td>
<td>43.2</td>
<td>-</td>
<td>2.6</td>
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<td>4</td>
<td>31.7</td>
<td>-</td>
<td>2.8</td>
<td>88.8</td>
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<tr>
<td>$P_v$</td>
<td>40.0</td>
<td>-</td>
<td>3.0</td>
<td>120.0</td>
</tr>
<tr>
<td>$P_n$</td>
<td>110.0</td>
<td>2.07</td>
<td>227.7</td>
<td></td>
</tr>
</tbody>
</table>
Overturning Moment = 227.7 kNm
Resisting Moment = 727.2 kNm

\[ \sum V = R_v = 417.3 \text{ kN} ; \quad \sum H = 110.0 \text{ kN} ; \quad \sum M = 509.9 \text{ kNm} \]

Factor of safety against overturning

\[ \frac{727.2}{227.7} = 3.19 \]

If Rv acts at a distance x from B, then

\[ R_v \times x = 499.5 \]

\[ x = \frac{499.5}{417.3} = 1.20 \text{ m} \]

eccentricity of Rv from Centre of foundation = 1.50 - 1.22 = 0.30 m

1/6B = 1/6 \times 3 = 0.50 m

Hence R is well within the middle third of the foundation

Maximum bearing pressure on the soil

\[ P_{\text{max}} = \frac{R_v}{B} \left[ 1 + \frac{6 \times e}{B} \right] \]

\[ = \frac{417.3}{3.0} \left[ 1 + \frac{6 \times 0.30}{3} \right] = 222.6 \text{ kN/m}^2 \]

This is less than 250 kN/m²

Hence acceptable

Factor of safety against sliding

\[ \frac{\mu R_v}{P_n} = \frac{0.60 \times 417.3}{110.0} = 2.28 \text{ Acceptable} \]

Now let us compare the results obtained by Coulomb's theory

Assume angle of friction = 2/3\(\phi\) = 2/3 \times 33 = 22 deg.

Let the total active thrust acts an angle of 20 deg to the horizontal.

\[ K_a = \left[ \frac{\text{cosec 95. sin (95.7 - 33)} \sqrt{\sin (95.7 + 22) + \sin (33+22) \sin (33-20)}}{\sin (95.7 - 20)} \right]^2 \]

\[ = \frac{1.005 \times 0.886}{0.941 + 0.436} = 0.421 \]

\[ P_a = 1/2 \times 0.421 \times 7.0 \times 2 = 128.8 \text{ kN/m} \]

\[ P_n = 128.8 \cos 20 = 121 \text{ kN/m} \]

\[ P_v = 128.8 \sin 20 = 44.1 \text{ kN/m} \]

Weight of wall = 43.2 + 259.2 + 432 = 345.6 kN

Factor of safety against sliding = \[ \frac{0.60 (345.6 + 44.1)}{120.2} = 1.95 \]

The other alternative assumptions are \(\delta = \phi/2 = 16.5^\circ\) and that the resistant acts an angle of 22° to the normal to the back of the wall.

Example 5.15

The dimensions of a cantilever retaining wall are shown in Figure 5.47. The relevant properties of the foundation soil and the backfill material are as follows:

Foundation soil : \(\phi = 36^\circ\); \(c = 0\); \(\gamma = 19 \text{ kN/m}^3\)
Backfill : $\phi = 33^\circ$; $c = 0$; $\gamma = 18$ kN/m$^3$; 
The unit weight of concrete can be taken as 24 kN/m$^3$; 
The coefficient of friction between the base of the wall and the foundation can be assumed to be equal to tan 36. 
Determine the magnitude of factors of safety against: 
i) overturning 
ii) sliding and 
iii) bearing capacity failure assuming the ultimate bearing capacity as 500 kN/m$^2$. 
Use Rankine's theory.

Solution

$$BC = H = 5.5 + 2.5 \tan 25 = 6.67 \text{ m}$$

$$K_a = \frac{\cos 25 - \sqrt{\cos^2 25 \times \cos^2 33}}{\cos 25 - \cos^2 25 \times \cos^2 33}$$

$$= \frac{0.9063 - 0.3435}{0.9063 + 0.3435} \times 0.9063 = 0.408$$

Active earth thrust $P = 1/2 \ K \gamma H^2 = 1/2 \times 0.408 \times 18 \times 6.67^2 = 163.36 \text{ kN/m}$

$$P_n = 163.36 \times \cos 25 = 148.05 \text{ kN/m}$$

$$P_v = 163.36 \times \sin 25 = 69.04 \text{ kN/m}$$

<table>
<thead>
<tr>
<th>Part</th>
<th>Vertical Force (kN)</th>
<th>Horizontal Force (kN)</th>
<th>Leverarm from A (m)</th>
<th>Moment about A (kN m)</th>
</tr>
</thead>
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<tr>
<td>$P$</td>
<td>148.05</td>
<td>2.22</td>
<td></td>
<td>328.7</td>
</tr>
</tbody>
</table>

Overturning moment = 328.7 kNm
Resisting Moment = 1169.8 kNm
\[ \sum v = 436.0 \text{kN} : \sum H = 148.1 \text{kN} : \sum M = 841.1 \text{kNm} \]

i) Factor of safety against overturning

\[
\frac{1169.8}{328.7} = 3.56
\]

ii) Factor of safety against sliding

Resisting Force against sliding \( 436.0 \times \tan 36 = 316.8 \text{kN} \)

Factor of safety \( \frac{316.8}{148.1} = 2.14 \)

iii) Factor of safety against bearing capacity

Let the resultant vertical force \( R_v \) act at a distance \( x \) from \( A = 436.0 \times x = 841.1 \)

\( x = 1.93 \text{ m} \)

Eccentricity \( e = 2 - 1.93 = 0.07 \text{ m} \)

Hence \( R_v \) is well within the middle third of the foundation.

The maximum bearing pressure on the soil

\[
P_{\text{max}} = \frac{R_v}{B} \left( 1 + \frac{6e}{B} \right) = \frac{436.0}{4} \left( 1 + \frac{6 \times 0.07}{4} \right) = 120.4 \text{kN/m}^2
\]

Factor of safety against bearing capacity failure

\[
\frac{500}{120.4} = 4.15
\]

Table 5.2: Typical Values of \( K_s \) for Cohesionless Soils (Coulomb’s Theory)

(Vertical Back And Level Backfill)

<table>
<thead>
<tr>
<th>( K_v )</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0.41</td>
<td>0.33</td>
<td>0.27</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>0.37</td>
<td>0.31</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>20</td>
<td>0.34</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>30</td>
<td>–</td>
<td>0.26</td>
<td>0.21</td>
<td>0.17</td>
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</tbody>
</table>

5.15 SUMMARY

The importance of movement of retaining walls relative to backfill in the development of active, passive and at-rest conditions is delineated. The theoretical background of Rankine and Coulomb theories and estimation of lateral earth force by the two theories are explained in detail. It is shown that the influence of surcharge loads, full and partial submergence of backfill, inclination of surface of backfill and back of wall and tension cracks on the lateral earth force can be conveniently included in the analysis. The merits of different backfill materials and drainage systems are discussed. Different types of retaining walls and their common causes of failure—sliding and bearing capacity are described. The procedures to check the stability of gravity and cantilever retaining walls against sliding and bearing capacity failure are demonstrated. Numerous examples have been worked out to enable the students to understand the theoretical portions.

Self-assessment questions are given so that students can test his grasp of the subject.

5.16 ANSWER TO SAQs

SAQ 1

i) Refer to Figure 5.48
Consider a semi-infinite elastic half space. At any depth $z$

$$\sigma_z = \gamma \varepsilon \quad \sigma_x = \sigma_y = K_o \gamma \varepsilon$$

Also $\varepsilon_x = \varepsilon_y = 0$. This satisfies condition.

Simplifying, $\sigma_z (1 - \mu) = \mu \sigma_z$

or $\frac{\sigma_x}{\sigma_z} = K_0 = \frac{\mu}{1 - \mu}$

3) For water, Poisson's ratio = 0.5

Hence

SAQ 2

i) We can write $\tan^2 (45 - \phi/2) = \frac{\sin^2 (45 - \phi/2)}{\cos^2 (45 - \phi/2)}$

You recollect the trigonometric relations

$$1 - \cos 2A = 2 \sin^2 A$$

$$1 - \cos 2A = 2 \cos^2 A$$

Also $\sin (90 - A) = \cos A$

Hence, $2 \sin^2 (45 - \phi/2) = 1 - \cos (90 - \phi)$

$$2 \cos^2 (45 - \phi/2) = 1 - \cos (90 - \phi)$$

$$\tan^2 (45 - \phi/2) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

ii) For $K_o = \tan^2 (45 - \phi/2) = \frac{1}{3} \quad K_o = 1 - \sin \phi = 0.5$

$$K_p = \tan^2 (45 + \phi/2) = 3$$

iii) For water and soft clay $\phi = 0$.

Hence $K_h = K_p = 1.0$

SAQ 3

i) Let us assume suitable values as given in Examples 5.1 and 5.5

$\phi = 30^\circ$; $\gamma_{dry} = 16$ kN/m$^3$; $\gamma_{sub} = 10.2$ kN/m$^3$ $K_s = 1/3$

When sand is dry, active lateral earth force

$$P_a = 1/3 \times 1/2 \times 16 \times 25 = 66.7 \text{ kN/m}$$
When water level rises to the ground level and there is no drainage, the total lateral force is

\[ P_a = \frac{1}{3} \times \frac{1}{2} \times 10.2 \times 25 + \frac{1}{2} \times 9.5 \times 25 = 165.0 \text{ kN/m} \]

You can clearly observe that the lateral force has increased by a factor of about 2.4. This is greater than the factor of safety of 1.5. So the wall will fail.

ii) If the wall remains rigid instead of rotating about the base, earth pressure at the rest conditions prevail. So to calculate lateral earth pressure \( K_o \) have to be used. If \( \phi = 30^\circ \), \( K_o = 0.5 \) and \( K_s = 0.33 \). The lateral earth force will increase by a factor of 1.5. Hence the present factor of safety will be 1.0

The wall is unsafe.

iii) a) Suitable arrangements for drainage have to be provided. You will study about different systems of drainage in section 5.12

b) You have to ascertain what are the movements of the wall before designing the structure. Appropriate coefficients have to be used.

SAQ 4

1. Refer Figure 5.49

[Diagram of a failure plane, force diagram, and space diagram]

\[ L = \tan (45 - \phi/2) \]
\[ W = \frac{\gamma H^2}{2} \tan (45 - \phi/2) \]

Applying the same rule in the force diagram

\[ \frac{W}{\sin (45 + \phi/2)} = \frac{P_a}{\sin (45 + \phi/2)} \]
\[ P_a = W \tan (45 - \phi/2) \]
\[ = \frac{1}{2} \tan^2 (45 - \phi/2) \gamma H^2 \]
\[ = \frac{1}{2} K_s \gamma H^2 \]

Please refer to sub-section 5.5.1, Equation 5.15. This is the same result obtained by Rankine’s method.

SAQ 5

i) Read section 5.11.

ii) Refer Figure 5.50
In a cantilever retaining wall, the failure plane passes through the heel of the wall as shown. Hence Rankine's theory is a realistic assumption.

iii) The buttress is constructed at the face of the wall while the counterfort is constructed at the back of the wall. Hence buttress is structurally more efficient. But in actual practice counterfort is preferred since buttress occupies valuable space in front of the wall. Generally buttress is constructed of masonry rather than reinforced concrete.