UNIT 5 BASIC SYSTEM CONCEPTS

Structure

5.1 INTRODUCTION

We had so far dealt with the steady state analysis of electrical circuits excited by d.c. or sinusoidal sources. In this Unit, we wish to familiarise you with certain common concepts, terminologies and techniques that are used in different branches of Engineering. All these come under the umbrella of what is now widely known as *System Theory*. Though the study of system theory was first perhaps taken up by Electrical Engineers, it is now being used as well by those belonging to other disciplines like mechanical, chemical or aerospace engineering.

A system is said to be under steady state when the variables associated with it are either constant or vary periodically in time. Voltages and currents constitute these variables in the case of an electrical circuit, which can be regarded as a particular type of a general system. When a switch is operated, altering the circuit parameters/configuration or when an excitation function undergoes a change, the circuit moves from one steady state to another. In the intervening time, called the transient interval, the circuit variables are neither constant nor periodic functions of time. Study of transients is often important in certain applications. While being introduced to system concepts, you will also learn in this unit, the basic characteristics of the transient behaviour of systems and circuits.

You will also be introduced to certain important system related concepts like system function, frequency response function, block diagram representation and feedback. You will also learn the basic principles of feedback control systems.

Objectives

At the end of this unit, you should be able to

- describe what constitutes a system and classify systems as static or dynamic, linear or nonlinear, time varying or time invariant,
- form the input-output equations of simple single input-single output systems,

- distinguish between natural response and forced response and between transient response and steady state response,
- explain the meaning of and formulate system function, frequency response function, and characteristic equation of a given simple system,
- set up block diagram representation of systems and determine the overall system function of a configuration of several blocks,
- explain the concept of feedback and arrive at the block diagram of a system with feedback,
- distinguish between positive and negative feedback and appreciate the advantages of negative feedback, and
- understand the working of a simple 'control system' and evaluate transfer functions of simple feedback control systems.

5.2 SYSTEM CHARACTERISTICS

5.2.1 What is a System ?

A system may be regarded as a set of components put together to serve a specific purpose. Electric circuits constitute a particular category of systems, where the variables are all electrical, viz., voltages and currents. A general system is characterised by variables of different kinds. For instance in a flour mill viewed as a system, the variables of interest may be the current, voltage, load torque and speed of the driving motor, the rate of production of flour, etc.

A system is often characterised by its input-output relationship. Let us take a spring balance. With one end of the spring fixed, a load applied to the other end causes the free end to undergo a linear displacement. We have an *input* which is a force and an *output* which is a displacement. Suppose the spring is enclosed in a black box and we have access only to its free end where the force is applied and displacement measured. We would not know that a spring is inside. But we know that there is a device which has the property of providing a displacement varying linearly with the *input* force. We would be able to represent the *unknown* spring inside a box as given in Figure 5.1, with the force as input coming into the box from the one end and displacement going out as *output* at the other end.

Fig. 5.1(a): Representation of a linear spring as a 'black-box'

What is inside the box is called a *system*, which can now be diagrammatically represented as in Figure 5.1(b).

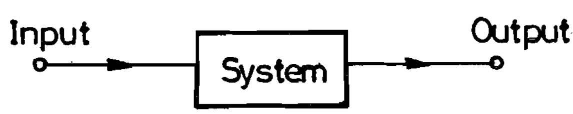


Fig. 5.1(b): General representation of a system

Example 5.1

An ideal voltage source $v(t)$ is applied to a pure resistor R and the current $i(t)$ is measured. View this as a system and draw the system-representation.

Solution

The input is voltage and output is current. Hence the desired representation is as in Figure 5.2.

The simple examples of spring and pure resistor are chosen just to bring home the method of representing physical devices as systems. In actual practice, a system can be

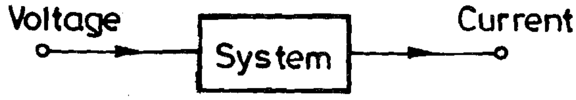


Fig. 5.2 for solution to Example 5.1

quite complex. There could also be systems as shown in Figure 5.3, where there are *multiple inputs* and *multiple outputs.* A rocket, a power station, a cement plant can all be regarded as systems, each with appropriately defined inputs and outputs.

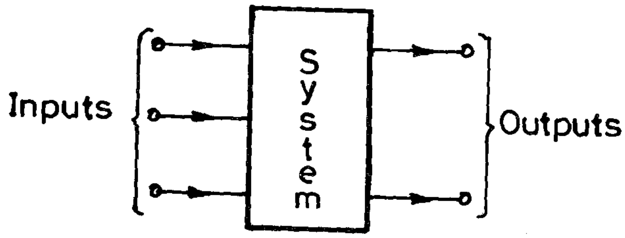


Fig. 5.3: Representation of a system with three inputs and two outputs.

An automobile can be looked upon as a system with three inputs, namely, the accelerator pedal position, the brake pedal position, and the steering wheel angular position. The outputs could be speed and direction of motion of the vehicle. We shall confine our study here to *single-input, single-output systems.*

The input is usually represented by $x(t)$ and the output by $y(t)$, where x and y are functions of time. $x(t)$ and $y(t)$ are also referred to as *excitation* and *response* of the system. The behaviour of the system is characterised by a mathematical equation relating $y(t)$ to $x(t)$. For example, in the case of the spring, $x(t)$ is the force and $y(t)$ is the displacement of the free end. The '*system equation*' or '*the input-output relation*' is

$$y(t) = Kx(t), \tag{5.1}$$

where K is the spring-compliance.

Example 5.2

Identify the input and output for the case of the pure resistor considered earlier and write down the system equation.

Solution

The input $x(t)$ is the voltage and the output $y(t)$ is the current. The system equation is

$$y(t) = \frac{1}{R} x(t) \tag{5.2}$$

where R is the resistance-value.

In system studies, we idealise the internal elements constituting the system and arrive at its characteristic in terms of its input-output equation. You will recall that a similar method of idealisation is adopted in circuit theory, where devices are represented by suitable interconnection of idealised circuit elements.

5.2.2 Classification of Systems

In the case of both the spring and the resistor, the input-output relationship is described by an algebraic equation. Such systems are called **static systems.** On the other hand, let us consider as our system, a mass M (vide Figure 5.4) to which the input is an applied force $f(t)$. If the velocity u is regarded as output, we have,

$$f(t) = M \frac{du}{dt}$$

$$u = \frac{1}{M} \int f(t)\, dt$$

With the convention of adopting the symbol x for input and y for output, the relevant equation is

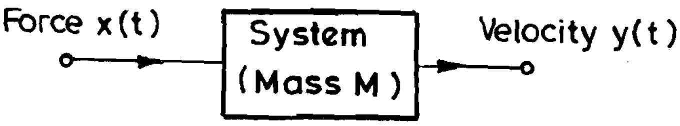


Fig. 5.4: A mechanical system constituted by a mass M

$$y = \frac{1}{M}\int x\,dt \tag{5.3}$$

Similarly, if we consider a pure inductor L as a system, with voltage as input and current as output, the input-output equation is:

$$\frac{di}{dt} = \frac{1}{L}v$$

or in system-theoretic notation, the relationship is

$$\frac{dy}{dt} = \frac{1}{L}x \tag{5.4}$$

Note that we are expressing the relation between v and i in the inductor in a general situation and not under steady state d.c. or sinusoidal regime.

The output-input relationship is now a differential equation. Systems of this type are called **dynamic systems.** The differential equation in such a case is called the system differential equation or system equation. For a more complex system, the system equation could be

$$b_2\frac{d^2y}{dt^2} + b_1\frac{dy}{dt} + b_o y = x(t) \tag{5.5}$$

The order of the system is the order of the differential equation, which is the order of the highest derivative of the output in the latter. In this case it is 2. The coefficients b_2, b_1, b_0 in Eq. (5.5) would depend on the physical parameters inside the system. If the system differential equation is linear, the system is also said to be linear.

Example 5.3

Considering the equation

$$5t\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3t^2y = x(t)$$

illustrate that a superposition principle similar to the one formulated in Section 2.6.1 for circuits is valid for a linear system.

Solution

Let x_1 and x_2 be two inputs and y_1 and y_2 the corresponding outputs. The following equations are valid.

$$5t\frac{d^2y_1}{dt^2} + 4\frac{dy_1}{dt} + 3t^2y_1 = x_1 \tag{5.6}$$

$$5t\frac{d^2y_2}{dt^2} + 4\frac{dy_2}{dt} + 3t^2y_2 = x_2 \tag{5.7}$$

Adding the two equations and rewriting, we have

$$5t\frac{d^2}{dt^2}(y_1 + y_2) + 4\frac{d}{dt}(y_1 + y_2) + 3t^2(y_1 + y_2) = (x_1 + x_2) \tag{5.8}$$

From Eq. (5.8), we see that an input $(x_1 + x_2)$ applied to the system causes an output $(y_1 + y_2)$. Hence, the principle of superposition as learnt in Section 2.6.1 for electrical circuits is found to be valid for more general linear systems as well.

If in the system equation Eq. (5.5), the coefficients b_o, b_1, b_2 ($b_0 \ldots b_n$, for a system of order 'n') are not functions of t or x, then the system is called a **linear time-invariant system.** If they are functions of t, the system is time-varying.

The examples of systems we considered in the earlier section and at the beginningof this section are simple ones, constituted by a single element. A slightly more complex system may involve interconnection of two or more elements. Consider the electrical example shown in Figure 5.5 where the output is the voltage v_o across R and input is the voltage v_i.

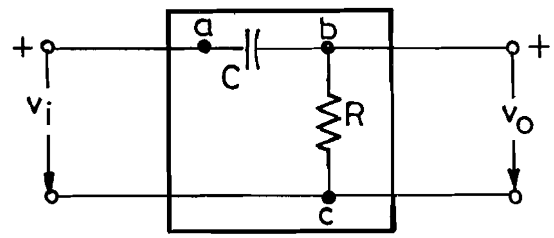


Fig. 5.5: Example of an electrical system with interconnection of elements shown.

Example 5.4

Using the node method of circuit-analysis, show that for the circuit of Figure 5.5, v_o and v_i are related by the differential equation

$$RC\frac{dv_o}{dt} + v_o = RC\frac{dv_i}{dt}$$

Solution

With the node c as reference,

$v_a = v_i$

$v_b = v_o$

Writing KCL at node b, we have

$i_{bc} + i_{ba} = 0$

We should like to express these currents in terms of v_o and v_i, keeping in mind that we are treating a general situation and not steady state d.c. or a.c. conditions. Accordingly we have

$$i_{bc} = \frac{v_o}{R} \text{ and } i_{ba} = C\frac{d}{dt}(v_o - v_i)$$

Thus, $$\frac{1}{R}v_o + C\frac{d}{dt}(v_o - v_i) = 0$$

Rearranging the terms, we get,

$$\frac{dv_o}{dt} + \frac{v_o}{RC} = \frac{dv_i}{dt} \qquad (5.9)$$

If we represent the circuit arrangement of Figure 5.5 as a single- input, single-output system as in Figure 5.6, v_i has the status of the input $x(t)$ and v_o would be the output $y(t)$.

The system differential equation would be

$$b_1\frac{dy_o}{dt} + b_o y = a_1\frac{dx}{dt},$$

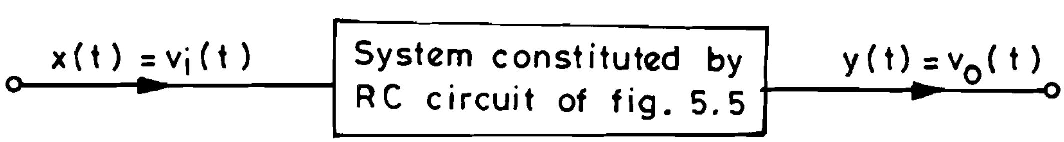


Fig. 5.6: Input and output of the electrical system of Fig. 5.5

where $$b_1 = 1, b_o = \frac{1}{RC} \text{ and } a_1 = 1 \qquad (5.10)$$

A general form of *system differential equation* could be

$$b_n \frac{d^n y}{dt_n} + b_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + b_1 \frac{dy}{dt} + b_0 y = a_m \frac{d^m x}{dt^m} + a_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \cdots a_1 \frac{dx}{dt} + a_0 x \qquad (5.11)$$

As already indicated, the order of the differential equation, n in this case, would be referred to as the *order of the system*. The order of the electrical system of Example 5.4 is 1.

Example 5.5

What is the order of the system whose differential equation is

$$5 \frac{d^3 y}{dt^3} + 4 \frac{dy}{dt} = 3 \frac{d^2 x}{dt} + 2 \frac{dx}{dt} + 6x \, ?$$

Solution

Since the highest order output-derivative-term is $5 \frac{d^3 y}{dt^3}$, the order of the system is 3.

SAQ 1

What is the order of the system represented in Figure 5.1(a)?

5.2.3 Natural and Forced Response

The spring considered at the beginning of Section 5.2 is an element which is capable of storing energy. The inductor and capacitor are energy-storage elements in an electrical system. A system may contain a large number of energy-storage elements. Instead of *forcing* an input x(t) into the system, we may put some initial energy in the energy-storage elements of the system and leave the system *to itself* or in its "natural" state. Since the input $x(t)$ is equal to zero, the system equation Eq. (5.11) under these conditions would be

$$b_n \frac{d^n y}{dt^n} + b_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + b_1 \frac{dy}{dt} + b_o y = 0. \qquad (5.12)$$

The solution of this homogeneous differential equation will depend on the value of y and its first $(n-1)$ derivatives at some instant, usually prescribed at $t = 0$. Taking the example of the system of Figure 5.5, the system differential equation with zero-input would be

$$\frac{dv_o}{dt} + \frac{1}{RC} v_o = 0,$$

whose solution is of the form $A\, e^{-t/RC}$, where A depends on the initial conditions prescribed.

The response $y_o(t)$ with zero-input is called the **natural response** or **free response** of the system. It is given by the complementary solution of the system differential equation i.e., solution to the homogeneous differential equation in $y(t)$ obtained by setting $x(t)$ to zero. For a given system, this response is decided by the *initial condition* of the energy storage elements in the system (inductors and capacitors in the electrical system and springs and masses in the mechanical system). In a system which contains only passive elements including some energy dissipating elements (like resistors in an electric circuit), the free response tends to zero as time progresses and hence represents only a transient phenomenon.

If the input $x(t)$ is specified as say

$$x(t) = e^{st} \text{ or } x(t) = X \cos wt ,$$

the particular integral solution $y_f(t)$ of the system differential equation would be called the **forced response** of the system. For a constant or periodic $x(t)$, the forced response also is either a constant or is periodic. In this event, the forced response is also called **steady state response.** The total response $y(t) = y_o(t) + y_f(t)$ i.e., the sum of the natural and the forced responses, is the complete solution to the differential equation.

SAQ 2

Give one example each, of the system equations governing a second order linear, dynamic system which is

a) time-varying

b) time-invariant.

5.3 SYSTEM FUNCTION

5.3.1 The System Function : Forced Response to an Exponential Input

Consider again the system of Figure 5.5 constituted by the RC-combination. The system equation is

$$\frac{dv_o}{dt} + \frac{1}{RC} v_o = \frac{dv_i}{dt},$$

or in our system-theoretic notation,

$$b_1 \frac{dy}{dt} + b_o y = a_1 \frac{dx}{dt} .$$

Using the differential operator 'D', we have:

$$(b_1 D + b_o) \; y = a_1 D x$$

$$y(t) = \left\{ \frac{a_1 D}{b_1 D + b_o} \right\} x(t),$$

where the flower bracket { } signifies that the quantity inside is an operator. In a general case (vide Eq. (5.11)), we have

$$y(t) = \left\{ \frac{a_m D^m + a_{m-1} D^{m-1} + \cdots + a_1 D + a_o}{b_n D^n + b_{n-1} D^{n-1} + \cdots + b_1 D + b_o} \right\} x(t)$$

$$= \left\{ \frac{Nr(D)}{Dr(D)} \right\} x(t), \qquad (5.13)$$

where $Nr(D)$ and $Dr(D)$ are the two *polynomials in operator D*. If the input $x(t)$ is an exponentially varying function e^{st}, we know from principles of differential equations that the particular integral solution would be given by

$$y_p = \frac{Nr(s)}{Dr(s)} \cdot e^{st} . \qquad (5.14)^*$$

* We confine ourselves to cases where $Dr(s) \neq 0$.

The *operation* has become just an algebraic manipulation. Since the particular integral solution of the differential equation is the forced response (Section 5.2.3), we have

$$y_f(t) = \frac{Nr(s)}{Dr(s)} \cdot e^{st}$$

$$= H(s)\ e^{st} \cdot \qquad (5.15)$$

$\frac{Nr(s)}{Dr(s)}$ is denoted by *H(s)* and is called the *system function.*

The reason for assigning this name would be clear to you when you complete the study of this Section.

Example 5.6

Find the system function for the mass shown in Figure 5.4.

Solution

The input-output equation is

$$y = \frac{1}{M} \int x\, dt$$

$$= \left\{\frac{1}{MD}\right\} x$$

$$Nr\ (D) = 1$$

$$Dr\ (D) = MD$$

$$\text{Therefore } H(s) = \frac{Nr(s)}{Dr(s)} = \frac{1}{Ms}.$$

Example 5.7

Find the forced response of the RC-network of Figure 5.5 to an input $v_i = e^{st}$ V. Evaluate the same for $R = 1\ \Omega$, $C = 2F$, $s = \sqrt{3}$.

Solution

$$y(t) = \left\{\frac{D}{D + 1/RC}\right\} x(t)$$

$$Nr(D) = D$$

$$Dr(D) = D + 1/RC$$

$$H(s) = \frac{s}{s + 1/RC} = \frac{RCs}{RCs + 1}$$

$$y_f(t) = \frac{RCs}{RCs + 1}\ e^{st}.$$

Inserting the numerical values, we get

$$y_f(t) = \left(\frac{1 \times 2 \times \sqrt{3}}{1 \times 2 \times \sqrt{3} + 1}\right) e^{\sqrt{3}t} = \left(\frac{2\sqrt{3}}{1 + 2\sqrt{3}}\right) e^{\sqrt{3}t}.$$

To ensure dimensional consistency, *st* should be dimensionless. *s* has therefore the dimensions of $(\text{time})^{-1}$ or units of *per second.* To prevent possible confusion, we avoided mentioning the unit of *s* in this Example.

5.3.2 Frequency Response

Looking at the forced response of Eq. (5.11), namely,

$$y_f(t) = \frac{Nr(s)}{Dr(s)} \cdot e^{st},$$

we can see that if $s = jw$, we would get

$$y_f(t) = \frac{Nr(jw)}{Dr(jw)} e^{jwt}$$

$$= H(jw)\, e^{jwt}.$$

Let us again take the example of the RC-network and find the output for $R = 1/2\ \Omega$, $C = 2F$ and $s = j\sqrt{3}$,

i.e.,
$$x(t) = e^{j\sqrt{3}t}.$$

$$y_f(t) = \frac{\frac{1}{2} \times 2(j\sqrt{3})}{\frac{1}{2} \times 2(j\sqrt{3}) + 1} e^{j\sqrt{3}t}$$

$$= \frac{\sqrt{3}}{2} e^{j[(\pi/2) - (\pi/3)]} e^{j\sqrt{3}t} = \frac{\sqrt{3}}{2} e^{j(\sqrt{3}t + \pi/6)}$$

If $x(t) = \cos\sqrt{3}t$, i.e., a sinusoid of unit amplitude and radian frequency $\sqrt{3}$ rad / s, we have

$\cos\sqrt{3}t$ = Real part of $(e^{j\sqrt{3}t})$. Then,

$$y_f(t) = \text{Real part of} \left[(\sqrt{3}/2)\, e^{j(\sqrt{3}t + \pi/6)}\right]$$

$$= (\sqrt{3}/2) \cos[\sqrt{3}t + (\pi/6)].$$

The output is sinusoidal, has an amplitude $(\sqrt{3}/2)$ times that of the sinusoidal input, its phase being advanced by an angle of 30° with respect to the input. Notice that $(\sqrt{3}/2)$ is the magnitude of the system function, $H(s)$ evaluated at $s = j\sqrt{3}$ and that $\pi/6$ is its angle at the same value of s.

We also note that for *sinusoidal driving functions, $y_f(t)$, the forced component of the response is also the steady state response $y_{ss}(t)$.*

Taking the general case where the system function is $H(s)$, we have for an $x(t) = \cos wt$,

$$y_{ss}(t) = \text{Re}[H(s)\, e^{st}]\,|_{s = jw}$$

$$= \text{Re}[H(jw)\, e^{jwt}]\cdot$$

$$= |H(jw)| \cos(wt + \varphi),$$ where φ is equal to arg $[H(jw)]$.

The steady state response to an input in the form of a sinusoid of unit amplitude and angular frequency w is thus another sinusoid of amplitude equal to $|H(jw)|$ having a phase angle arg $[H(jw)]$ with respect to the input. In $x(t) = \cos wt$, if w, the frequency of the input is varied keeping the amplitude constant at unity, we can determine and plot the variation of the output amplitude and phase with respect to the input frequency. For the system constructed by the RC-network with numerical values of the elements chosen in Example 5.7, these two curves would be as shown in Figure 5.7. They have been obtained by substituting various values of w in the expression

$$H(jw) = \frac{jwCR}{1 + jwCR} = \frac{j2w}{1 + j2w}$$ and determining $|H(jw)|$ and arg $[H(jw)]$.

Curves of the type of Figure 5.7 plotted for a linear time-invariant system are called **frequency response** curves. The curve showing variation of $|H(jw)|$ is called **amplitude response** and that of arg $[H(jw)]$ is known as **phase response**. The theoretical study of the frequency response curve of a system at the design stage will enable the system-designer to verify how the system would respond to input signals of different frequencies and to identify if unduly large responses which will overload the components are likely to occur at certain input frequencies. In the latter event, he would make appropriate modification in the design. $H(jw)$ is called the **frequency response function**.

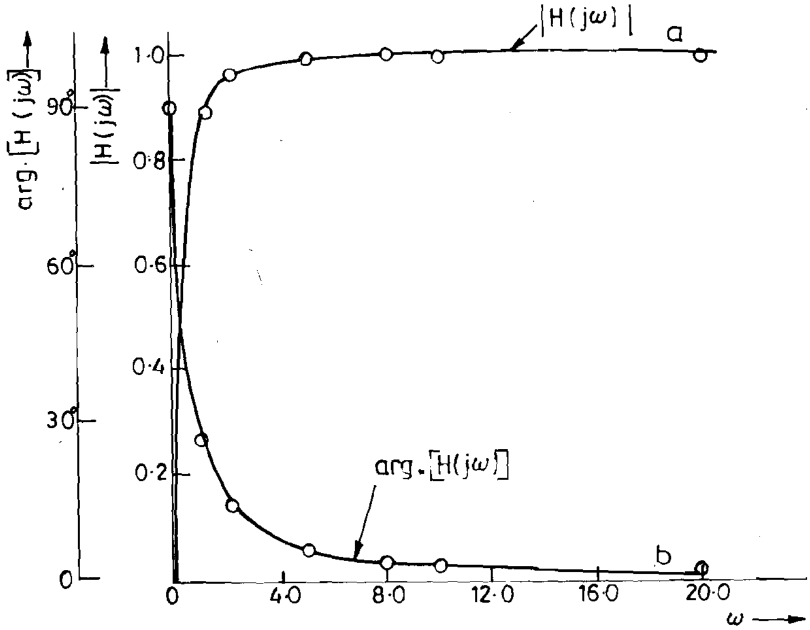


Fig. 5.7 : Frequency response curves of the RC-network of Fig. 5.5 with R = 1 Ω, C = 2 F
(a) Magnitude response (b) Phase response.

SAQ 3

For the range $w = 0$ to 10, plot the frequency response curves of a system for which the system function $H(s) = 1 / (s + 2)$

5.3.3 Step Response

An input $x(t)$ having the time-variation shown in Figure 5.8 is described by the following time-function.

$$x(t) = 0 \text{ for } t < 0$$
$$x(t) = 1 \text{ for } t \geq 0$$

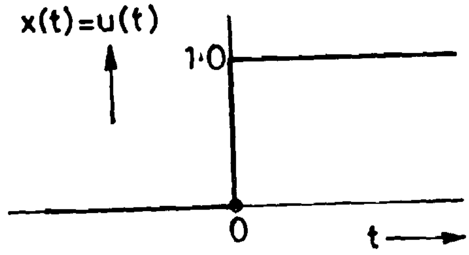


Fig. 5.8: A unit-step function u(t)

This function as you know from Section 1.6.2, is the unit-step $u(t)$. If such an input is fed to a system, the output obtained is referred to as **step-response**. Consider as an example, a first order system having the system differential equation

$$3\frac{dy}{dt} + 2y = x(t). \tag{5.16}$$

If $x(t) = u(t)$, the system equation is

$$3\frac{dy}{dt} + 2y = u(t).$$

The system function

$$H(s) = \frac{1}{3s + 2}\cdot$$

$u(t)$ can be considered as e^{ot} for $t \geq 0$. Hence the forced response

$$y_f(t) = \frac{1}{3 \times 0 + 2} e^{ot} = \frac{1}{2}\cdot$$

The natural response component is got from the solution of the homogenous equation

$$3\frac{dy}{dt} + 2y = 0\cdot$$

$$y_o(t) = A e^{-(2/3)t}.$$

Total response to the step-input is $y(t) = y_f(t) + y_o(t)$

$$= 1/2 + Ae^{-(2/3)t}$$

* If $y(0) = 0, A = -\frac{1}{2}$.

$$y(t) = \frac{1}{2}\left[1 - e^{-(2/3)t}\right]\cdot$$

The total response of an initially relaxed system to a unit-step input is called *step-response*. For the example just considered, the step-response is sketched in Figure 5.9.

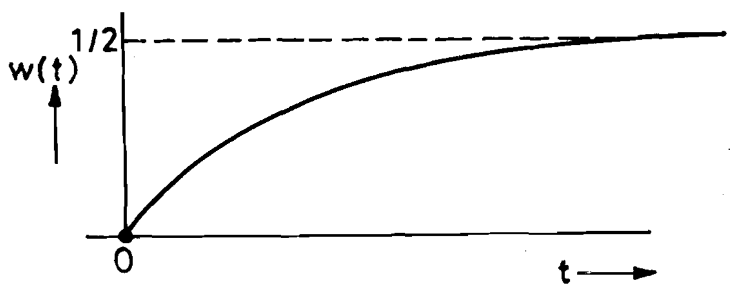


Fig. 5.9 : Step-response $w(t)$ of the system with $H(s) = \frac{1}{s + 2}$

The response increases exponentially and reaches the steady state value as t tends to infinity. As you are by now aware, Eq. (5.16) represents a first-order system and hence the curve of Figure 5.9 is the step-response of a typical first-order system. For a higher-order system, the response to a step-input (with the system being initially relaxed) can have an oscillatory character. This is depicted in Figure 5.10 for a second order system operating under certain conditions.

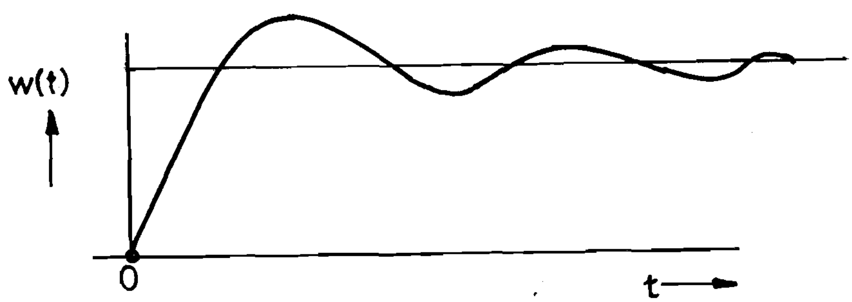


Fig. 5.10: Typical step-response w(t) of a second-order system.

5.3.4 Characteristic Equation

As learnt in Section 5.3.1, the system function has the form

* If $y(0) = 0$, the system is said to be initially relaxed.

$$H(s) = \frac{Nr(s)}{Dr(s)}.$$

The solution to the equation $Dr(s) = 0$ enables us to determine the natural response of the system. **This equation, namely** $b_n s^n + b_{n-1} s^{n-1} \ldots + b_1 s + b_o = 0$ **is also called the characteristic equation of the system.** The coefficients b_o, b_1, ... b_n and the order n depend only on the characteristics of the physical elements that go to make up the system and not on the input functions. The roots of the equation, say, s_1, s_2, ... are called the characteristic roots. From differential equation study, you know that the natural response is of the form

$$y_0(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \ldots \tag{5.17}$$

If s_1, s_2, ... are real and unequal, $\frac{-1}{s_1} = \tau_1$, $\frac{-1}{s_2} = \tau_2$ ... are called the **time-constants** present in the natural response. These represent the rate of decay of the natural response. The smaller a τ_i is, the faster is the decay of the particular component of the response.

Example 5.8

Find the time-constants present in the natural response of a second-order system having a system differential equation

$$2\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 6y = x(t)$$

Solution

$$H(s) = \frac{1}{2s^2 + 7s + 6}$$

The characteristic equation is :

$$2s^2 + 7s + 6 = 0.$$

$$(s + 2)(2s + 3) = 0$$

The characteristic roots are :

$$s_1 = -2$$

$$s_2 = -3/2.$$

The time constants are $\tau_1 = \frac{1}{2}$ sec, $\tau_2 = \frac{2}{3}$ sec.

Suppose s_1 and s_2 of a second-order system are not real. You know from the theory of quadratic equations that s_1 and s_2 will be complex conjugates. As an example, take the equation

$$s^2 + s + 1 = 0.$$

Its roots are :

$$s_{1,2} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}.$$

The natural response here will be of the form

$$y_n(t) = Be^{-(1/2)t} \cos[(\sqrt{3}/2)t + \varphi],$$

where B and φ are constants.

For the case when $s_1$and s_2 are real and equal, the natural response will be of the form

$$y_n(t) = (C + Dt)e^{s_1 t} \tag{5.18}$$

The characteristic equation is arrived at from the parameter values and nature of the elements constituting the system. It gives us complete information on the type of natural response. It is obtained equating to zero the denominator polynomial $Dr(s)$ of the system function $H(s)$. The numerator and denominator polynomials together give us the forced response. Hence the name *system function* is quite appropriate.

SAQ 4

Summarise the properties of the system function $H(s)$.

5.4 BLOCK DIAGRAM

5.4.1 Symbols and Defining Equations

If you look at the Figures 5.1 to 5.4 and 5.6, you notice that each one of them is a rectangular block, containing mostly the word *system* inside. In every case, we ignored all the physical components and devices present inside and viewed it as a system with an input and an output. Since there is a system function $H(s)$ defining the input-output relationship of any linear time-invariant system, it suggests to us that we can place inside the box, the function $H(s)$. Together with the input $x(t)$ and output $y(t)$ the block diagram representation would then be complete. The spring of Figure 5.1 will now be represented as in Figure 5.11.

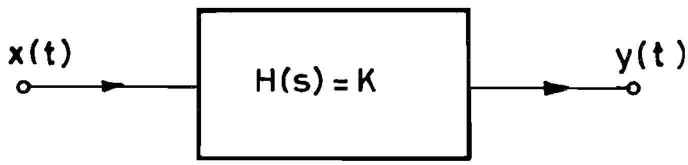


Fig. 5.11 : Complete block-diagram representation of a spring

The electrical system constituted by the *RC*-network of Figure 5.5 will have a block-diagram representation as in Figure 5.12. (Refer to step 4 in the solution of Example 5.7).

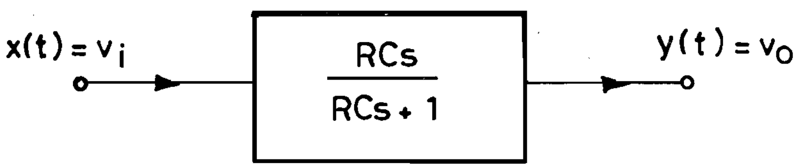


Fig. 5.12 : Block diagram of the RC-network of Fig. 5.5

We shall take one more example.

Consider as a system, the voltage-controlled voltage source discussed in Section 1.3.4. Its input $x(t)$ is the controlling voltage and output $y(t)$ is the dependent voltage source. The block diagram of the system can now take the form as in Figure 5.13.

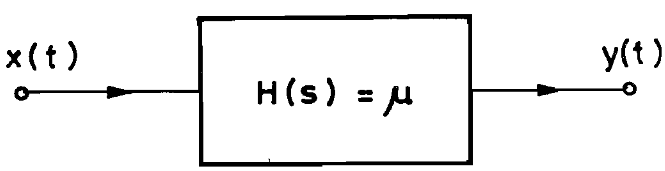


Fig. 5.13: A VCVS represented as a system block

where $H(s) = \mu$ and the system equation is :

$$y(t) = \mu \cdot x(t) \tag{5.19}$$

Example 5.9

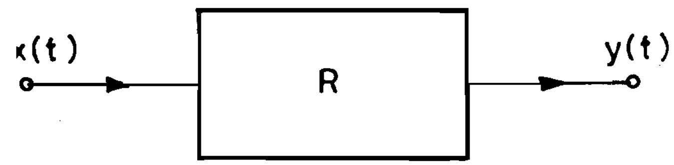
Write down the block-diagram representation of the CCVS viewed as a system.

Solution

For a CCVS, the defining equation is

$v_o(t) = R\ i_i(t)$

Hence the block-diagram is



The system equation is $y(t) = R\,x(t)$.

Notice that the block-diagram of a CCVS is the same as that of a resistor R, which is energised by a current source as input, the voltage across R being taken as output.

SAQ 5

Shown in Figure 5.14 is a mass attached to a fixed wall through a spring of compliance K. Assume that the contact between the mass and the floor offers viscous damping with a damping coefficient B. With the applied force as input and velocity u of the mass as output, arrive at the block-diagram of the system.

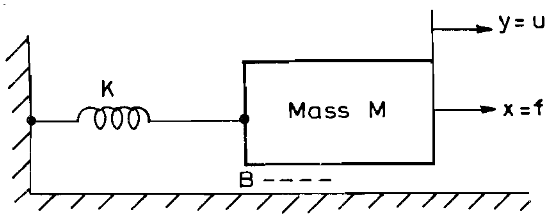


Fig. 5.14 : A translational mechanical system

5.4.2 Interconnection of Blocks

In Section 1.1, you learnt that an electrical network is formed by the interconnection of various electrical circuit elements. In the same way, a complex or big system may involve the interconnection of a number of individual smaller systems. Each small system has a block-diagram and hence the block diagram of the overall big system would involve the interconnection of the blocks of individual systems.

In an electrical network, we have end-nodes for each element. In the block diagram of the system we have the input and output points as the end-points of the block.

Consider the arrangement in Figure 5.15. The block with $H_1(s)$ represents system 1 with its input and output points, and the one with $H_2(s)$ stands for system 2. The output end-point of 1 is connected to the input end-point of system 2. This type of connection is called **cascading** of system 2 to system 1.

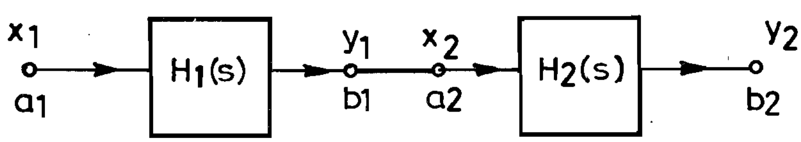


Fig. 5.15 : An important type of interconnection of two systems.

There are two other important types of interconnection employed in the study of systems. You should know about them even at this stage. The operations performed by these interconnections are *summing* and *branching*. The summing operation is carried out in a system at a *summing point* as illustrated in Figures 5.16 (a), (b) and (c).

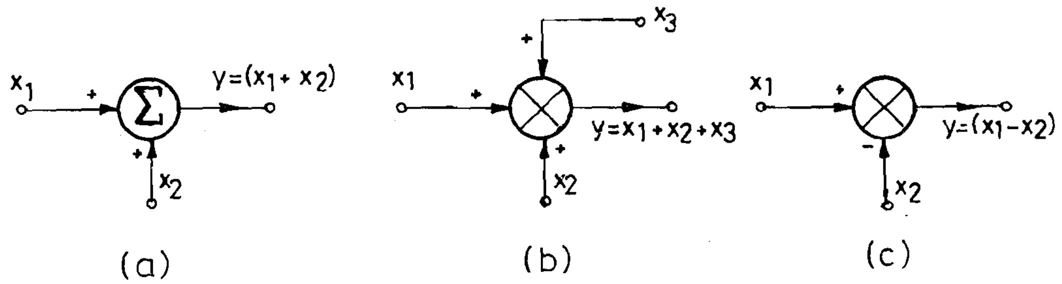


Fig. 5.16: Summing point in a system

The summing point has two or more inputs and one output. It is represented in the form of a circle with either a "Σ" sign (sigma) or a cross inside. Addition or subtraction of the different input signals is performed to yield the output. Thus we have for the Figures 5.16(a) to (c), the relationships

$y = x_1 + x_2$ for Figure 5.16(a),

$y = x_1 + x_2 + x_3$ for Figure 5.16(b), and

$y = x_1 - x_2$ for Figure 5.16(c).

The second important type of connection is called *pick-off* or *branching*. At a pick-off point, the same variable is made available without modification at a number of points or along a number of other *paths*. This point should be clear to you from Figure 5.17.

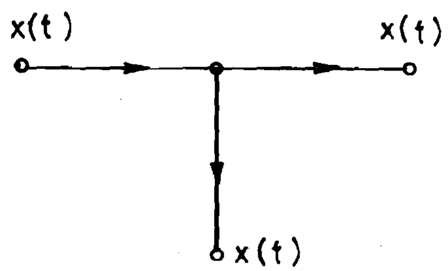


Fig. 5.17: Pick-off point in a system

You should not confuse it with a node in an electrical system and think of applying KCL. If you wish to understand it in terms of system-representation, you may think of it as a single-input, multi-output system, where each of the outputs is equal to the input.

5.4.3 Block Diagram Algebra

If a complex system involves the interconnection of a number of small systems using summing points and pick-off points, we would like to know the input-output relationship or the system function of the overall system in terms of those of the smaller blocks. It is at this stage that we need to go back to the interconnection scheme of Figure 5.15. A simple look tells us that the following relations are valid for excitations of the types e^{st}.

$$y_1 = H_1(s)\,x_1$$

$$x_2 = y_1$$

$$y_2 = H_2(s)\,x_2. \quad (5.20)$$

Simple algebraic substitution yields

$$y_2 = H_1(s) \cdot H_2(s)\,x_1. \quad (5.21)$$

This is an important relation and can be stated as follows:

Let two system-blocks be connected in **cascade** such that the output of the first block forms the input of the second block. If the input to the first block be regarded as the overall input and the output of the second block be considered as the overall output, the net system function is the product of the system functions of the individual blocks. This is represented in Figure 5.18, where the block of Figure 5.18(b) with a system function

$$H(s) = H_1(s) \cdot H_2(s) \tag{5.22}$$

is the equivalent of the two cascaded blocks H_1 and H_2 of Fig. 5.18(a)

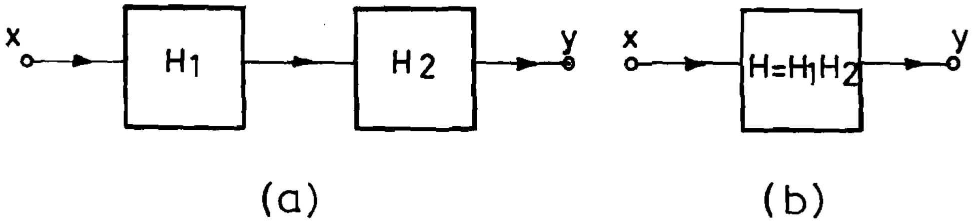


Fig. 5.18 : System function of cascaded blocks.

We are now in a position to find the overall system functions of systems involving the above three interconnections.

Example 5.10

Find the overall system function of the cascaded system of Figure 5.19.

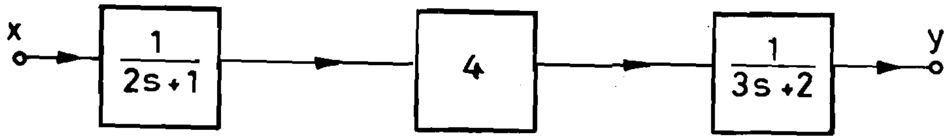


Fig. 5.19: The cascaded system for example 5.10

Solution

The blocks having the system functions $1/(2s + 1)$ and 4 are equivalent to a single block whose system function is $4/(2s + 1)$. This is once again cascaded to a block $1/(3s + 2)$. Hence the overall system function is

$$H(s) = \frac{4}{(2s+1)\ (3s+2)}.$$

SAQ 6

Find the system function $H(s)$ for the interconnected system shown in Figure 5.20.

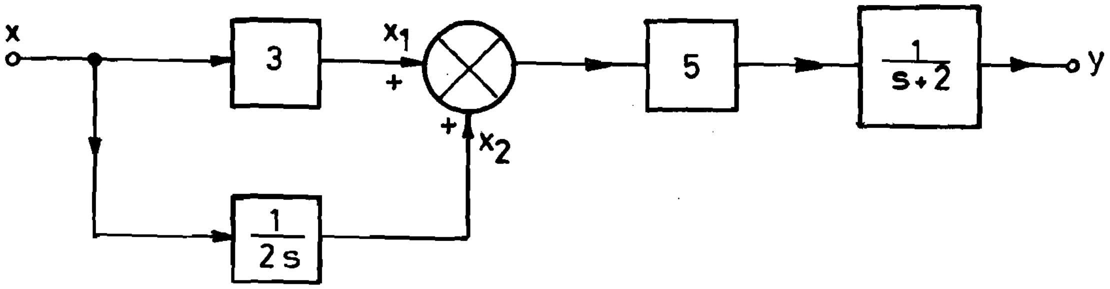


Fig. 5.20: Figure for SAQ 6

5.5 FEEDBACK

5.5.1 A Feedback System and Its Block Diagram

Feedback is an important process that can be used with respect to any task involving one or more number of steps or stages. Based on prior planning, we may desire a certain event to occur or a certain result to be achieved at an important stage in a task-chain. In actual practice, other factors that are either unforeseen or unpredictable may have produced a different result or event. Provision of the information on the status at any location at any time, to an earlier point in the chain would be called feedback. Incorporation of proper feedback facility would help in achieving the desired result, by corrective action.

We have learnt about the *pick-off*-point in Section 5.4.2, where a certain output is made available along more than one path. We also know about the "summing point" in a system.

Now look at Figure 5.21. At the pick-off point Q, a signal $y(t)$ coming from left is taken out along QS (shown by an additional dotted line drawn alongside). This signal is given as input to a block, whose system function is β. For the present, we shall take β to be a constant and positive. The output of the block is therefore $\beta\, y(t)$. This output forms one of the inputs to a summer, which gets $x(t)$ as the other input. Between the output of the summer and the pick-off point Q is a block of system function $H(s)$. Figure 5.21 constitutes the block diagram of a system with feedback. If there were no feedback, the summer, the pick-off point and the *feedback-block* of system function β would have been absent.

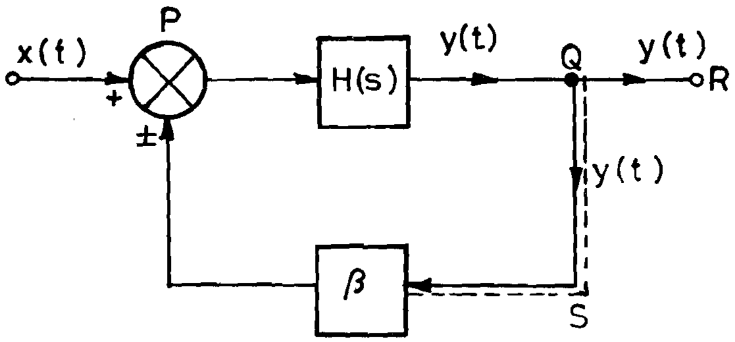


Fig. 5.21: Incorporation of feedback in the block-diagram of a system

With β being positive, Figure 5.22 shows two possible ways of connection at the summer. In Figure 5.22(a), the output of the summer is $x(t) + \beta\, y(t)$, while it is $x(t) - \beta\, y(t)$ in Figure 5.22(b). The connection of Fig. 5.22(a) is referred to as **positive feedback** and that of Figure 5.22(b) as **negative feedback.** We sometimes say *'The output is negatively fedback to the input'*, when we have the configuration corresponding to Figure 5.22(b). In Figure 5.21, the path from P to the output through $H(s)$ is called forward path and the path from Q to P via the β-block is known as feedback path.

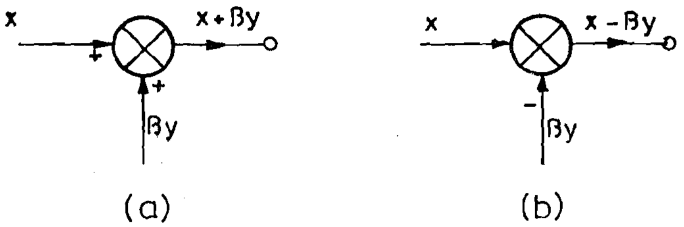


Fig. 5.22: (a) Positive and (b) negative feedback

5.5.2 System Function with Feedback

Going back to Figure 5.21, we can see that the block $H(s)$ has as its input, the signal $x(t) \pm \beta\, y(t)$, according as whether the feedback is positive or negative. Since the output of this block is $y(t)$, it follows that the output $y(t)$ and the input $x(t) \pm \beta\, y(t)$ are related by the system function $H(s)$.

Assuming that we are dealing with signals of the form e^{st}, we can use simple algebraic manipulation and write

$$(1 \pm \beta H)\, y(t) = H(s)\, x(t)$$

$$y(t) = \left(\frac{H(s)}{1 \pm \beta H(s)}\right) x(t).$$

With feedback introduced, the system function therefore becomes

$$H'(s) = \frac{H(s)}{1 \pm \beta H(s)}. \tag{5.23}$$

The '+' sign holds good in the denominator for negative feedback and the '–' sign for positive feedback. β is called *'feedback factor'*.

$$\frac{H(s)}{1 \pm \beta H(s)}.$$

Example 5.11

A system has a $H(s) = \frac{1}{s+2}$. Negative feedback is employed, with a feedback factor 0.5. Find the system function with feedback.

Solution

$$H(s) = \frac{1}{s+2}$$

$$\beta = 0{\cdot}5$$

$$H'(s) = \frac{H(s)}{1 + \beta H(s)} \quad \text{(negative feedback)}$$

$$= \frac{\frac{1}{s+2}}{1 + \frac{0 \cdot 5}{s+2}}$$

$$= \frac{1}{s + 2 \cdot 5}.$$

In the feedback system of Figure 5.21, we see a closed loop, if we start from the output of the summer go through $H(s)$, take the path QS and come back to the summer. The system with feedback is therefore, often referred to as **a closed-loop system**. If the path QS back to the summer is 'cut', the loop gets opened and we have an **open-loop system**.

Without feedback, the open-loop system has a system function $H(s)$. Eq. (5.23) shows that the closed-loop system with a feedback factor β is equivalent to an open-loop one with a system function

Figure 5.23 shows a closed-loop system where the β-block is absent. It corresponds to a case where β equals unity.

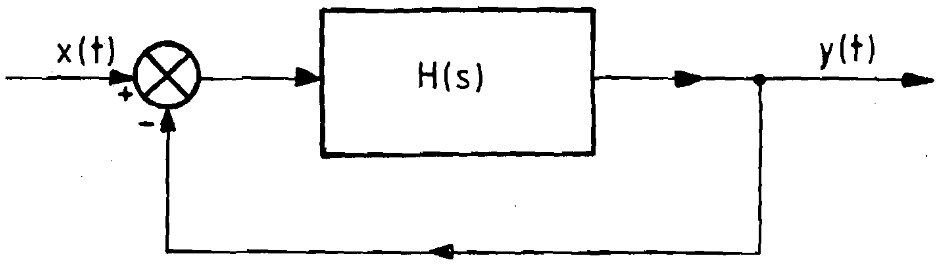


Fig. 5.23 : Unity feedback system

For such a 'unity feedback system',

$$H'(s) = \frac{H(s)}{1 + H(s)} \qquad (5.24)$$

Example 5.12

Identify the *RC*-network of Figure 5.5 as a closed-loop system and hence obtain its system function.

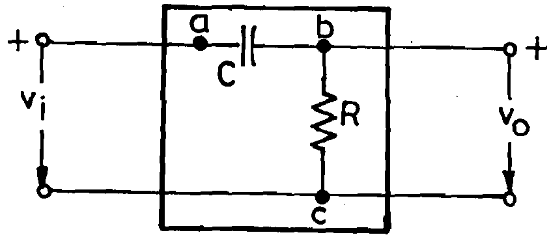


Fig. 5.5: Example of an electrical system with interconnection of elements shown.

Solution

Let us regard each element, namely *C* and *R* as a block. We see that $(v_i - v_o)$ gets applied to *C*. This voltage acts as an input to *C* and produces a current *i* as an intermediary output variable. *i* in turn forms the input to the resistor *R* and causes v_o as the output. It is this v_o which is connected in a negative feedback fashion to the input v_i, with $\beta = 1$. Hence, the overall block diagram is as in Figure 5.24.

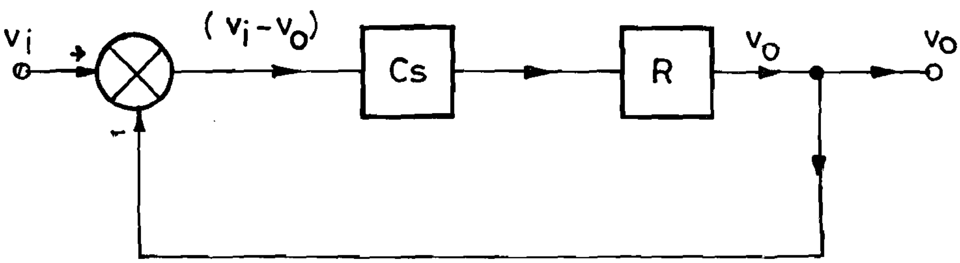


Fig. 5.24 : System corresponding to Example 5.12

Open-loop system function $H(s) = (Cs)R = RCs$.

Feedback factor $\beta = 1$;

therefore,

$$H'(s) = \frac{RCs}{RCs + 1}.$$

Compare the above with the solution to Example 5.4 and Example 5.7.

You would note that in the foregoing derivation, the system function of a capacitor *C* with its applied voltage as the input and the current through it as the output was taken as *Cs*. Prove this.

As mentioned earlier, a system configuration may involve the interconnection of a number of blocks through summers, pick-off points and feedback paths. To determine the overall system function for analysis purposes, one would need to reduce the system step by step and arrive at a single block having an overall system function *H(s)*. Let us take an example.

Example 5.13

Reduce the system shown in Figure 5.25 to an open-loop one having a single block and hence determine the system function.

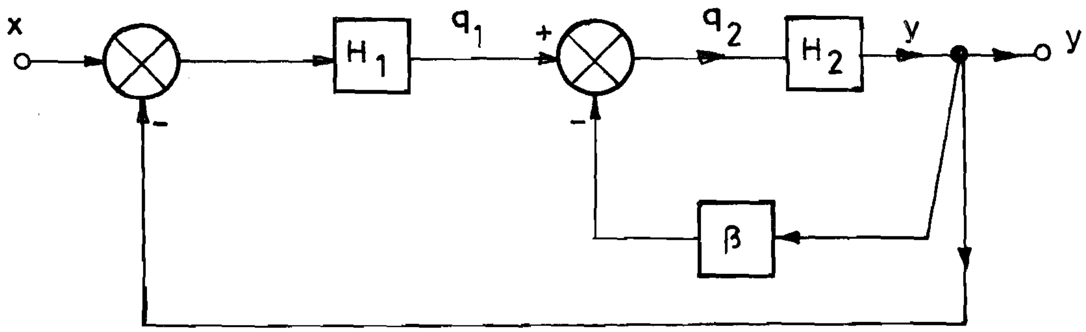


Fig. 5.25: Configuration of system corresponding to Example 5.13.

Solution

Let q_1 and q_2 be the intermediary variables marked on the figure. Notice that there are two feedback paths, one of which has a feedback factor of unity. We can write

$$q_1 = H_1 (x - y)$$

$$q_2 = q_1 - \beta y$$

$$y = H_2 \, q_2.$$

Starting from the left and proceeding right, up to the output point, we can see that

$$[\, H_1 (x - y) - \beta y \,] \, H_2 = y$$

Collecting terms, we have :

$$(1 + H_1 H_2 + \beta H_2) \, y = H_1 H_2 \, x$$

$$y = \left(\frac{H_1 H_2}{1 + H_1 H_2 + \beta H_2} \right) x.$$

The given system with multiple feedback is equivalent to a single-block open-loop system having a system function

$$H(s) = \frac{H_1(s) \, H_2(s)}{1 + H_1(s) \, H_2(s) + \beta H_2(s)}.$$

5.5.3 Advantages of Negative Feedback

You would have noticed that though mention was made in Section 5.5.1 about positive as well as negative feedback, most of the discussion in Section 5.5.2 was confined to systems employing negative feedback. This is because, introduction of negative feedback into a system brings in a number of benefits. We shall learn about a few of them.

Consider Example 5.11. The first order open-loop system had a

$$H(s) = \frac{1}{s + 2}.$$

With feedback ($\beta = 0.5$), the new system function is

$$H'(s) = \frac{1}{s + 2{\cdot}5}.$$

From Section 5.3.3, we know that $\frac{1}{2}$ and $\frac{1}{2{\cdot}5}$ are time constants.

Hence the time constant with feedback would be less than that without feedback. Qualitatively speaking, the response can be made faster with negative feedback.

To appreciate the next advantage, consider the following example.

Example 5.13

An open-loop system has a system function $H(s) = 10{,}000$. Negative feedback is employed with a $\beta = 0.005$. (a) Find the closed-loop system function $H'(s)$ (b) If, due to internal parameter changes, $H(s)$ changes by 10 percent, estimate the relative change in $H'(s)$.

Solution

$$H'(s) = \frac{H}{1+\beta H}$$

$$= \frac{10000}{1 + 0{\cdot}005 \times 10000}$$

$$= \frac{10000}{51}$$

$$= \frac{10000}{50\,(1 + 0{\cdot}02)}$$

$$\approx 200\,(1 - 0{\cdot}02)$$

$$= 196 \cdot$$

Differentiating the expression for H' with respect to H, we get :

$$\frac{dH'}{dH} = \frac{(1+\beta H) \times (1) - H \times (\beta)}{(1+\beta H)^2}$$

$$= \frac{1}{(1+\beta H)^2} = \frac{H}{(1+\beta H)} \times \frac{1}{H(1+\beta H)}$$

$$= H' \times \frac{1}{H(1+\beta H)}$$

or

$$\frac{dH'}{H'} = \frac{dH}{H} \times \frac{1}{1+\beta H}. \qquad (5.25)$$

A change of 10 % in H means that $dH/H = \pm\, 0.1$.

The relative change in H' due to this is

$$\frac{dH'}{H'} = \pm\, 0{\cdot}1 \times \frac{1}{1 + 0{\cdot}005 \times 10{,}000}$$

$$= \pm\, 0{\cdot}1 \times \frac{1}{1+50}$$

$$= \pm \frac{0{\cdot}1}{51}$$

$$\approx (0.02) \times 0.1$$

$$= 0.002 \text{ i.e., } 0.2 \text{ percent}$$

A second advantage of negative feedback system is thus seen, namely, that its adoption makes the system function less sensitive to parameter changes in the system.

Let us say that instead of using negative feedback, we retain the open-loop nature of the system, and cascade a block of $H_2(s) = 0.0196$ to the block $H(s) = 10{,}000$. We would get a

$$H' = H_2 \times H$$

$$= 0.0196 \times 10{,}000 = 196,$$

which is the same as what was got by using negative feedback. But a 10 percent change in H would then produce a 10 percent change in H' as well.

SAQ 7

A first order system function having a time content of 1 second is given by $1/s + 1$. Negative feedback is employed, β being equal to 0.1. Find the time constant with feedback.

5.6 BASICS OF CONTROL SYSTEMS

5.6.1 Control System Characteristics

A system with an input and an output variable gets the name *control system* , when a particular output is 'desired' or looked for with a specified input. In an automobile for example, the accelerator pedal position, acting via the fuel-valve controls the speed of the vehicle. Hence it is a control system. An electrical furnace used for heating a metal would be a control system, where the temperature of the furnace is the output and current on the electrical side would be the input. Control systems could be open-loop or closed loop. If a blind person were at the wheel of the automobile and pressed the accelerator pedal, the vehicle would move with increasing speed, regardless of other traffic on the road. This would represent an open-loop system. On the other hand, a conscious driver, watching the traffic and regulating the pedal position brings in feedback into this system which now becomes a closed-loop one.

5.6.2 Feedback Control System

Most of the control systems designed in practice are of the feedback or closed-loop type. They would therefore possess a system block diagram similar to Figure 5.21. It is redrawn in the same form in Figure 5.26 below, with some new nomenclature commonly used by control system engineers.

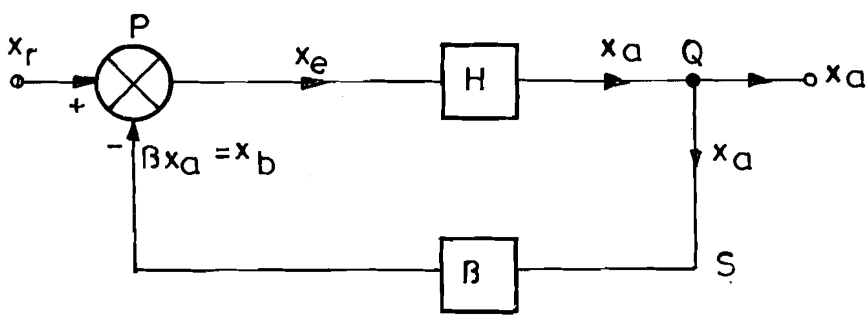


Fig. 5.26 : Block diagram of a feedback control system

The block in the forward path has a system function H and the feedback factor is β.

In a closed-loop control system, we have the actual output x_a and an input x_r which corresponds to the desired output and forms the reference. The inputs to the summer, instead of being x and $-\beta y$, are here x_r and $-\beta x_a$. What we have so far called system function is termed *transfer function* by control system engineers. With x_r being of the form e^{st} the relation between x_a and x_r would be similar to Eq.(5.23) and would be given by

$$\frac{x_a}{x_r} = \frac{H}{1 + \beta H}. \tag{5.26}$$

It is also called *closed-loop gain* and may be denoted by G_f. One more term is often used in control engineering. The output of the summer in Figure 5.26 which is $(x_r - \beta x_a)$ is called the **error signal** x_e and βx_a, the **feed-back signal** x_b. The relation between x_b and x_e is given by $x_b = \beta x_a = \beta H x_e$, where βH is called the loop gain (vide Example 5.14 for a justification of the term). Also from Eq. (5.26) and $x_a = H x_e$, we have

$$\frac{x_e}{x_r} = \frac{1}{1 + \beta H} \qquad (5.27)$$

Example 5.14

Find the loop gain of the feedback control system shown in Figure 5.27.

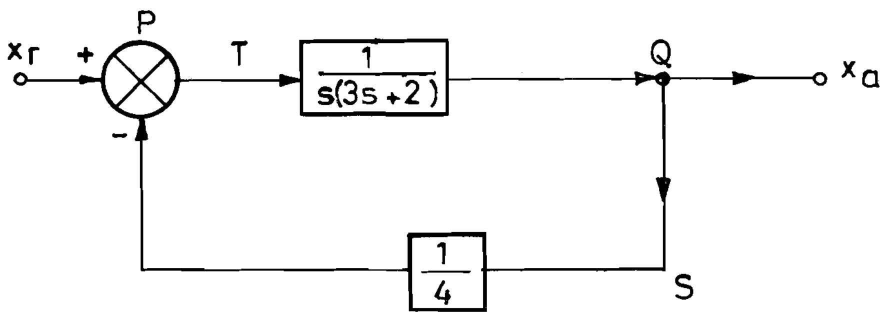


Fig. 5.27 : System for Example 5.14 and SAQ 8.

Let the closed loop *P-T-Q-S-P* constituted by the forward and feedback paths be opened at a point just before the summer. The loop now becomes a path. If you go round this path, you find two blocks namely, $\frac{1}{s(3s+2)}$ and $\frac{1}{4}$ cascaded. The open-loop transfer function of this path also called loop gain is :

$$\frac{1}{4} \frac{1}{s(3s+2)} = \frac{1}{4s(3s+2)}.$$

SAQ 8

Find the closed-loop transfer function of the system of Figure 5.27.

In a feedback control system, the block H usually contains a number of sub-blocks. An amplifier (Unit 12) and a "controller" are two important elements forming part of H in most control systems. The controller often has a transfer function of the form.

$$K_P + K_I / s$$

The feedback block β is often used to convert the controlled variable x_a (e.g. speed) into another variable of appropriate dimension (e.g., voltage) and level suitable for comparison with the reference signal.

5.6.3 Dynamic Response and Stability

Two main advantages of feedback have been mentioned, namely, improvement of speed of response and decreased sensitivity to variations in system parameters. When feedback is employed in a control system, the performance is usually judged in terms of accuracy and stability. The provision of amplification in the forward path (Section 5.5.1) implies a smaller value of x_e and hence increases accuracy.

There could be sudden changes occurring in a control system either due to a change in the input variable or due to a disturbance at some point in the loop. Given the system configuration consisting of the elements of the H-block and the feedback β, one would like to study the behaviour of the closed loop system under 'dynamic' conditions. The concepts of step response and frequency response discussed in Sections 5.3.3 and 5.3.2 are quite useful here. The Control Engineer makes such studies on the open-loop and closed-loop transfer functions. Let us take the closed-loop transfer function

$$G_f = \frac{H}{1 + H\beta}$$

We know that H is a function of s. To understand the behaviour at a frequency w, we substitute jw for s resulting in

$$G_f(jw) = \frac{H(jw)}{1 + H(jw)\beta}.$$

If it so happens that at some frequency, the complex number $H(jw)\,\beta$ becomes $(-1 + j0)$, the denominator will tend to zero. An output at this frequency can occur without any input. The system is said to be set into instability or oscillations.

Accuracy and stability are usually conflicting factors. Provision of amplification to improve accuracy may lead to the possibility of oscillations and instability. The task of the control engineer involves the judicious choice of trade off between these two mutually opposing requirements.

SAQ 9

How does amplification or increase of gain in the forward path improve accuracy in a feedback control system?

Example 5.15

In a certain feedback control system having a block diagram similar to Figure 5.26,

$$H(jw) = \left(\frac{H_0}{1 + j\ (w/10^3)}\right)^3.$$

Assuming a feedback factor $\beta = 0.005$, find the value of w at which the feedback system may oscillate and the minimum value of H_0 that could cause such oscillations.

Solution

$$H(jw)\cdot\beta = \frac{H_0^3}{[\,1 + (jw/10^3)\,]^3} \times 0\cdot 005$$

Phase angle of $[H(jw)\,\beta]$ $$= -\ 3 \text{ arc tan} \left(\frac{w}{10^3}\right).$$

For oscillations to occur, this angle should become $-180°$, so that $[H(jw).\ \beta]$ becomes $(-1 + j0)$.

$$\text{arc tan} \frac{w}{10^3} = 60°$$

$$\frac{w}{10^3} = \sqrt{3}$$

$$w = 10^3\sqrt{3} = 1732.\ \text{rad/sec}$$

At $w = 1732$,

$$|H(jw)\cdot\beta| = \frac{0.005 \times H_0^3}{|1+j\sqrt{3}|^3}$$

$$= 0.005\,\frac{H_0^3}{8}$$

For oscillation to occur,

$$|H(jw)\cdot\beta| = 1$$

i.e.,

$$0.005\,\frac{H_0^3}{8} = 1$$

$$H_0 = \left(\frac{8}{0.005}\right)^{\frac{1}{3}} = 11.696$$

5.7 SUMMARY

Let us recapitulate briefly what we have learnt in this Unit. We first learnt what constitutes a system and familiarised ourselves with different classifications of systems. The discussion in this unit was limited to linear time-invariant systems with a single input and a single output. We saw that the input-output relation of such a system can be described in the form of a differential equation or a system function. The system can also be represented symbolically by a block. We learnt the distinction between natural response and forced response, as also between transient response and steady-state response. We noted that the frequency response function specifies the steady state output of the system for a sinusoidal input of varying frequency. We saw that a complex system can be represented in terms of interconnected blocks, each block representing a subsystem. We also acquainted ourselves with the algebra of block diagrams which is useful for the analysis of complex systems.

The concept of feedback was then introduced. We saw two main advantages of feedback, namely, reduction of time constant and sensitivity. Finally, we learnt the basic principles of a feedback control system. The existence of two mutually opposing factors of accuracy and stability in a closed-loop control system has been identified.

System Theory is a vast subject. If, after finishing the course in Electrical Sciences, you wish to learn more about System Theory, you need to refresh your knowledge of Mathematics, learn Laplace Transform Techniques etc. The books listed at 2, 7 and 8 at the end of this block and will be useful for further reading.

5.8 ANSWERS TO SAQs

SAQ 1 :

The relationship between output and input is an algebriac equation. Hence the order of the system is ZERO.

SAQ 2 :

Both the equations should be differential equations of order 2.

a) If the system is time-varying, the coefficients can be functions of time, i.e., the equation could be, say,

$$4\frac{d^2y}{dt^2} + 3t\,\frac{dy}{dt} + 2y = t^2x$$

b) If the system is time-invariant, the equation could be

$$\frac{d^2y}{dt^2} + 2\,\frac{dy}{dt} + 3y = 4x$$

SAQ 3 :

The system function is $H(s) = \frac{1}{2+s}$.

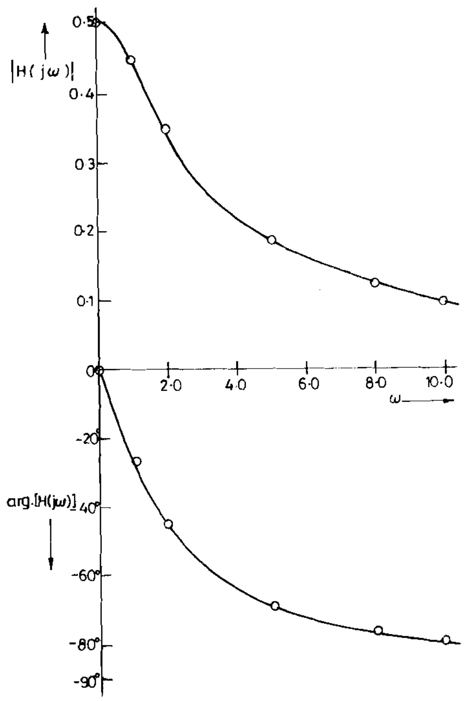


Figure for Answer to SAQ 3 : Frequency response curves for the system

$$H(jw) = \frac{1}{2+jw}.$$

$$|H(jw)| = \frac{1}{\sqrt{4 + w^2}}$$

$$\text{Arg}\,[H(jw)] = -\tan^{-1}(w/2)$$

The table below gives the $|H(jw)|$ and arg $[H(jw)]$ values for the range 0 to 10 of w

w	$\lvert H(jw) \rvert$	arg $[H(jw)]$
0	0.5	0°
1	0.447	- 26.57°
2	0.354	- 45°
5	0.186	- 68.2°
8	0.121	- 75.96°
10	0.098	- 78.69°

The amplitude and phase response curves for this system are plotted in the figure.

SAQ 4 :

1(a) $H(s)$ is arrived at from the system equation

$$Dr(D)\ \ y(t) = \ Nr(D)\ \ x(t)$$

(b) If the input is $x(t) = e^{st}$ the forced response

$$y_f(t) = \frac{Nr(s)}{Dr(s)} \cdot e^{st}$$

$$= H(s)\, e^{st}$$

'D' in (a) gets replaced by 's' and the "operation" becomes an algebraic manipulation for exponential inputs.

2 For a general input $x(t)$, we can say : $y(t) = \{H(D)\} \cdot x(t)$.

3 If $s = s_i$ is a root of $Dr(s) = 0$, its contribution to the natural response is $A_i\, e^{s_i t}$

SAQ 5 :

By d'Alembert's principle which you have studied in Mathematics,

$$\Sigma f = 0$$

$$f(t) - f_M - f_B - f_K = 0$$

where f_M, f_B, f_K are respectively the inertia, damping and spring forces.

$$f_M + f_B + f_K = f(t)$$

The individual force-velocity relationships are :

$$f_M = M\frac{du}{dt}$$

$$f_B = Bu$$

$$f_K = \frac{1}{K}\int u dt \cdot$$

The system equation is therefore :

$$M\frac{du}{dt} + Bu + \frac{1}{K}\int u\,dt = f(t)\cdot$$

Using the symbol $x(t)$ for the input and $y(t)$ for the output, the system equation in operator 'D'-form is

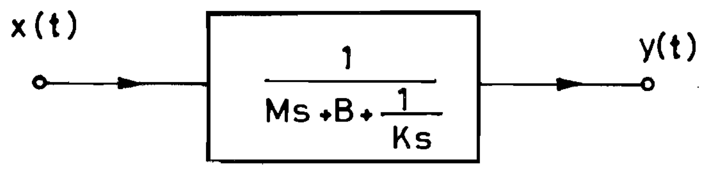
$$\left\{MD + B + \frac{1}{KD}\right\} y(t) = x(t)$$

Here again the flower bracket signifies that what is inside is an *operator function*.

The system function is

$$H(s) = \frac{1}{Ms + B + 1/Ks}\cdot$$

Hence the block diagram for the mass-spring-damper combination is as follows.



SAQ 6 :

The system may appear to be complex, but evaluation of the overall system function is quite easy. The inputs at the summer are :

$$x_1 = 3x$$

$$x_2 = \frac{1}{2s}x$$

Output of the summer is

$$x_1 + x_2 = \left(3 + \frac{1}{2s}\right)x = \left(\frac{6s+1}{2s}\right)x$$

The blocks having system function 5 and $1/(s + 2)$ are in cascade. Hence their combined system function is $5/(s + 2)$. The input to this block is

$\left(\frac{6s+1}{2s}\right)x$ · Hence the output $y = \frac{5(6s+1)}{2s\,(s+2)}x$

$$H(s) = \frac{5(6s+1)}{2s\,(s+2)}\cdot$$

SAQ 7 :

$$H(s) = \frac{1}{s+1}$$

$$H'(s) = \frac{H(s)}{1 + \beta H(s)} = \frac{\frac{1}{s+1}}{1 + \frac{0.1}{s+1}}$$

$$= \frac{1}{s+1.1}$$

The new time constant is $= \frac{1}{1.1} = 0.9$ s.

SAQ 8 :

$$G_f = \frac{H}{1 + H\beta}$$

$$= \frac{1/(3s+2)s}{1 + 1/4s(3s+2)}$$

$$= \frac{4}{4s(3s+2)+1}$$

$$= \frac{4}{12s^2 + 8s + 1}$$

$$= \frac{4}{(6s+1)(2s+1)}.$$

SAQ 9 :

From Eq. (5.27), $\frac{x_e}{x_r} = \frac{1}{1 + \beta H}$

When H increases, x_e the error signal decreases for a given x_r and a given β.

BLOCK 1 FURTHER READING

1. K.V.V. Murthy and M.S. Kamath : "Basic Circuit Analysis", Tata McGraw Hill, New Delhi, 1989
2. Ralph J. Smith: "Circuits, Devices and Systems", John Wiley, New York, 1976.
3. V. Del Toro : "Principles of Electrical Engineering", Prentice Hall of India, New Delhi, 1975
4. A.E. Fitzgerald, D.E. Higginbotham and A. Grabel : Basic Electrical Engineering, McGraw Hill, 1981.
5. E. Hughes, "Electrical Technology", Longmans, ELBS Edition, 1987.
6. S. Parker Smith : "Problems in Electrical Engineering", C.B.S. Publishers, Delhi, 1984 (contains a wealth of numerical problems in all topics of Electrical Sciences ranging from the routine to those of a complex nature. You will profit by attempting the solution of as many problems as you can on the topics covered in this Block.)
7. D.K. Cheng, Analysis of Linear Systems, Narosa Publishing House, New Delhi, 1990.
8. Alan V. Oppenheim, Alan S. Willsky and Ian T. Young: "Signals and Systems", Prentice Hall of India, 1990.