UNIT 4 THREE PHASE CIRCUITS

Structure

4.1 INTRODUCTION

A sinusoidal voltage source with 2 terminals having a single voltage output is termed a single phase source. Circuits incorporating such sources are called single phase (1-phase) circuits and formed the subject of our study in Unit 3. In contrast, a polyphase system contains sources each of which has several voltage outputs with a fixed phase difference between them. The three-phase (3-phase) system is the most common example of a polyphase system.

The generation and transmission of electrical energy and its utilization in bulk form is effected through 3-phase systems. In this Unit, you will first learn the precise nature of a 3-phase system and the advantages it provides relative to a single-phase system. You will then be introduced to the terminology, classification and characteristics of 3-phase circuits. Analysis of balanced 3-phase circuits will be taken up next. The representation of a balanced 3-phase system by a single line diagram and issues related to power will also be considered. You will observe that the analysis methods used in this unit are straightforward extensions of those employed for single phase circuits.

Objectives

After completing a study of this unit, you should be able to

- describe the features of a 3-phase system,
- distinguish between delta and star-connections of sources and loads,
- distinguish between 3-wire and 4-wire systems,
- explain the meaning of phase sequence,
- distinguish between balanced and unbalanced systems,
- differentiate between phase and line quantities and calculate one set from the other,
- analyse balanced 3-phase circuits comprising loads connected in star or delta,
- represent a 3-phase balanced circuit by a single-phase circuit and inter-relate the variables in the two systems,
- make calculations relating to power, reactive power, apparent power and p.f. for a given 3-phase circuit, and
- design the capacitor sizes needed for p.f. improvement in a 3- phase circuit.

4.2 THE THREE PHASE SYSTEM

4.2.1 The Nature of a 3-Phase System

A single phase a.c. generator consists of a rotating magnet driven by a prime mover and a winding embedded in the stationary part of the machine called the stator. Figure 4.1 shows an elementary form of the generator with a single turn coil AA' on the stator. As the magnet rotates, the flux lines linking with the coil undergo a periodic variation and hence induce a periodic emf in the latter. The frequency of this emf is fixed by the speed of rotation of the magnet. Special steps are taken in the design and construction of the machine to make the waveform of the induced voltage sinusoidal. Thus *the coil functions as a single-phase a.c. voltage source* with terminals A_1 and A_2, to which a load may be connected.

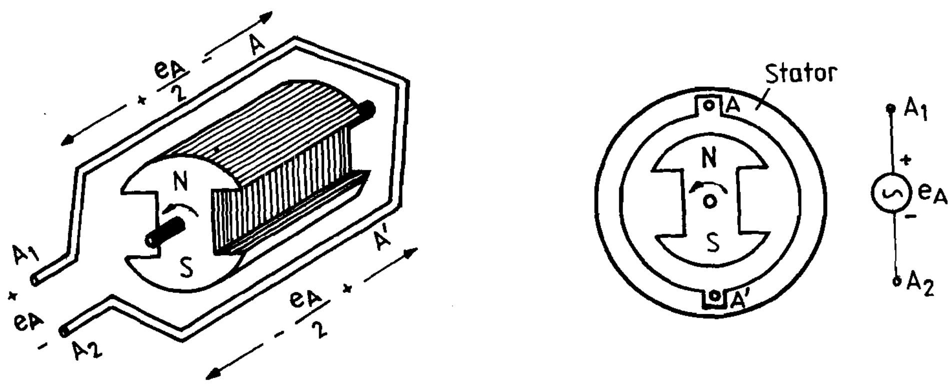

Fig. 4.1 : Elementary single phase generator and its circuit representation

Figure 4.2 illustrates the construction of an elementary 3-phase generator. Here we have 3 identical coils AA', BB', CC' placed on the stator with a displacement of 120° from one

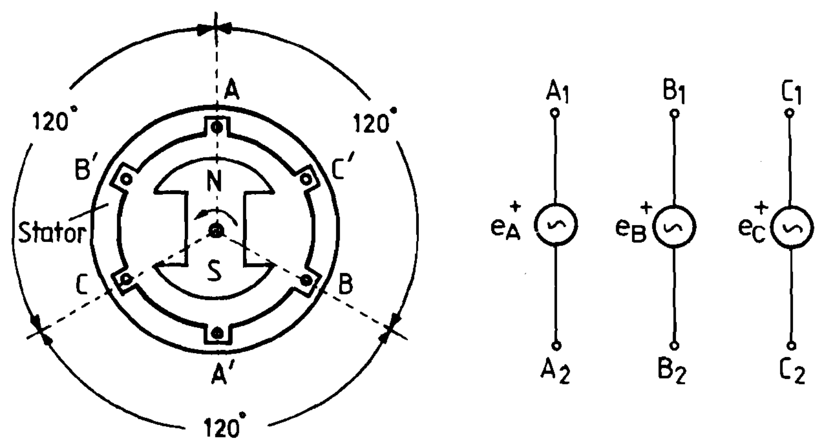

Fig. 4.2 : Elementary 3-phase generator and its circuit representation

another. The three emf's e_A, e_B, e_C generated in the coils therefore have the same rms value but have a phase difference of 120° from one another as shown in Figure 4.3. A ·

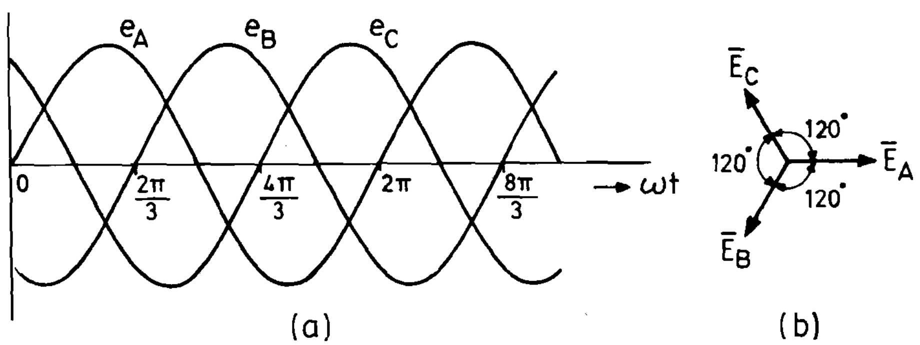

Fig. 4.3 : Voltages produced in a 3-phase generator (a) Waveforms (b) Phasors

3-phase generator can therefore be viewed as a composite unit comprising 3 single phase voltage sources with a fixed phase difference of 120° between any two of them. In practice, it is rare for a 3-phase generator to have all the six terminals brought out. The three coils are connected either in star or delta and only 3 or 4 terminals are brought out, as we shall see later.

A three-phase system is one which contains 3-phase sources besides 3-phase load impedances and feeder lines interconnecting them.

The three individual sections which constitute a 3-phase arrangement are referred to as Phase *A*, Phase *B*, and Phase *C* respectively. Another common practice is to label them as *R*(red), *Y*(yellow) and *B*(blue) phases. We shall follow the former convention in our work.

4.2.2 Merits of a 3-phase System

Let us now look at the advantages provided by 3-phase systems relative to single-phase systems.

A 3-phase a.c. generator utilises the available space on the stator more effectively than a 1-phase generator and has 50% more kVA rating for the same physical size. All commercial power stations therefore employ 3-phase generators as they cost less than single-phase generators for the same kVA rating.

The cost of electrical transmission and distribution lines used to carry bulk power from generating stations to receiving substations and distribute power from the substations to different load centres depends substantially on the volume of conducting material (usually aluminium) required for constructing these lines. It turns out that a 3-phase arrangement for transmission and distribution requires less conductor material for the lines and is therefore less expensive than a 1-phase system for handling the same amount of power at a given system voltage.

In power utilization, a 3-phase motor develops essentially a constant output torque whereas a single-phase motor can inherently provide only a pulsating torque. Not only is the three-phase motor consequently quieter in operation but it also provides better starting characteristics, higher efficiency of power conversion from electrical to mechanical form and better p.f. It is also in general cheaper than a 1-phase motor of the same power rating.

The foregoing economic and technical advantages have led to the universal adoption of the 3-phase system for the generation, transmission and utilization of bulk power. Small electrical loads, of typically less than 3 kW power rating, are however designed and built for single -phase operation. These include electric lights, fans, heaters and small motors needed for various domestic appliances, machine tools, pump sets and the like. The benefits that may stem from 3-phase operation of these loads are not commensurate with the additional cost of manufacturing them to be suitable for 3-phase use and of running 3-phase lines to each individual item. In practice, these small loads are fed from single phase supplies available from a 3-phase distribution system.

SAQ 1

Distinguish between a 3-phase generator and a single-phase generator.

SAQ 2

Fill up the blanks :

A 3-phase generator has more ________________ [1] than a 1-phase generator of the same physical size. A 3-phase transmission line employs less ________________ [2] than a 1-phase transmission line for the same power transmitted and the same system voltage. The torque developed by a 3-phase motor is ________________ [3] while the torque developed by a 1-phase motor is ________________ [4]

4.3 CHARACTERISTICS OF 3-PHASE SYSTEMS

After having been acquainted with the nature of 3-phase systems and their advantages, you will study in this section the characteristics of 3-phase sources, loads and associated systems in greater detail.

4.3.1 Balanced Sets of Voltages and Currents

At any section of the circuit representing a 3-phase system, there exist three voltages and three currents which constitute the variables of interest. **Three such voltages or currents are said to form a balanced set if they have equal effective values and if the phase difference between any two is 120°.** A balanced set of three voltages $\bar{V}_A$, $\bar{V}_B$ and $\bar{V}_C$ is depicted in Figure 4.4.

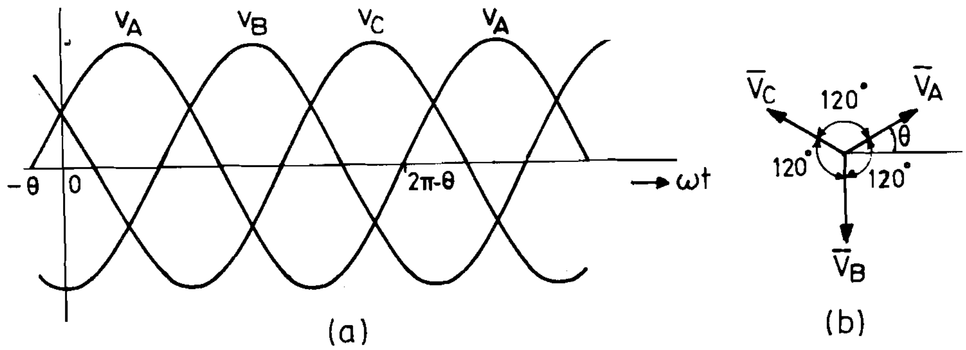

Fig. 4.4: A balanced set of three voltages with A B C phase sequence (a) Waveforms (b) Phasors

Note that similar events in the three waveforms (e.g., positive peak values) occur in the sequence *ABCABC*..... For this reason, the three voltages are said to have the ***ABC* phase sequence.** (We could as well have called it the *BCA* or *CAB* phase sequence but, by convention, choose the natural alphabetical order). Referring to the phasor diagram in Figure 4.4(b), if one were to imagine the three phasors to rotate in the anticlockwise direction, they sweep past a stationary point in the sequence *ABC*. This alternative way of judging the phase sequence from a phasor diagram would be useful when the waveforms are not explicitly plotted.

There exists a second possible phase sequence for a balanced voltage set, as depicted in Figure 4.5. Here similar events in the three signals occur in the sequence *ACBACB* ..... This sequence is called the ***ACB* phase sequence** (it could as well have been called *CBA* or *BAC* phase sequence). Note that the three related phasors now sweep past a stationary observer in the order *ACB*.

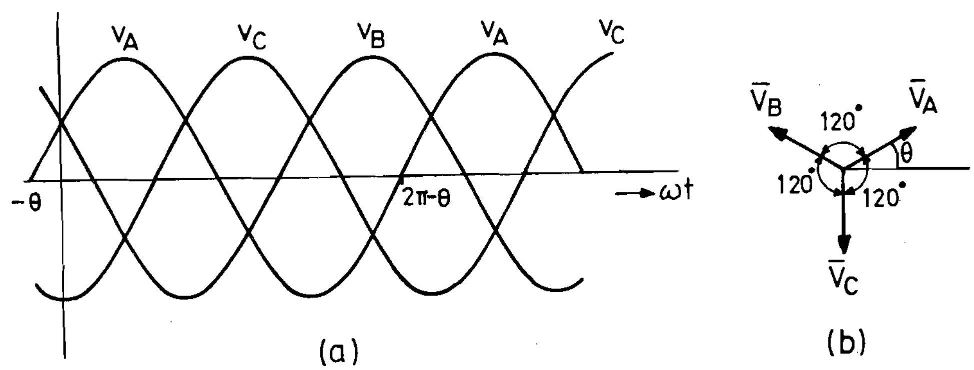

Fig. 4.5: A balanced set of voltages with ACB phase sequence (a) Waveforms (b) Phasors

In normal practice, the individual phases are so labelled as to correspond to the *ABC* phase sequence. We shall assume this to be the phase sequence in all our further work unless the contrary is specifically indicated. What has been discussed above with respect to a set of balanced voltages holds equally well with respect to a set of 3 currents. A set of balanced 3 phase voltages or currents would then have the following expressions (with *ABC* phase sequence assumed).

$$v_A = \sqrt{2}\ V\ \sin(wt + \theta)$$
$$v_B = \sqrt{2}\ V\ \sin(wt + \theta - 2\pi/3) \qquad (4.1)$$
$$v_C = \sqrt{2}\ V\ \sin(wt + \theta + 2\pi/3)$$
$$i_A = \sqrt{2}\ I\ \sin(wt + \beta)$$
$$i_B = \sqrt{2}\ I\ \sin(wt + \beta - 2\pi/3) \qquad (4.2)$$
$$i_C = \sqrt{2}\ I\ \sin(wt + \beta + 2\pi/3)$$

The corresponding phasors would be

$$\bar{V}_A = V \angle \theta\ ;\ \bar{V}_B = V \angle (\theta - 2\pi/3)\ ;\ \bar{V}_C = V \angle (\theta + 2\pi/3) \qquad (4.3)$$
$$\bar{I}_A = I \angle \beta\ ;\ \ \bar{I}_B = I \angle (\beta - 2\pi/3)\ ;\ \ \bar{I}_C = I \angle (\beta + 2\pi/3) \qquad (4.4)$$

The following properties of balanced voltages (or currents) are noteworthy:

With *ABC* phase sequence, v_A leads v_B by 120°, v_B leads v_C by 120° and v_C leads v_A by 120°. With *ACB* phase sequence, v_A leads v_C by 120°, v_C leads v_B by 120° and v_B leads v_A by 120°.

If the set of voltages (or currents) is known to be balanced and the phase sequence is fixed, the data pertaining to one voltage or current would suffice to deduce the other two. For example, if it is known then $\bar{V}_B = 100 \angle 30°$ and that the phase sequence is *ABC*, it follows that $\bar{V}_A = 100 \angle 150°$ and $\bar{V}_C = 100 \angle -90°$.

The sum of three balanced quantities is identically zero in time domain.

$$v_A + v_B + v_C = 0 \qquad (4.5)$$

$$i_A + i_B + i_C = 0 \qquad (4.6)$$

The above results can be proved through manipulation of the trigonometric expressions in Eq. (4.1) and (4.2). They can also be verified by observing that the ordinates of the three pertinent waveforms like those in Figure 4.4(a) add up to zero at every instant of time.

The equivalent results in phasor domain are

$$\bar{V}_A + \bar{V}_B + \bar{V}_C = 0 \qquad (4.7)$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \qquad (4.8)$$

To check the validity of Eq.(4.7) please refer to Figure 4.6. Since $\bar{V}_A$ and $\bar{V}_B$ have equal magnitudes and are 120° apart, their resultant $\bar{V}_A + \bar{V}_B$ is at 60° from $\bar{V}_A$ and has the same magnitude. $\bar{V}_A + \bar{V}_B$ is therefore equal and opposite to $\bar{V}_C$. Hence $\bar{V}_A + \bar{V}_B + \bar{V}_C = 0$. If drawn from end to end, the three directed line segments $\bar{V}_A$, $\bar{V}_B$ and $\bar{V}_C$ form a *closed* triangle as seen in Figure 4.6(b). This is an alternative way of showing that $\bar{V}_A$, $\bar{V}_B$ and $\bar{V}_C$ add up to zero.

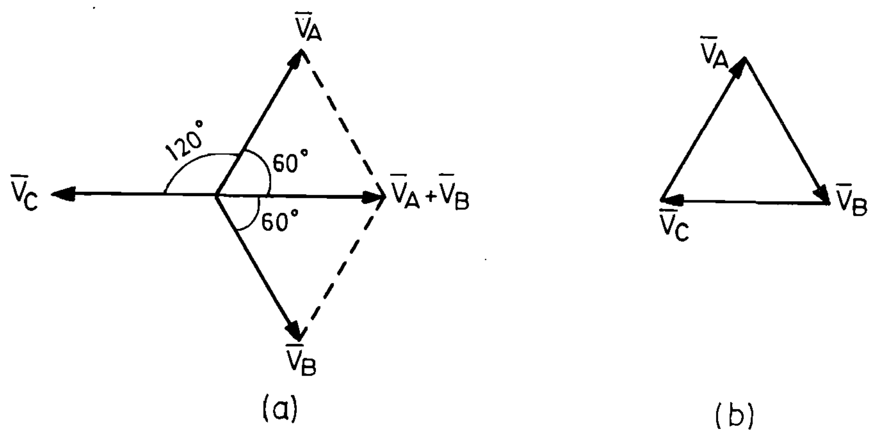

Fig. 4.6 : Demonstration of the result, $\bar{V}_A + \bar{V}_B + \bar{V}_C = 0$.

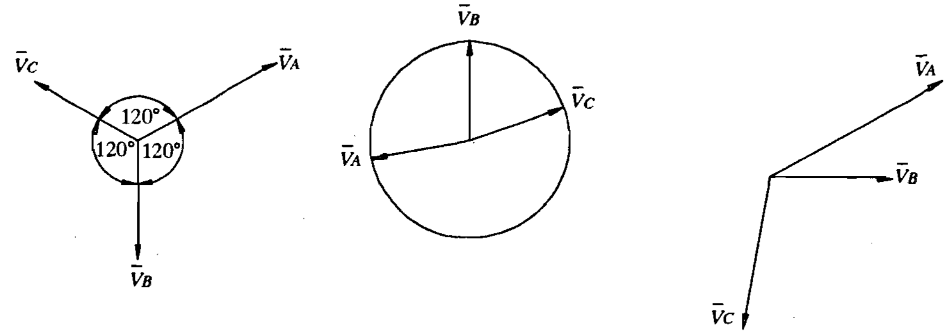

Fig. 4.7 : Examples of sets of unbalanced voltages.

Finally, you should note that three voltages / currents are unbalanced if their effective values are not equal or their phase differences are not 120° or both. Figure 4.7 gives examples of sets of unbalanced voltages.

Example 4.1

At a certain section in a 3-phase circuit, $\bar{V}_B = 120 \angle 60°$ and $\bar{I}_C = 4 \angle 180°$. If the voltages and currents are balanced and the phase sequence is *ABC*, deduce $\bar{V}_A$, $\bar{V}_C$, $\bar{I}_A$ and $\bar{I}_B$.

Solution

With *ABC* sequence, $\bar{V}_B$ leads $\bar{V}_C$ by 120° and lags $\bar{V}_A$ by 120°. Thus

$\bar{V}_A = 120 \angle 180°$; $\bar{V}_C = 120 \angle -60°$.

$\bar{I}_C$ leads $\bar{I}_A$ by 120° and lags $\bar{I}_B$ by 120°.

Hence, $\bar{I}_A = 4 \angle 60°$; $\bar{I}_B = 4 \angle 300° = 4 \angle -60°$.

SAQ 3

State if the following assertions are true or false.

1 Three currents in a 3-phase system are balanced if their phasors are equal.

2 In a set of balanced 3-phase voltages with *ABC* phase sequence $\bar{V}_A$ leads the other two voltages $\bar{V}_B$ and $\bar{V}_C$.

3 If $i_A + i_B + i_C = 0$, then i_A, i_B and i_C form a balanced set of 3-phase currents.

SAQ 4

Taking v_A and i_A as in Eq. (4.1) and (4.2), write the expressions for the other quantities if phase sequence is *ACB*.

SAQ 5

Taking the following to be balanced sets of voltages / currents with ABC phase sequence, fill the blanks

1 $v_A = \sqrt{2} \times$ ....[1] $\sin (wt +$ ....[2]);
$v_B = \sqrt{2} \times 100 \sin (wt +$ ....[3])
$v_C = \sqrt{2} \times$ ....[4] $\sin (wt + 45°)$.

2 $\bar{I}_A =$ ....[5] $\angle$ ....[6] ; $\bar{I}_B =$ ....[7] $\angle 60°$; $\bar{I}_C = 4 \angle$ ....[8].

4.3.2 Delta and Star Connections

You would recall that a 3-phase generator essentially consists of 3 single-phase sources, having output voltages say, e_A, e_B and e_C. A **balanced 3-phase source** is one in which these three voltages form a balanced set. All commercial 3-phase generators are built in this manner. The three single phase sources are connected internally either in *delta* or in *star* as shown in Figure 4.8 and terminals brought out for connection to external loads.

Note the symmetrical way of connecting the three single-phase sources. In the delta connection, A_2 is connected to B_1, B_2 is connected to C_1 and C_2 is connected to A_1. In the star connection A_2, B_2 and C_2 are joined together. Such an orderly method of connections is needed to ensure the balanced condition of the voltages available between the terminals A, B and C of the 3-phase generator.

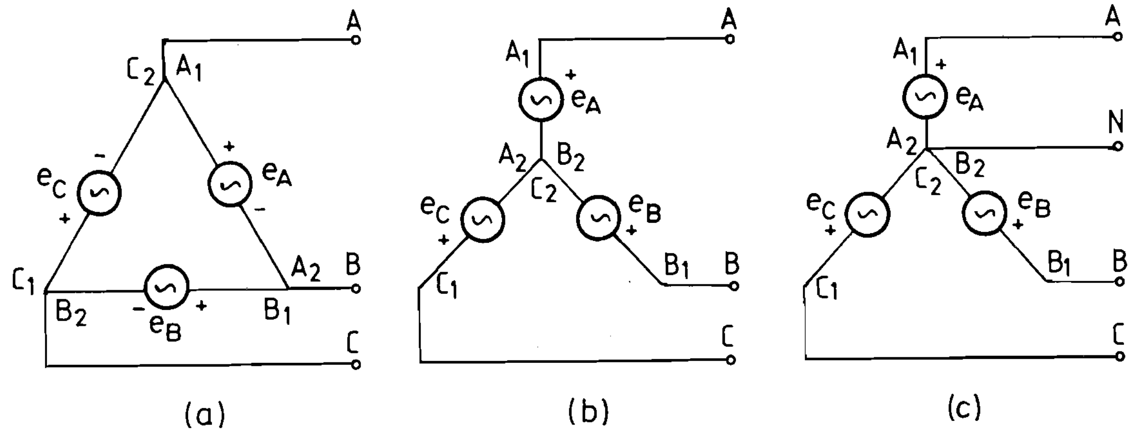

Fig. 4.8 : Internal connections of a 3-phase voltage source
(a) Delta connection (b) Star connection with 3 terminals (c) Star connection with 4 terminals.

In the delta connection, the effective emf of the three series connected sources around the closed circuit is $e_A + e_B + e_C$. If the three voltages do not add up to zero there would be a large circulating current in the delta even with no load connected to terminals A, B, C and this is clearly an undesirable situation. However, for a balanced source this contingency does not arise as $e_A + e_B + e_C = 0$ (vide Eq. (4.5)). It is this fact which makes the delta connection of a 3-phase source feasible.

In the star connection, the common terminal of the 3 sources (star point) is called the **neutral point**. Here there exist two possible arrangements. Where a separate terminal is not provided for the neutral point as in Figure 4.8(b), the generator forms part of what is known as a **3-wire 3-phase system**. On the other hand, the arrangement shown in Figure 4.8(c) permits connection of the generator in a **4-wire 3-phase system**. The terminals A, B, C are called the line terminals and N is called the neutral terminal.

A **3-phase load** generally comprises three impedances in a configuration suitable for connection in a 3-phase circuit. Similar to the connections in a 3-phase generator, here also we have 3 possible connections as shown in Figure 4.9. Notice that the configuration in Figure 4.9(c) is suitable for connection only in a 4-wire 3-phase system.

A 3-phase load is balanced if the three complex impedances are equal i.e.,

$$\bar{Z}_A = \bar{Z}_B = \bar{Z}_C \text{ for a star-connected load} \quad (4.9a)$$

and

$$\bar{Z}_{AB} = \bar{Z}_{BC} = \bar{Z}_{CA} \text{ for a delta-connected load} \quad (4.9b)$$

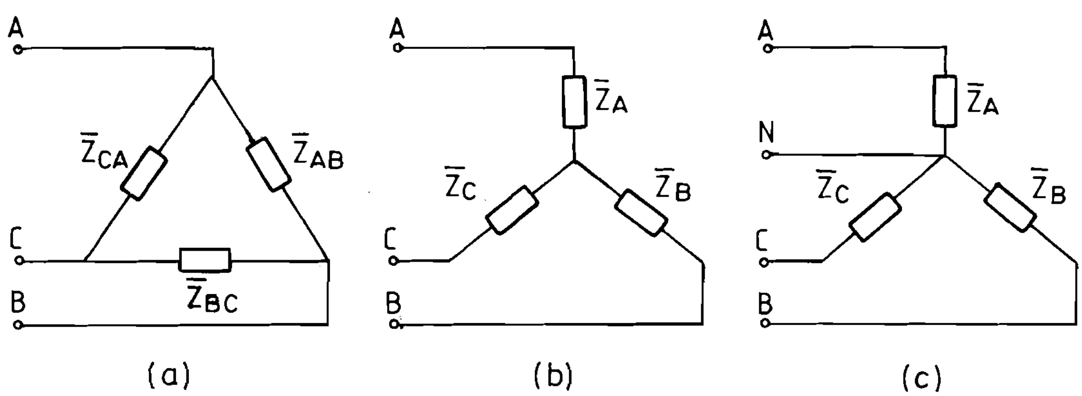

Fig. 4.9: 3-phase loads.
(a) Delta connection (b) Star connection with 3 terminals (c) Star connection with 4 terminals

Not only the magnitudes but also the angles of the impedances should be equal for a 3-phase load to be balanced. We can therefore use a common symbol $\bar{Z}_Y$ for the three impedances in star and $\bar{Z}_\Delta$ for the three impedances in delta.

You may recall the star-delta conversion formulas for resistances discussed in Unit 2. The same formulas hold for complex valued impedances as well. Thus a delta-connected 3-phase load has an equivalent star-connected configuration and vice-versa. If the load in Figure 4.9(b) is to be equivalent to that in Figure 4.9(a), we require

$$\bar{Z}_A = \frac{\bar{Z}_{AB}\,\bar{Z}_{CA}}{\bar{Z}_{AB}+\bar{Z}_{BC}+\bar{Z}_{CA}}; \quad \bar{Z}_B = \frac{\bar{Z}_{BC}\,\bar{Z}_{AB}}{\bar{Z}_{AB}+\bar{Z}_{BC}+\bar{Z}_{CA}};$$
$$\bar{Z}_C = \frac{\bar{Z}_{CA}\,\bar{Z}_{BC}}{\bar{Z}_{AB}+\bar{Z}_{BC}+\bar{Z}_{CA}} \qquad (4.10)$$

For balanced loads, the formulas reduce to

$$\bar{Z}_Y = (1/3)\ \bar{Z}_\Delta \qquad (4.11)$$

A 3-phase circuit is formed through the interconnection of 3- phase sources and 3-phase loads. If all the sources are balanced and have the same phase sequence and all the loads are also balanced, then the 3-phase circuit is said to be balanced. A characteristic of a **balanced 3-phase circuit** is that the voltages and currents at any arbitrary location are balanced. In our study we shall be concerned only with *balanced systems.*

SAQ 6

Fill up the blanks :

The ________________ [1] connection of sources / impedances is suitable either for 3-wire or for 4-wire three-phase systems but the ________________ [2] connection of sources/impedances is suitable only for ________________ [3] three-phase systems.

SAQ 7

State if the following assertions are true or false.

(a) Three impedances $\bar{Z}_A$, $\bar{Z}_B$ and $\bar{Z}_C$ form a balanced 3-phase load if $\bar{Z}_A + \bar{Z}_B + \bar{Z}_C = 0$.

(b) The neutral point is not available in a 3-phase delta-connected source.

Example 4.2

A balanced 3-phase load is formed by three impedances of 60 +*j*90 ohms each, connected in delta. If this load is equivalent to a star-connected load having $\bar{Z}_Y$ in each leg of the star, calculate $\bar{Z}_Y$.

Solution

$$\bar{Z}_Y = \frac{1}{3}\ \bar{Z}_\Delta = \frac{1}{3}(60 + j90) = 20 + j30\ \Omega$$

4.3.3 Relations between Line and Phase Quantities

In 3-phase circuits one distinguishes between *line voltages and currents* on one hand and *phase voltages and currents* on the other. Phase quantities are the internal voltages or currents associated with the single phase sources constituting a 3-phase source or the three impedances constituting the 3-phase load. Line quantities, on the other hand, are those which can be measured at the terminals. These are the voltages between and the currents in the external supply lines connected to the terminals. We now proceed to show the relationship between the two sets. We use the double subscript notation in the following analysis. Recall that v_{xy} represents the voltage drop from x to y and i_{xy} represents the current from x to y.

Star Connection

Referring to the circuits of Figure 4.10, the following constitute the line quantities and phase quantities in a star-connected source/load.

Phase currents : $(\bar{I}_A)_P$, $(\bar{I}_B)_P$, $(\bar{I}_C)_P$

Phase voltages : $\bar{V}_{AN}$, $\bar{V}_{BN}$, $\bar{V}_{CN}$

Line currents : $(\bar{I}_A)_L$, $(\bar{I}_B)_L$, $(\bar{I}_C)_L$

Line voltages : $\bar{V}_{AB}$, $\bar{V}_{BC}$, $\bar{V}_{CA}$

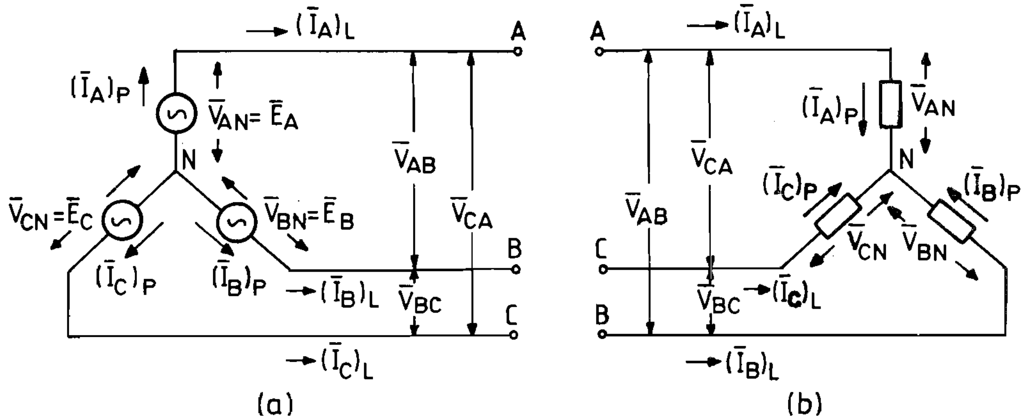

Fig. 4.10: Phase and line quantities in a star-connected (a) source and (b) load.

We straightaway note that

$$(\bar{I}_A)_P = (\bar{I}_A)_L, \quad (\bar{I}_B)_P = (\bar{I}_B)_L, \quad (\bar{I}_C)_P = (\bar{I}_C)_L$$

and

$$\bar{V}_{AB} = \bar{V}_{AN} + \bar{V}_{NB} = \bar{V}_{AN} - \bar{V}_{BN}$$

$$\bar{V}_{BC} = \bar{V}_{BN} + \bar{V}_{NC} = \bar{V}_{BN} - \bar{V}_{CN}$$

$$\bar{V}_{CA} = \bar{V}_{CN} + \bar{V}_{NA} = \bar{V}_{CN} - \bar{V}_{AN} \qquad (4.12)$$

In a balanced system, each one of the above four sets of phase and line voltages and currents is balanced. Let us designate the rms values of the phase currents and voltages as I_P and V_P and the rms values of the line currents and voltages as I_L and V_L respectively. With this notation,

$$(I_A)_P = (I_B)_P = (I_C)_P = I_P$$

and

$$(I_A)_L = (I_B)_L = (I_C)_L = I_L.$$

We therefore have

$$I_L = I_P \qquad (4.13)$$

To find the relation between V_L and V_P, let us start taking $\bar{V}_{AN}$ as the reference phasor. Since the phase voltages form a balanced set,

$$\bar{V}_{AN} = V_P \angle 0° \ ; \quad \bar{V}_{BN} = V_P \angle -120° \ ; \quad \bar{V}_{CN} = V_P \angle 120°$$

We have

$$\bar{V}_{AB} = \bar{V}_{AN} - \bar{V}_{BN} = V_P \angle 0° - V_P \angle -120°$$
$$= V_P [1 - (-0.5 - j\sqrt{3}/2)]$$
$$= (\sqrt{3}/2)\, V_P (\sqrt{3} + j1) = \sqrt{3} V_P \angle 30°$$

$$\bar{V}_{BC} = \bar{V}_{BN} - \bar{V}_{CN} = V_P \angle -120° - V_P \angle 120°$$
$$= \sqrt{3} V_P \angle -90°$$

$$\bar{V}_{CA} = \bar{V}_{CN} - \bar{V}_{AN} = V_P \angle 120° - \bar{V}_P \angle 0° = \sqrt{3} V_P \angle 150°$$

The disposition of the phasors of the phase voltages in the complex plane and the construction of the line voltages therefrom are illustrated in Figure 4.11. Note for example that $\bar{V}_{AB}$ is the sum of $\bar{V}_{AN}$ and $-\bar{V}_{BN}$. Since the latter two have the same magnitude and are displaced from each other by 60°, their resultant has a magnitude $2\cos 60° = \sqrt{3}$ times each and an angle exactly midway between them.

Hence $\bar{V}_{AB} = \sqrt{3}V_P \angle 30°$. You are advised to make use of this graphical deduction of line voltages from phase voltages wherever needed, as it is simpler and more illustrative than purely analytical methods.

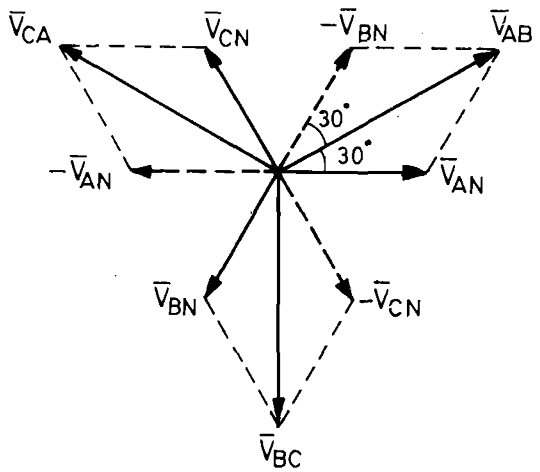

Fig. 4.11: Derivation of line voltages from phase voltages in a balanced star configuration

From the foregoing analysis, we have the important result,

$$V_L = \sqrt{3}V_P \tag{4.14}$$

Note that if the three line voltages phasors were to rotate in the anticlockwise direction, their first subscripts as well as the second subscripts appear to a stationary observer in the order *ABCABC....* indicating that the phase sequence of the line voltages is also *ABC*.

The following important characteristics of a *star configuration* in a balanced system emerge from the foregoing derivations :

- Line currents have the same effective value as phase currents. $I_L = I_P$.
- Line voltages have $\sqrt{3}$ times the effective value of phase voltages. $V_L = \sqrt{3}V_P$
- For a balanced set of phase voltages with *ABC* phase sequence, the line voltages are also balanced, have the same phase sequence and are displaced from the phase voltage set by 30° ($\bar{V}_{AB}$ leads $\bar{V}_{AN}$ by 30°).

Delta connection

For the delta connected source and load illustrated in Figure 4.12, we make the following identification of the line and phase quantities.

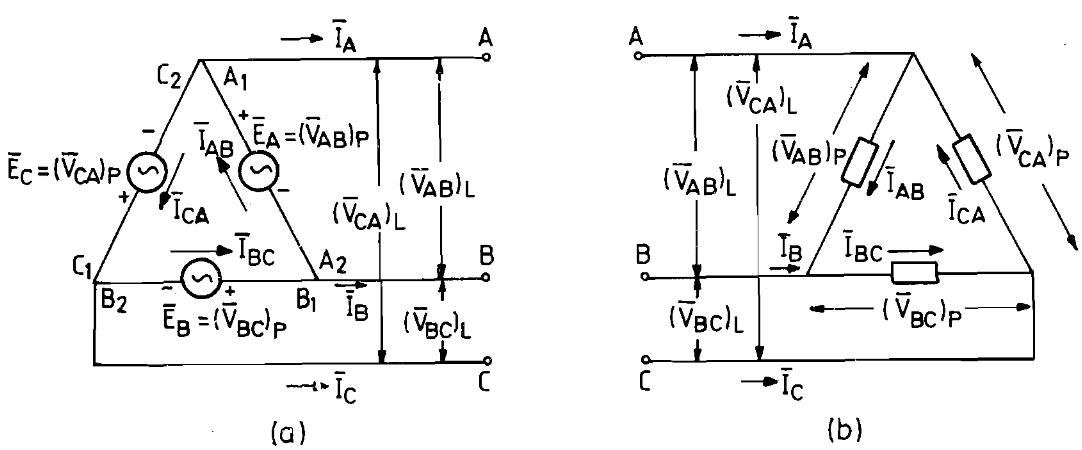

Fig. 4.12: Phase and line quantities in a delta-connected (a) source and (b) load.

Phase currents : $\bar{I}_{AB}$, $\bar{I}_{BC}$, $\bar{I}_{CA}$

Phase voltages : $(\bar{V}_{AB})_P, (\bar{V}_{BC})_P, (\bar{V}_{CA})_P$

Line currents : $\bar{I}_A, \bar{I}_B, \bar{I}_C$

Line voltages : $(\bar{V}_{AB})_L, (\bar{V}_{BC})_L, (\bar{V}_{CA})_L$

Here we straightaway note that

$$(\bar{V}_{AB})_P = (\bar{V}_{AB})_L \,;\, (\bar{V}_{BC})_P = (\bar{V}_{BC})_L \,;\, (\bar{V}_{CA})_P = (\bar{V}_{CA})_L \tag{4.15}$$

For the currents we have

$$\begin{aligned} \bar{I}_A &= \bar{I}_{AB} - \bar{I}_{CA} \\ \bar{I}_B &= \bar{I}_{BC} - \bar{I}_{AB} \\ \bar{I}_C &= \bar{I}_{CA} - \bar{I}_{BC} \end{aligned} \tag{4.16}$$

For a balanced system, let us designate the rms values of phase and line voltages as V_P and V_L respectively and of phase and line currents as I_P and I_L respectively. We then note

$$V_P = V_L \tag{4.17}$$

To deduce the corresponding relation for currents, let us refer to the phasor diagram of Figure 4.13, which has been drawn taking the phase current $\bar{I}_{AB}$ as reference and using the relations in Eq. (4.16).

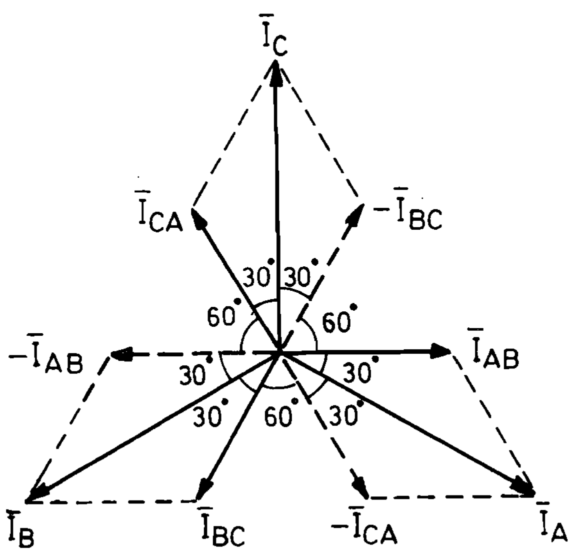

Fig. 4.13 : Deduction of line currents from phase currents in a balanced delta configuration.

For instance,

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = I_P \angle\, 0° + I_P \angle -60°$$

$$= 2I_P \cos 30° \angle -30° = \sqrt{3} I_P \angle -30°$$

Since $|\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = I_L$ in a balanced configuration, we have

$$I_L = \sqrt{3} I_P \tag{4.18}$$

A balanced delta configuration then has the following important characteristics :

- Phase voltages and line voltages have the same effective value. $\boldsymbol{V_L = V_P}$
- Line currents have $\sqrt{3}$ times the effective value of phase currents. $\boldsymbol{I_L = \sqrt{3} I_P}$
- The line currents and phase currents have the same phase sequence and the set of line current phasors is displaced by 30° from the set of phase current phasors (line current $\bar{I}_A$ lags $\bar{I}_{AB}$ by 30°)

SAQ 8

Fill up the blanks:

In a 3-phase system ____________________ [1] are the voltages that can be measured between the supply lines, whereas ____________________ [2] are the voltages associated with the individual legs of a 3-phase source or load.

In a star configuration the ____________________ [3] is $\sqrt{3}$ times the ____________________ [4], whereas in the ____________________ [5] configuration, the ____________________ [6] is $\sqrt{3}$ times the ____________________ [7].

Example 4.3

The line voltage $\bar{V}_{AB}$ across a star-connected load has a phasor $400 \angle 70°$. Find $\bar{V}_{AN}$ of the A phase voltage.

Solution

If $\bar{V}_{AN} = V_P \angle \theta$, then $\bar{V}_{BN} = V_P \angle \theta - 120°$

We have

$$\bar{V}_{AB} = \bar{V}_{AN} - \bar{V}_{BN} = V_P \angle \theta - V_P \angle \theta - 120° = V_P \angle \theta + V_P \angle \theta + 60°$$

$$= \sqrt{3} V_P \angle \theta + 30° = 400 \angle 70°$$

Therefore $V_P = 400/\sqrt{3} = 231 V, \ \ \theta = 40°$

Hence $\bar{V}_{AN} = 231 \angle 40°$

SAQ 9

What do you understand by

(a) a balanced 3-phase voltage,

(b) a balanced 3-phase load,

(c) a balanced 3-phase circuit?

SAQ 10

In the balanced delta configuration of Figure 4.8(a), let the phase voltage be 100 V. If now the terminals of the generator A are interchanged, what would be the net voltage acting around the loop?

SAQ 11

Two three-phase loads, connected in star and delta, are in parallel as shown, where all values are impedances in ohms. Find an equivalent delta-connected configuration.

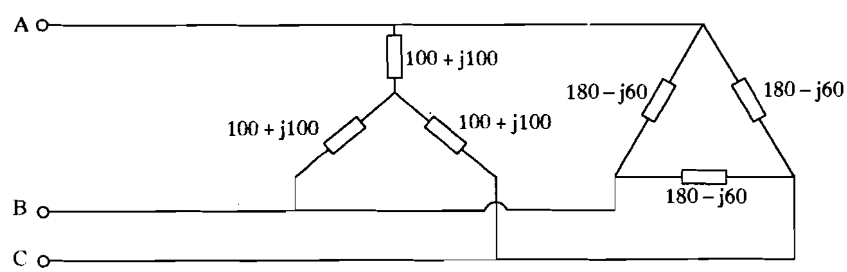

Fig. 4.14 for SAQ 11

4.4 ANALYSIS OF BALANCED 3-PHASE CIRCUITS

You would recall from the previous section that a balanced 3-phase circuit comprises only balanced sources feeding balanced loads. In this section you will learn how to analyse a few typical 3-phase balanced circuits.

The supply lines *A, B, C* feeding a 3-phase load in a commercial power distribution network do not emanate from an actual 3-phase generator, but from a 3-phase transformer. However for our purposes, we may visualize the transformer to function as a 3-phase source.

A 3-phase supply is conventionally designated by its line voltage since it is this voltage that can be measured at the supply terminals. Thus the term *a 3-phase 400-V supply* is intended to convey that the line voltage of the supply is 400 V. If the source is delta-connected, the phase voltage of the source is also 400 V. If however the source is star-connected the phase voltage of the source is $400/\sqrt{3} \approx 230$V. Now in a 3-phase 4-wire system, we have not only the line voltages V_{AB}, V_{BC} and V_{CA} available but also the phase voltages V_{AN}, V_{BN}, V_{CN}. The 3-phase 400 V, 4-wire supply system is the standard adopted in our country and many others for power distribution. Apart from 3-phase loads, this system caters to single phase 230 V loads (e.g., lights and fans) which are connected between a line (*A*, *B* or *C*) and the neutral. These single-phase loads are distributed evenly between the three phases as far as possible, so that the overall load on the system is nearly balanced.

4.4.1 Star-connected Load Fed from a Balanced Supply

Figure 4.15 shows a 3-phase star-connected load comprising three impedances $Z \angle \alpha$, connected to a 3 phase star-connected source in a 4-wire system. Let the source have a line voltage of V_L volts. Let us analyse the circuit, taking $\bar{V}_{AN}$ as reference.

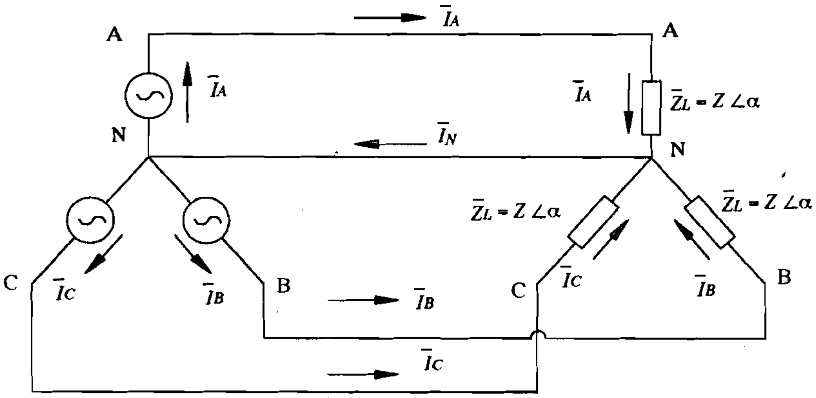

Fig. 4.15 : Star-connected load in a 4-wire system

$$\bar{V}_{AN} = \frac{V_L}{\sqrt{3}} \angle 0 ; \quad \bar{V}_{BN} = \frac{V_L}{\sqrt{3}} \angle -2\pi/3 ; \quad \bar{V}_{CN} = \frac{V_L}{\sqrt{3}} \angle 2\pi/3 \qquad (4.19)$$

Assuming that the impedances of the feeder lines are negligible, the phase voltages at the load are also the same as at the source. We then have,

$$\bar{I}_A = \frac{\bar{V}_{AN}}{\bar{Z}_L} = \frac{V_L \angle 0}{\sqrt{3}} \times \frac{1}{Z \angle \alpha} = \frac{V_L}{\sqrt{3}Z} \angle (-\alpha)$$

$$\bar{I}_B = \frac{\bar{V}_{BN}}{\bar{Z}_L} = \frac{V_L \angle -2\pi/3}{\sqrt{3}} \frac{1}{Z \angle \alpha} = \frac{V_L}{\sqrt{3}Z} \angle (-\alpha - 2\pi/3)$$

$$\bar{I}_C = \frac{\bar{V}_{CN}}{\bar{Z}_L} = \frac{V_L}{\sqrt{3}} \angle 2\pi/3 \frac{1}{Z \angle \alpha} = \frac{V_L}{\sqrt{3}Z} \angle (2\pi/3 - \alpha)$$

Note that after having found $\bar{I}_A$, we could have straightaway deduced $\bar{I}_B$ and $\bar{I}_C$ invoking the properties of a balanced set of currents. Separate calculations for $\bar{I}_B$ and $\bar{I}_C$ as shown above are therefore not necessary in a balanced system. Figure 4.16 is the phasor diagram showing all the relevant quantities. The line voltages at the load are:

$$\bar{V}_{AB} = V_L \angle \pi/6 ; \quad \bar{V}_{BC} = V_L \angle -\pi/2 ; \quad \bar{V}_{CA} = V_L \angle 5\pi/6 \qquad (4.20)$$

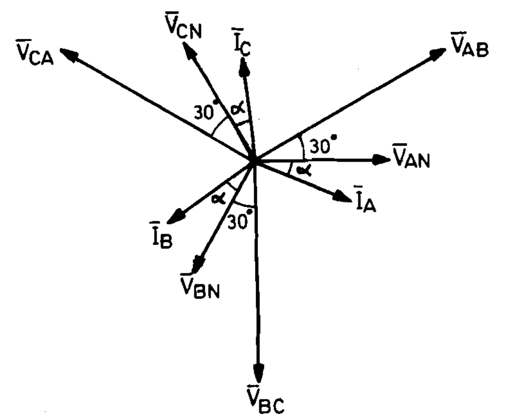

Fig. 4.16 : Phasor diagram for the circuit of Fig. 4.15

The current in the neutral line is $\bar{I}_N = \bar{I}_A + \bar{I}_B + \bar{I}_C$. This is zero for a balanced load fed from a balanced supply. Thus the neutral wire does not carry any current. Even if it is omitted (i.e., even if the balanced 3-phase load is fed from a 3-wire source) the line currents as calculated above do not undergo any change. It would thus appear that the provision of a neutral wire is not necessary. However, in practice, there is no assurance that the loads would be perfectly balanced on the three phases. In the event of an unbalance in the load impedances, the neutral wire serves to maintain the balance of the load phase voltages and carries the out of balance current $\bar{I}_N = \bar{I}_A + \bar{I}_B + \bar{I}_C$.

Let us now consider a delta-connected source connected to the same star-connected load of Figure 4.15 in a 3-wire configuration. If a delta-connected source has the same line voltages $\bar{V}_{AB}$, $\bar{V}_{BC}$ and $\bar{V}_{CA}$, it can be deduced that the load phase voltages and hence the load currents will remain the same as in the earlier analysis. In other words, such a delta connected source is equivalent to a 3-terminal star-connected source with phase voltages given by Eq. (4.19). This equivalence comes about because both have the same line to line terminal voltages.

Example 4.4

Three impedances of 100 +*j*80 ohms each are connected in star across a balanced 400 V, 3-phase, 3-wire supply. Find the line currents taken by the load and the voltage across each impedance. Draw a phasor diagram.

Solution

Since we are interested only in the load currents and voltages, it is immaterial to us whether the source is connected in delta or star. We are given that V_L = 400 V. Hence $V_P = 400/\sqrt{3} = 231$ V. This is the voltage across each impedance.

$I_P = V_P/Z = 231/\sqrt{100^2 + 80^2} = 1.8\,A$. The line currents also have a value of 1.8 A. The phasor diagram taking $\bar{V}_{AN}$ as reference is given in Figure 4.17(b). Note that each phase current (e.g. $\bar{I}_{AN}$) lags the respective phase voltage (e.g. $\bar{V}_{AN}$) by $\tan^{-1}(80/100) = 39°$.

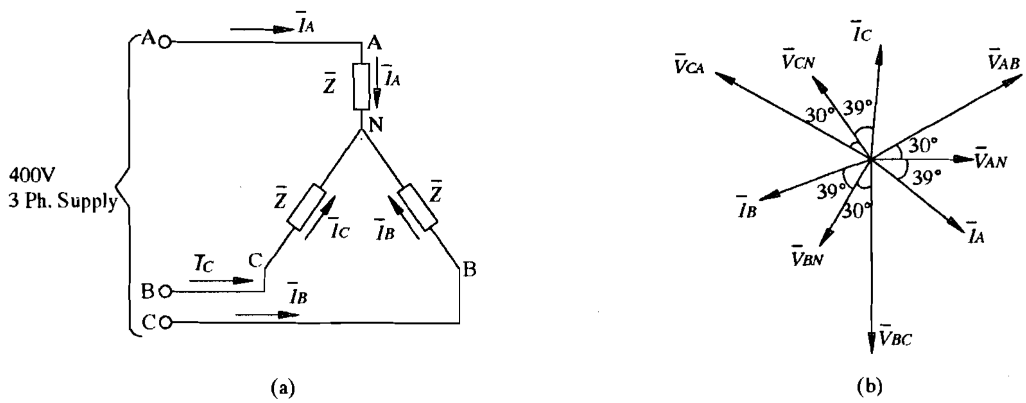

Fig. 4.17 for Example 4.4 (a) Circuit diagram (b) Phasor diagram

4.4.2 Delta Connected Load Fed from a Balanced Supply

Figure 4.18 shows a balanced delta-connected load fed from a star-connected balanced source in a 3-wire system. Let the line voltages have an rms value of V_L volts and let $\bar{V}_{AB}$ be the reference. Thus

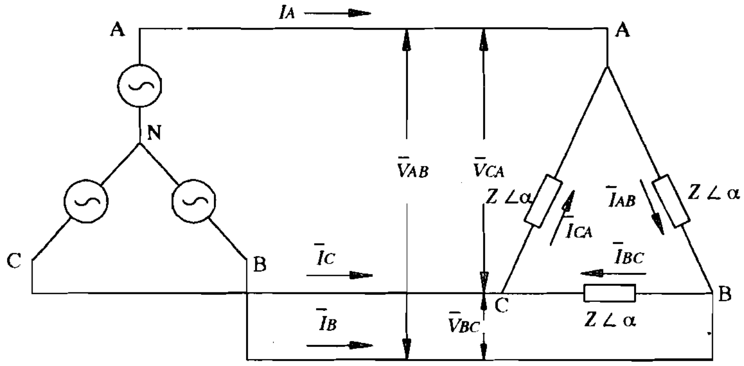

Fig. 4.18 : Delta-connected load fed from a balanced supply

$$\bar{V}_{AB} = V_L \angle 0\,;\ \bar{V}_{BC} = V_L \angle -2\pi/3\,;\ \bar{V}_{CA} = V_L \angle 2\pi/3 \tag{4.21}$$

The line voltages are also the phase voltages, directly appearing across the three impedances. We then have,

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z \angle \alpha} = \frac{V_L}{Z} \angle(-\alpha)\,;\ \bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z\angle\alpha} = \frac{V_L}{Z}\angle(-\alpha - 2\pi/3)$$

$$\bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z\angle\alpha} = \frac{V_L}{Z}\angle(2\pi/3 - \alpha) \tag{4.22}$$

We therefore have for the phase current, $I_P = V_p / Z$

Now $\quad \bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = I_P \angle(-\alpha) - I_P \angle(2\pi/3 - \alpha)$

$$= \sqrt{3} I_P \angle(-\alpha - \pi/6)$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB} = \sqrt{3} I_P \angle(-\alpha - 5\pi/6)$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC} = \sqrt{3} I_P \angle(\pi/2 - \alpha)$$

Note that the three line currents have an effective value

$$I_L = \sqrt{3} V_L / Z$$

The phasor diagram showing the voltages as well as the phase and line currents at the load is given in Figure 4.19. The phase voltages of the star-connected source have a magnitude $V_L/\sqrt{3}$ and have a displacement of 30° from the pertinent line voltages as already seen in Section 4.3.3.

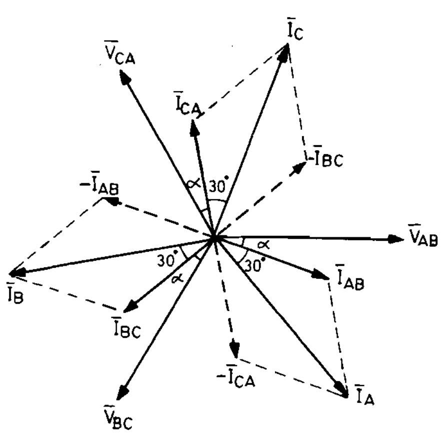

Fig. 4.19: Phasor diagram for the circuit of Fig. 4.18

Thus $\bar{V}_{AN} = V_L/\sqrt{3} \angle -\pi/6$

$\bar{V}_{BN} = V_L/\sqrt{3} \angle -5\pi/6$

$\bar{V}_{CN} = V_L/\sqrt{3} \angle \pi/2.$

These are not drawn in Figure 4.19.

Obviously, a delta connected source with the same line voltages as the star-connected source would produce the same set of currents in the load.

Example 4.5

If the three impedances of Example 4.4 are reconnected in delta and fed from the same 400 V, 3-phase balanced supply, what would be the line and phase currents? Draw a phasor diagram.

Solution

So as to compare the answers with those in Example 4.4, let us take $\bar{V}_{AB}$ to have the same phase in both.

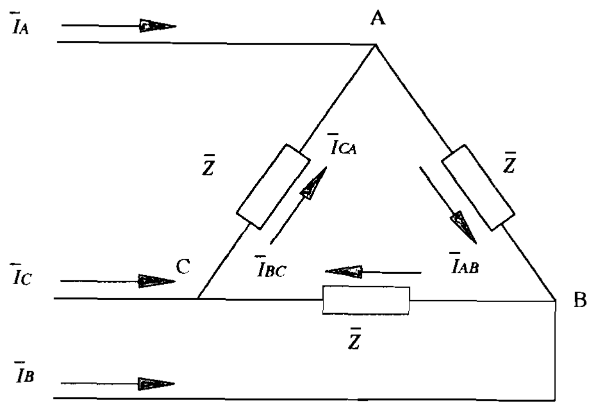

Fig. 4.20 : for Example 4.5

Thus $\bar{V}_{AB} = 400 \angle 30°$

$\bar{V}_{BC} = 400 \angle -90°$

$\bar{V}_{CA} = 400 \angle 150°$

We have $\bar{I}_{AB} = \dfrac{\bar{V}_{AB}}{\bar{Z}} = \dfrac{400 \angle 30°}{100 + j80} = \dfrac{400 \angle 30°}{128 \angle 39°} = 3.1 \angle -9°$

$\bar{I}_{BC} = 3.1 \angle -129°$; $\bar{I}_{CA} = 3.1 \angle 111°$

Thus $I_P = 3.1\,A$

$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = 3.1 \angle -9° - 3.1 \angle 111° = 5.4 \angle -39°$

$\bar{I}_B = 5.4 \angle -159°$; $I_C = 5.4 \angle 81°$

Thus $I_L = 5.4\,A$

Note that all line currents are trebled in comparison with those in Example 4.4. To maintain the same current, the delta-connected impedances should have three times the value of star-connected impedances. This is in keeping with Eq. (4.11).

SAQ 12

What is the function of a neutral wire in a 3-phase 4-wire system? What current does it carry in a balanced system?

SAQ 13

A balanced star-connected load draws 5 A line currents from a 1000 V 3-phase source. If the current in a particular phase leads the corresponding phase voltage by 60°, find the complex value of impedance in each phase of the load.

SAQ 14

A star-connected source feeds a delta-connected balanced load containing 60 + j90 ohms in each phase. If the line currents in the circuit have 4 A effective value, find the phase voltage of the source.

4.4.3 Power Considerations

Having considered the methods of analysis of simple balanced three-phase circuits, let us now examine how the power in a 3-phase circuit may be computed.

The total power/reactive power supplied by a 3-phase source can be computed as the sum of the powers/reactive powers supplied by the three single phase sources. In fact the complex power supplied by the 3-phase source is equal to the sum of the complex powers supplied by the three individual single phase sources. A similar statement is true of loads as well. The total power/reactive power/complex power drawn by a 3-phase load is the sum of individual values of power/reactive power/complex power drawn by the three impedances constituting the 3-phase load.

Let P_A, P_B and P_C be the values of power consumed in the three phases A, B, C of the balanced star connected load of Figure 4.15.

Then the total power P_T consumed by the load would be :

$$P_T = P_A + P_B + P_C = V_{AN} I_A \cos\alpha + V_{BN} I_B \cos\alpha + V_{CN} I_C \cos\alpha$$
$$= 3\, V_P I_P \cos\alpha\,,$$

as $\quad V_{AN} = V_{BN} = V_{CN} = V_P \quad$ and $\quad I_A = I_B = I_C = I_P$.

But since in a star-connected system $V_P = V_L/\sqrt{3}$ and $I_P = I_L$, an alternative expression for P_T would be

$$P_T = 3(V_L/\sqrt{3})\, I_L \cos\alpha = \sqrt{3} V_L I_L \cos\alpha.$$

Similarly, the power consumed by the balanced delta-connected load of Figure 4.18 would be

$$P_T = P_{AB} + P_{BC} + P_{CA} = V_{AB} I_{AB} \cos\alpha + V_{BC} I_{BC} \cos\alpha + V_{CA} I_{CA} \cos\alpha = 3\, V_P I_P \cos\alpha\,,$$

as $\quad V_{AB} = V_{BC} = V_{CA} = V_P \quad$ and $\quad I_{AB} = I_{BC} = I_{CA} = I_P$.

In a delta-connected circuit, $V_L = V_P$ and $I_L = \sqrt{3} I_P$. Hence an alternative expression for P_T would be

$$P_T = 3V_L(I_L/\sqrt{3}) \cos\alpha = \sqrt{3}\, V_L I_L \cos\alpha.$$

In summary, we have the following two equivalent expressions for total power consumed by a 3-phase load, irrespective of whether the load is connected in star or delta.

$$\mathbf{P_T = 3P_{ph} = 3V_P I_P \cos\alpha} \qquad (4.23a)$$
$$\mathbf{= \sqrt{3} V_L I_L \cos\alpha\,,} \qquad (4.23b)$$

where P_{ph} is the power in one phase. The formula in Eq. (4.23b) is often used since it expresses P_T in terms of externally measurable quantities V_L and I_L. It is important to note that even in this expression, α is the angle of the balanced load impedances or the angle of lag of a phase current with respect to the associated phase voltage. It does not represent the angle between a line voltage and a line current.

It can be similarly shown that the total *reactive power* Q_T and *apparent power* S_T of the balanced 3-phase load are given by

$$Q_T = 3\ V_P\ I_P \sin\alpha = \sqrt{3} V_L I_L \sin\alpha\ ;\ S_T = 3\ V_P\ I_P = \sqrt{3}\ V_L\ I_L \qquad (4.24)$$

Thus

$$P_T = S_T \cos\alpha\ ;\ Q_T = S_T \sin\alpha = P_T \tan\alpha \qquad (4.25)$$

The *power factor (p.f.) of the balanced load* is defined in a similar manner as in single phase circuits.

P.F. of a balanced load = $(P_T/S_T) = \cos\alpha$, (4.26)

where α is the angle of the balanced load impedances.

When several 3-phase balanced loads are connected on a system the aggregate active power P_a is the sum of the individual three-phase powers. The aggregate reactive power Q_a is the sum of the individual three-phase reactive powers. However the aggregate apparent power S_a is not the sum of the individual apparent powers as the p.f. of each balanced load may be different. S_a and the overall p.f. are given by

$$S_a = \sqrt{P_a^2 + Q_a^2} \qquad (4.27)$$

$$\text{Overall p.f.} = \frac{P_a}{S_a} \qquad (4.28)$$

SAQ 15

Fill up the blanks

The p.f. of a balanced 3-phase load is cosine of the .... [1] of impedance in each phase. It is also equal to cosine of the angle between a phase voltage and the corresponding .... [2]. Furthermore it is the ratio of total power to .... [3] of the load.

Example 4.6

Find P_T, Q_T and S_T taken by the load in Example 4.4. What is the value of p.f. of the balanced load.

Solution

$$P_T = 3P_{ph} = 3V_P I_P \cos\alpha = 3 \times 231 \times 1.80 \times \frac{100}{(100^2 + 80^2)^{1/2}} = 974\ \text{W}$$

$$Q_T = 3Q_{ph} = 3\ V_P I_P \sin\alpha = 3 \times 231 \times 1.80 \times \frac{80}{(100^2 + 80^2)^{1/2}} = 779\ \text{VAR}$$

$$S_T = \sqrt{P_T + Q_T^2} = \sqrt{974^2 + 779^2} = 1247\ \text{VA}$$

Alternative :

$$P_T = \sqrt{3} V_L I_L \cos\alpha = \sqrt{3} \times 400 \times 1.80 \times \frac{100}{(100^2 + 80^2)^{1/2}} = 974\ \text{W}$$

$$Q_T = \sqrt{3}\ V_L I_L \sin\alpha = \sqrt{3} \times 400 \times 1.80 \times \frac{80}{(100^2 + 80^2)^{1/2}} = 779\ \text{VAR}$$

$$S_T = \sqrt{3}\ V_L I_L = \sqrt{3} \times 400 \times 1.8 = 1247\ \text{VA}$$

P.F. of the balanced load = $100/\sqrt{100^2 + 80^2}$ = 0.78 lagging

Example 4.7

A 3-phase motor takes 5 kW at 0.8 p.f lagging from a 3-phase 400 V supply. If the motor is represented by a balanced delta- connected set of impedances, find the impedance in each phase. What is the reactive power taken by the motor?

Solution

$P_T = 3V_P I_P \cos\alpha$

$5000 = 3 \times 400 \times I_P \times 0.8 \Rightarrow I_P = 5.21$ A

$Z_P = V_P/I_P = 400/5.21 = 76.78\ \Omega$

$\bar{Z}_P = 76.78 \cos\alpha + j76.78 \sin\alpha = 61.42 + j46.07\ \Omega$

$Q_T = 3V_P I_P \sin\alpha = P_T \tan\alpha = 5000 \tan(\cos^{-1} 0.8) = 3750$ VAR

SAQ 16

Find P_T, Q_T and S_T taken by the load in Example 4.5.

SAQ 17

Three impedances each of $100 + j100$ ohms are connected in delta across a balanced 400 V, 3-phase supply. Find the total power consumed by the load and its p.f.

Example 4.8

If three capacitors of C farads each are connected in delta across a 3-phase balanced supply of line voltage V_L, find the power, reactive power and apparent power taken by the three capacitors.

Solution

In delta connection $V_L = V_P$

$I_P = (wC)V_P = wCV_P = wCV_L$

$\cos\alpha = 0$ and $\sin\alpha = -1$ for purely capacitive impedances.

$P_T = 3V_P I_P \cos\alpha = 0$

$Q_T = 3V_P I_P \sin\alpha = 3V_L (wCV_L)(-1) = -3wCV_L^2$ VAR

$S_T = |Q_T| = 3wCV_L^2$ VA

Note that the reactive power taken by the capacitors is taken as negative by convention.

4.4.4 Power Factor Correction

The necessity for and the principle of power factor correction have been explained in Unit 3. We now examine this process in relation to 3-phase circuits. In fact it is mainly 3-phase installations that call for power factor correction since they consume bulk quantities of electrical power and since the tariffs for supply of electricity to such installations penalise low power factor operation. As most industrial loads operate at lagging power factors, the process of power factor improvement amounts to connecting a 3-phase load comprising capacitors in parallel with the existing load. Such a capacitive load takes the form of several small capacitors of practical size connected in parallel in each phase, the entire assembly being referred to as a *3-phase bank of capacitors*.

It has already been remarked in Section 4.4.3 that when several 3-phase loads are in parallel, the powers and reactive powers of individual loads can be added to obtain the aggregate values of power and reactive power. This principle can be used as the key to the calculation of the required capacitor values for power factor improvement.