UNIT 14 REINSURANCE

Structure

14.0 OBJECTIVES

After going through this unit you will be able to:

- understand the underlying idea behind reinsurance;
- identify the forms of reinsurance operation;
- estimate reinsurance claims and losses; and
- analyse reinsurance pricing provisions contained in different models.

14.1 INTRODUCTION

Reinsurance is a means by which an insurance company shares the risk of loss with another insurance company. It is not difficult to see that the demand for reinsurance emerges as a part of the responsibility of the insurance company to manage a portfolio of risks for the benefit of its policy holders. Reinsurance allows the insurance (ceding) company to assume individual risks greater than its size would otherwise allow. Moreover it protects the cedent against catastrophic losses.

The major functions performed by reinsurance are:

Risk Transfer: The insurance company can absorb larger losses and would reduce the amount of capital needed to provide coverage.

Surplus Relief: Reinsurance effectively reduces the amount of net liability and helps build increase company's surplus.

Arbitrage: An insurance company may be motivated to go for arbitrage by way of purchasing reinsurance coverage at a lower rate than the expected cost of the underlying risk. Viewed in the above context, the demand for reinsurance brings forth a case of more general issue concerning how one has designed contract so as to allow for efficient allocation, pricing and management of risk.

Remember that the traditional approach to risk management is typically built upon an analysis of specific type operating or financial risk. If you do not remember, operating risks include uninsurable business risk such as loss of competitive position, product substitution and obsolescence as well as risk for which insurance is often available (cases such as property-liability risks or workplace risks that can adversely effect employee life and health). Financial risks, on the other hand, include credit risk as well as risks resulting from market fluctuations in the commodity prices, currencies and financial assets. Insurance and derivatives are commonly used to control the cost of 'insurable' operating and financial risks. In case of the class of uninsurable business risks that are usually difficult to quantify and manage, one has to resort to alternative contract designs and organizational structures.

In contrast to the traditional approach, there has emerged an integrated approach to risk management in recent years. This approach is basically adopted by brining together the insurance and financial markets along with the increasing frequency and severity of natural and man-made catastrophes.

Once we accept that risk is costly, our primary concern is to find out how reinsurance can be used to reduce the cost of risk for insurance companies. We will have to address this question by analysing the economic and financial conditions that are necessary in order for reinsurance to add value to the insurance enterprise.

Discussion in the following on the section of reinsurance pricing draws heavily from the work of Kreps (1998). The equations and their sequencing have been adopted without change. Keeping in view the overall level of exposition of the

themes in the present course we have selected the basic concepts of the above work. However, it will be useful to see the entire work once you have understood the arguments employed in analysing the reinsurance pricing.

14.2 TYPES AND FORMS OF REINSURANCE

For the purposes of this unit, it will be assumed that the reinsurance considered is of two forms, viz., proportional and non-proportional

14.2.1 Proportional Reinsurance

Under a proportional reinsurance arrangement, the original insurance company and the reinsurer share the cost of all claims for each risk. The direct writer must pay a premium to effect this reinsurance. This premium may, or may not, be calculated on the same basis as the premiums charged by the insurer.

Proportional reinsurance operates in two forms, quota shares and surplus. With quota share reinsurance, the proportions are the same for all risks. On the other hand, with surplus reinsurance, the proportions can vary from one risk to another.

14.2.2 Non-Proportional Reinsurance

Under a non-proportional reinsurance, the direct writer pays a fixed premium to the reinsurer. The reinsurer will be required to make payments where part of the claim amount falls in a particular reinsurance layer. The layer will be defined by a lower limit, and an upper limit (or "infinity" if the cover is unlimited).

Two forms of non-proportional reinsurance are:

Individual excess of loss (XOL) reinsurance: In this the reinsurer is required to make a payment when the claim amount for any individual claim exceeds a specified excess point or retention.

Stop loss reinsurance: With this form the reinsurer makes payments when the total claim amount for a specified group of policies exceeds a specified amount (usually expressed as a percentage of the gross premium). For example, the reinsurer might agree to pay 95% of the excess when the total claim amount for all car insurance policies exceeds 102% of the total gross premium, with no upper limit.

14.3 REINSURANCE ARRANGEMENT

Let us start with some helpful terms to be used in the discussion that follows:

1) Net Claim

The actual claim amount that the direct insurer has to pay after allowing for payments under the reinsurance arrangements is called the net claim amount.

2) Net Premium

The actual premium that the direct writer keeps after making any payments for reinsurance is called the net premium.

3) Gross Claim/Premium

The original amounts without adjustment for reinsurance are referred to as the gross claim amount and the gross premium.

14.3.1 Excess of Loss Reinsurance –The Insured

In excess of loss reinsurance, the insurer pays any claim in full up to an amount M, the retention level; an amount above M is borne by the reinsurer. Symbolically, we can write

$$Y = X \text{ if } X \leq M$$

$$Y = M \text{ if } X > M \text{ i.e., } Y = \min(X,M)$$

where X denotes the claim amount and Y denotes the amount paid by the insurer.

The reinsurer pays the amount $Z = Y - Y$, i.e., $Z = \max(X - M,0)$.

when a layer between M and 2M is reinsured, the amount paid by the insurer is

$$Y = X \text{ if } X \leq M$$

$$Y = M \text{ if } M < X \leq 2M$$

$$Y = X - M \text{ if } X > 2M$$

The insurer's liability is affected in two obvious ways by reinsurance:

1) the mean amount paid is reduced;

2) the variance of the amount paid is reduced.

Both the mean and the reduction in the mean of the amount paid by the insurer under excess of loss reinsurance can be obtained.

Observe that the mean amount paid by the insurer without reinsurance is

$E(X) = \int_0^\infty x f(x)\,dx$ where $f(x)$ is the PDF of the claim amount X.

With a retention level of M the mean becomes

$$E(Y) = \int_0^M x f(x)\,dx + M\ \mathrm{P}(X > M)$$

The main difficulty in dealing with excess of loss reinsurance is that the integral is incomplete.

With excess of loss reinsurance there is a method that converts the incomplete integral to a complete integral.

$$\begin{aligned} E(Y) &= \int_0^\infty xf(x)dx - \int_M^\infty xf(x)dx + M\int_M^\infty f(x)\,dx \\ &= E(X) - \int_M^\infty (x-M)f(x)dx \\ &= E(X) - \int_0^\infty zf(z+M)dz \text{ by setting } z = x-M. \end{aligned}$$

If $X \sim Pareto(\alpha,\lambda)$, and the retention level is M, then

$$\begin{aligned} E(Y) &= \frac{\lambda}{\alpha-1} - \int_0^\infty z\frac{\alpha\lambda^\alpha}{(\lambda+z+M)^{\alpha+1}}dz \\ &= \frac{\lambda}{\alpha-1} - \left(\frac{\lambda}{\lambda+M}\right)^\alpha \int_0^\infty z\frac{\alpha(\lambda+\alpha)^\alpha}{(\lambda+z+M)^{\alpha+1}}dz \\ &= \frac{\lambda}{\alpha-1} - \left(\frac{\lambda}{\lambda+M}\right)^\alpha \frac{\lambda+M}{\alpha-1} \end{aligned}$$

For cases where this integral is not easy to calculate, the formula has limited use.

If $X \sim Gamma(\alpha,\lambda)$, and the retention level is M, then

$$E(Y) = \frac{\alpha}{\lambda} - \int_0^\infty z\frac{1}{\Gamma(\alpha)}\lambda^\alpha (z+M)^{\alpha-1} e^{-\lambda(z+M)}dz$$

This is not going to be easy to evaluate either.

14.3.2 Excess of Loss Reinsurance-The Reinsurer

The reinsurer may have a record only of claims that are greater than M. If a claim is for less than M, then the reinsurer may not even know a claim has occurred. The reinsurer thus has the problem of estimating the underlying claims distribution when only those claims greater than M are observed.

The statistical terminology is to say that the reinsurer observes claims from a truncated distribution.

In this case, the values observed by the reinsurer relate to a conditional distribution, since the numbers are conditional on the original claim amount exceeding the retention limit.

We write the random variable for the amount paid by the reinsurer as

$$Z = X - M \mid X > M\,.$$

Suppose that the underlying claim amounts have $PDF - f(x)$ and CDF $- F(x)$. Suppose that the reinsurer is only informed of claims greater than the retention

M and has a record of $z = x - M$. What is the PDF g(z) of the amount, z, paid by the reinsurer?

The argument goes as follows:

$$P(Z < z) = P(X < z + M \mid X > M)$$
$$= \int_M^{z+M} \frac{f(x)}{1 - F(M)} dx$$
$$= \frac{F(z+M) - F(M)}{1 - F(M)}$$

the PDF of the reinsurer's claims is

$$g(z) = \frac{f(z+M)}{1 - F(M)} \quad z > 0.$$

Note that the original PDF applied to the gross amount z + M, divided by the probability that the claim exceeds M.

It is also possible that the reinsurer is aware of all the claims that are made under the original policies. In this case the distribution of the reinsurer's outgo may include those claims on which it does not in fact pay anything.

In such a case we are looking at the unconditional distribution of the reinsurer's outgo Z, where:

$$Z = 0 \text{ if } X < M$$

$$Z = X - M \text{ if } X \geq M$$

The calculation of the mean and variance of the reinsurer's outgo in this case will be similar to the corresponding calculations for the insurer.

If we use W to denote the amount payable by the reinsurer on claims in which it is involved, i.e.,

$$W = Z \mid Z > M$$
$$= X - M \mid X > M,$$

then

$$g(w) = \frac{fz(w)}{1 - Fz(0)}$$

$$E(W) = \frac{E(Z)}{P(Z > 0)} = \frac{E(Z)}{P(X > M)}$$

14.3.3 Proportional Reinsurance

In proportional reinsurance the insurer pays a fixed proportion of the claim, whatever the size of the claim. Using the same notation as above, the proportional reinsurance arrangement can be written as follows:

If the claim is for an amount X, then the company will pay Y where

$$Y = \alpha X \qquad 0 < \alpha < 1$$

The parameter α is known as the retention level.

As the amount paid by the insurer on a claim X is $Y = \alpha X$ and the amount paid by the reinsurer is $Z = (1-\alpha)X$, the distribution of both of these amounts can be found by a simple change of variable.

$$f_Y(y) = \frac{1}{\alpha} f_X\left(\frac{y}{\alpha}\right)$$

$$f_Z(z) = \frac{1}{1-\alpha} f_X\left(\frac{z}{1-\alpha}\right)$$

If X represents the size of an individual claim, the retention level α is 0.8, then the amount paid by the insurer is $Y = 0.8X$, and $E(Y) = 0.8\,E(X)$, $Var(Y) = 0.64\,Var(X)$. The amount paid by the reinsurer is $Z = 0.2X$, and $E(Z) = 0.2E(X)$, $Var(Z) = 0.04\,Var(X)$.

14.4 REINSURANCE CALCULATION

There are a number of useful integral formulae that simplify reinsurance calculations when working with particular distributions. Two useful distributions for this purpose could be lognormal and normal distributions.

14.4.1 Inflation

The examples so far have assumed that claim distribution remain constant over time. In fact claims are likely to increase because of inflation, at least in the longer term. Thus, we need to adjust out claim distributions to allow for inflation.

In this section we will look at how claims in inflation affect reinsurance arrangements.

Suppose that the claims X are inflated by a factor of k but the retention M remains fixed. The amount claimed is kX, and the amount paid by the insurer, Y, is

$$Y = kX \text{ if } kX \le M$$
$$Y = M \text{ if } kX > M$$

The mean amount paid by the insurer is

$$E(Y) = \int_0^{M/k} kxf(x)dx + M\ \mathrm{P}(X > M/k)$$

which can be written as

$$\begin{aligned} E(Y) &= \int_0^{\infty} kxf(x)dx - \int_{M/k}^{\infty} kxf(x)dx + M\int_{M/k}^{\infty} f(x)dx \\ &= kE(X) - k\int_{M/k}^{\infty}(x - M/k)f(x)dx \\ &= k\left(E(X) - \int_0^{\infty} yf(y + M/k)dy\right) \text{ where } \left(y \equiv x - \frac{M}{k}\right) \end{aligned}$$

One important general point that can be made is that the new mean claim amount is not k times the mean claim amount without inflation.

Of course if the retention limit increases by a factor k as well, both mean claim amounts for the insurer and reinsurer will increase by the same factor.

When examining the details of a reinsurance arrangements in real life it is very important to check whether the retention limits are fixed or linked to an agreed inflation index.

14.5 REINSURANCE ESTIMATION

Consider the problem of estimation in the presence of excess of loss reinsurance. Suppose that the claims record shows only the net claims are paid by the insurer.

A typical claims record might be

$$x_1,\ x_2,\ M\ x_3,\ M\ x_4,\ x_5,\ \ldots$$

and an estimate of the underlying gross claims distribution is required.

The statistical terminology for a sample of this form is censored. In general, a censored sample occurs when some values are recorded exactly and the remaining values are known only to exceed a particular value.

The method of moments is not available since even the mean claim amount cannot be computed.

It may be possible to use the method of percentiles without alteration; this would happen if the retention level M is high and only the higher sample percentiles were affected by the (few) reinsurance claims.

Maximum likelihood can be applied to censored samples.

The likelihood is made up of two parts. The complete likelihood is

$$L(\underline{\theta}) = \prod_{1}^{n} f(x_i;\underline{\theta}) \times \left[1 - F(M;\underline{\theta})\right]^m$$

Policy Excess

Insurance policies with an excess are common in motor insurance and many other kinds of property and accident insurance.

Under this kind of policy, the insured agrèes to carry the full burden of the loss up to a limit, L, called excess. If the loss is an amount X, greater than L, then the policyholder will claim only $X - L$. If Y is the amount actually paid by the insurer, then

$$Y = 0 \text{ if } X \leq L$$

$$Y = X - L \text{ if } X > L.$$

Clearly, the premium due on any policy with excess will be less than that on a policy without an excess.

The position of the insurer for a policy with excess is exactly the same as that of the reinsurer under excess of loss reinsurance. The position of the policyholder as far as losses are concerned is exactly the same as that of an insurer with an excess of loss reinsurance contract.

In practice, expenses form a significant part of the insurance cost. So the presence of an excess might not affect the premium as much as might be expected.

Check Your Progress 1

1) What are the main functions of reinsurance?

.....

.....

.....

.....

.....

2) Explain the meaning of proportional and non-proportional reinsurance.

.....

.....

.....

.....

.....

3) What is excess of loss insurance from the view point of an insured?

.....

.....

.....

.....

.....

4) Why do you need to take account of inflation while calculating reinsurance?

.....

.....

.....

.....

.....

5) What technique do you want to use for an estimation of reinsurance in the presence of excess of loss?

.....

.....

.....

.....

.....

14.6 REINSURANCE PRICING

We may think that reinsurance pricing is market-driven. However, to appreciate its basis some description of the economic origin of reinsurance risk load needs to be given. Let us look at the return from the investment to be made when there is insurance risk. Here we have to take decision considering a combination of the reinsurance contract and financial techniques comes to play. The inputs are the loss distributions and reinsurer's underwriting constraints.

Although the forces of market will determine the equilibrium price to be charged for contracts, both insurer and reinsurer have to use an economic pricing model to help decide whether to write the contract. Correspondingly the combination of a reinsurance arrangement and suitable financial techniques will become a basis for making investment choice.

Let us start with the assumption that investment criteria is based on the form of a target mean return and risk measure thereon.

When the reinsurer accepts a contract, she arranges the availability of sufficient liquefiable assets to cover possible losses up to some safety level. These assets arise from premium and assets allocated from surplus, both of which are invested in appropriate financial instruments. So the reinsurer attempts to have at least as favourable return and risk over the period of the contract as possible when setting the target investment.

Such a behaviour looks at risk load as an opportunity cost and represents it as a (partially offset) cost of liquidity.

The loss safety level is essentially a measure of reinsurer constraint in the insurance business. The more weightage attached to it the less probable it is that the safety level will be exceeded. However, higher safety levels will typically result in more expensive contracts.

To understand the idea consider the following example (due to Kreps, 1998) Consider a person who decides to build a house in snow country. The question is, how strong to build the roof for snow load? If it is a cabin for only a few years, perhaps building to survive the 10 year storm will be enough. If it is meant for the grandchildren, perhaps the 200 year storm is more appropriate. It is, of course, more expensive to build it stronger. In any case some level is chosen depending on the builder's criteria.

Thus, risk load involves the cost of capital and measures of investment return and risk in particular. If capital were freely available, insurance, much less reinsurance, would be unnecessary since a firm would borrow to overcome difficulties. We will consider here standard deviation (or variance) measure of investment risk although there are more sophisticated strictly downside measures, such as a semi-variance or the average value of the (negative) excess of return. In the cases where very large losses may generate negative results, such a downside risk measure may be more desirable.

We consider two types of simple financial techniques: First, risk-free instrument such as government securities. This is called a swap in finance literature. Cost associated with it is the loss of investment income. There is a gain in terms of reduced risk, however.

This technique offers a simple formula. But often it turns out to create a higher risk load. Hence it is more expensive to the cedent.

The second technique is buying "put" options. You may recall that in put option there is the right to sell the underlying target investment at a predetermined strike price at maturity (European options). Here the strike price will be equal to investment in risk-free securities. So the reinsurer can buy the right to sell the target investment at a return not less than the risk-free rate.

The Black-Scholes formula is used to price the option. The distribution of investment returns underlying this formula is assumed for the reinsurer's target investment. The cost of these options will contribute to the risk load, but this is

partly offset because the options both increase the return and decrease the variance of the target investment.

14.6.1 Single Payment at One Year

We follow Kreps (1998)'s formulation and let the principal determinants of interest to be

s = the dollar safety level associated with the loss distribution.

L = the amount of the loss.

μ_L = the mean value of the loss.

σ_L = the standard deviation of the loss.

r_f = the risk-free rate.

y = the yield rate of the target investment.

σ_y = the standard deviation of the investment yield rate.

P = the premium net to the reinsurer after expenses.

Quantities derived from the above are

A = the assets allocated by the reinsurer.

F = the funds initially invested: premium and assets less option cost, if applicable.

R = the risk load in the premium: the premium less the discounted expected loss.

In all cases the premium comprises the risk load the expected loss discounted at the risk-free rate. Note that the premium above does not include any reinsurer expenses. Then for a single payment at one year,

$$p = R + \frac{\mu_L}{1 + r_f} \tag{14.6.1}$$

The constraints are: (1) the investment result from F as input must be at least s, and (2) the standard deviation of the overall result must be no larger than σ_y.

The cash flow relations are stochastic. However, it is possible to obtain explicit formulae for the mean and variances involved and get explicit forms for the risk load. Let us work to obtain mean and variance.

14.6.1.1 Swap Case

Let the reinsurer have an inflow of P and an outflow of

$$F = (P + A) \tag{14.6.2}$$

Since the investment is in risk-free securities, at the end of the year she has an inflow of $(1+r_f)F$ and an outflow of the loss L. The internal rate of return (IRR) on these cash flows is defined by the stochastic relation

$$(1+\text{IRR})\text{A} = (1+r_f)F - L \tag{14.6.3}$$

where L and IRR are stochastic variables. Take the mean value of this equation and the mean value of the IRR to be the yield rate y to get,

$$(1+y)A = (1+r_f)F - \mu_L \tag{14.6.4}$$

which may be expressed (See Appendix 3)as

$$R = \frac{(y-r_f)}{(1+r_f)}A \tag{14.6.5}$$

We need another equation to solve the system with two constraints, a loss safety constraint and an investment variance constraint. In general, it is by making the asset base large enough, we can make the fractional variability as small as desired and the funds available as large as desired. Hence there is always a solution. Both constraints may be placing lower limits on the allocated assets.

Safety constraint: funds available at the year end has to be greater than or equal to the safety level. So

$$(1+r_f)F \geq s\,. \tag{14.6.6}$$

Combining Eqs. (14.6.4) and (14.6.6) to eliminate F, we have

$$A \geq \frac{(s-\mu_L)}{1+y} \tag{14.6.7}$$

From Eqs. (14.6.5) and (14.6.7) the risk load at the equality is

$$R = \frac{(y-r_f)(s-\mu_L)}{(1+r_f)(1+y)} \tag{14.6.8}$$

From Equation (14.6.1) premium before expenses is

$$P = R + \frac{\mu_L}{1+r_f} \tag{14.6.9}$$

which gives the result for the safety constraint.

Variance Constraint:

Since there is no variability in the investment return because it is risk-free, the standard deviation of the IRR is

$$A\sigma_{IRR} = \sigma_L \tag{14.6.10}$$

from Equation (14.6.3).

Investment Constraint: The IRR should have variance less than or equal to that of the target investment, so that

$$A \geq \sigma_L / \sigma_y \tag{14.6.11}$$

and from equation (14.6.5), we get

$$R = \frac{(y - r_f)}{(1 + r_f)}(\sigma_L / \sigma_y). \tag{14.6.12}$$

Given the values for the loss distribution and the target investment the latter is likely to be the more stringent constraint will hold when

$$(s - \mu_L)/\sigma_L < (1 + y)/\sigma_y. \tag{14.6.13}$$

14.6.1.2 Option Case

Consider the case where the reinsurer receives the premium at time zero, but keeps the initial assets invested in the target investment. She also buys an option to sell the target investment at the end of the year for the value that the risk-free technique would have achieved. By doing so she has obtained an instrument that eliminates a portion of the investment return distribution which lies below the risk-free rate. This will have two effects: increasing the mean return from the investment and decreasing its standard deviation.

Let

r = the rate (cost per dollar of investment protected) of a put option.

I = investment return

i = mean investment return (see Appendix 2).

The value of r depends upon the underlying investment parameter σ, which is determined by y and σ_y, (see Appendix 1). For small values of the ratio of σ_y, to $(1+y)$, it is approximately true that

$$\sigma = \frac{\sigma_y}{(1+y)} \tag{14.6.14}$$

and

$$r = \frac{1}{\sqrt{2\pi}}\sigma\left(1 - \frac{\sigma^2}{24}\right). \qquad (14.6.15)$$

While at time zero the reinsurer has an inflow of P and an outflow of $(P + A)$, the funds available for investment have decreased by the cost of the option. so, Eq. (14.6.2) indicates

$$F = P + A - rF \qquad (14.6.16)$$

or $$F = \frac{(P + A)}{(1 + r)}. \qquad (14.6.17)$$

We are considering investment in risky securities (hedging at the bottom end so that these to do not drop below the risk-free rate). At the end of the year, the reinsurer has an inflow of $(1 + I)F$ and an outflow of the loss, L. The internal rate of return on these cash flows is defined by the stochastic relation

$$(1 + IRR)A = (1 + I)F - L. \qquad (14.6.18)$$

Since we must have mean value of IRR = target yield rate,

$$(1 + y)A = (1 + i)F - \mu_L. \qquad (14.6.19)$$

While it is not easy to simplify this we have two unknowns R and A at our disposal and this is one equation relating them. The other equation will come from constraints, the one which is more restrictive.

Take the loss safety constraint on the funds available:

$$(1 + r_f)F \geq S.$$

Note that the actual funds available are likely to be larger than this, since r_f, represents the minimum value of the realisable investment return, due the option.

Combining Eqs. (14.6.6) and (14.6.19) to eliminate F, the allocated assets are obtained such that

$$\mathrm{A} \geq \frac{1}{1 + y}\left\{\frac{(1 + i)}{(1 + r_f)}s - \mu_L\right\}. \qquad (14.6.20)$$

This is larger than in the swap case since $i > y > r_f$.

The risk load at equality is given by

$$\mathrm{R} = \frac{1}{(1 + r_f)}\left[s\{(1 + y)(1 + r) - (1 + i)\} - \mu_L(y - r_f)\right]. \qquad (14.6.21)$$

Using the above two formulae and putting $i = r_f$ and $r = 0$, the results of the previous section can be obtained.

For formulating the investment variance constraint it is necessary to obtain the correlation between the loss and the investment return. The linkage by inflation suggests that there may be a negative correlation-if inflation rises, (claims costs rise and bond values fall). We make a simplifying assumption that the correlation is zero. The standard deviation of the investment return is derived in Appendix 2 and written as σ_i. When the variance of the IRR is required to be that of the target investment, we get

$$\left(A\sigma_y\right)^2 = \left(F\sigma_i\right)^2 + \left(\sigma_L\right)^2 . \tag{14.6.22}$$

On substituting the value of the initial fund F from the equation for the mean, we have

$$-aA^2 + 2bA + c = 0 \tag{14.6.23}$$

with

$$a = \sigma_y{}^2\left(1+i\right)^2 - \sigma_i{}^2\left(1+y\right)^2 \tag{14.6.24}$$

$$b = L\left(1+y\right)\sigma_i{}^2 \tag{14.6.25}$$

$$c = L^2\sigma_i{}^2 + \sigma_L{}^2\left(1+i\right)^2 \tag{14.6.26}$$

The coefficients, a, b, and c are positive: last two due to their explicit construction and the first because the option decreases the variance and increases the mean of the investment return compared to the target values.

Thus,

$$A = \frac{b + \sqrt{b^2 + ac}}{a} \tag{14.6.27}$$

and

$$R = A\frac{\left(1+r\right)\left(1+y\right) - \left(1+i\right)}{1+i} + L\left[\frac{1+r}{1+i} - \frac{1}{1+r_f}\right]. \tag{14.6.28}$$

In the limit as $\sigma_i \to 0$ the solution for A is same as the ratio of standard deviations; with $i = r_f$ and $r = 0$ the risk load is the same as earlier form found in case of swap.

14.6.1.3 High Excess Layer and Minimum Premium

The above formulae can be applied to the case of a high excess layer or any similar finite rare event cover. A non-zero rate on line (ratio of premium to limit) is predicted even for cases where the loss probability goes to zero.

Consider the loss distribution to be binomial. The probability, p, is hitting the layer, and if it hits there is a total loss. The safety level, S, is the limit (total amount payable) of the layer.

The mean loss μ_L is ps and the variance of the loss is $p(1-p)s^2$. As p gets smaller, corresponding to higher and higher layers, in both the swap and option cases the variance constraint yields A and R to be zero. However, the safety constraint in both cases is linear in p with a non-zero intercept. In the option case, the rate on line (ROL) in the limit $p \to 0$ is

$$ROL = \frac{(1+y)(1+r)-(1+i)}{(1+r_f)(1+y)}. \tag{14.6.29}$$

We get this when in Eq. (14.6.21) $L = 0$ and recalling that ROL is the ratio of R to S.

See that the swap version may be obtained by letting $r = 0$ and $i = r_f$, which results in

$$ROL = \frac{1}{1+r_f} - \frac{1}{1+y} = \frac{y-f}{(1+r_f)(1+y)}. \tag{14.6.30}$$

Such a from suggests that the minimum ROL is of the order of the real target return i.e., the excess of the return over the risk-free rate.

14.6.2 Single Payment at Arbitrary Time

If we went to express the returns in the preceding discussion returns in terms of the equivalent annualized returns, the results hold after the following replacements are made:

$$(1+i) \to (1+i)' \tag{14.6.31}$$

$$(1+y) \to (1+y)' \tag{14.6.32}$$

$$(1+r_f) \to (1+r_f)' \tag{14.6.33}$$

The forms for the option rate and the standard deviations given in Appendix 2 contain the time dependence.

14.6.3 Multiple Payments

In case of possible multiple loss payments, the same formulation in case single payment can be used. However, it needs a more complex formulation. In the single payment case, simultaneously enforcing the rate of return (through the mean value of the stochastic equation) and the safety constraint yielded an easy solution. However, in multiple payments, the variance constraint cannot be derived easily and it can be evaluated only for any given level of allocated assets.

Thus general procedure to follow is: (1) express the fundamental stochastic process on a spreadsheet. It is now more than a simple equation because of the interaction of the fund levels at different times, but it is still easily expressed; (2) define the safety levels; (3) use those to define what funds are needed and what options need to be bought; (4) find the risk load corresponding to the target return for the indicated safety constraint by putting all the stochastic variables at their mean values; (5) simulate to see if the variance constraint is satisfied; (6) if it is not, then add "excess capital" and simulate again; (7) repeat step until both constraints are satisfied.

Check Your Progress 2

1) Which measures of risk, you would like to use while taking decision on investment in reinsurance?

.....

.....

.....

.....

.....

2) What is the difference between swap and option?

.....

.....

.....

.....

.....

3) What do mean by single payment at one year in reinsurance pricing?

.....

.....

.....

.....

.....

4) Distinguish between single payment and multi-payment cases in reinsurance pricing.

.....

.....

.....

.....

.....

14.7 LET US SUM UP

In this unit we have discussed the functions and types of reinsurance along the pricing options in it. We defined reinsurance as a means by which an insurance company shares risk of loss with another insurance company. Its functions are concerned with risk transfer, surplus relief and arbitrage. Reinsurance could be proportional and non-proportional. The proportional reinsurance is also known as quota share reinsurance. The reinsurance here takes a stated percent share of each policy written by the insurer and presents the premium and losses in that same proportion. The non–proportional (or excess of loss) reinsurance helps only if the loss suffered by the insured exceeds a certain amount (retention). Excess of loss reinsurance can be of two forms – per risk or per occurrence (catastrophe). In per risk case, the insurance policy limits are exposed within the reinsurance limits. On the other hand, in catastrophe excess loss, the insurance policy limits must be less than the reinsurance retention.

While calculating reinsurance premium, it helps to consider distribution such as log normal and normal. Presence of inflation may have to be considered in reinsurance provisions. An estimation of reinsurance claims with provision of excess loss is considered. The censored sample methods with some value recorded exactly and remaining known only to exceed a particular value has been pointed out.

In the last part of the discussion, we have presented the pricing options for reinsurance. The major focus of discussion in respect of pricing is the presence of investment return and risk. It was pointed out that in single payment case IRR was used because it was unequivocally defined. In multiple payment case the IRR might not even be definable. In order to consider further value of the cash flow, it was found to be necessary to set up some description of the investment policy on the released funds.

In the single payment at one year framework we have considered there cases, viz., option and high excess layer and minimum premiums, and discussed the required changes to be introduced in multiple payments.

14.8 KEY WORDS

Censored Sample: Case where the variable of interest is only observable under certain conditions.

Exposure: Possibility of loss.

Loss Development: The difference between the estimated amount of loss as initially reported to the reinsurer and the amount of an evaluation at a late amount paid in final settlements(s).

Loss Loading: A factor applied to pure loss cost or losses to produce a reinsurance rate or premium.

Loss Ratio: The ratio of incurred losses and loss-adjustment expenses to net premiums earned. This ratio measures the company's underlying profitability, or loss experience, on its total book of business.

Losses Incurred (Pure Losses): Net paid losses during the current year plus the change in loss reserves since the prior year-end.

Net Loss: The amount of loss sustained by an insurer after giving effect to all applicable reinsurance, salvage, and subrogation recoveries.

Net Retention: The amount of insurance which a ceding company keeps for its own account and does not reinsure in any way.

Reinsurance: In effect, insurance that an insurance company buys for its own protection. The risk of loss is spread, so a disproportionately large loss under a single policy doesn't fall on one company. Reinsurance enables an insurance company to expand its capacity; stabilise its underwriting results; finance its expanding volume; secure catastrophe protection against shock losses; withdraw from a line of business or a geographical area within a specified time period.

Reinsurance Ceded: The unit of insurance transferred to a reinsurer by a ceding company.

Reinsurance Premium: The consideration paid by the ceding company to the reinsurer for the reinsurance afforded by the reinsurer.

Reinsurance: Assumption by one insurance company of all or part of a risk underwritten by another insurer.

Reinsurer: The insurer, which assumes all or a part of the insurance or reinsurance written by another insurer.

Retention: Net retention plus any amounts of other cedes all of the reinsurance it has assumed to another reinsurer.

Stop Loss: Any provision in a policy designed to cut off an insurer's losses at a given point.

Stop Loss Reinsurance: A form of excess of loss reinsurance which indemnifies the ceding company against the amount by which the ceding company's losses incurred during a specific period (usually 12 months) exceed either (1) a predetermined dollar amount or (2) a percentage of the company's subject premiums (loss ratio) for the specific period.

Swaps: The simultaneous buying, selling or exchange of one security for another among investors to change maturities in a bond portfolio.

Truncated Sample: Case where whole observations are missing.

14.9 SOME USEFUL BOOKS

Deykin, C. D. Pentiken, T., Pesonan, M. (1994), *Practical Risk Theory for Actuaries,* Chapman and Hall, London

M. Goovaerts & D. Vyncke (2004), *Encyclopedia of Actuarial Science, Wiley,* Vol III, 1403-1404

Kreps, Rodney (1998), Investment-Equivalent Reinsurance pricing, proceedings of the Casualty Actuarial Society, 85, PP. 101-131 (see Internet)

Richard, A. Brealey and Stewart. C. Myers (1996), *Principles of Corporate Finance,* 5th edition, Irwin-McGraw-Hill

Rolski, T., Schmidli, H., Schmidt, V., Teugels, J. (1998), Stochastic Processes for Insurance and Finance. J. Wiley & Sons, Chichester

14.10 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) See Section 14.1 and answer

2) See Section 14.2 and answer

3) See Section 14.3 and answer

4) See Section 14.4 and answer

5) See Section 14.5 and answer

Check Your Progress 2

1) See Section 14.6 and answer

2) See Sub - section 14.6.1.1 and 14.6.1.2 and answer

3) See Sub - section 14.6.1 and answer

4) See Sub – section 14.6.1 and 14.6.1.3 and answer

APPENDICES

(Taken from Kreps, Rodney)

Appendix 1

The form of the Black-Scholes formula for the price of a European call option on a security is

$$\text{call price} = \Phi(\Delta_1)P_o - \Phi(\Delta_2)PV(E)$$

where $P(E)$ = present value of the exercise price discounted at the risk-free rate,

P_o = price of the security at time zero,

$$\Delta_1 = \frac{In\left(\frac{p_o}{PV(E)}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$\Delta_2 = \Delta_1 - \sigma\sqrt{t}$$

where σ is a parameter of the distribution of the underlying security and is $\Phi(x)$ the cumulative distribution function for the normal distribution, that is

$$\Phi(x) = \int_{-x}^{x} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz .$$

This function is available in at least one standard spreadsheet program.

The option is the right to buy the underlying security at the exercise price at the time t.

The logarithm of the value of the security is assumed to follow a normal distribution with parameters μt and $\sigma\sqrt{t}$ for the mean and standard deviation, respectively. Given the expected annual yield rate y and its standard deviation σ_y,

$$\sigma^2 = In\left\{1 + \left(\frac{\sigma_y}{1+y}\right)^2\right\}$$

and,

$$\mu = \ln(1+y) - \frac{\sigma^2}{2}$$

The price for a put option, which is actually what is of interest here, is given by put-call parity as

$$\text{put price} = \text{call price} + \text{P}V(E) - \text{P}_0,$$

In the case of interest here $PV(E) = P_0$ since we want the exercise price to be the growth at the risk-free rate. Hence the put price is equal to the call price, and for either option the

$$\text{option cost} = P_0\Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - P_0\Phi\left(-\frac{\sigma\sqrt{t}}{2}\right)$$

so the

$$\text{option rate} = \Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{t}}{2}\right)$$

or,

$$\text{Option rate} = \sqrt{\frac{2}{\pi}}\int_0^{\sigma\sqrt{t}/2} e^{-z^2/2}dz.$$

The exponential may be expanded to first order in a Taylor series to get the approximation quoted, which is actually rather good for the order of magnitude of numbers used here.

Appendix 2

As stated in appendix 1, the probability density function for the investment value (which is 1 + return) is lognormal with parameters μt and $\sigma\sqrt{t}$. That is,

$$f(x)=\frac{1}{\sigma x\sqrt{2\pi t}}\exp\left\{\frac{-(In(x)-\mu)^2}{\sigma^2 t}\right\}.$$

The investment hedged with the option to time t has the characteristics (f is the risk-free rate)

$$\text{investment} = x \text{ for } x \geq (1+f)^t$$

$$= (1+f)^t \text{ for } x < (1+f)^t$$

What is needed are the moments of the investment; in particular its mean and standard deviation.

Define

$$F_n = \int_0^{(1+f)^t} x^n f(x)dx$$

$$= \Phi(\varsigma - n\sigma\sqrt{t})\exp\{n\mu t + n^2\sigma^2 t/2\}$$

where

$$\xi = \sqrt{t}\left\{\frac{\ln(1+f)-\mu}{\sigma}\right\}.$$

In general,

$$\text{moment}(n) = \int_0^{\alpha} \text{investment}^n f(x)dx$$

$$= (1+f)^{nt}\int_0^{(1+f)^t} f(x)dx + \int_{(1+f)^t}^{x} x^n f(x)\,dx$$

Using the results for F_n above, the moment of order n of the investment is

$$\text{moment}(n) = (1+f)^{nt} F_o + \exp(n\mu t + n^2\sigma^2 t/2) - F_n$$

$$= (1+f)^{nt}\Phi(\varsigma) + \exp(n\mu t + n^2\sigma^2 t/2)\left[1-\Phi(\varsigma - n\sigma\sqrt{t})\right]$$

The mean value is just moment (1) and the variance of the investment is (moment (2) - moment$(1)^2$ }.

Appendix 3

Derivation of Eq. (14.6): Substitute for μ_L and F in Eq. (14.6):

Eq. (14.6) may be solved for μ_L as

(A.1) $$\mu_L = (1+r_f)(p-R)$$

Substitute F from Eq. (14.6) and L from Eq. (A.l) into Eq. (14.6):

$$(1+y)A = (1+r_f)(\mathrm{P}+A)-(1+R_f)(\mathrm{P}-r)$$

(4) $$=(1+r_f)A+(1+r_f)R$$

Solving for R gives Eq. (14.6).

Derivation of Eq. (14.6):

Eq. (14.6) can be written

$$(1+r)F = p+A = A+\frac{\mu_L}{(1+r_L)}+R$$

from Eq. (14.6). Rearranging to solve for R, and subsequently using Eq. (6) for F and Eq. (14.6) for A,

$$R=(1+r)F-A-\frac{\mu_L}{(1+r_f)}$$

$$=(1+r)\frac{S}{1+r_f}-\frac{1}{1+y}\left(\frac{1+i}{1+r_f}S-\mu_L\right)-\frac{\mu_L}{1+r_f}$$

$$=\frac{S}{1+r_f}\left[(1+r)-\frac{1+i}{1+y}\right]+\mu_L\left[\frac{1}{1+y}-\frac{1}{1+r_f}\right]$$

$$=\frac{1}{(1+r_f)(1+y)}\left[S\{(1+r_f)(1+r_f)-(1+i)\}-\mu_L\{y-r_f\}\right]$$

Derivation of Eqs. (14.6)-(14.6):

Eq. (14.6) can be written

$$F=\frac{(1+y)A+\mu_L}{1+i}$$

Substituting for F in Eq. (14.6) gives

$$A^2\sigma^2 = \left[(1+y)^2 A^2 + 2A\mu_L(1+y) + \mu_L^2\right]\frac{\sigma_i^2}{(1+i)^2} + \sigma_L^2$$

Multiplying through by the denominator and collecting terms,

$$0 = A^2\left[(1+y)^2\sigma_i^2 - \sigma_y^2(1+i)^2\right] + 2A\mu_L(1+y)\sigma_i^2 + \mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2$$

This is Eqs. (14.6)-(14.6). If there is a correlation ρ_{iL} between investment and loss. then this equation becomes

$$0 = A^2\left[(1+y)^2\sigma_i^2 - \sigma_y^2(1+i)^2\right] + 2A(1+y)\sigma_i\left[\mu_L\sigma_i + \sigma_{iL}(1+i)\right]$$
$$+\mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2 + 2\mu_L\sigma_i\sigma_L(1+i)\rho_{iL}$$

Derivation of Ea. (14.6):

By substituting for F from Eq. (14.6) into Eq. (14.6)

$$(1+y)A = (1+i)\frac{p+A}{1+r} - \mu_L$$

Multiplying through by the denominator and using Eq. (14.6) for P,

$$A(1+y)(1+r) = (1+i)\left(R + \frac{\mu_L}{1+r_f} + A\right) - \mu_L(1+r)$$

Rearranging terms,

$$(1+r) = A\left[(1+y(1+r) - (1+i)\right] + \mu_L\left[(1+r) - \frac{1+i}{1+r_f}\right]$$

Eq. (14.6) for R results immediately.