UNIT 10 MONEY AND THE ROLE OF MONETARY POLICY

Structure

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10.0 OBJECTIVES

After going through this Unit you should be able to explain:

- the role of money in facilitating the transaction in a standard two period overlapping generations model of exchange;
- why money has positive value in an economy where money is the only medium of exchange; and
- the role of money in a model where people derive direct utility by holding money.

1 1 INTRODUCTION

As you know, money is the medium of exchange. In old days gold and silver were used as the medium of exchange. These were commodity money, which had some value apart from their role as medium of exchange. However, in the modern world, money is typically paper money which does not have any worth on its own: its only value is in terms of the commodities that you can buy in exchange.

Money also serves a second purpose. In so far as money is needed to carry out any exchange, it is also used as a store of value. However, as a store of value, money is typically dominated by all other assets in the sense that holding money involves zero return in nominal terms, whereas all other assets carry some positive return in nominal terms. Then why is paper money valued at all? Here we shall seek the answer to this question.

The neoclassical growth models that you have studied so far are based on the assumption that there is a single final commodity. Hence all payments–including the wage rate and the rate of interest – are made in terms of this final commodity. There is no money. In this chapter we shall see how the presence of money affects the real decisions and the dynamic equilibria in these models.

10.2 MONEY IN THE OVERLAPPING GENERATIONS MODEL

For money to be valued it must facilitate the exchange process. Samuelson's overlapping generations (henceforth OLG) set up can illustrate the usefulness of money as a medium of exchange.

Let us look at an OLG model of exchange. The structure of the model is very similar to the OLG growth model that you have studied earlier. The only difference is that the present model is an OLG model of exchange'–there is no production.

The household side of the story is exactly same as before. Each generation lives exactly for two periods. Thus for the cohort of individuals who are born at the beginning of period 't', they are alive in period 't' (when they are young) and in period '$t+1$' (when they are old). On the other hand, a new set of individuals are born at the beginning of period $t+1$, who would be alive in period '$t+1$' and period '$t+2$' respectively. The set of people who are born at the beginning of period t will be called 'generation t'. Population in successive generations grows at the rate n. We shall assume that the initial stock of population (L_0) was unity, so that number of people belonging to generation t is given by: $L_t = (1+n)^t$.

The representative member of generation t is endowed with one unit of a consumption good when young and receives no endowment when old. This consumption good is perishable; hence it cannot be stored for future consumption. On the other hand money can be stored. Thus anybody who intends to consume in the next period must sell the commodity in exchange of money in the first period and then again buy back the commodity against money in the next period. The two-period utility function of the representative member is given by $U(c_{1t}, c_{2t})$, where the utility function has all the standard properties.

At any period the consumption possibilities from the society's point of view is shown in the following diagram. The horizontal axis measures the per capita consumption of young and the per capita consumption of the old. Note that at any point of time t, the total endowment of goods is given by L_t (since each young person receives 1 unit and there are L_t number of young persons in the economy at point t). If the entire endowment is consumed by the young, then each of them consume one unit, which is denoted by point A on the horizontal axis. If, on the other hand, the entire amount is consumed by the old, then each of them consume $(1+n)$ units, which is denoted by point B on the vertical axis. The straight line joining these two points represents the society's consumption possibility frontier: each point on this line represents a certain distribution of the total endowment between the young and the old (see Fig. 10.1).

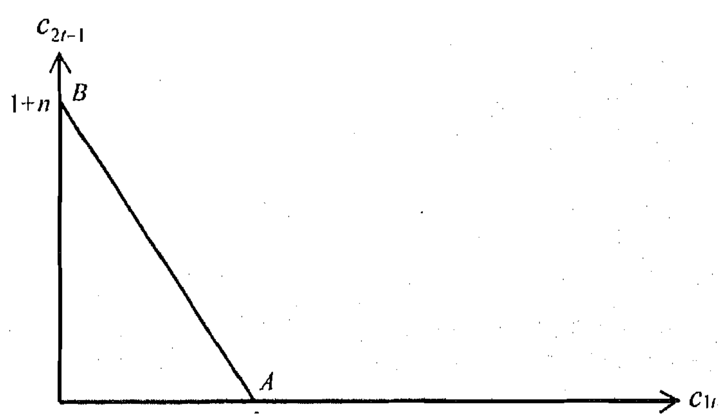

Fig. 10.1: Consumption Possibility Frontier

Let us now see **what** is the best point from an individual's point of view. The individual has an utility function $U(c_{1t}, c_{2t})$. If he could choose between all the points on the consumption possibility frontier then he would have chosen the point which would have maximized his life-time utility. This point is represented by point E in Fig. 10.2.

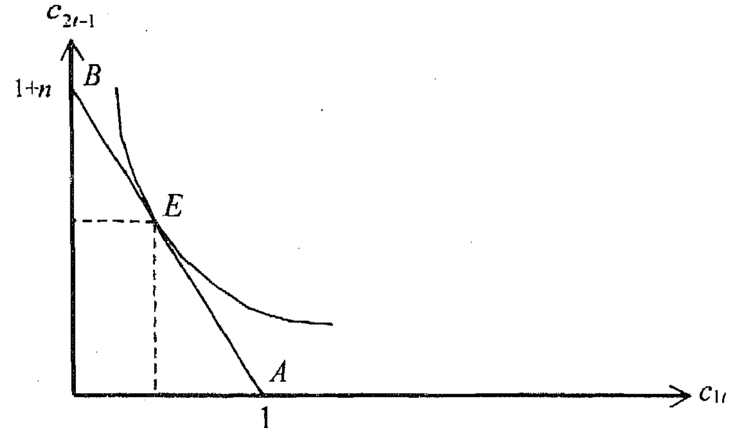

Fig. 10.2: Lifetime Utility Maximisation

It is however important to note that **without money, this point is not reachable by any individual.** In other words, a simple trade between the current young and the current old cannot ensure that point E is reached. Why not? The reason is the following. In order to reach the point E the current young will have to give a part of their endowment to the current old. In exchange they will have to be given part of the endowment of tomorrow's young generation. Since tomorrow's young generation is not present'yet, such a contract cannot be enforced on them tomorrow. So no trade can take place.

Let us now see how introduction of money can help everybody to reach the optimal point. Suppose at time *t* the government gives the current old generation an amount of money, $\overline{M}$, which is equally distributed among all the members of the current old generation. Suppose everybody in the economy (including the future generations) believes that they will be able to exchange money for goods at a certain price. The price level can vary from time to time depending on the demand for and supply of money. Let P_t denote the price level at time *t*. It is obvious that the current old generations are the suppliers of money. But who demands money? The demanders are the current young. Why do they demand money? Since money is **not** perishable (while the consumption good is perishable), the young generation can store the money and in the next period (when they are old) can exchange it for goods so as to reach the optimal point. To *see* the argument more clearly, let us consider the maximization problem of a representative member of generation *t*. He will maximize his utility $U(c_{1t}, c_{2t})$ subject to the following two constraints:

i) $P_t(1 - c_{1t}) = M_t^d$; and

ii) $P_{t+1} \cdot c_{2t} = M_t^d$,

where M_t^d is the demand for money by the representative member of generation ***t***. The underlying intuition behind these two constraints are quite straight forward. The first constraint says that out of his endowment of one unit of final good, the representative member of generation *t* saves an amount $(1 - c_{1t})$ when young. This amount he exchanges for money at period *t* at the current price P_t. Thus $P_t(1 - c_{1t})$ constitute his total demand for money in period ***t***. In the next period he sells the stored money M_t^d to the young members of generation *t+1* at a price P_{t+1}, to get a second period consumption $c_{2t} = M_t^d / P_{t+1}$. Rearranging the terms we get the second constraint. Eliminating M_t^d from the above two constraints, we can generate a single constraint:

$$P_t(1 - c_{1t}) = P_{t+1}.c_{2t}.$$

Maximizing utility subject to this single constraint one can derive the following first order condition: $\frac{U_1(c_{1t}, c_{2t})}{P_t} = \frac{U_2(c_{1t}, c_{2t})}{P_{t+1}}$. This first order condition along with one of the constraints in turn defines the optimal demand for money function in real terms as a function of P_t and P_{t+1}: $\frac{M_t^d}{P_t} = F\left(\frac{P_t}{P_{t+1}}\right)$.

Next we characterize the equilibrium in the money market. At time ***t*** each of the current young demands M_t^d amount of money. Hence the total demand for money at time t is given by $L_t M_t^d$, or $(1+n)^t L_0.F\left(\frac{P_t}{P_{t+1}}\right).P_t$. ***On*** the other **hand**, the entire **old** generation receives a money endowment equal to $\overline{M}$. Thus the total supply of money at period **t** is given by $\overline{M}$. Equating demand and supply, we get the money market equilibrium .condition at period ***t*** as:

$$(1+n)^t L_0.F\left(\frac{P_t}{P_{t+1}}\right)P_t = \overline{M}$$ [1]

Note that $\left(\frac{P_t}{P_{t+1}}\right)$ is the rate of return on money in real terms. It has close **link** with the rate of deflation. The latter concept is defined **as** π_t, where $(1+\pi_t) \equiv \left(\frac{P_t}{P_{t+1}}\right)$. Thus we can write the money market equilibrium condition at any period ***t*** in terms of the current rate of deflation and the current price level in the following way: $(1+n)^t L_0.F(1+\pi_t)P_t = \overline{M}$.

Notice that the money market equilibrium condition defines a short run **equilibrium rate** of deflation for every point of time. At time ***t*** **the** equilibrium **rate** of deflation is given by $(1+n)F(1+\pi_t) = \overline{M}$, **and** at time ***t+1*** **the** equilibrium rate of deflation is **given** by $(1+n)^{t+1}L_0.F(1+\pi_{t+1})P_{t+1} = \overline{M}$. Comparing the **two** we get: $F(1+\pi_t)P_t = (1+n)F(1+\pi_{t+1})P_{t+1}$. **Rearranging terms:** $\frac{P_t}{P_{t+1}} = (1+n)\frac{F(1+\pi_{t+1})}{F(1+\pi_t)}$,

[1] **Since there are only two commodities – the final good and money – money market equilibrium in any period, by Walras' Law, implies that the goods market is also in equilibrium.**

i.e., $(1+\pi_t) = (1+n)\frac{F(1+\pi_{t+1})}{F(1+\pi_t)}$. At the steady state (long run equilibrium) the rate of deflation is constant which, from the above equation, implies that $\pi_t = \pi_{t+1} = n$. That is, at the long run equilibrium, the rate of deflation is equal to the rate of growth of population.

With a rate of deflation equal to n, the budget line of an individual (which is represented by the equation, $P_t(1 - c_{1t}) = P_{t+1}.c_{2t}$) coincides with line AB in fig. 10.2 and therefore the representative individual chooses the optimal point E, which maximizes his utility, Thus introduction of money in this overlapping generations model enables the members to reach the optimal point which was earlier unattainable in the barter economy (without money).

While the overlapping generations model highlights the usefulness of money, the model is inadequate as a theory of money. In the OLG model discussed above, money is the only asset which can be used as a store of value. Had there been any other asset (say, physical capital) with a return higher than money, money would have ceased to have any positive value in this framework. A proper theory of money must explain why money continues to be valued despite being dominated in terms of return by other assets. We now turn to other theories of money which address this issue.

Check Your Progress 1

1) In a barter economy introduction of money may enable people to reach the socially optimal outcome – do you agree? Elaborate in the context of a two-period overlapping generations model of exchange.

10.3 CASH-IN-ADVANCE MODEL

The cash-in-advance model emphasizes the role of money as a medium of exchange. The model starts with the assumption that "money buys goods, goods buy money but goods do not buy goods". This assumption is known as the 'Clower' constraint or the 'cash-in-advance' constraint. The term 'cash-in-advance' implies that to buy goods one must accumulate some cash in advance; one cannot directly exchange goods for goods.

In its simplest form the cash-in-advance model specifies that purchase of goods must be paid for with money held at the beginning of the period. The formal presentation of the model is as follows.

Let the utility function of an infinitely lived representative agent be

$$\sum_{t=0}^{\infty} u(c_t^1, c_t^2, c_t^3, \ldots\ldots, c_t^n),$$

where $c^1, c^2, c^3, \ldots\ldots c^n$ represent the consumption of good1, good2, good 3, ..., good *n* respectively. The agent can keep his savings either in the form of money, which earns zero nominal return, or in the form of bond which yields a positive nominal return denoted by ρ. The agent starts with a certain stock of money and a certain stock of bond. He also has an income Y at each period Hence the period *t* budget constraint of the agent is given by:

$$\sum_{i=1}^{n} P_t^i c_t^i + M_{t+1} + B_{t+1} = Y_t + M_t + (1+\rho_{t-1})B_t .$$

It is easy to see that since bonds earns positive return while money earns zero return (that is, money is dominated by bond), maximization of the utility function subject to this budget constraint will generate an optimal solution where holding of money will be zero.

Now suppose we add a cash-in advance constraint to this framework. That is, we specify an additional constraint of the form: $\sum_{i=1}^{n} P_t^i c_t^i \le M_t$. This constraint says that all the purchases must be carried out from the stock of money. One can show that the solution to the Kuhn-Tucker maximization problem with this additional constraint will imply the optimal money holding is no longer zero. In other words, adding a cash-in-advance constraint immediately generates a demand for money.

Check Your Progress 2

1) What is Clower constraint? How does it ensure that money has positive value?

10.4 MONEY IN THE UTILITY FUNCTION

The cash-in-advance model described above highlights the transaction demand for money. There exists a second approach which emphasizes the fact that money might provide some utility on its own. In the latter approach money, or more precisely the amount of real balances, enters directly into individual's utility function. We describe below a model which uses this framework. The specific model that we shall consider here is due to Sidrauski. The model is an extension of the Ramsey model which allows for holding of money.

As is the Ramsey model we assume that at each point of time t the society consists of L_t number of infinitely lived individuals who have identical preferences. The utility function of the representative individual is given by $W = \int_0^{\infty} u(c_t, m_t) \exp^{-\rho t} dt$, where c_t and m, are respectively the per capita consumption and the per capita holding of real balances in period t, $u(c_t)$ is the associated instantaneous utility in period t, and ρ is the subjective discount rate of the agent. The instantaneous utility function is well-behaved and has all the standard properties: $u_c, u_m > 0; u_{cc}, u_{mm} > 0$. The population in this economy grows at a constant exogenous rate n.

People can hold their wealth either in the form of money or in the form of physical capital. At each point of time the government distributes certain amount of new money stock equally among the households. These constitute transfers which in real terms is denoted by X_t. Accordingly the society's budget constraint at period t is given by:

$$C_t + \frac{dK_t}{dt} + \frac{1}{P_t}\frac{dM_t}{dt} = w_t L_t + r_t K_t + X_t.$$

Noting that $\frac{1}{L}\frac{dK}{dt} = \frac{dk}{dt} + nk$ and $\frac{1}{L}.\frac{1}{P}\frac{dM}{dt} = \frac{dm}{dt} + nm + \pi m$, where k is the per capita capital stock and π is the rate of inflation, we can write the budget constraint in per capita terms as:

$$c_t + \frac{dk_t}{dt} + nk_t + \frac{dm_t}{dt} + (n + \pi_t)m_t = w_t + r_t k_t + x_t.$$

Note that per capita household wealth is given by $a = m + k$. Therefore we can write the above budget constraint in terms of per capita wealth as:

$$c_t + \frac{da_t}{dt} = w_t + (r_t - n)a_t + x_t - (\pi_t + r_t)m_t.$$

As in the Ramsey model, the complete dynamic optimization problem for the household can now be written in terms of the three time dependent variables per capita consumption (c_t), per capita real money balances holding (m,) and per capita asset stock (a,):

$$\text{Maximize } W = \int_0^{\infty} u(c_t, m_t) \exp^{-\rho t} \text{dt}$$

subject to $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t ; a_0$ ***given.*** Also the household treats the wage rate (w_t), the rate of interest (r_t), the rate of inflation (π_t) and the transfer from government (x_t) as exogenously given. Thus for the household now there are two control variables, c_t and m_t, and one state variable a_t. The corresponding Hamiltonian function and the first order conditions in terms of the control, state and co-state variables are given by:[2]

$$H = u(c_t, m_t)\exp^{-\rho t} + \lambda_t\left[w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t\right]$$

i) $\quad u_c(c_t, m_t)\exp^{-\rho t} = \lambda_t$

ii) $\quad u_m(c_t, m_t)\exp^{-\rho t} = \lambda_t(\pi_t + r_t)$

iii) $\quad \frac{d\lambda}{dt} = -\lambda_t(a_t - n)$

iv) $\quad \frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t$

v) $\quad \lim_{t\to\infty} \lambda_t a_t = 0$.

Additionally there is the No-Ponzi-Game condition given by

vi) $\quad \lim_{t\to 0} a_t \exp^{-\int_0^t (r_\tau - n)d\tau} \geq 0$.

Finally note that while the household treats: w_t, r_t, π_t and x_t as exogenous to its decision making process, it nonetheless has perfect foresight so that it can exactly ***guess*** the correct values of these variables at every point of time. Accordingly in equations (i)-(vi) we can put $w_t = f(k_t) - k_t f'(k_t)$ and $r_t = f'(k_t)$. Also as we have mentioned before, the new money is distributed as government transfer (in real terms) to the households, so that $x_t = \frac{1}{L_t}\left(\frac{1}{P_t}\frac{dM}{dt}\right)$. We can write this latter term as

$x_t = \frac{1}{L_t}\frac{M_t}{P_t}\left(\frac{1}{M}\frac{dM}{dt}\right) \equiv m_t\sigma$, where σ is the rate of growth of nominal money stock.

Let us now look at the steady state of this economy. At steady state $\frac{da}{dt} = \frac{dm}{dt} = \frac{d\lambda}{dt} = 0$. The condition $\frac{dm}{dt} = 0$ implies $\pi = \sigma - n$. Putting these and the other steady state conditions in equations (i)-(iv), we get the steady state per capita capital stock and the steady state per capita consumption as: $f'(k^*) = \rho + n$; and $c^* = f(k^*) - nk^*$. Comparing *these* values with the corresponding steady state

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2 To know the mathematical technique go back to the Mathematical Appendix of Unit 9 where the infinite horizon Ramsey model has been discussed.

10.4 MONEY IN THE UTILITY FUNCTION

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People can hold their wealth either in the form of money or in the form of physical capital. At each point of time the government distributes certain amount of new money stock equally among the households. These constitute transfers which in real terms is denoted by X_t. Accordingly the society's budget constraint at period *t* is given by:

$$C_t + \frac{dK_t}{dt} + \frac{1}{P_t}\frac{dM_t}{dt} = w_t L_t + r_t K_t + X_t .$$

Noting that $\frac{1}{L}\frac{dK}{dt} = \frac{dk}{dt} + nk$ and $\frac{1}{L}\cdot\frac{1}{P}\frac{dM}{dt} = \frac{dm}{dt} + nm + \pi m$, where k is the per capita capital stock and π is the rate of inflation, we can write the budget constraint in per capita terms as:

$$c_t + \frac{dk_t}{dt} + nk_t + \frac{dm_t}{dt} + (n + \pi_t)m_t = w_t + r_t k_t + x_t .$$

Note that per capita household wealth is given by $a = m + k$. Therefore we can write the above budget constraint in terms of per capita wealth as:

$$c_t + \frac{da_t}{dt} = w_t + (r_t - n)a_t + x_t - (\pi_t + r_t)m_t .$$

As in the Ramsey model, the complete dynamic optimization problem for the household can now be written in terms of the three time dependent variables per capita consumption (c_t), per capita real money balances holding (m_t) and per capita asset stock (a_t):

$$\text{Maximize } W = \int_0^{\infty} u(c_t, m_t)\exp^{-\rho t}\, dt$$

subject to $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t ; a_0$ given. Also the household treats the wage rate (w_t), the rate of interest (r_t), the rate of inflation (π_t) and the transfer from government (x_t) as exogenously given. Thus for the household now there are two control variables, c_t and m_t, and one state variable a_t. The corresponding Hamiltonian function and the first order conditions in terms of the control, state and co-state variables are given by:[2]

$$H = u(c_t, m_t)\exp^{-\rho t} + \lambda_t\left[w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t\right]$$

i) $u_c(c_t, m_t)\exp^{-\rho t} = \lambda_t$

ii) $u_m(c_t, m_t)\exp^{-\rho t} = \lambda_t(\pi_t + r_t)$

iii) $\frac{d\lambda}{dt} = -\lambda_t(a_t - n)$

iv) $\frac{da}{dt} = w_t + (r_t - n)a_t + x_t - c_t - (\pi_t + r_t)m_t$

v) $\lim_{t\to\infty} \lambda_t a_t = 0$.

Additionally there is the No-Ponzi-Game condition given by

vi) $\lim_{t\to 0} a_t \exp^{-\int_0^t (r_\tau - n)d\tau} \geq 0$.

Finally note that while the household treats: w_t, r_t, π_t and x_t as exogenous to its decision making process, it nonetheless has perfect foresight so that it can exactly ***guess*** the correct values of these variables at every point of time. Accordingly in equations (i)-(vi) we can put $w_t = f(k_t) - k_t f'(k_t)$ and $r_t = f'(k_t)$. Also as we have mentioned before, the new money is distributed as government transfer (in real terms) to the households, so that $x_t = \frac{1}{L_t}\left(\frac{1}{P_t}\frac{dM}{dt}\right)$. We can write this latter term as

$x_t = \frac{1}{L_t}\frac{M_t}{P_t}\left(\frac{1}{M}\frac{dM}{dt}\right) \equiv m_t\sigma$, where σ is the rate of growth of nominal money **stock.**

Let us now look at the steady state of this economy. At steady state $\frac{da}{dt} = \frac{dm}{dt} = \frac{d\lambda}{dt} = 0$. The condition $\frac{dm}{dt} = 0$ implies $\pi = \sigma - n$. Putting these and the other steady state conditions in equations (i)-(iv), we get the steady state per capita capital stock and the steady state per capita consumption as: $f'(k^*) = \rho + n$; and $c^* = f(k^*) - nk^*$. Comparing these values with the corresponding steady state

[2] **To know the mathematical technique go back to the Mathematical Appendix of Unit 9 where the infinite horizon Ramsey model has been discussed.**

values of the original Ramsey model without money (described in unit 9), we find that they are exactly identical, Thus introduction of money to the Ramsey model does not affect the real equilibrium.

10.5 POLICY IMPLICATIONS

We have so far discussed three different dynamic frameworks with money. Note that the stock of money often constitutes an important policy variable in the hand of the government. By varying the stock or the rate of growth of money the government may influence the real decisions of the agents. Let us now examine the effectiveness of monetary policy in these different frameworks.

Let us first consider the OLG framework. If the government increases the stock of money in each period at a constant rate and distributes it as lumpsum transfer to the old generation, then that increases the second period endowment of the old. An increase in the second period endowment will unambiguously decrease savings which in turn will affect the price level and therefore the rate of inflation. Note that the negative of the rate of inflation (i.e., the rate of deflation) is the real return on money holding. Hence a change in the inflation rate implies a change in the real return to money, which will affect the optimal decision about the other real variables as well. Thus in the OLG framework money is not neutral.

In the Cash-in-advance model also increasing the supply of money eases the Clower constraint and enables the households to buy more goods. Thus as long as the Clower constraint is binding, money affects the real variables.

In the money-in-the-utility-function approach, on the other hand, the steady state values of the real variables are unaffected by the introduction of money. To be more precise, the ***rate of growth of money*** stock does not affect the real equilibrium. This result is known as the 'supemeutrality' of money. Thus monetary policy has no role to play in this model.

Check Your Progress 3

1) If one introduces money in the Ramsey model how does the long run equilibrium values of the real variables change?

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2) Can change in the money supply have any impact on the real variables? Discuss your answer in the context of the different model of money that you have **studied** here.

10.6 LET US SUM UP

In this unit, we have looked at the role of money in an economy. First we have introduced money in the standard overlapping generations framework of exchange. We have seen that in this framework money allows the economy to reach its optimal point. We also discuss the role of money in the presence of Clower constraint when all transactions must be carried out in terms of money. We also discuss the role of money in a model where people derive direct utility from money. This latter model is an extension of the Ramsey model. We find that introduction of money has no impact on the real variables in this model.

10.7 KEYWORDS

Clower Constraint : A situation where money is the only medium of exchange so that any transaction requires money.

Super-neutrality of Money : A situation where a change in the rate of growth of money stock does not have any impact on the equilibrium values of the real variables in the economy.

10.8 SOME USEFUL BOOKS

Blanchard, Olivier and Stanley Fischer, 1989, ***Lectures on Macroeconomics, MIT*** Press, chapter 2.

10.9 ANSWER/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Read Section 10.2 and write your answer.

Check Your Progress 2

1) Read Section 10.3 and write your answer.

Check Your Progress 3

1) Read Section 10.4 and write your answer.

2) Read Section 10.5 and write your answer.