UNIT 4 THEORY OF PRODUCTION

Structure

4.0 OBJECTIVES

After studying this unit, you will be able to:

- appreciate the relationship between input and output in production;
- measure productivities of inputs;
- analyse laws of production with single input variation and multiple inputs variation;
- assess of production in short-and long-run; and
- measure to degree of substitution between factors of production.

4.1 INTRODUCTION

Production activities related to goods and services require inputs. Typically, the set of inputs includes labour, capital equipments and raw materials. The producing unit usually has to solve the choice problem as a given amount of output can be produced from various combinations of inputs. Firms, therefore, look for production possibilities that are technologically feasible. A production function describes the relation between input and output with a given the technology. More formally, it shows the maximum amount of output that can be produced from any specified set of inputs, given the existing technology. If we assume that there are only two factors of production – labour (L), capital (K) and a single output q, mathematically a production function can be written as,

$$q = F(L, K)$$

4.2 SHORT PERIOD ANALYSIS

Short period in production refers to a time when some inputs remain fixed. A fixed input is one, whose quantity cannot be changed readily, whereas, a variable input varies with production. Inputs like land, building and major pieces of machinery cannot be varied easily and, therefore, can be called fixed inputs. On the other hand, inputs like labour (labour hours), raw materials, and processed materials can be easily increased or decreased. Therefore, these are categorised as variable inputs. Depending on whether inputs can be kept fixed or not, we have a short period or a long period. To put this more precisely, if inputs being used in the production process have just enough time such that they cannot be varied, then the analysis pertains to the short-run. On the other hand, if the inputs employed have enough time such that they are amenable to variation, then the analysis is based on the frame work of long-run.

Generally, the firms do not readily change their capital, which could be land, machinery, managerial and technical personnel. Therefore, these are fixed input in the short-run. When we treat these under $\bar{K}$, the production function can be written as

$$q = F(L, \bar{K})$$

where L = Labour, a variable factor

$\bar{K}$ = Capital, a fixed factor

Marginal Product (MP) of a Factor

From the above mentioned production function, immediately we can study the effect on total output when there is a variation in labour utlilisation, keeping the other factor $\bar{K}$, fixed. Thus, we have the marginal physical product, which shows the change in output quantity for a unit change in the quantity of an input, (L), when all other inputs (K) are held constant. Mathematically, it is given by the first partial derivative of a production function with respect to labour. Thus,

$$q = F(L, \bar{K}) = \text{Total Product (TP)}.$$

$$\therefore \text{ MP of Labour} = MP_L = \frac{\partial q}{\partial L} = f_L$$

$$\text{If } L = \bar{L}, \text{ then marginal product of capital} = MP_K = \frac{\partial q}{\partial K} = f_K$$

It is reasonable to expect that the marginal product of an input depends on the quantity used of that input. In the above example, use of labour is made keeping the amount of other factors (such as equipments and land) fixed. Continued use of labour would eventually exhibit deterioration in its productivity. Thus, the relationship between labour input and total output can be recorded to show the declining marginal physical productivity. Mathematically, the diminishing marginal physical productivity is assessed through the second-order partial derivative of the production function.

Thus, change in labour productivity can be presented as:

$$\frac{\partial MP_l}{\partial L} = \frac{\partial^2 q}{\partial L^2} = f_{ll} < 0$$

Similarly, change in productivity of capital is denoted by

$$\frac{\partial MP_k}{\partial K} = \frac{\partial^2 q}{\partial K^2} = f_{kk} < 0$$

Average Product (AP) of a Factor

The productivity of a factor is often seen in terms of its average contribution. Although not very important in the theoretical discussions, where analytical insight is tried to be drawn from marginal productivities, average productivity finds a platform in empirical evaluations. Deriving it from the total product is relatively easy. It is the output per unit of a factor.

So, AP of labour = $AP_L = \frac{TP}{L} = \frac{q}{L}$. Similarly,

AP of capital = $AP_K = \frac{TP}{K} = \frac{q}{K}$.

Relation between TP and MP

Graphically, given the total product curve, MP is the slope of the tangent at any point on the TP curve. This is shown in Figure 4.1.

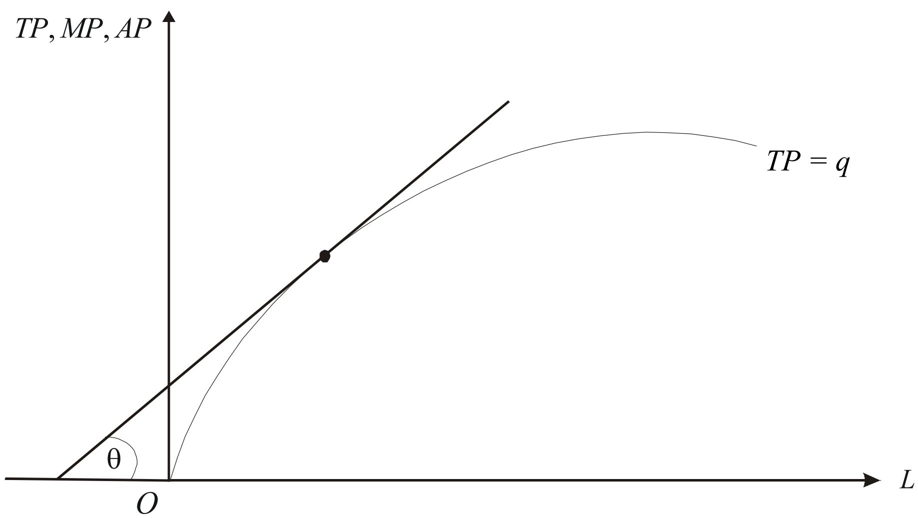

Fig. 4.1: Total Product Curve and Marginal Product

See that $MP_L = \tan\theta = \frac{\partial TP}{\partial L}$

Relation between TP and AP

Given the TP curve, AP is the slope of a ray from the origin to any point on the TP curve.

$$AP_L = \tan\alpha = \frac{QL}{OL} = \frac{q}{L}$$

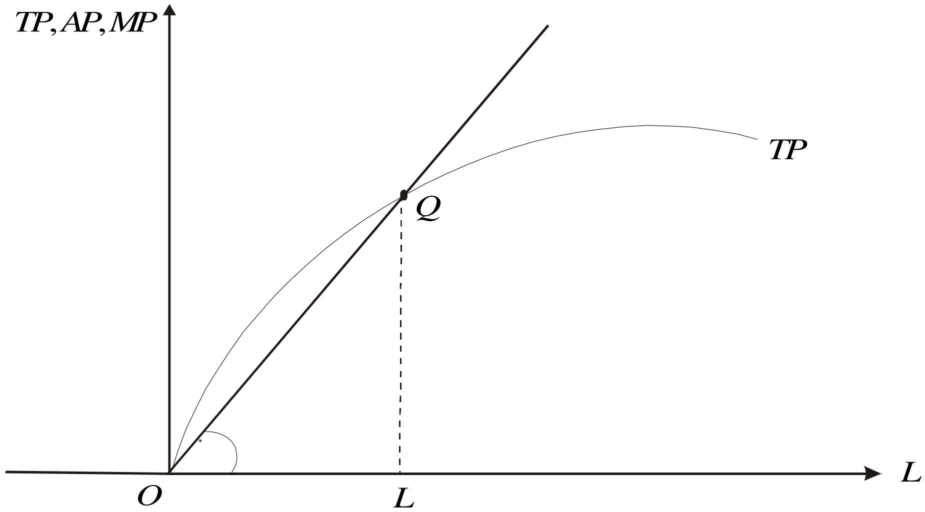

Fig. 4.2: Derivation of AP from TP Curve

Relation between AP and MP

This relation between AP and MP is true for all "average" and "marginal" productivity conditions. When AP is rising, MP > AP; when AP is maximum, MP = AP and when AP is falling, MP < AP.

Proof:

Since $AP_L = \frac{q}{L}$, differentiating AP_L partially with respect to L, we get,

$$\frac{\partial}{\partial L}(AP_L) = \frac{\partial}{\partial L}\left(\frac{q}{L}\right) = \frac{L\frac{\partial q}{\partial L} - q\frac{\partial L}{\partial L}}{L^2} = \frac{L\frac{\partial q}{\partial L} - q}{L^2}$$

or, $$\frac{\partial}{\partial L}(AP_L) = \frac{\partial q}{\partial L}.\frac{1}{L} - \frac{q}{L^2}$$

or, $$\frac{\partial}{\partial L}(AP_L) = \frac{1}{L}\left(\frac{\partial q}{\partial L} - \frac{q}{L^2}\right) = \frac{1}{L}\left(MP_L - AP_L\right) \qquad \ldots(1)$$

From equation 1,

$\frac{\partial}{\partial L}(AP_L) > 0$ when $MP_L > AP_L$ (since $L > 0$);

$\frac{\partial}{\partial L}(AP_L) = 0$ when $MP_L = AP_{L;}$

$\frac{\partial}{\partial L}(AP_L) < 0$ when $MP_L > AP_L$ (since $L > 0$)

It is an empirically observed feature that all inputs have positive but diminishing marginal products. If L and K are the only factors of production, then $F_L > 0$, $F_K > 0$, $F_{LL} < 0$, $F_{KK} < 0$.

Thus, for any factor, initially, MP is positive. Then a situation arises when MP is zero, and further on it falls to negative as more of the factor is employed in production.

Check Your Progress 1

1) What is a production function?

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2) In short period why you cannot increase the managerial inputs?

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3) Define marginal productivity of labour

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4.2.1 Returns to a Factor

This is a short-run concept which deals with the variability of only one factor keeping the others constant. There are three kinds of returns:

i) Increasing returns: when the AP of a factor rises and MP > AP

ii) Constant returns: when AP is constant and MP = AP

iii) Diminishing returns: when AP is falling MP < AP

The concept of returns to a factor can also be expressed in terms of the "partial input elasticity of output".

Partial Input Elasticity of Output

Partial input elasticity of output is also called elasticity of output with respect to a factor. It is the percentage change in output quantity for one per cent change in the quantity of a factor when all other factors remain constant.

Elasticity of q with respect to L is given by,

$$E_{q,L} = \frac{\frac{\partial q}{q}}{\frac{\partial L}{L}} = \frac{\partial q}{\partial L}.\frac{L}{q} = \frac{MP_L}{AP_L}$$

$$\text{Similarly, } E_{q,K} = \frac{\partial q}{\partial K}.\frac{K}{q} = \frac{MP_K}{AP_K}$$

Let us now relate this to returns to scale.

$$\text{Since } AP_L = \frac{q}{L},$$

$$\therefore \frac{\partial}{\partial L}(AP_L) = \frac{\partial}{\partial L}\left(\frac{q}{L}\right) = \frac{L\frac{\partial q}{\partial L} - q.\frac{\partial L}{\partial L}}{L^2} = \frac{L\frac{\partial q}{\partial L} - q}{L^2}$$

$$= \frac{1}{L}\frac{\partial q}{\partial L} - \frac{q}{L^2}$$

$$= \frac{q}{L^2}\left(\frac{\partial q}{\partial L}.\frac{L}{q} - 1\right) = \frac{q}{L^2}\left(E_{q,L} - 1\right) \qquad \ldots(2)$$

As $\frac{q}{L^2} > 0$ from equation (2),

increasing returns = $\frac{\partial}{\partial L}\left(AP_L\right) > 0 = E_{q,L} > 1$

constant returns = $\frac{\partial}{\partial L}\left(AP_L\right) = 0 = E_{q,L} = 1$

diminishing returns = $\frac{\partial}{\partial L}\left(AP_L\right) < 0 = E_{q,L} < 1$

Besides the above mentioned three returns, there can be another type known as the ‘non-proportional returns to a variable factor’. Under it, initially, there is increasing returns to a factor up to a certain level beyond which there is diminishing returns.

Graphical Representation of Various Returns

Diminishing Returns: If the TP curve is as shown in the adjacent Figure 4.3, then the MP_L given by $\tan\theta$ is throughout less than the AP_L given by tan □.

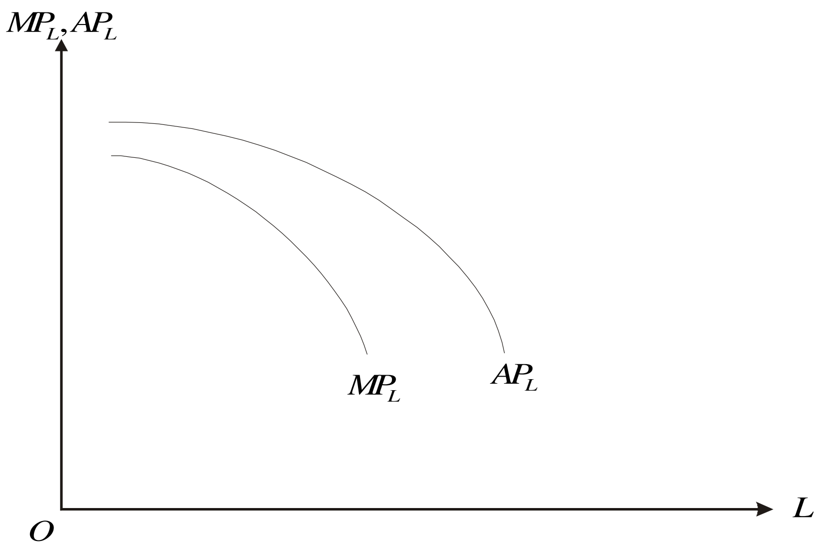


Fig. 4.3: Derivation of MP_L from TP

As AP_L is falling from the relation between MP and AP, MP < AP we have the adjoining Figure 4. 4.

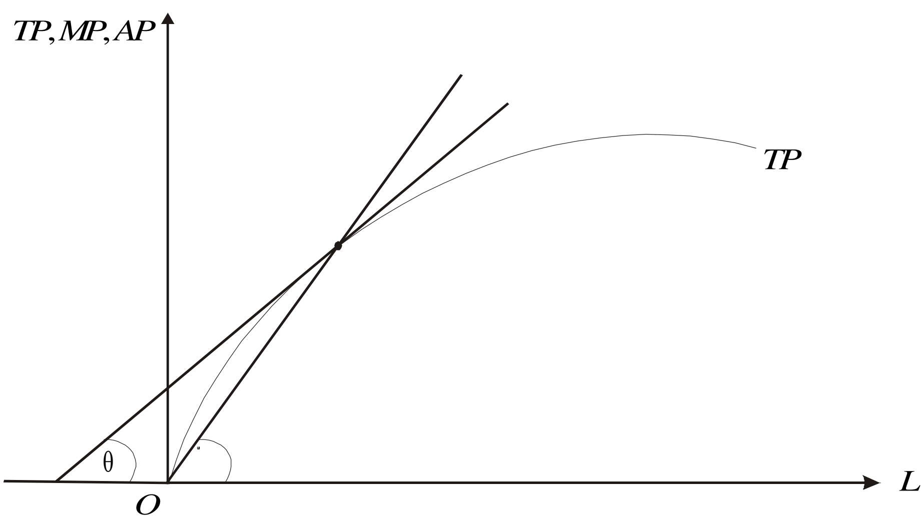
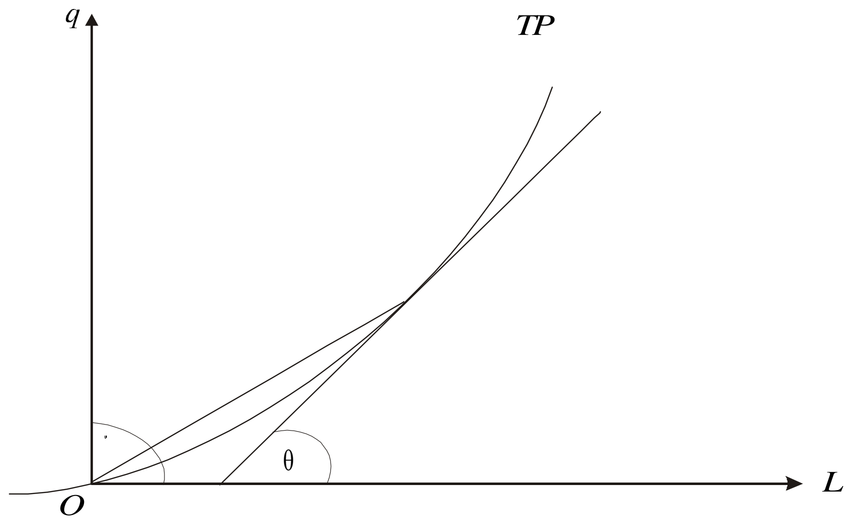


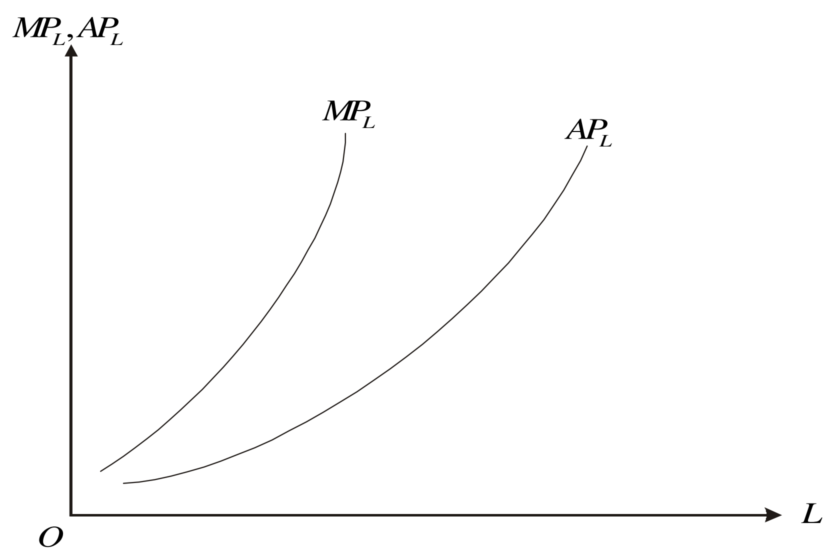
Fig. 4.4: Diminishing MP and AP

Increasing Returns

Here AP_L rises and $\tan \square < \tan\theta$, for all L. Therefore, MP > AP. This is shown in the adjacent Figure 4.5.



(a)

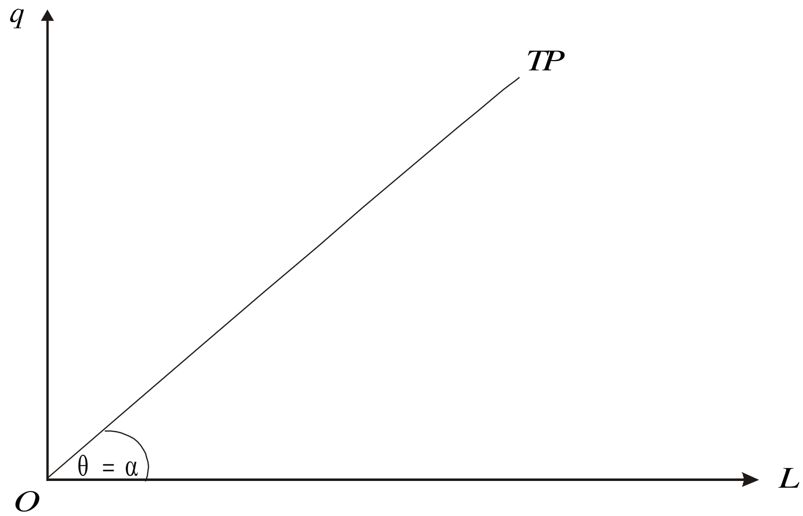


(b)

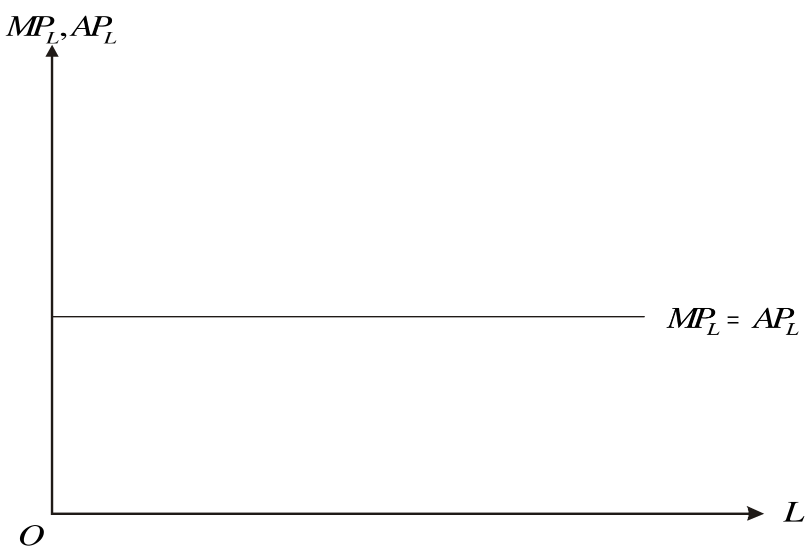
Fig. 4.5: Increasing Returns as given by TP, MP and AP

Constant Returns

Here, AP_L is constant and $\tan\theta$ = tan □, therefore, $MP_L = AP_L$ as is shown by a horizontal straight line in the next Figure 4.6(a,b)



(a)



(b)

Fig. 4.6: Constant Returns as given by TP, MP and AP

Non-Proportional Returns

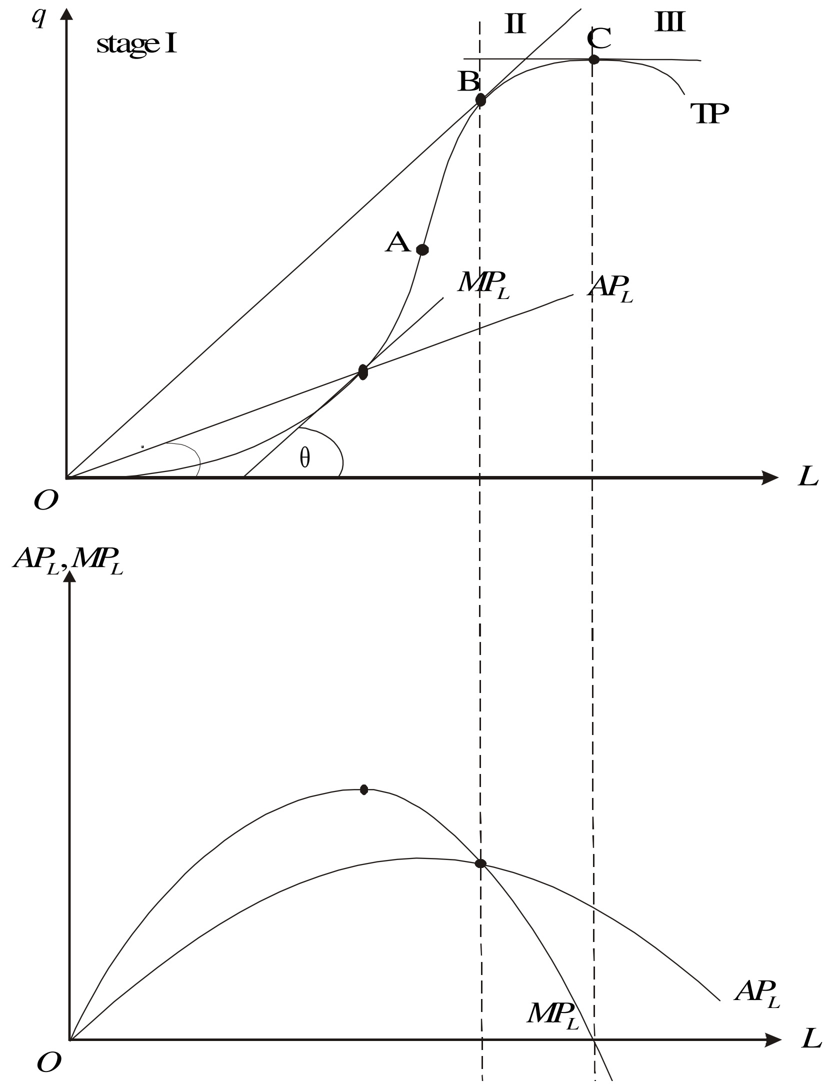


Fig. 4.7: Derivation of Stages of Production

The TP curve is such that upto point A, MP is rising and so is AP and MP > AP, as shown in the diagram below. Beyond point A, MP falls but AP rises, till the two are equated at point B. At B, AP is maximised. AP falls beyond the point B. At point C, the TP curve flattens out and therefore, MP = 0. Beyond C, MP is negative and AP is falling. Therefore, in the case of non-proportional return, both MP and AP rise, initially. MP reaches a maximum earlier than AP. When they both are equated, AP is maximised. Finally, there is a situation where both are falling.

Depending on the nature of MP and AP, the production process can be divided into three stages – I, II, and III, as shown in Figure 4.7.

Characteristics of the three stages are :

Stage I: MP > 0, AP rising, thus MP > AP

Stage II: MP > 0, AP falling, thus MP < AP

Stage III: MP < 0, AP falling

In stage I, by adding one more unit of L, the producer can increase the average productivity of all the units. Thus, it would be unwise for the producer to stop production in this stage.

In stage III, MP < 0, so that by reducing the L input, the producer can increase the total output and save the cost of a unit of L. Therefore, it is impractical for a producer to produce in this stage.

Hence, stage II represents the economically meaningful range. This is so because here MP > 0 and AP > MP. So that an additional L input would raise the total production. Besides, it is in this stage that the TP reaches a maximum.

Check Your Progress 2

1) Why marginal productivity of labour declines?

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2) The producer must choose the second stage from the total production curve. Why?

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3) What is partial input elasticity of output?

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4.3 LONG PERIOD ANALYSIS

Long period refers to a time when all the factors are variable. Earlier in the short period analysis, we had considered capital (K) to be fixed factor. Here in this part, we assume both L and K to be variable factors. Therefore, the production function would be:

$$q = F(L, K)$$

The producer can now employ L and K units at her will to produce output q as per the production technology. Therefore, in the K – L input space the producer can choose any combination of K and L to produce output.

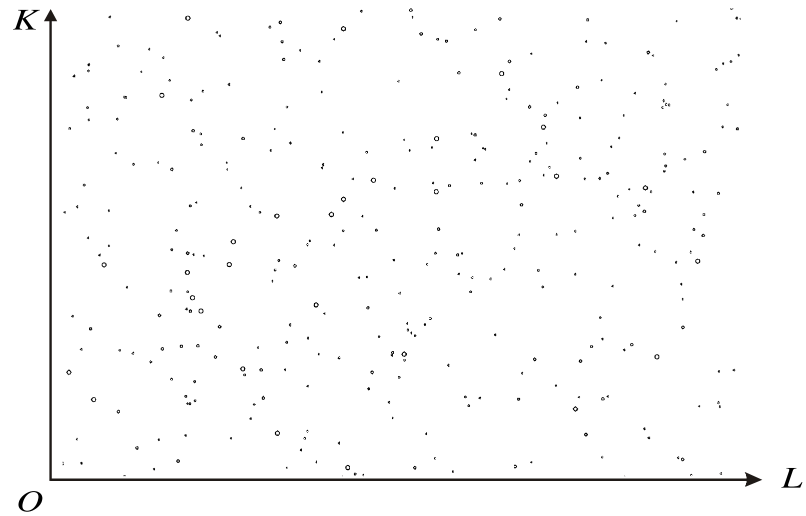

Fig. 4.8: K-L Input Space Available to Producer

4.3.1 Iso-quant

The dots in the above Figure 4.8 denotes the various combinations of (L, K) that the producer can pick up from form to produce. Among these combinations, there can be those, which produce the same level of output. Herein comes the concept of an iso-quant. An iso-quant is a locus of combinations of (L, K) that produce the same level of output ‘q’ (see Figure 4.9).

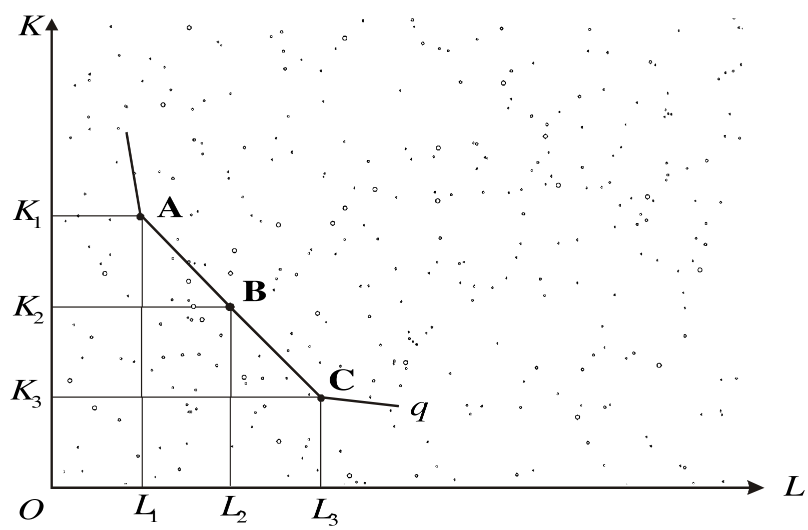

Fig. 4.9: Shape of Iso-quant

In the above Figure 4.9, $\overline{q}$ amount of output can be produced by the input combinations — (L_1, K_1), (L_2, K_2), (L_3, K_3). Joining these, we get an iso-quant, which is also denoted by $\overline{q}$. Therefore, we see that the same level of output $\overline{q}$ can be produced using different techniques – either more K, less L (e.g., technique A), or more L and less K (e.g., technique C).

Slope of an Iso-quant

Since along an iso-quant the level of output remains the same, if ΔL units of L are substituted for ΔK units of K, the increase in output due to ΔL units L (namely, $\Delta L.MP_L$) should match the decrease in output due to a decrease of ΔK units of K (namely $\Delta K.MP_K$). In other words,

$$|\Delta L.MP_L| = |\Delta K.MP_K|$$

$$\Rightarrow \left|\frac{\Delta K}{\Delta L}\right| = \frac{MP_L}{MP_K}$$

or, $$\frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$$

when Δ is a very small amount we can write $\frac{\Delta K}{\Delta L}$ as $\frac{dK}{dL}$. In the K – L input space, $\frac{dK}{dL}$ represents the slope of the iso-quant at any point on it.

$\therefore$ Slope of the iso-quant = $\frac{dK}{dL} = -\frac{MP_L}{MP_K}$.

Check Your Progress 3

1) Define an iso-quant.

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2) Derive the slope of an iso-quant mathematically, if the production function is q = F(L,K).

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The absolute value of $\frac{dK}{dL}$, denoted by $\left|\frac{dK}{dL}\right|$ is known as the marginal rate of technical substitution of L for K. ($MRTS_{LK}$). By definition, it measures the reduction in one input per unit increase in the other that is just sufficient to maintain a constant level of output. It is equal to the ratio of the marginal product of L to the marginal product of K.

Curvature of the Iso-quant

An iso-qunat is convex to the origin. This is so because as more and more units labour are employed, the producer would prefer to give up less and less of the other input to produce the same amount of output. This is shown in the following Figure 4.10:

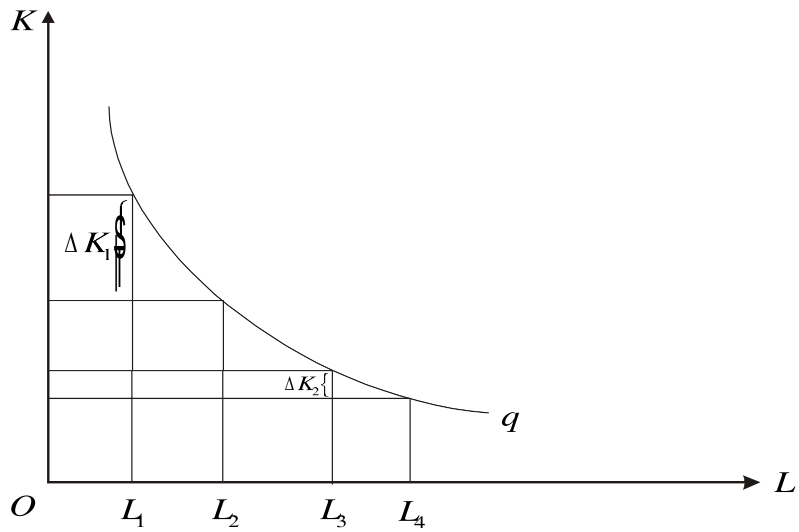

Fig. 4.10: Curvature of an Iso-quant

For a rise in L from L_1 to L_2, the producer gives up ΔK_1 amount of K. For the same output level $\bar{q}$, as L increases from L_3 to L_4, she gives up ΔK_2 amount of K. As $\Delta K_2 < \Delta K_1$, it implies that for more units of L, the producer is willing to give up less of K.

A convex iso-quant implies a diminishing $MRTS_{LK}$. As $MRTS_{LK} = \frac{MP_L}{MP_K}$, a diminishing MRTS means as L increases, MP_L decreases and MPK increases.

Economic Region of Iso-quants

With help of production function, we generate iso-quant maps as shown in the following Figure 4.11. The only difference these have with the previous iso-quants is that presently we have positively sloped segments.

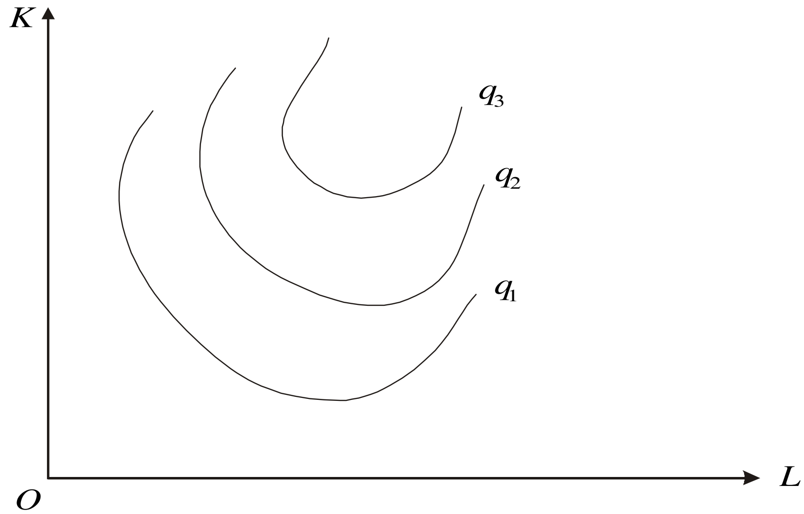

Fig. 4.11: Economic Region of Production and Iso-quants

Let us examine the characteristic of one such iso-quant with the help of the following Figure 4.12.

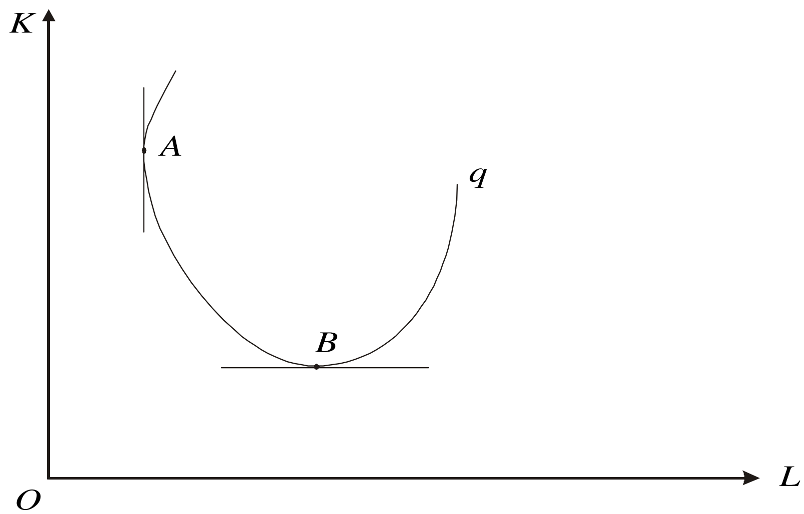

Fig. 4.12: Economic Region of Production

We know that the slope of an iso-quant is given by, $\frac{dK}{dL} = \frac{MP_L}{MP_K}$.

From the figure 4.12, at point A, the slope is infinite or undefined. This implies that at A, $MP_K = 0$. Beyond A, $\frac{dK}{dL} > 0$, implying $MP_K < 0$. As we had seen in the short-run analysis, no producer has an incentive to undertake production at this portion (this zone is similar to stage III of short run analysis).

At point B, $\frac{dK}{dL} = 0$, implying $MP_L = 0$. Beyond point B, MP_L falls. By similar logic, no producer would want to employ L beyond point B.

Therefore, production beyond points A and B are irrelevant. Hence, segment AB of the iso-quant is the economically feasible region. This is true for all iso-quants belonging to a family with positively sloped portions.

For these types of iso-quants we can obtain an *economic region of production* comprising the economically feasible portions of the iso-quants. This is obtained by constructing *ridge lines*, which are loci of points where $MP_K = 0$. The ridge lines provide a boundary to the economic region of production. This is shown in the following Figure 4.13.

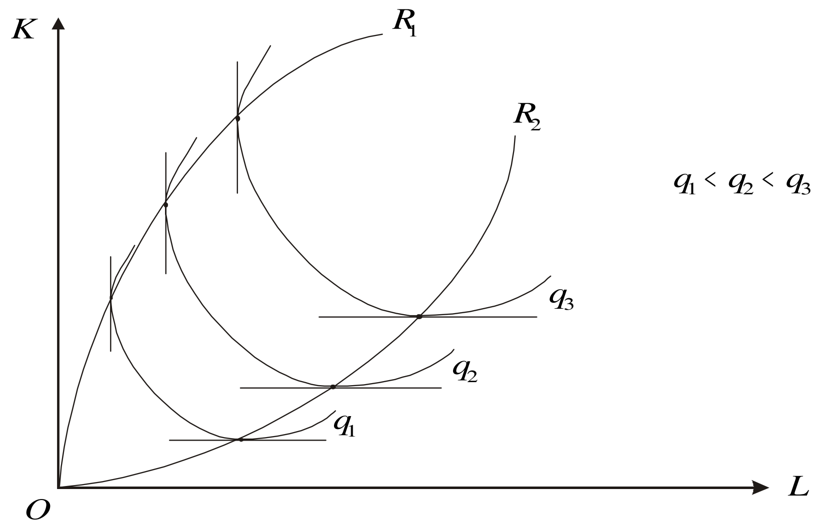

Fig. 4.13: Selection Economic Region of Production

In Figure 4.13, R_1 represents the ridgeline which is a locus of points where $MP_K = 0$. R_2 represents the ridge line where $MP_L = 0$.

Properties of Iso-quants

1) Iso-quants are non-intersecting, so that one and only one iso-quant will pass through a given point.

2) If both the inputs have positive marginal products, then the iso-quants are negatively sloped.

3) Iso-quants are convex to the origin.

4) An iso-quant lying away from the origin represents higher quantity of output.

Types of Iso-quants

Besides the smooth convex iso-quants assuming continuous substitutability, we get other types as follows:

1) **Linear:** This type of iso-quant assumes perfect substitutability between the factors of production. A given output quantity may be produced by using either only L or only K or by a combination of both (see Figure 4.14).

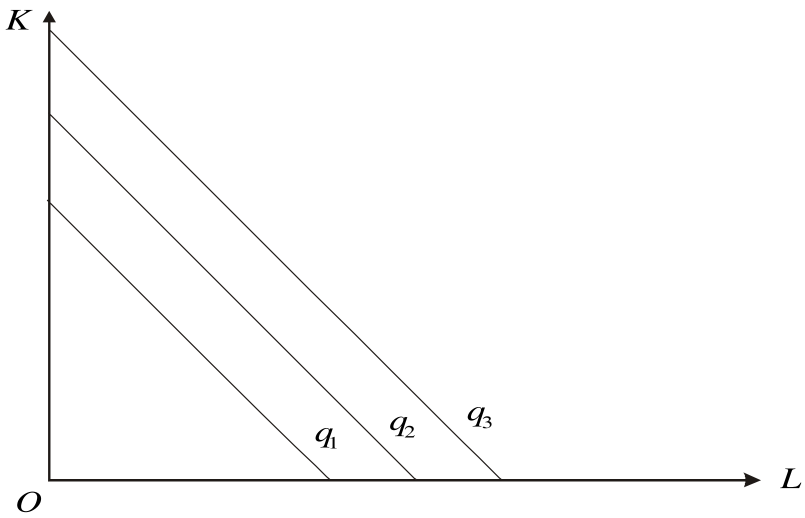

Fig. 4.14: Linear Iso-quant

2) **L-shaped:** Assuming strict complementarity between the inputs i.e., no substitutability of factors of production, we get L-shaped iso-quant (see Figure 4.15). It implies there is only one method of production.

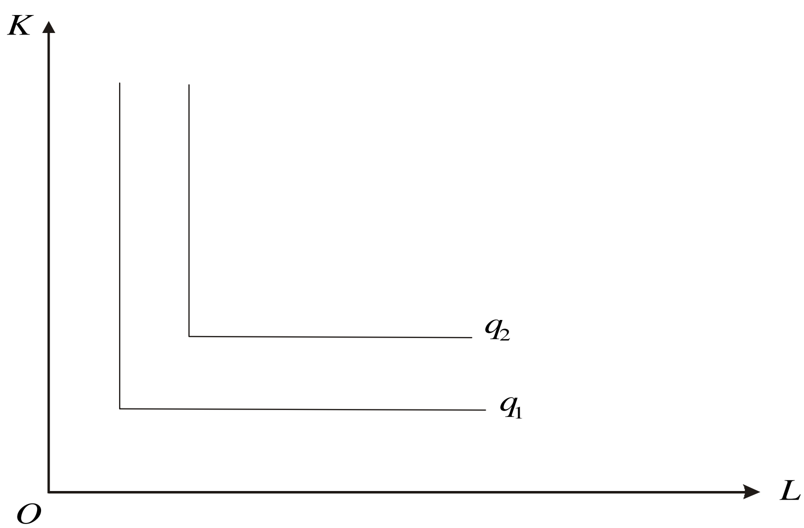

Fig. 4.15: L-shaped Iso-quants

4.3.2 Elasticity of Substitution

MRTS has a drawback as a measure of the degree of substitutability because it depends on the units of measurement of factors. Therefore, we need to consider the concept of elasticity.

The elasticity of substitution denoted by σ, is a pure number that measures the rate at which substitution takes place. It is defined as the percentage change in K – L ratio for one percentage change in the MP_L – MP_K ratio (or $MRTS_{LK}$).

Thus, $$\sigma = \frac{\text{\% change in K/L}}{\text{1\% change in MRTS}}$$

In terms of differential calculus,

$$\sigma = \frac{\dfrac{d(K/L)}{(K/L)}}{\dfrac{d(f_L/f_K)}{(f_L/f_K)}} = \frac{d(K/L)}{d(f_L/f_K)} \cdot \frac{f_L/f_K}{K/L}$$

$$\sigma = \frac{d \log\left(\dfrac{K}{L}\right)}{d \log\left(\dfrac{f_L}{f_K}\right)}$$

Generally, σ varies from point to point on an iso-quant. But in some special cases $\sigma = 0$ or σ □ □.

Check Your Progress 4

1) What is MRTS ?

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2) What is elasticity of substitution?

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3) Find for which iso-quants $\sigma = 0$ and for which σ □ □.

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4) What do you mean by returns to scale?

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5) What is economic region of production?

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4.4 RETURNS TO SCALE

It is a typical long-run concept and involves the effect of change in inputs on the quantity of output produced. There are three types of returns to scale.

For 1% change in all the factors, if correspondingly, output changes by

a) 1%, then there is said to be Constant Returns to Scale (CRS)

b) Less than 1%, then there is decreasing Returns to Scale (DRS)

c) More than 1%, then there is Increasing Returns to Scale (IRS).

Diagrammatic Representation

Constant Returns to Scale

As there is CRS, along a ray through the origin, the distance between the consecutive iso-quants remains the same (see Figure 4.16).

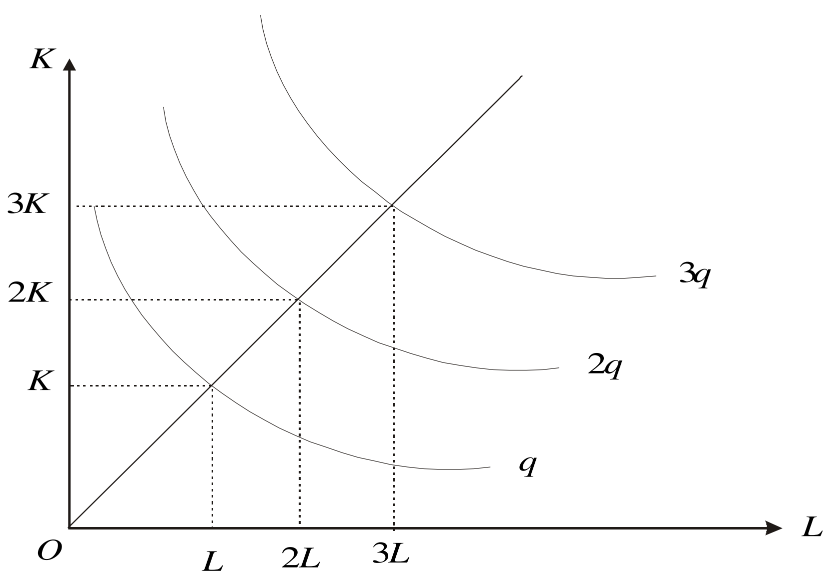

Fig. 4.16: Derivation of Line of Scale

Decreasing Returns to Scale

As there is DRS, along a ray through the origin, the distance between the consecutive iso-quants increases (see Figure 4.17).

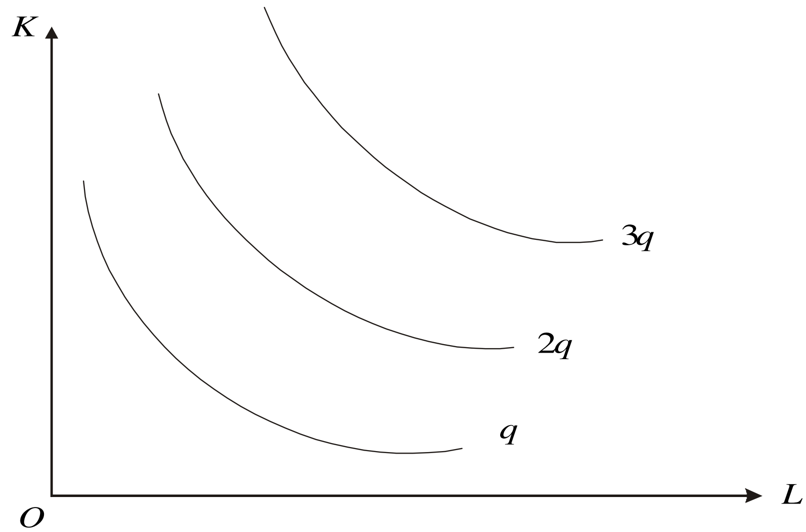

Fig. 4.17: DRS and Distance between Iso-quants

Increasing Returns to Scale

As there is IRS, along a ray through the origin, the distance between the consecutive iso-quants is less (see Figure 4.18).

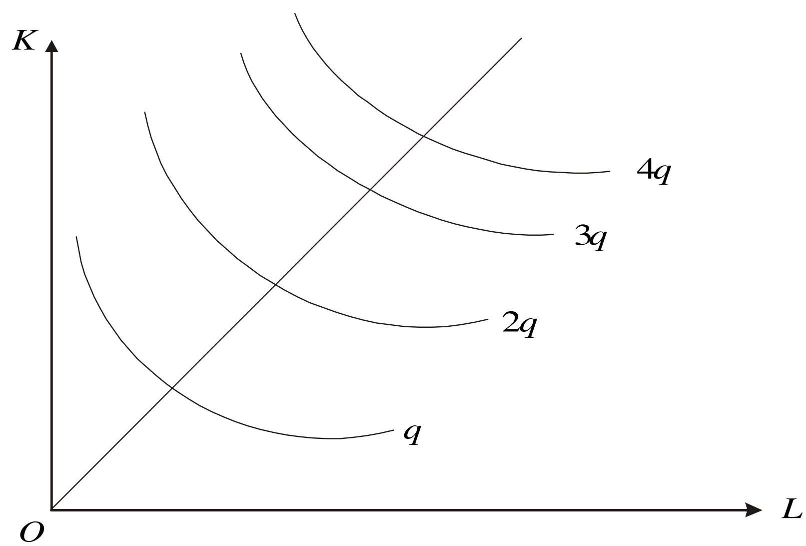

Fig. 4.18: IRS and Distance between Iso-quants

4.5 HOMOGENEOUS PRODUCTION FUNCTION

The concept of returns to scale can be captured asily using the concept of homogeneity of production functions.

A production function say, z = f(x, y) is said to be homogeneous of degree n in x and y, if and only if,

$f(tx, ty) = t^n z$ for any $t > 0$.

If n = 1, then f(tx, ty) = tz, then the function is said to exhibit CRS. It is also known as a linearly homogeneous production function.

If n > 1, then the function exhibits IRS and when n< 1, it exhibits DRS.

Example:

Suppose we have a production function,

$q = AL^{\alpha}K^{\beta}$

If L and K are increased by a factor t(>0) then, we have, $A(tL)^{\alpha}(tK)^{\beta}$

$= At^{\alpha}L^{\alpha}t^{\beta}K^{\beta}$

$= t^{(\alpha+\beta)}AL^{\alpha}K^{\beta}$

$= t^{(\alpha+\beta)}q$

 The function is homogeneous of degree (α+β). It is said to exhibit

CRS, if α+β = 1; DRS if α+β < 1; IRS if α+β > 1

When α+β = 1, the function is also called a linearly homogeneous function.

Properties of a homogeneous production function.

1) If q = F(K, L) is a homogeneous production function, then we can write it as,

   $F(tK, tL) = t^{n}q$ for any t > 0.

   Let $t = \frac{1}{L}$ for any L > 0.

Then the function can be written as,

$$F(tK, tL) = F\left(\frac{K}{L},\ 1\right) = \frac{q}{L^{n}}$$

Let $K = \frac{K}{L}$ and $F = \left(\frac{K}{L},\ 1\right) = f(k)$.

$\therefore$ we have $f(k) = \frac{q}{L^{n}}$

or, $q = L^{n}f(k)$

2) $q = L^{n}f(k)$

$$MP_{L} = \frac{\partial q}{\partial L} = nL^{n-1}f(k) - KL^{n-2}f'(k)$$

$$MP_{K} = \frac{\partial q}{\partial K} = L^{n-1}f'(k).$$

Both MP_L and MP_K are functions of $k\left(=\frac{K}{L}\right)$.

It may be useful to remember that homogenous production function is a special case of **homothetic production** functions. To take note of the concept, you must look for the ratio MP_L/MP_K, which does not change with any proportionate change in L and K in case of homothetic production function. The difference this formulation has with that of homogenous production function can be seen from the iso-quants given in Figure 4.19. The production function is homothetic if

slope of Q_1 at A_1 = slope of Q_2 at A_2

and

slope of Q_1 at B_1 = slope of Q_2 at B_2

In contrast to this feature, the homogenous production function would have $OA_2/OA_1 = OB_2/OB_1$.

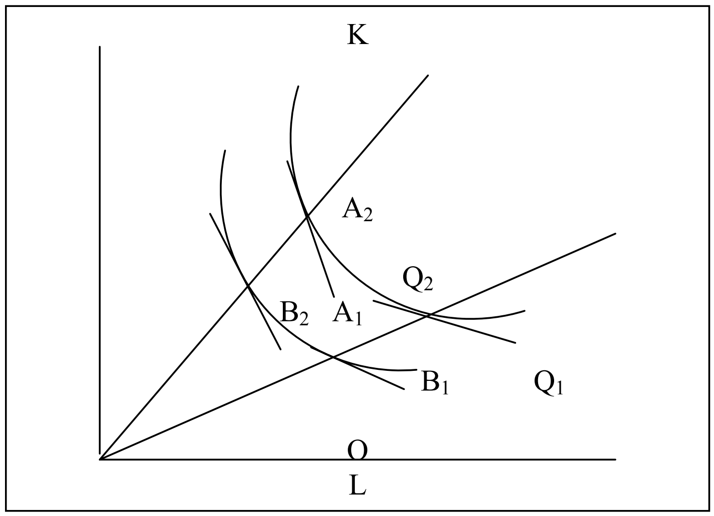

Fig. 4.19: Homothetic Production Function

Check Your Progress 5

1) What is a homogeneous production function?

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2) Prove for $q = L^n f(k)$ $MP_L = nL^{n-1} f(k) - KL^{n-2} f'(k)$; $MP_K = L^{n-1} f'(k)$.

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3) When MP_L and MP_K are functions of (K/L)?

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4.6 LET US SUM UP

This unit covers theoretical insights on production process. It starts with production function and points out that it a technical relation between inputs and output. Production decisions are based on short-and long-run considerations. While the short-run offers enough time to a producer to change the variable inputs like labour and raw materials, the long-run allows changing all inputs, including machinery and building etc. To carry on production in both the periods, it helps understand the productivity concepts of average and marginal products. The average product is the output produced per unit of an input. Marginal product, on the other hand, gives change in the total product due to a unit change in one of the inputs. It is convenient to speak in terms of average produce of labour (AR) and marginal product of labour $\left(MP_2\right)$. The production process brings out the relationship between AP and MP. It is seen that when AP is rising MP>AP, when AP is maximum, MP = AP and when AP is falling MP<AP. The total production passes through three distinct stages (I, II and III). In stage –I, the total product curve shows an increasing trend, while in stage –III it declines. A producer usually chooses to operation in stage-II of the product curve when AP reaches the maximum point.

A single level of output can be produced by different combinations of inputs. The curve that captures this feature is called an iso-quant. The slope of an iso-quant allows one to analyse the process of input substitution for a given level of output. When there are two inputs, labour and capital, the slope of the iso-quant is given by $\frac{dK}{dL} = -\frac{MP_L}{MP_K}$, where $\frac{\mathrm{dK}}{\mathrm{dL}}$ is called marginal rate of technical substitution of L for K

$\left(MRTS_{LK}\right)$ and MP_L and MP_K are marginal productivities of labour and capital. The rate at which the substitution between the factors takes place is given by the elasticity of substitution. It is usually devoted by σ**.** Thus

$$\sigma = \frac{\text{\% change in K/L}}{\text{\% change in MRTS}}$$

Simultaneous change of all inputs results in a change in output in proportion to greater, equal or lower scale. Such a feature is studied through returns to scale. The last part of the discussion is focussed on returns to scale through homogenous production function. The degree of homogeneity is related to an assessment of returns to scale and the properties of the homogenous production function is highlighted to analyse the partial productivities of capital and labour.

4.7 KEY WORDS

Average Product: Total product per unit of an input

Cobb-Douglas Production Function: A production function of the form Q= f(aK^bL^c) where a, b and c are constants, Q is output, and L and K are inputs

Constant Returns to Scale: The case where a proportionate change in all inputs changes output by the same proportion.

Decreasing Returns to Scale: The case where a proportionate increase in all inputs leads output to increase by a small proportion

Diminishing Marginal Rate of Substitution: The declining marginal rate of substitution as one input is substituted for another.

Economic Region of Production: The downward sloping segment of an iso-quant

Elasticity of Substitution: A measure of the responsiveness of the input ratio to a change in the input-price ratio.

Homogeneous Production Function: A special case of homothetic production function in which a proportionate change in inputs causes output to change by a proportion which does not vary changes in the inputs.

Homothetic Production Function: A production function where the ratio of marginal product is unaffected by a proportionate change in inputs**.**

Increasing Returns to Scale: A situation where proportionate increase in all inputs causes output to increase by a large proportion.

Iso–quant Line: The locus of points representing various combinations of inputs yielding a specified and of output.

Long Period Production: A period of time sufficient for altering the quantities of all inputs into the production process.

Marginal Rate of Technical Substitution: The rate at which one input can be substituted for another without affecting the level of output.

Production Function: The functional relationship between inputs and output.

4.8 SOME USEFUL BOOKS

Ferguson and Gould (1989), *Microeconomic Theory*, Irwin Publications in Economics; Homewood, IL: Irwin.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillian.

Ferguson, C. E. (1969), *The Neoclassical Theory of Production and Distribution*, Cambridge: Cambridge University Press.

4.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) A technical relation between inputs and output.

2) It is likely to conflict with the scale of production and may contribute to the increased cost.

3) Change in output due to a unit change in labour with level of other factors kept constant.

Check Your Progress 2

1) Because contribution of other factors to production cannot be perfectly substituted by labour.

2) At this state AP reaches the maximum.

3) It is elasticity of output with respect to a factor (show the derivation).

Check Your Progress 3

1) Different combinations of inputs producing a fixed level of output.

2) See section on slope of an iso-quant

Check Your Progress 4

1) Rate of substitution between inputs allowed by the production function.

2) Define as given in the text and explain the meaning.

3) Consider the iso-quants which are not of regular shape, e.g., complementary inputs.

4) See the change in output by changing all inputs simultaneously.

Check Your Progress 5

1) Change in inputs changes the value of function to a certain degree.

2) See the derivation given the text in section on Homogenous Production Function.

3) In Homogenous Production Function (see, the derivation in the text on homogenous production function).