UNIT 6 PRODUCTION ECONOMICS

Structure

6.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the important theoretical insights in looking at production process empirically;
- formulate cost and production functions for purpose of estimation;
- appreciate the idea of technical process; and
- derive profit maximisation and cost minimisation conditions as a part of dual problem.

6.1 INTRODUCTION

An examination of production processes of a firm requires understanding of the relationship between input and output specified by the technology along with cost conditions that would decide the feasibility of a production plan. Therefore, it is necessary to understand the empirical formulation of production functions and relation between the conditions of profit maximisation and cost minimisation. In view of this, the following discussion focuses on (i) specifications of production and cost functions, (ii) conditions of profit maximisation and (iii) cost minimising input demands that satisfy the profit maximisation requirements. In the process, some important issues related to technical progress, constrained optimisation and duality between cost minimisation and output maximisation would be looked into.

6.2 PRODUCTION FUNCTIONS

In this section, we present four simple production functions that are widely used in empirical estimation. In each case, the discussion aims at depicting the features of input-output functional relation, returns scale envisaged and scope for factor substitution. Two-input cases have been considered in each production function although generalisation to many inputs can easily be accomplished and we will present these separately.

i) **Linear Production Function**

Suppose that the production function is given by:

$q = f(K, L) = aK + bL.$ …..(6.0)

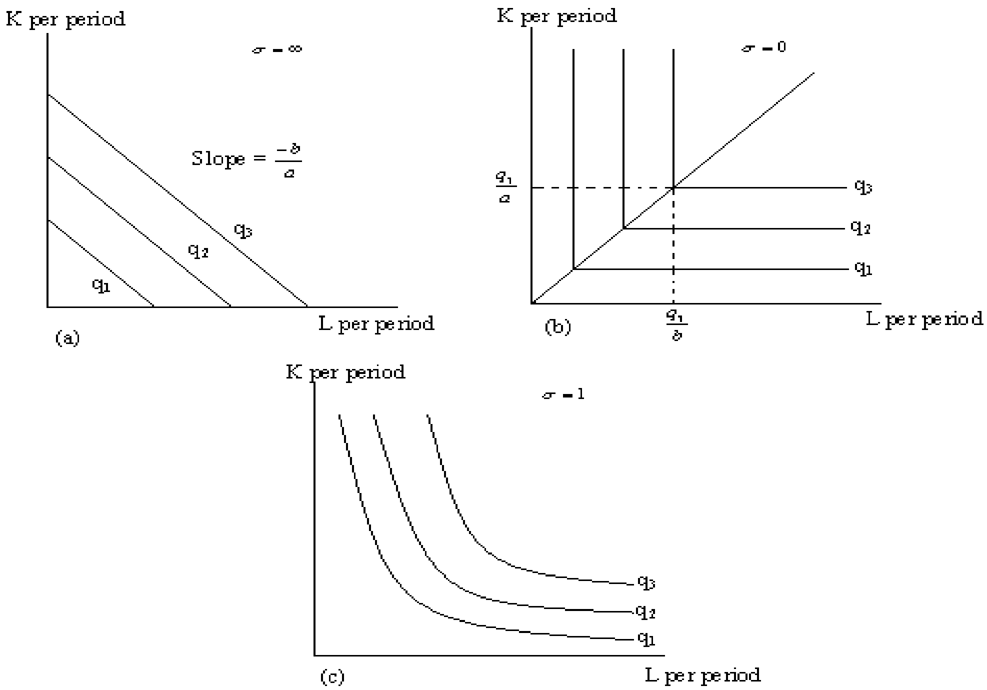

Fig. 6.1: Iso-quant Maps for Production Functions

The production function exhibits constant returns to scale. It can be seen that for any m > 0,

$f(mK, mL) = amK + bmL = m(aK + bL) = mf(K, L).$ ….. (6.1)

The iso-quants of the production function are parallel straight lines with slope (–b/a) (see Figure 6.1(a)). Since along any straight-line iso-quant, the RTS is constant, the denominator of the elasticity of factor substitution σ is equal to 0, and hence σ is equal to infinity. The feature of infinity factor substitution possibility indicates that you can substitute labour for capital or capital for labout without any limit. Such flexibility makes the functional formulation counter intuitive. Perhaps you will appreciate that we require both of capital and labour for any production and cannot think of ignoring either irrespective of their prices.

ii) **Fixed Proportions Production Function**

The fixed-proportions production function is given by:

$q = \min(aK, bL) \; a, b > 0$ ….. (6.2)

where the operator "min" indicates that q is given by the smaller of the two values in parentheses. See that in this formulation capital and labour are used in a fixed ratio. Consequently, its iso-quants are L-shaped (see Figue 6.1(b)).

A firm characterised by this production function will always operate along the ray where the ratio K/L is fixed at b/a and any point of operation other than at the vertex of the iso-quants would be inefficient. Because K/L is a constant, it is easy to see from the definition of the elasticity of substitution that σ must equal 0.

With the form reflected by Equation 6.2, the fixed proportion production function exhibits constant returns to scale since

f(mK, mL) = min (amK, bmL) = (m)min (aK + bL) = mf(K, L)

for any m > 0, increasing or decreasing returns can be easily incorporated into the function by using a nonlinear transformation of the functional form.

iii) **Cobb-Douglas Production Function**

The Cobb-Douglas production function is given by:

$q = f(K, L) = AK^aL^b,$ ….. (6.3)

where A, a, and b are all positive constants.

Iso-quants resulting from the functional form have convex shape (see Figure 6.1(c)). The Cobb-Douglas function can exhibit any degree of returns scale depending on the values of a and b. Suppose all inputs were increased by a factor of m. Then,

$$f(mK, mL) = A(mK)^a\,(mL)^b = Am^{a+b}\,K^aL^b \qquad \text{….. (6.4)}$$

$$= m^{a+b} f(K, L).$$

Hence, in the Cobb-Douglas function

- a + b = 1 implies constant returns to scale
- a + b > 1 implies increasing returns to scale
- a + b < 1 implies decreasing returns-to-scale case.

To determine the elasticity of substitution in Cobb-Douglas production function, let us use Allen's definition $\sigma = \dfrac{\frac{\partial q}{\partial L}\cdot\left(\partial q/\partial k\right)}{q\cdot\left(\partial^2 q/\partial L\cdot\partial K\right)}$.

When $q = AK^a\,L^{1-a}$,

$$\sigma = \frac{(1-a)\left(q/L\right)\cdot a\left(q/K\right)}{\dfrac{q^2\,(1-a)(a)}{KL}} = 1$$

Note that the Cobb-Douglas function is linear in logarithms, i.e.,

$\ln q = \ln A + a \ln K + b \ln L.$ ….. (6.5)

As a result, the constant a in (eqn. 6.5) is the elasticity of output with respect to capital input, and b is the elasticity of output with respect to labour input. To get the result, take

$e_{q,k} = \dfrac{\partial q}{\partial K}\cdot\dfrac{K}{q} = \dfrac{\partial \ln q}{\partial \text{ in } K}$ such that

$e_{q,k}$ = a from Equation 6.5. Similarly, we get $e_{q,L}$ = b.

iv) **CES Production Function**

The constant elasticity of substitution (CES) production function is given by

$$q = \gamma \left[\delta K^{\rho} + (1-\delta) L\rho\right]^{e/\rho} \quad \text{….. (6.6)}$$

where $\gamma > 0$, $0 \le \delta \le 1$, $\rho \le 1$ and $\varepsilon \ge 0$.

It is important to note that

γ shifts the production function and is called the efficiency parameter;

δ allows is K and L to vary and is called a distribution parameter and

ρ is the substitution parameter. In case of higher the elasticity of substitution, ρ is equal to its maximum value of 1. It can be shown that for the constant returns-to-scale case

$$\sigma = \frac{1}{1-\rho}$$

so that CES function incorporates the linear, fixed-proportions and Cobb Douglas functions as special cases (for ρ = 1, – ∝, and 0 respectively). In particular, for $\rho = 0$, CES production function approaches a Cobb-Douglas function of the form

$$q = \gamma K^{\delta} L^{1-\delta}$$

Further, it explains the use of the term distribution parameter for the parameter δ, since the exponents of the Cobb-Douglas function with constant returns to scale equal competitively determined factor income shares.

It can be shown that CES function exhibits increasing, constant or decreasing returns to scale depending on the parameter $\varepsilon > 1$, $\varepsilon = 1$, or $\varepsilon < 1$:

$$f(mK, mL) = \gamma [\delta(mK)^{\rho} + (1-\delta)\, mL^{\rho}]^{\varepsilon\rho}$$
$$= \gamma [m^{\rho}]^{\varepsilon/\rho} + (\delta K^{\rho} + (1-\delta) L^{\rho})^{\varepsilon/\rho} \quad \text{….. (6.7)}$$
$$= m^{e} f(K, L)$$

for any m > 0.

Many-Input Cases

As we have stated earlier, the two-input cases of production functions can be generalised to many-input ones. In the following, we present Cobb-Douglas and CES formulations along with Generalised Leontif and Translog production functions. In all of these examples, β's are nonnegative parameters, and the n inputs are represented by $X_1 \ldots X_n$.

Cobb-Douglas

A many-input Cobb-Douglas production function is given by:

$q = \prod_{i=1}^{n} X_i^{\beta i}$ where X_i for i = 1, 2………n represents input and parameter β_i, the non-negative input coefficient.

The function would exhibit constant returns to scale if $\sum_{t=i}^{n} \beta_i = 1$. Any degree of increasing returns to scale can be incorporated into this function depending on $\sum_{t=i}^{n} \beta_i$.

CES

The many-input CES production function is given by

$$q = \left[\sum \beta_i X^{\rho} \right]^{\varepsilon/\rho} \qquad \rho \le 1.$$

If ε = 1, the function is under constant returns to scale while for ε > 1 it would be under increasing returns to scale.

Generalised Leontief

The many-input Leontief production function is given by

$q = \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \sqrt{X_i X_j} \qquad \beta_{ij} = \beta_{ji}$. We need to impose the symmetry condition of $\beta_{ij} = \beta_{ji}$ to ensure the fulfillment of second-order partial derivatives.

The function exhibits constant returns to scale and increasing returns to scale can be incorporated into the function by using the transformation

$q' = q^{\varepsilon}$, $\varepsilon > 1$.

Translog

The translog production function is written as

$$\log q = \beta_o + \sum_{i=1}^{n} \beta_i \ln X_i + 0.5 \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln X_i \ln X_j \quad \text{with} \quad \beta_{ij} = \beta_{ji} \quad .$$

The condition is $\beta_{ij} = \beta_{jt}$ is required to assure the equality of cross-partial derivatives.

The important features of the function are:

- flexibility of capturing any degree of returns to scale. If $\sum_{i=1}^{n} \beta_i = 1$ and $\sum_{i=1}^{n} \beta_{ij} = 0$ for all i, it exhibits constant returns to scale
- depiction of Cobb-Douglas function as a special case if $\beta_0 = \beta_{ij} = 0$ for all i, j. n

6.2.1 Technical Progress

Using the concept of production function, it is possible to assess improvement in the methods of production over time. In order words, we can examine if the same level of output is produced with less input or higher levels of output are obtained with the same level of inputs. Essentially our concern is to derive the change in output per unit of input within the framework of production function over time. The possible variations in the production that may be observed are called technical progress.

Note that the technical progress attempts to evaluate the rate of growth of output over time, which might have exceeded the growth rate that can be attributed to the growth in conventionally defined inputs. To evaluate such a feature, the production function is written in a modified form as follows:

$q = A(t) f(K, L)$ ….. (6.9)

Look at the term A(t) in the above function. It represents influences that go into the determination of q besides the inputs K and L. Changes in A over time represent technical progress.

Differentiating Equation 6.9 with respect to time gives:

$$\frac{dq}{dt} = \frac{dA}{dt} \cdot f(K,L) + A.\frac{df(K,L)}{dt}$$

$$= \frac{dA}{dt} \cdot \frac{q}{A} + \frac{q}{f(K,L)}\left[\frac{\partial f}{\partial K} \cdot \frac{dK}{dt} + \frac{\partial f}{\partial L} \cdot \frac{dL}{dt}\right]. \qquad \text{….. (6.10)}$$

Dividing by q, we get

$$= \frac{dq/dt}{q} = \frac{dA/dt}{A} + \frac{\partial f/\partial K}{f(K,L)} \cdot \frac{dK}{dt} + \frac{\partial f/\partial L}{f(K,L)} \cdot \frac{dL}{dt}. \qquad \text{….. (6.11)}$$

or, $$\frac{dq/dt}{q} = \frac{dA/dt}{A} + \frac{\partial f}{\partial K} \cdot \frac{K}{f(K,L)} \cdot \frac{dK/dt}{K} + \frac{\partial f}{\partial L} \cdot \frac{L}{f(K,L)} \cdot \frac{dL/dt}{L}.$$

For any variable x, (dx/dt)/x is the rate of growth of x per unit of time (G_x). We write Equation 6.11 as

$$G_q = G_A + \frac{\partial f}{\partial K} \cdot \frac{K}{f(K,L)} \cdot G_K + \frac{\partial f}{\partial L} \cdot \frac{L}{f(K,L)} \cdot G_L \qquad \text{….. (6.12)}$$

Since $\frac{\partial f}{\partial K} \cdot \frac{K}{f(K,L)} = \frac{\partial q}{\partial K} \cdot \frac{K}{q} =$ elasticity of output with respect to capital input

$= e_{q.K}$

and

$\frac{\partial f}{\partial L} \cdot \frac{L}{f(K,L)} = \frac{\partial q}{\partial L} \cdot \frac{L}{q} =$ elasticity of output with respect to labour input

$= e_{q.L}$.

Thus, Equation 6.11 can be shown as

$G_q = G_A + e_{qK}G_K + e_{qL}G_L$. ….. (6.13)

It is not difficult to see that the rate of growth in output is decomposed into the sum of two components: growth attributed to changes in inputs (K and L) and other "residual" growth due to technical progress clubbed together to changes in A.

6.2.2 Classifying Technical Progress

When technical progress enters the production, we get more output from a given combination of inputs. With the on set of such a process, the technical change factor A(t) can have three possible effects, viz.,

1) *Neutral technical progress:* The change affects all the inputs equally
2) *Capital augmenting technical progress:* The change affects only capital
3) *Labour augmenting technical progress:* The change affects only labour.

Check Your Progress 1

1) What is the basic difference between Cobb-Douglas and CES production functions?

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2) How do you determine the returns to scale of a Cobb-Douglas production functions?

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3) How would you measure technical progress?

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6.3 COST FUNCTIONS

The total cost function depicts the functional relationship set of input costs for any output level. We present three important cost functions often used in empirical estimation. The following discussion starts with two-input case of Cobb-Douglas and CES; generalises these to many–input framework and adds the translog cost function.

Cobb-Douglas

The Cobb-Douglas cost function for two inputs (K and L) is given by

$TC = (q)(r^{\alpha} w^{\beta})$ where $\alpha + \beta = 1$.

The elasticity of substitution between K and L of the function is measured by

$$s = \frac{\partial(K/L)}{\partial(w/r)} \cdot \frac{(w/r)}{(K/L)} = \frac{\partial \ln(K/L)}{\partial \ln(w/r)}$$

CES

The CES cost function for two inputs is given by:

$TC = q[\delta^{\sigma} r^{1-\sigma} + (1-\delta)^{\sigma} w^{1-\sigma}]^{1/1-\sigma}$.

The elasticity of substitution of the function is the same and constant for all values of K and L as found for its production function.

Translog

The translog cost function is given as

$\ln TC = \ln q + \beta_0 + \beta_1 \ln w + (1-\beta_1) \ln r + \beta_2 (\ln w)^2 + \beta_3 (\ln r)^2 + \beta_4 \ln w \ln r$.

See that it is formulated from an approximation to any arbitrary cost function. It reduces to Cobb-Douglas cost function if $\beta_2 = \beta_3 = \beta_4 = 0$ and homogeneous of degree l in w and r, with $\beta_2 + \beta_3 + \beta_4 = 0$.

Cost Functions with Many-Inputs

The three cost functions above can be generalized to many inputs (X_i …. X_n) with prices w_1 …. w_n.

i) **Cobb-Douglas:** $TC = q\prod_{i=1}^{n} w_i^{\beta_i}$ where $\sum_{i=1}^{n} \beta_i = 1$

ii) **CES:** $TC = q\left[\sum_{i=1}^{n} \beta_1 w_i^{1-a}\right]^{1/1-\sigma}$

iii) **Translog:** $\ln TC = \ln q + \beta_0 + \sum_{i=1}^{n} \beta_i$, $\ln w_i + 0.5 \sum_i \sum_j \beta_n \ln w_i \ln w_j$

where $\sum \beta_i = 1$ and $\beta_{ij} = \beta_{ji}$.

The partial elasticity of substitution (s_{ij}) is computed from this cost function as

$$s_{ij} = \frac{TC \cdot TC_{ij}}{TC_i \cdot TC_j}$$

The advantage of the above formulation of s_{ij} is that for the Cobb-Douglas cost function $s_{ij} = 1$ and for that of CES, s_{ij} = a constant.

a) cost shares for the many-input translog cost function are given by

$$s_i = \frac{w_i X_i}{TC} = \beta_i + \sum_{i=1}^{n} \beta_{ij} \ln w_i .$$

b) For the translog case, the partial elasticity of substitution is given by

$s_{ij} = (\beta_{ij} + s_i s_j)/s_i s_j$.

Check Your Progress 2

1) A cost function is given by $TC = .2\, qw^{.5}\, r^{.5}$. Name which cost function it describes. Why do you think so?

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2) Write a CES cost function. Find its elasticity of substitution.

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3) Write a translog cost function and show the condition in which it will be homogenous of degree 1.

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6.4 PROFIT MAXIMISATION

A profit–maximising firm chooses both its inputs and output such that it reaps maximum economic profits. In order to see the underlying process, let us start with profit as the difference between revenue earned from selling output and costs incurred in the process of its production.

So, π = TR (q) – TC (q),

= p(q)q – TC (q).

To maximise π, we different with respect to q so that MR = MC.

The second order condition requires that

$$\frac{d^2\pi}{dq^2}\Big|_{q=q^*}$$

$$= \frac{d\pi'(q)}{dq}\Big|_{q=q^*} < 0$$

i.e., marginal profit must be decreasing at the optimal level of q.

6.4.1 Profit Maximisation and Input Demand

Until now we have considered firm's decision to maximise profit from a given output level. Since output level itself is determined by the inputs chosen through a production function, say q = f(K, L), it seems appropriate to include inputs in the profit maximisation. So, we write

π (K, L) = pq – TC (q)

= pf(K, L) – (rK + wL).

The profit maximisation exercise, therefore, is firm's decision problem of selecting appropriate levels of capital and labour inputs. Let us assume that the producer is a price taker so that the output and input prices are parameters in the maximisation problem.

The first order conditions (FOCs) of the maximisation problem are:

$$\frac{\partial \pi}{\partial \mathrm{K}} = p\frac{\partial f}{\partial \mathrm{K}} - r = 0$$

$$\frac{\partial \pi}{\partial \mathrm{L}} = p\frac{\partial f}{\partial \mathrm{L}} - w = 0$$

Solving these two equations, we get

$$\frac{\mathrm{MP}_L}{\mathrm{MP}_\mathrm{K}} = \frac{w}{\mathrm{r}}$$

The second order conditions of the maximisation need to satisfy such that:

$\pi_{KK} < 0$, $\pi_{LL} < 0$ and $\pi_{KK}\pi_{LL} - \pi^2_{KL} > 0$

6.4.2 Supply Function

The supply function shows the quantity of output produced by the firm at various prices for its output as well as input costs. Thus, from the first order condition of profit maximisation, we solve for optimal inputs functions of the parameter p, r and w. If

$K^* = K^*$ (p, r, w) and

$L^* = L^*$ (p, r, w), these inputs can be included in the production q = *f*(K, L) to obtain the profit-maximising output (q^*) to be chosen. So,

$q^* = f(K^*, L^*)$

$= f\,[K^*(p, r, w), L^*(p, r, w,)]$

$= q^*(p, r, w)$

Thus, the profit function shows that firm's profit depends on market prices, p, r and w. We will derive the supply function for the firm, given the profit function. To understand the underlying reasoning, consider the case of a single output produced through a single input. In this case

$\pi = pf(x) - wx$, the FOC is

$$\frac{d\pi}{dx} = p\frac{\partial f(x)}{\partial \mathrm{x}} - w = 0 \qquad \text{….. (6.14)}$$

The factor demand function x(p, w) must satisfy the FOC. Therefore, we write the profit function as

$\pi(p, w) = pf(x(p, w)) - wx(p, w)$

Differentiating the profit function with respect to w, we have

$$\frac{\partial \pi}{\partial w} = p\frac{\partial f(x(p,w))}{\partial \mathrm{x}} - \frac{\partial x}{\partial w} - w\frac{\partial x}{\partial w} - x(p,w)$$

$$= \left[p\frac{\partial f(x(p,w))}{\partial \mathrm{x}} - w\right]\frac{\partial x}{\partial w} - x(p,w)$$

Substituting from (1), we derive

$$\frac{\partial \pi}{\partial w} = -x(p, w) \quad \text{….. (6.15)}$$

The minus sign indicates that increasing price of an input leads to decrease in profit.

Return to the equation above and see that change in the input price has two part-effects on the changed profit. First, there is a direct effect of increasing profit because of the price increase even when the firm produces the same level of output. Secondly, there is an indirect effect induced by the increased output price; the firm moves to change its level of output by a small amount. However, the change in profits due to an infinitesimal change in output must be zero since production is already in the profit-maximizing level.

6.4.3 Hotelling Lemma

Equation (6.15) can be related to the method used in the Hotelling Lemma to derive firm's net supply function from the profit function by differentiating it.

Let $y_i(p)$ be the firm's net supply function for good i. Then

$$y_i(p) = \frac{\partial \pi(P)}{\partial p_i} \text{ for } i = 1, 2, \ldots, n$$

assuming that the derivative exists and that $p_i > 0$.

6.4.4 The Envelope Theorem

The envelope theorem shows the change in the optimal value of a particular function when a parameter of the function changes. In the context of our present discussion, we would like to see the change in profit function due to change in input prices. The envelope theorem provides a short route to solve the problem. It states that the change in the optimal value of a function f (x, a) with respect to a parameter (a) of that function can be derived by partially differentiating the objective function while holding the optimal value of x constant. To see the underlying process, take f (x, a) as the function of both x and a.

Let a be the parameter in the function above. We choose x to maximise the function. However, for each value of a, there will be a different optimal choice of x. Therefore, in sufficiently regular cases, it will be possible to write the function x(a), which gives us the optimal choice of x for each different value of a.

There are two equivalent ways to solve the optimisation problem, provided by the envelope theorem. First, define a value function M(a) = f (x(a),a) which gives the optimal value of f for different choices of a.

So with optimised value of x, we write

$$M(a) \equiv f(x(a),a) \quad \text{….. (6.16)}$$

Differentiating both the sides of the identity (6.16), we have

$$\frac{dM(a)}{da} = \frac{\partial f(x(a),a)}{\partial x} \cdot \frac{\partial x(a)}{\partial a} + \frac{\partial f(x(a),a)}{\partial a}$$

Since x(a) is the choice of x that maximises f, we know that

$$\frac{\partial f(x(a),a)}{\partial x} = 0.$$

So $\dfrac{dM(a)}{da} = \dfrac{\partial f(x(a),a)}{\partial a}$

or, $\dfrac{dM(a)}{da} = \dfrac{\partial f(x,a)}{\partial a}\Big|_{x=x(a)}$

which shows that the derivative is taken holding x fixed at the optimal value x(a).

Note that the envelope theorem states: the total derivative of the value function with respect to the parameter is equal to the partial derivative when the derivative is evaluated at the optimal choice. This result is based on the idea that there would be two effects direct and indirect of change in a.

The change in `a` directly affects *f*. Again the change in a affects x which in turn affects *f*. Since x is chosen optimally, a small change in x has a zero effect on *f*. So the indirect effect is eliminated and the direct effect remains.

Example:

Let $f(x, a) = \log x - ax$ ….. (6.17)

To get the optimal value of x, differentiate, $f(x, a)$ with respect to x, so that

$$f'(x, a) = \frac{1}{x} - a$$

For maximum, $\dfrac{1}{x} - a = 0$

or, $\dfrac{1}{x} = a$

or, $x = \dfrac{1}{a}$

Thus, the optimal value of x, say, x^* is $\dfrac{1}{a}$.

Using this result write the value function

$$M(a) = f(x(a), a)$$

$$= \log\left(\frac{1}{a}\right) - a \cdot \frac{1}{a}$$

$$= \log(a) - 1$$

Now, $\dfrac{dM(a)}{da} = -\dfrac{1}{a}$

In the second method, take

$f(x, a) = \log x - ax$ and get

$$\frac{df(x,a)}{da} = -x$$

Setting x to its optimal value, which is $\dfrac{1}{a}$, obtain

$$\frac{df(x,a)}{da} = -\frac{1}{a}$$

Let us return to our profit maximisation problem with one-input and one-output and apply the result of envelope theorem.

The profit maximisation problem is:

$\pi(p, w) = p f(x) - wx$.

The parameter a in the envelope theorem we saw can be extended to either p or w in the profit function above. So

$$\frac{\partial \pi(p,w)}{\partial p} = f(x) \Big|_{x=x(p,\,w)}$$

$= f(x(p, w))$, which gives the profit maximising, supply of the firm at prices (p, w). Similarly,

$$\frac{\partial \pi(p,w)}{\partial w} = -x \Big|_{x=x(p,\,w)}$$

gives the profit maximising net supply of the factor x.

Check Your Progress 3

1) Take an example and solve it according to the method of envelope theorem.

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2) Describe Hotelling Lemma.

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3) What is the first order condition for profit maximisation?

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6.5 COST MINIMISATION AND PROFIT

Continuing with two inputs, capital and labour, the total cost of the firm can be written as

TC = wL + rk where L and K represent labour and capital with prices w and r. Assuming that the firm produces one output, its total revenue is given by price (p) of its product and quantity of output (q), produced by the production function $q = f(K, L)$. Then profit (π) of the firm is given by the difference between total revenue (TR) and total costs (TC) for an output level q^0.

Thus, $\pi = TR - TC = pq - wL - rk$.

6.5.1 Cost-Minimising Inputs

The objective of a firm if it behaves rationally is to maximise profit. Therefore, cost of production has to be minimised by selecting appropriate inputs and without violating the requirements of production function. In the process, the firm has to solve a constrained cost minimisation problem. In order words, we seek to minimise total costs, given $q = f(K, L) = q^0$. Setting up the Lagrangian format

$$l = wL + rK - \lambda\,[q^0 - f(K, L)]$$

the first order conditions for a constrained minimum are:

$$\frac{\partial l}{\partial L} = w - \lambda \frac{\partial f}{\partial L} = 0$$

$$\frac{\partial l}{\partial K} = r - \lambda \frac{\partial f}{\partial L} = 0$$

$$\frac{\partial l}{\partial \lambda} = q^0 - f(K, L) = 0$$

For a solution, divide the first two equations to get

$$\frac{w}{r} = \frac{\partial f / \partial L}{\partial f / \partial K} = \text{RTS (L for K), i.e.,}$$

the cost-minimising firm should equate the RTS for the two inputs to the ratio of their prices.

6.5.2 Cost-Minimising Factor Demand Functions

From the FOC we can get the *cost-minimising factor demand functions* or *conditional factor demand functions.* To derive these, let us consider from the three equations of FOC above K, L and λ as three unknowns. Given any parameter values for r, w, and q, we can solve these equations for the unknowns. It may be necessary to note that these functions are different from the factor demand functions derived from the profit maximisation problem where output was produced keeping total cost fixed.

Shephard's Lemma

There is an alternative method of deriving the input demand from the cost minimisation objective of a firm. The compensated demand functions can be computed directly from the expenditure function for inputs by partial differentiation of the expenditure with respect to input prices, which is known as the Shephard's Lemma.

Since output is held constant in the cost minimisation problem and the firm is a price taker, so that r and w are given; from the constrained optimisation problem

$1 = wL + rK - \lambda\,[q^0 - f(K, L)]$, we get from the method offered by envelope theorem,

$\delta 1 / \delta w = L$ and

$\delta 1 / \delta r = K$.

Again, the total cost function yields

$\delta(TC)/\delta w = L$ and $\delta(TC)/\delta r = K$. Therefore,

$\delta(TC)/\delta w = \delta 1 / \delta w = L$ and $\delta(TC)/\delta r = \delta 1 / \delta r = K$.

The input demand functions are also constant output demand functions.

6.5.3 Dual Problem: Output Vs Cost in Profit Maximisation

Output Maximisation

The result discussed above can also be obtained by considering the dual formulation of firm's primal cost-minimisation problem, that is, it can maximise the level of output for a given total cost of inputs (TC_I). Mathematically, the Lagrangian expression for this problem is

$1 = f(K, L) - \lambda\,(wL + rK - TC_I)$

Differentiating with respect to K and L for output maximisation, the necessary FOCs can be derived as under:

$MP_K - \lambda r = 0$

$MP_L - \lambda w = 0$

$wL + rK - TC_I = 0$

By solving the first two equations we get

$MP_K / r = MP_L / w$, which is identical to the condition that was necessary for cost minimisation.

Cost Minimisation and Profit Maximisation Duality

If we compare the FOCs for the profit-maximization problem with the FOCs for the cost minimisation problem, we can see that they will give the same solution values for K and L if the value of the Lagrange multiplier is $\lambda = p$. It is not difficult to get this condition in the presence of competitive market conditions as in the equilibrium there is MC = p. Consequenly, λ can be related to MC through cost optimising conditions.

Check Your Progress 4

1) What is the first order condition of cost minimising output?

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2) How would you establish the duality between cost minimisation and output maximisation of a firm?

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3) Derive the Shephard's Lemma from a cost equation.

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6.6 LET US SUM UP

In this unit, we have discussed the production related issues delineated by microeconomic theory. The important types of production functions usually employed by empirical studies have been pointed out by considering two- and multi-input cases. The scope of substitution between inputs and returns to scale in each production function has been highlighted. The technical change over time, which is responsible for producing more output with a given set of inputs or yielding the same level of output with fewer inputs, is pointed out. Some of the important cost functions estimated in empirical studies are discussed after the production functions. The profit maximising conditions, which highlight the MR and MC equality are derived as a constrained optimisation exercise. Since profit maximisation has a bearing on the choice of input demand through cost minimisation for a given level of output, we have discussed the conditions of compensated input demand function. The use of results offered by Envelope Theorem, Hotelling Lemma and Shephard's Lemma in choosing the optimal input demand, deriving supply functions from profit functions and input demand from cost functions has been pointed out. The last part of the unit is devoted to establish the duality between cost minimisation and profit maximising output decisions.

6.7 KEY WORDS

Cobb- Douglas production Function: A production function of the form Q = $aL^b K^c$ where c are constants, Q is output, and K are the Quantities of input employed.

Compensated Demand Curve: Curve showing relationship between the price of a good and the quantity consumed while holding real income (or utility) constant. denoted by $h(P^x, P^y, U)$.

Duality: The relationship between any constrained maximisation problem and its related "dual" constrained minimisation problem.

Envelope Theorem: a mathematical result: the change in the maximum value of a function brought about by a change in a parameter of the function can be found by partially differentiating the function with respect to the parameter (when all other variables take on their optimal values).

Fixed Proportions production Process: A production process characterised by an elastitution in this case are right angles.

Homogeneous Production Function: A special case of homothetic production function where a proportionate change in input causes output to change by a proportion which does not vary with changes in the input bundle.

Input Demand Function: Function showing the firm's demand for an input (say, labor) that depends on put costs (w, v) and on the level of output (a):

$$L = L(w, v, q).$$

Marginal Cost: The cost of an additional unit of output. Graphically, marginal cost is the slope of the total coat curve.

Marginal Rate of Substitution: The rate at which one commodity can be substituted for another without changing total utility. The marginal rate of substitution or x or y is equal to the absolute value of the slope of the indifference curve.

Marginal Rate of Technical Substitution: The rate at which one input can be substituted for another input in the production process without affective total output. Graphically, the marginal rate of technical substitution is equal to the absolute value of the slope of the isoquant. Mathematically, it is equal to the ratio of marginal products for the two inputs.

Marginal Revenue Product: The change in total revenue resulting from a one-unit increase in input usage, ceteris paribus.

Profit Function: The relationship between a firm's maximum profits (π^*) and the output and input prices it faces:

$$\pi^* = \pi^* \text{ (P, v, w)}.$$

6.8 SOME USEFUL BOOKS

Ferguson, C.E., *The Neoclassical Theory of Production and Distribution*. Cambridge: Cambridge Univeristy Press, 1969.

Fuss, M., and McFadden, D., eds. *Production Economics: A Dual Approach to Theory and Applications*. Amsterdam: North-Holland Publishing Co., 1978.

Nicholson, Walter (1992), *Microeconomic Theory*, Harcourt Brace Jovanovich, Orlando, FL, USA.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

6.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) Elasticity of substitution is one in C-D and constant in CES
2) Take the powers of the inputs and add.
3) Insert a term with time, say A(t), in the production function.

Check Your Progress 2

Hint Q1) C-D. Consider its elasticity of substitution.

Hint Q2) Same as in CES production function

Q3) Sum of coefficients associated with the square and interactive terms of input prices = 0

Check Your Progress 3

1) Do Yourself
2) Do Yourself
3) $\frac{MP_L}{MP_K} = \frac{w}{r}$

Check Your Progress 4

1) $\frac{w}{r} = \frac{\partial f / \partial L}{\partial f / \partial K} =$ RTS (L for K).
2) The order conditions of both would be the same.
3) Do yourself by reading Sub-section 6.5.2.

6.10 EXECISES

1) A BPO firm in Delhi operates with a production function

   $q = 10e^{.05t} K^{5} L^{5}$ It succeeds in increasing its production by 5% per time period t using the same level of inputs. Obtain the type of technical change this firm experiences.

   Hints: In (-1) the nature technical change is indistinguishable.

2) You are given a production function of a firm represented by

   $q = \beta_0 + \beta_1 (KL)^{.5} + \beta_2 K + \beta_3 L$ with $0 \le \beta_i \le 1$ and i=1, 2, 3.

   If this function is supposed to be experiencing constant returns to scale, what restriction you should impose on the parameters $\beta_0,...,\beta_3$?

   Use Allen's formula for elasticity of substitution to calculate σ. Is σ constant?

   (Hints $\beta_0 = 0$, σ is not constant)

3) The total cost function of a firm is given as

$$TC = \left[.5r + (rw)^{.5} + .5w\right]q$$

a) Get the output demand function for K and L using Shepherd's Lemma.

b) Derive the underlying production function.

Hint: $L = .5q\left[1 + \left(\frac{r}{w}\right)^{.5}\right]$

$$K = .5q\left[1 + \left(\frac{w}{r}\right)^{.5}\right]$$

$$q = .5\overset{-1}{K} + .5\overset{-1}{L}\Big]^{-1}$$

4) The production function of a firm producing polished diamond products is given by q = $2L^{.5}$. The firm is a price taker in the market.

Drive the supply function of the product. If the prices of output and labour are P and W, show that such a function if homogenous of degree zero in P and W.

Hints: $f = \frac{2P}{W}$