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## **UNIT 3 RECENT DEVELOPMENT OF DEMAND THEORY**

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### **3.0 OBJECTIVES**

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In this unit, we will discuss some of the recent development in demand analysis. First, we will look at an important implication of utility maximisation exercise viz., linear expenditure system. Then we move on to another important theory in consumer behaviour called consumer surplus, where we introduce three different types of definition with their graphical interpretation. In the next section, we introduce a more advance theory of consumer behaviour where consumers present decision depend on her future concerns. The price determination in the market is covered next. Then we move on to explaining a dynamic model called Cobweb model, which will explain the dynamic stability property of the equilibrium of Demand-Supply analysis. Finally, we will discuss a model related to lagged adjustment in interrelated markets.

This unit will enable you to:

- Determine the optimum choice of a consumer under linear expenditure system;
- Evaluate consumer surplus in different markets;
- Decide the optimum choice under two period analysis of consumer behaviour;
- Determine price under Demand-Supply analysis;
- Find the nature of equilibrium under Demand-Supply analysis; and
- Assess the equilibrium under lagged adjustment in interrelated markets.

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### **3.1 INTRODUCTION**

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The basic theory of consumer behaviour discussed in the previous unit can be extended in many directions, and can be applied to cover optimal behaviour

for a variety of specific types of utility functions. Some of these extensions and specific applications are discussed here. In the market, prices of all goods are given to the consumers. They can't influence the price by changing their own decisions. Some times prices are also given to the individual firm i.e., in some cases, firms also are not able to charge prices that they want and have to settle with the price prevailing in the market. There was considerable interest therefore among the economists to explain the price formation in different types of markets. The Demand-Supply analysis is the most important among them. Once the equilibrium is achieved the second most important question came to mind is the question of stability of that equilibrium. There are many approaches to determine the stability property of equilibrium. Among them Cobweb model is simplest and quite elegant in nature.

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### 3.2 RECENT DEVELOPMENT IN DEMAND ANALYSIS: LINEAR EXPENDITURE SYSTEMS

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For many years economic theorists analysed the optimal behaviour of consumers while econometricians estimated consumer demand and expenditure relations, with little communication between the two. Theorists would provide examples that were of little aid for empirical work, and econometricians would estimate relations that had little connection with the theory of utility maximisation. Fortunately, as days passed on, the gap between theory and empirical evidence has lessened, and a number of theoretically strong examples that allow empirical estimation have been developed. In this section we present one of such examples.

Consider the utility function

$$U = \alpha_1 \ln(q_1 - \gamma_1) + \alpha_2 \ln(q_2 - \gamma_2)$$

with the domain  $q_1 > \gamma_1$  and  $q_2 > \gamma_2$ . The  $\gamma$ 's may be interpreted as minimum subsistence quantities and are positive. The  $\alpha$ 's are also positive. Applying the positive monotonic transformation  $U' = U/(\alpha_1 + \alpha_2)$  we get,

$$U' = \beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2)$$

The coefficients  $\beta_1$  and  $\beta_2$  ( $\beta_1 + \beta_2 = 1$ ) are called "share" parameters.

The consumer's objective is to maximise her utility subject to budget constraint. So, she will try to solve the problem given below.

Maximise  $\beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2)$

$$\begin{aligned} \text{Subject to } & q_1 > 0 \\ & q_2 > 0 \\ & y \geq p_1 q_1 + p_2 q_2 \end{aligned}$$

We set Lagrange function of the above maximisation exercise as

$$Z = \beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2) + \lambda(y - p_1 q_1 - p_2 q_2)$$

and set its first partial derivatives equal to zero (we assume interior solution of this maximisation problem):

$$\frac{\partial Z}{\partial q_1} = \frac{\beta_1}{q_1 - \gamma_1} - \lambda p_1 = 0$$

$$\frac{\partial Z}{\partial q_2} = \frac{\beta_2}{q_2 - \gamma_2} - \lambda p_2 = 0$$

$$\frac{\partial Z}{\partial \lambda} = y - p_1 q_1 - p_2 q_2 = 0$$

It can be easily verified that the second order condition for the maximisation is satisfied. By evaluating the above three equation one can also find out that the marginal utility of income is decreasing.

Solving the above equations for optimal quantities gives the demand functions,

$$q_1 = \gamma_1 + \frac{\beta_1}{p_1}(y - p_1 \gamma_1 - p_2 \gamma_2)$$

and  $q_2 = \gamma_2 + \frac{\beta_2}{p_2}(y - p_1 \gamma_1 - p_2 \gamma_2)$

Multiplying the first equation of the above two demand functions by  $p_1$  and the second by  $p_2$  we get the expenditure functions

$$p_1 q_1 = p_1 \gamma_1 + \beta_1 (y - p_1 \gamma_1 - p_2 \gamma_2)$$

and  $p_2 q_2 = p_2 \gamma_2 + \beta_2 (y - p_1 \gamma_1 - p_2 \gamma_2)$

which are linear in income and prices, and thus suitable for linear regression analysis.

**Check Your Progress 1**

- 1) Consider the utility function  $U = q_1^{\delta_1} . q_2^{\delta_2}$ . Find out the linear expenditure function.

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### 3.3 THEORY OF CONSUMER SURPLUS

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In this section, we discuss the basic concept of consumer surplus and its derivation. A consumer normally pays less for a commodity than the maximum amount that she would be willing to pay rather than forego its consumption. Consumer surplus therefore in crude sense is the difference between what consumer willing to pay and what she actually pays. Several measures of such consumer’s surplus have been proposed. We will discuss three of them. Attention is limited to a consideration of the good under investigation and a composite commodity called “money”, with consumption

quantities of  $q$  and  $M$  respectively. Let the distance  $OA$  in Figure 3.3.1 represents the consumer's income. She achieves a tangency solution at point  $D$  on indifference curve  $I_2$ . If she were unable to consume  $Q$ , she would be at  $A$  on the lower indifference curve  $I_1$ . She would have to be given an income increment of  $AB$  dollars to restore her to indifference curve  $I_2$ . This increment, called compensating income variation, is denoted by  $c$ , and provides a measure of consumer's surplus.

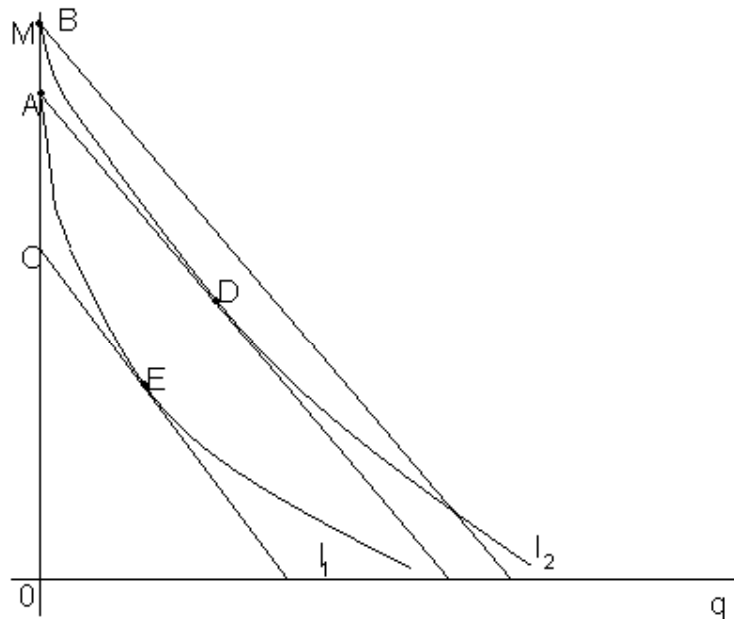


Fig. 3.1: Consumer Surplus

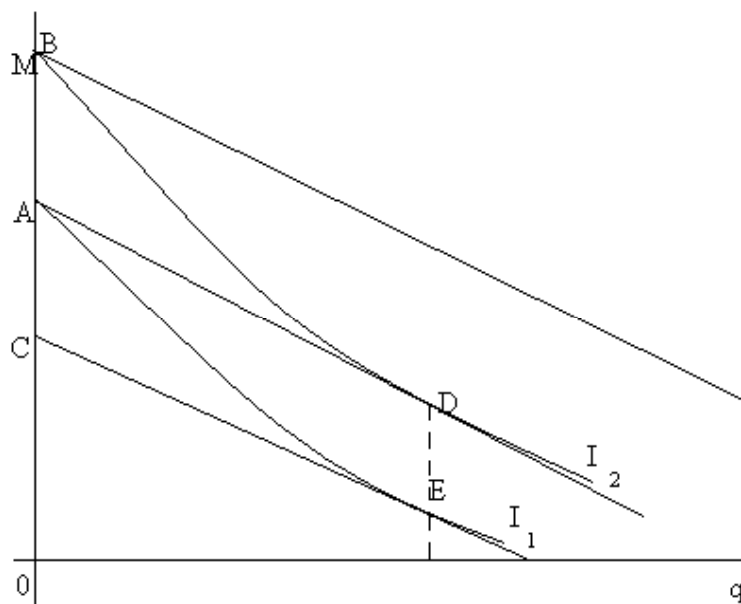


Fig. 3.2: Consumer Surplus

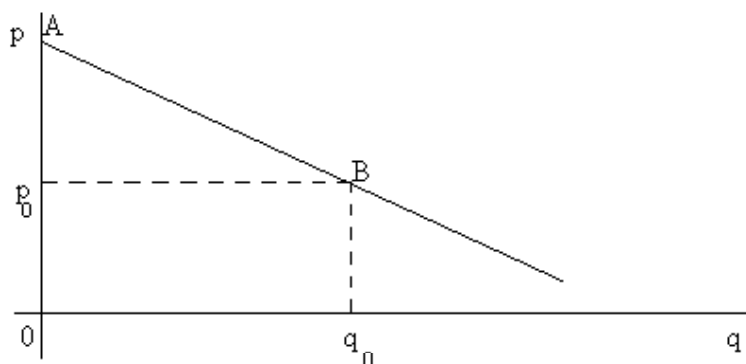


Fig. 3.3: Consumer Surplus

At the given prices, the consumer would be willing to forgo  $AC$  dollars of income rather than lose her opportunity to consume good  $Q$ . With income  $OC$ , her consumption is at  $E$ , which is on the same indifference curve as  $A$ . The amount corresponding to  $AC$  is called equivalent income variation and is denoted by  $e$ . It provides an alternative measure of consumer's surplus. A third measure is provided by the demand curve in Figure 3.3.3 for the price-quantity combination  $p_0q_0$ . It equals the area  $ABp_0$ , which is the difference between the area lying under the demand curve  $OABp_0$  and the consumer's expenditure  $Op_0Bq_0$ , and is denoted by  $s$ .

It can be shown that  $c \geq s \geq e$ . The strict inequalities hold for the case pictured in Figure 3.3.1 as a consequence of the income effect. If the consumer were to pay more to consume the good, her demand would decline because of her lower effective income, and the area under the demand curve would exceed the amount that she would pay rather than forego consumption of the good. Figure 3.3.2 depicts a case in which the income effect is zero throughout. A perpendicular such as the line through  $D$  and  $E$  connects points with the same marginal rate of substitution. The indifference curves are "parallel" with a constant vertical distance between a pair of indifference curves. In this case  $AB=AC$  and the three measures of consumer's surplus are the same.

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### 3.4 THEORY OF INTER-TEMPORAL CONSUMPTION

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In the previous two units, we have been concerned with choices among contemporaneous commodities. An important class of choices made by consumers, however, relates to consumption over time, that is, how one allocates income earned in different time periods to consumption. It seems that when income is earned in an uneven pattern, individuals attempt to "smooth out" their consumption through borrowing and lending. In this way, people's consumption varies less than their income.

We began this discussion by considering consumption in just two-time period. Denote the present as period 1 and the future (next year) as period 2, and consumption in period 1 and 2 as  $x_1$  and  $x_2$ . Suppose a person earns  $x_1^0$  in the present (this year) and  $x_2^0$  in the future (next year). Suppose also that this individual can borrow and lend in the "capital market" at rate of interest  $r$ . What this means is any income  $y$  not spent this year can be loaned to others, in return for which the consumer receives some greater amount  $y + r y = y(1 + r)$  next year. Alternatively, the consumer can increase present consumption by

some amount  $y$  and repay  $y(1+r)$  next year. The opportunity cost of consuming income  $y$  this year is thus forgoing consumption of  $y(1+r)$  next year.

The price of present consumption is thus  $(1+r)$  units of future consumption; alternatively, the price of future consumption is  $(1/(1+r))$  units of present consumption. We commonly say that the present value of Rs.  $Y$  one year from now is Rs.  $y/(1+r)$ ; this is merely the quantity,  $y$ , times its price in terms of present consumption. The interest rate is the “premium for earlier availability of goods”.

Wealth,  $W$ , in the present, is defined as the present value of current and future income. The consumer’s budget constraint is that she cannot spend more than her wealth, i.e.,

$$x_1 + \frac{x_2}{1+r} = x_1^0 + \frac{x_2^0}{1+r} = W \quad \text{----- (a)}$$

The consumer maximises  $U(x_1, x_2)$  subject to equation (a).

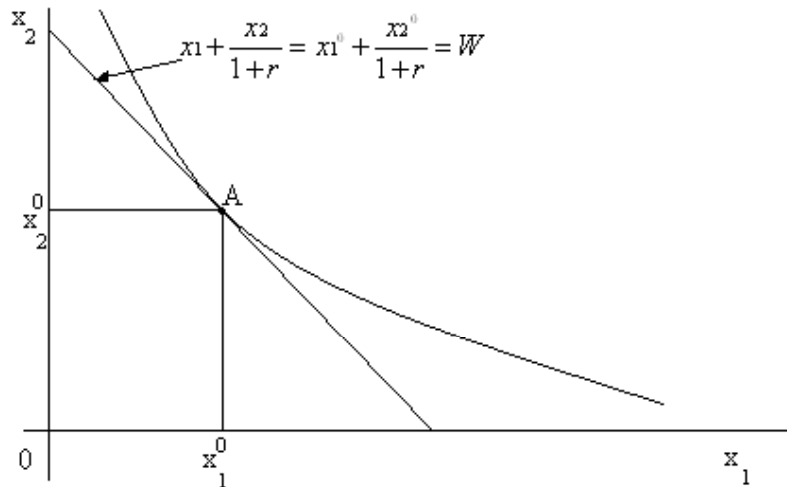


Fig. 3.4: Maximisation of Utility Subject to Wealth Constraint

Though we are using “income” and “consumption” interchangeably as arguments in the utility function, it is well to remember, as pointed out by economist I. Fisher, that “income” really consists of consuming something. “Saving” (or dissaving) is just a way of rearranging consumption over time. Income is realised when it is consumed.

The model is depicted in Figure 3.4.1. The budget line has slope  $\frac{dx_1}{dx_2} = -(1+r)$ , the price of  $x_1$  in terms of  $x_2$ , and passes through the endowment point A,  $(x_1^0, x_2^0)$ . An increase in the interest rate represents an increase in the price of the present consumption, and has the effect of rotating the wealth constraint clockwise through A.

The Lagrange function for this problem is

$$L = U(x_1, x_2) + \lambda[(x_1^0 - x_1) + \frac{(x_2^0 - x_2)}{(1+r)}] \quad \text{----- (b)}$$

assuming strict interior solution, producing the first order conditions

$$\frac{\partial L}{\partial x_1} = \frac{U(x_1, x_2)}{\partial x_1} - \lambda = 0 \quad \text{----- (b.1)}$$

$$\frac{\partial L}{\partial x_2} = \frac{U(x_1, x_2)}{\partial x_2} - \frac{\lambda}{(1+r)} = 0 \quad \text{----- (b.2)}$$

and the constraint

$$\frac{\partial L}{\partial \lambda} = x_1 + \frac{x_2}{1+r} - x_1^0 - \frac{x_2^0}{1+r} = 0 \quad \text{----- (b.3)}$$

Combining equations (b.1) and (b.2) yields

$$\frac{\frac{U(x_1, x_2)}{\partial x_1}}{\frac{U(x_1, x_2)}{\partial x_2}} = (1+r) \quad \text{----- (c)}$$

Equation (c) says that the consumer's marginal value of present consumption,  $U_1/U_2$  (where  $U_1 = \frac{U(x_1, x_2)}{\partial x_1}$  and  $U_2 = \frac{U(x_1, x_2)}{\partial x_2}$ ), equals the opportunity cost of present consumption, in terms of future consumption forgone. It will simplify the algebra if we let  $p = \frac{1}{(1+r)}$ , the price of future consumption.

Assuming the sufficient second-order conditions hold, the first-order conditions can be solved for the Marshallian demand functions

$$x_1 = x_1^M(p, x_1^0, x_2^0)$$

$$\text{and, } x_2 = x_2^M(p, x_1^0, x_2^0)$$

We can gain greater insight into the model by deriving the Slutsky equation, separating out the substitution effect and the wealth (income) effect.

The Hicksian demands can be derived minimising the endowment in either period so as to achieve some arbitrary indifference level  $U^0$ . We can therefore state the model as

Minimise

$$x_1^0 = x_1 + p(x_2 - x_2^0)$$

subject to

$$U(x_1, x_2) = U^0$$

The Lagrange function for this problem is then

$$L' = x_1 + p(x_2 - x_2^0) + \lambda(U^0 - U(x_1, x_2))$$

Assuming the first and sufficient second-order conditions hold, the implied first-order equations can be solved for the Hicksian demands

$$x_1 = x_1^U(p, U^0)$$

and  $x_2 = x_2^U(p, U^0)$

Substituting these demands into the objective function produces a minimum “expenditure” type of function

$$x_1^*(p, U^0) = x_1^U + p(x_2^U - x_2^0)$$

The fundamental identity linking between the Marshallian and Hicksian demands is therefore

$$x_i^U(p, U^0) \equiv x_i^M(p, x_1^*(p, U^0), x_2^0) \text{ for } i = 1, 2,$$

producing the famous Slutsky equation

$$\frac{dx_i^M}{dp} \equiv \frac{\partial x_i^U}{\partial p} + (x_2^0 - x_2^U) \left( \frac{\partial x_i^M}{\partial x_1^0} \right)$$

If the interest rate increases, the price of future consumption,  $p$ , decreases. This produces a pure substitution effect towards less present and greater future consumption:  $\frac{\partial x_2^U}{\partial p} < 0$ . However, a change in the interest rate produces an

attendant wealth effect. An increase in the endowment of present income is the same as an increase in wealth from any source, since income can be traded back and forth across time periods. Assume that consumption in both time periods enters the utility function as normal goods, so that  $\frac{\partial x_i^M}{\partial x_1} > 0$ . The

income or, more properly, the wealth term on the right hand side of the Slutsky equation, indicates that if, for example, the consumer is a net borrower in period 1, so that  $(x_2^0 - x_2^U) < 0$ , the substitution effect will be reinforced by the wealth effect. In this case, an increase in the interest rate, in addition to making present consumption relatively more expensive, also lowers the consumer’s wealth, producing an additional reduction in present consumption. If the individual is a net lender in period 1, the wealth and substitution effects oppose one another: an increase in the interest rate raises present wealth and leads to greater present consumption.

**Check Your Progress 2**

- 1) Suppose utility function of the consumer is  $U(x_1, x_2) = x_1^{0.5} x_2^{0.5}$ . Consumer lives two periods. In period 1 her income in Rs 1000 i.e.,  $x_1^0 = 1000$  and in period two she has no income i.e.,  $x_2^0 = 0$ . The market rate of interest is 50%. Find out the optimum consumption in each period.

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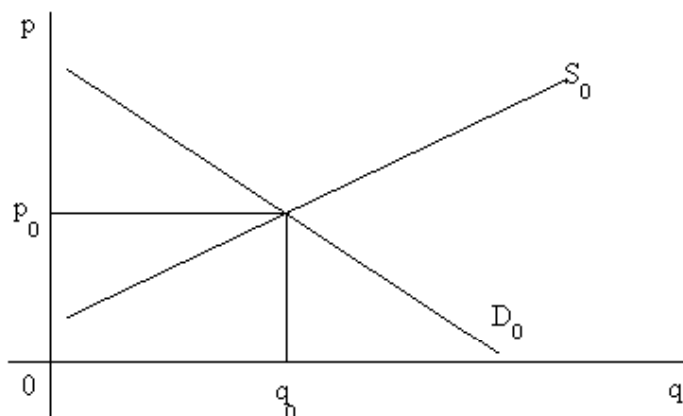
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### 3.5 ELEMENTARY THEORY OF PRICE FORMATION: DEMAND-SUPPLY ANALYSIS

In this section, we discuss the elementary theory of price formation. Demand curve in the market is derived from the aggregate consumer demand and supply curve is derived from the aggregate firms supply. Since market demand curve for a good is the sum total of demand for that good of all individual consumers and since demand curve for a good for an individual consumer is derived from its utility maximisation, so along the demand curve consumer's optimising behaviour is always fulfilled. That means each point on the demand curve represents that consumers are willing to purchase the corresponding demand quantity with corresponding price.

We consider perfect competition prevail in the market. In short, run perfectly competitive supply curve of a commodity in the market or industry is determined from supply curve of an individual firm, where supply curve of commodity of individual firm is derived from profit maximising objective of that firm. Hence, along the market supply curve-optimising behaviour of the firm is fulfilled. That means each point on the market supply curve represents that firms are willing to supply the corresponding supply quantity with corresponding price.



**Fig. 3.5: Demand Supply Equilibrium**

Clearly, at the point of intersection between market demand and supply curve, exchange will take place between consumers and producers, as both of them simultaneously fulfilled their optimising behaviour. Corresponding price and aggregate quantity are short run equilibrium price (say  $p_0$ ) and aggregate quantity (say  $q_0$ ) respectively, which is shown in Figure 3.5.1.

The process of adjustment of short run equilibrium of a competitive market takes place in the following way. Generally, by adjusting price of the commodity equilibrium in short run perfect competition is achieved as given below:

It is assumed that for any excess demand (or excess supply) prices will increase (or decrease). According to this behaviour of the market, price adjustment in disequilibrium will take place by a mechanism, which is known as auctioneer mechanism.

Suppose there is an invisible referee who controls the market price according to the above behavioural assumptions. Producers supply their quantity on the basis of existing market price. Suppose, the referee initially specifies a particular price on the basis of which producers and consumers specify their supply and demand respectively. Then suppose the referee observed that supply quantity is larger than the demand quantity i.e., we have excess supply of the commodity.

If producers fail to supply their entire supply quantity at the existing price, then according to the behavioural assumption, the referee specifies a lower price of that commodity. Producers will be discouraged and will supply lower quantity and consumers will be encouraged and will demand higher quantity. Thus, in both ways excess supply of the commodity decreases. Suppose it is observed that demand quantity is larger than the supply quantity in aggregate. We have excess demand for that commodity. The referee again specifies a larger price level. Hence, producers will increase their supply and consumers will decrease their demand. Thus, in both ways excess demand for that commodity goes down. This process will continue till the referee specifies a particular price at which corresponding demand and supply quantities are equal. That means supply quantity offered by the producer is demanded by the consumer at the corresponding price. So, both consumer and producer fulfilled their optimising behaviour simultaneously. The exchange of commodity will take place at this price and quantity. These price and quantity are the equilibrium price and quantity in the market respectively.

**Check Your Progress 3**

- 1) Describe the way of converging a disequilibrium point to equilibrium in a perfectly competitive market.

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**3.6 COBWEB MODEL**

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**Concept of dynamic stability:** A market equilibrium is said to dynamically stable only when disequilibrium price and quantity move and over time reach to any equilibrium, otherwise it is dynamically unstable. Movement of price and quantity in disequilibrium over time depends on behaviour of the market. Hence, analysis of dynamic equilibrium and stability depends on behavioural assumption of the market. Dynamic analysis is of two types. Here we consider discrete time analysis. Below we discuss Cobweb model in details.

Cobweb model analyses dynamic equilibrium and stability of a competitive market with the following behavioural assumptions:

- We consider linear demand and supply functions for simplicity.

- Both demand and supply functions depend on time, where time is a discrete matter (in that sense it is a dynamic model).
- Supply quantity at any time,  $t$ , depends on previous period's price and

$$S_t = c + d P_{t-1} \text{ ----- (i)}$$

But demand quantity at any time,  $t$ , depends on the price at that time

$$D_t = a + b P_t \text{ ----- (ii)}$$

That means demand quantity is instantaneously determined at the existing price but there is a lag in supply quantity with respect to price since supply of output requires some amount of time.

- Behaviour of the market is such that as soon as supply quantity comes into the market, entire quantity is demanded at that period by adjusting price so that market is clear in each period. Thus, at any time,  $t$ ,  $S_t = D_t$  -----(iii)

Since demand and supply functions are linear,  $a$ ,  $b$ ,  $c$  and  $d$  are constant. On the basis of these we now analyse dynamic equilibrium and stability.

At equilibrium of the market  $D_t = S_t$  holds where price doesn't change over time (i.e.,  $P_t = P_{t-1} = P_e$ , where  $P_e$  is the equilibrium price). Hence, at equilibrium

$$a + b P_e = c + d P_e$$

$$\text{or, } (a - c) = (d - b) P_e$$

$$\text{or, } P_e = \frac{(a - c)}{(d - b)}$$

This is the equilibrium price, which is assumed to be positive such that initially equilibrium exists. Now we find the time path of price from behavioural assumptions.

$$D_t = S_t$$

$$\text{or, } a + b P_t = c + d P_{t-1}$$

$$\text{or, } b P_t - d P_{t-1} = (c - a)$$

$$\text{or, } P_t - \frac{d}{b} P_{t-1} = \frac{(c - a)}{b} \text{ ----- (iv)}$$

The time path of price is a first order linear non-homogeneous difference equation. Its solution consists of a particular solution ( $P_p$ ) and a complementary solution ( $P_c$ ), viz.,

$$P_t = P_p + P_c \text{ ----- (v)}$$

To find out  $P_p$  we put  $P_t = P_{t-1} = \bar{P}$  (say) into the equation (iv), and have

$$\bar{P}(1 - \frac{d}{b}) = \frac{c - a}{b}$$

or, 
$$\bar{P}(\frac{b - d}{b}) = \frac{c - a}{b}$$

or, 
$$\bar{P} = \frac{a - c}{d - b} = P_e$$

Since  $P_e$  exists,  $P_p$  also exists and it is equilibrium price and independent of time.

To find out  $P_c$ , we put  $P_t = x^t$  in the homogeneous part of the difference equation (equation – (iv)), and write

$$x^t - \frac{d}{b} x^{t-1} = 0$$

or, 
$$\frac{x^t}{x^{t-1}} - \frac{d}{b} \frac{x^{t-1}}{x^{t-1}} = 0$$

or, 
$$x = \frac{d}{b},$$

which is the characteristic root. Therefore,  $P_c = m x^t$ , where  $m$  is any unknown integral constant. From equation (v) we get

$$P_t = P_e + m x^t \text{ ----- (vi)}$$

giving the general solution.

Suppose at initial time,  $t = 0$ ,  $P_t = P_0$  is known. Then from equation (vi), we have,

$$P_0 = P_e + m x^0$$

or, 
$$P_0 = P_e + m$$

or, 
$$m = P_0 - P_e,$$

So the definite solution of time path of price is

$$P_t = P_e + (P_0 - P_e)(\frac{d}{b})^t \text{ ----- (vii)}$$

Here  $P_0$ ,  $P_e$ ,  $d$  and  $b$  all are known and for each level of time price can be determined from equation (vii).

If  $P_t \rightarrow P_e$  as  $t \rightarrow \infty$  (i.e., if actual price,  $P_t$ , moves towards equilibrium price and overtime reaches to  $P_e$ ), then market equilibrium is dynamically stable (otherwise it is dynamically unstable). This requires  $(\frac{d}{b})^t \rightarrow 0$  as  $t \rightarrow \infty$

(since then only  $P_t \rightarrow P_e$  as  $t \rightarrow \infty$  where  $P_0 - P_e$  is constant and does not change over time). Now  $(\frac{d}{b})^t \rightarrow 0$  as  $t \rightarrow \infty$  only when

$$|\frac{d}{b}| < 1$$

$$\text{or, } |\frac{1}{b}| < |\frac{1}{d}| \text{ ----- (viii)}$$

This is the condition for dynamic stability (as in this case we have convergence to equilibrium from any disequilibrium over time). That means absolute slope of the demand curve  $|\frac{dP_t}{dD_t}| = |\frac{1}{b}|$  should be lower than the absolute slope of the supply curve  $|\frac{dP_t}{dS_t}| = |\frac{1}{d}|$ , i.e., demand curve should be flatter than supply curve for dynamic stability.

When  $|\frac{d}{b}| > 1$  then we have divergence from equilibrium over time (from equation (vii)) we have dynamically unstable equilibrium, and at that situation  $|\frac{1}{b}| > |\frac{1}{d}|$  i.e., demand curve is steeper than supply curve.

When  $|\frac{d}{b}| = 1$ , we have neither convergence to equilibrium nor divergence from equilibrium (from equation (vii)). So it is also dynamically unstable. At that situation  $|\frac{1}{b}| = |\frac{1}{d}|$  i.e., slope of the demand curve is equal to the slope of the supply curve in absolute sense.

If  $(\frac{d}{b}) > 0$ , then we have monotonic time path of price from equation (vii). If  $(\frac{d}{b}) < 0$ , then we have cyclical time path of price from equation (vii).

**Case 1:** Suppose demand curve is downward sloping and supply curve is upward sloping i.e.,  $d > 0$  and  $b < 0$ , therefore  $(\frac{d}{b}) < 0$ .

**Case A:** If  $|\frac{1}{b}| < |\frac{1}{d}|$ , then we have cyclical convergence to equilibrium. It's a dynamically stable equilibrium.

**Case B:** If  $|\frac{1}{b}| = |\frac{1}{d}|$ , then we have regular cycle around equilibrium. It's a dynamically unstable equilibrium.

**Case C:** If  $|\frac{1}{b}| > |\frac{1}{d}|$ , then we have cyclical divergence from equilibrium. It's a dynamically unstable equilibrium.

**Case 2:** Suppose demand curve and supply curve are upward sloping i.e.,  $d > 0$  and  $b > 0$  hence  $(\frac{d}{b}) > 0$ .

**Case D:** If  $|\frac{1}{b}| < |\frac{1}{d}|$  then we have monotonic convergence to equilibrium. It's a dynamically stable equilibrium.

**Case E:** If  $|\frac{1}{b}| = |\frac{1}{d}|$  then demand and supply curve are coincide if their intercepts are also same then we have infinite number of equilibrium and there is no need for dynamic stability analysis or demand and supply curve are parallel to each other (when intercepts are not equal) then equilibrium does not exist. Hence, there is no need for stability analysis in this case.

**Case F:** If  $|\frac{1}{b}| > |\frac{1}{d}|$  then we have monotonic divergence from equilibrium. It's a dynamically unstable equilibrium.

**Case 3:** Suppose demand curve is downward sloping and supply curve is upward sloping i.e.,  $d < 0$  and  $b < 0$ , therefore  $(\frac{d}{b}) > 0$ .

**Case G:** If  $|\frac{1}{b}| < |\frac{1}{d}|$  then we have monotonic convergence to equilibrium. It's a dynamically stable equilibrium.

**Case H:** If  $|\frac{1}{b}| = |\frac{1}{d}|$  then demand and supply curve are coincide if their intercepts are also same then we have infinite number of equilibrium and there is no need for dynamic stability analysis or demand and supply curve are parallel to each other (when intercepts are not equal) then equilibrium does not exist. Hence, there is no need for stability analysis in this case.

**Case I:** If  $|\frac{1}{b}| > |\frac{1}{d}|$  then we have monotonic divergence from equilibrium. It's a dynamically unstable equilibrium.

**Check Your Progress 4**

- 1) In Cobweb model under what condition equilibrium is dynamically stable when there is linear downward sloping demand and linear upward sloping supply curve.

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### 3.7 LAGGED ADJUSTMENT IN INTERRELATED MARKETS

Here we discuss the Walrasian dynamic stability analysis of a competitive commodity market. Let us consider the following assumptions:

- initially equilibrium exists short run competitive commodity market
- both demand and supply function are linear and depend on time where time is discrete.
- both demand and supply at any time,  $t$ , depend on price at that time and demand and supply functions are respectively given by

$$D_t = a + b P_t \text{ ----- (i)}$$

$$S_t = c + d P_t \text{ ----- (ii)}$$

- behaviour of the market is such that for positive (or negative) excess for commodity price rises (or falls) in next period. This can be captured by the following equation:

$$(P_t - P_{t-1}) = K E (P_{t-1}) \text{ ----- (iii)}$$

$$\text{where } E (P_{t-1}) = (D_{t-1} - S_{t-1}) = a + b P_t - c - d P_{t-1} = (a-c) + (b-d) P_{t-1} \text{ -- (iv)}$$

Note that (iv) is the excess demand at time  $(t-1)$  and  $K > 0$ . For simplicity,  $K$  is taken as a constant and it represents the speed of adjustment of price.

[Note: At time  $(t-1)$  if  $E (P_{t-1}) > 0$ , then price should increase in the next period according to the behavioural assumption i.e.,  $P_t > P_{t-1}$  i.e.,  $(P_t - P_{t-1}) > 0$  which is captured by equation (iii) with the restriction that  $K > 0$ ]

At equilibrium  $P_t = P_e$  at which  $D_t = S_t$  holds. Therefore,

$$a + b P_e = c + d P_e$$

$$\text{or, } (a-c) = (d-b) P_e$$

$$\text{or, } P_e = \frac{(a-c)}{(d-b)} \text{ [on the assumption that initial equilibrium exist}$$

where price must be positive].

Now we analyse whether the equilibrium price ( $P_e$ ) is dynamically stable or not. For that we find out the time path of price from the behavioural assumption of the market viz.,

$$(P_t - P_{t-1}) = K E (P_{t-1}) = K [(a-c) + (b-d) P_{t-1}]$$

$$\text{or, } (P_t - P_{t-1}) = K (a-c) + K(b-d) P_{t-1}$$

$$\text{or, } P_t - [1 + K (b-d)] P_{t-1} = K(a-c) \text{ --(v)}$$

The time path of price is represented by the first order linear non-homogeneous difference equation since by assumption  $a, b, c, d$  and  $K$  are constants. Solution of the time path of price ( $P_t$ ) consists of a particular solution ( $P_p$ ) and a complementary solution ( $P_c$ ). Thus,

$$P_t = P_p + P_c \text{ ----- (vi)}$$

To find out  $P_p$ , we put,  $P_t = P_{t-1} = P'$  (say), into the difference equation of price and get

$$P' [1 - \{1 + K (b-d)\}] = K (a-c)$$

or,  $P' (d-b) = (a-c)$

or,  $P' = P_e = \frac{(a-c)}{(d-b)} > 0$  since  $P_e > 0$ .

This is the particular solution; note that it is nothing but inter-temporal equilibrium price, which is a constant and independent of time.

[Note: In fact, here the inter-temporal price is equal to static equilibrium price where equilibrium does not change over time].

To find out  $P_c$ , we put  $P_t = x^t$ , into the homogeneous part of the difference equation of price and get

$$x^t - x^{t-1}[1 + K(b-d)] = 0$$

$$\frac{x^t}{x^{t-1}} - \frac{x^{t-1}}{x^{t-1}}[1 + K(b-d)] = 0$$

$$x = [1 + K(b-d)]$$

It is the characteristic root. Therefore, complementary solution is  $P_c = m x^t$ , where  $m$  is any unknown integral constant. So from equation (vi) we have

$$P_t = P_p + m x^t \text{ ----- (vii),}$$

which is the general solution.

Suppose at  $t = 0$  (i.e., at initial time)  $P_t = P_0$  is given. Hence, from equation (vii) we get

$$P_0 = P_e + m x^0$$

or,  $m = P_0 - P_e$

Therefore, definite solution of the time path of price is

$$P_t = P_p + (P_0 - P_e) [1 + K (b-d)]^t \text{ ----- (viii)}$$

When  $P_t \rightarrow P_e$  as  $t \rightarrow \infty$ , we have convergence to equilibrium from any disequilibrium price over time. So equilibrium is dynamically stable in Walrashian sense, which requires that

$$|1+K (b-d)| < 1$$

When  $[1 + K (b-d)] > 0$ , we have a monotonic time path of price. Alternatively, we get a monotonic convergence to equilibrium over time when  $0 < [1 + K (b-d)] < 1$ . Therefore,

$$K (b-d) < 0 \text{ or, } (b-d) < 0 \text{ or, } b < d,$$

which automatically holds when demand curve is downward sloping [i.e.,  $b < 0$ ] and supply curve is upward sloping [i.e.,  $d > 0$ ]. Equilibrium is dynamically unstable when demand curve is upward sloping and supply curve is downward sloping (as  $b > 0$  and  $d < 0$  so  $b < d$  does not hold)

When both demand and supply curves are upward sloping, the Walrashian dynamic stability requires  $b < d$  or,  $(\frac{1}{b}) > (\frac{1}{d})$ . That means  $|\frac{1}{b}| > |\frac{1}{d}|$  [since  $d > 0$  and  $b > 0$  and price is measured in vertical axis]. So, the supply curve is flatter than the demand curve. Otherwise, it would be dynamically unstable.

When both demand and supply curves are downward sloping, the Walrashian dynamic stability requires  $b < d$  or  $(\frac{1}{b}) > (\frac{1}{d})$ . That means  $|\frac{1}{b}| < |\frac{1}{d}|$  [since  $d < 0$  and  $b < 0$ ]. Hence, the demand curve should be flatter than the supply curve. Otherwise, it is dynamically unstable.

**Check Your Progress 5**

- 1) Derive the time path of a disequilibrium point under Walrashian dynamic stability analysis.

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**3.8 LET US SUM UP**

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In this unit, we learn some advance topics in consumer behaviour theory. We started with the theory of linear expenditure. It gives a demand function from which one can derive the expenditure function by minimising the expenditure subject to a given level of utility. The special features of this theory is the curvature of the expenditure function, which turns out to be linear. Linear expenditure function is extremely useful particularly when some one want to do an empirical study on utility maximisation. The usefulness of the linear expenditure system emanates from the fact that ordinary least square estimation needs linear function to estimate. Next, we move on to the theory of consumer surplus. Consumer surplus is broadly defined as the difference between what consumer willing to pay for a particular commodity bundle and what she actually pays for that commodity bundle. Here we discussed three different ways of finding out consumer surplus and their relationship to each other. In the next section, we analysed the theory of inter-temporal consumption. This theory is more realistic than the previous theory of consumer behaviour because in it, the consumer lives in two periods and there is a notion of saving. The theory can be easily upgraded to multi-period decision making under certainty. Elementary theory of price formation under demand-supply analysis is the next theme that was discussed. The concept of

demand and supply curve was introduced, where the demand curve was derived from utility maximising objective of the consumer and the supply curve from the profit maximising objective of the firm. Therefore, in the intersecting point between demand and supply curve both firm and consumer fulfill their objective. The market equilibrium point was thus presented. In the next section, we moved on to the dynamic stability analysis part and introduced the Cobweb model. A model for Walrathian dynamic stability analysis was taken up subsequently. In the Cobweb model, we derived the nature as well as the stability part of the equilibrium under different demand and supply curves. When demand curve was downward sloping and supply curve was upward sloping, the equilibrium was stated to be dynamically stable if absolute slope of the demand curve is less than the absolute slope of the supply curve (i.e., demand curve is relatively flatter than the supply curve). In the last section, we brought in a model of lagged adjustment in interrelated markets and discussed the Walrathian dynamic stability. In this model, we observed that under standard case (i.e., demand curve is downward sloping and supply curve is upward sloping) equilibrium was dynamically stable if slope of the demand curve was greater than the slope of the supply curve, which indeed happened for standard cases.

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### 3.9 KEY WORDS

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**Cobweb Model:** An analysis of competitive equilibrium attaining stability under dynamic conditions.

**Consumer Surplus:** Difference between the amount consumers are willing to pay for a good and the amount they actually pay.

**Dynamic Stability:** A market equilibrium which is reached by the movement of price and quantity from a situation of disequilibrium.

**Inter-temporal Consumption:** Choices made by consumer on consumption over time.

**Linear Expenditure System:** Econometric estimation of consumer demand through a system of expenditure equations postulating linear functional form.

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### 3.10 SOME USEFUL BOOKS

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Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillan.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

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### 3.11 ANSWER OR HINTS TO CHECK YOUR PROGRESS

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#### Check Your Progress 1

- 1) Hint: Set up the Lagrange function and solve the first order condition, assuming interior solution exists.

$$L = q_1^{\delta_1} \cdot q_2^{\delta_2} + \lambda(y - p_1q_1 - p_2q_2)$$

**Check Your Progress 2**

- 1) Hint: The Lagrange function of the problem is

$$L = x_1^{0.5} x_2^{0.5} + \lambda[(1000 - x_1) - (1/1.5)x_2].$$

The first-order conditions are

$$0.5x_1^{-0.5}x_2^{0.5} - \lambda = 0 \quad - (a)$$

$$0.5x_1^{0.5}x_2^{-0.5} - (1/1.5)\lambda = 0 \quad - (b)$$

$$1000 - x_1 = (1/1.5)x_2 \quad - (c)$$

Dividing equation (a) by (b) we get

$$x_2/x_1 = 1.5 \quad - (d)$$

Solving equation (c) and (d) we get the optimum consumption

$$x_1^* = 500 \text{ and } x_2^* = 750$$

**Check Your Progress 3**

- 1) See Section 3.5

**Check Your Progress 4**

- 1) See Section 3.6 Case 1  
2) See Section 3.6 Case 3  
3) See Section 3.6 Case 2

**Check Your Progress 5**

- 1) See Section 3.7