UNIT 11 LATIN SQUARE DESIGN

Structure

11.1 INTRODUCTION

We know that RBD is used when experimental material is heterogeneous with respect to one factor and this factor of variation is eliminated by grouping the experimental material into a number of homogeneous groups called blocks. This grouping can be carried one step forward and we can group the units in two ways, each way corresponding to a source of variation among the units, and get the LSD. In agricultural experiments generally fertility gradient is not always known and in such situations LSD is used with advantage. Then LSD eliminates the initial variability among the units in two orthogonal directions.

The latin square design represents, in some sense, the simplest form of a row-column design. It is used for comparing m treatments in m rows and m columns, where rows and columns represent the two blocking factors. Latin squares and their combinatorial properties have been attributed to Euler (1782). They were proposed as experimental designs by Fisher (1925, 1926), although De Palluel (l788) already utilized the idea of a 4×4 latin square design for an agricultural experiment (see Street and Street, 1987, 1988).

Layout and statistical analysis of latin square design are explained in Sections 11.2 and 11.3. Missing plots technique in LSD for one missing plot is described in Section 11.4 whereas the suitability of LSD is explored in Section 11.5.

Objectives

After studying this unit, you would be able to

- explain the latin square design;
- describe the layout of LSD;
- explain the statistical analysis of LSD;
- find out the missing plot in LSD; and
- explain the advantages and disadvantages of LSD.

11.2 LAYOUT OF LATIN SQUARE DESIGN (LSD)

Mathematically speaking, the latin square of order m is an arrangement of m latin letters in a square of m rows and m columns such that every latin letter occurs once in each row and once in each column, or more generally, the arrangement of m symbols in a m × m array such that each symbol occurs exactly once in each row and column. In the context of experimental design, the latin letters are the treatments. Latin squares exist for every m. A reduced latin square (or latin square in standard form) is one in which the first row and the first column are arranged in alphabetical order, for example, for m = 3,

A B C
B C A
C A B

This is the only reduced latin square. The number of squares that can be generated from a reduced latin square by permutation of the rows, columns, and letters is (m!). These are not necessarily all different. If all rows but the first and all columns are permuted, we generate m! (m − l)! different squares. From the reduced latin square of order 3 we can thus generate $3! \times (3-1)! = 12$ squares.

In LSD two restrictions are imposed by forming blocks in two orthogonal directions, row-wise and column-wise. Further in LSD the number of treatments equals the number of replications of the treatment. Let there are m treatments and each is replicated m times then the total number of experimental units needed for the designs are $m \times m$. These m^2 units are arranged in m rows and m columns. Then m treatments are allotted to these m^2 units at random subject to the condition that each treatment occurs once and only once in each row and in each column.

Selected Latin Squares

3×3	**4×4**			
	1	2	3	4
ABC BCA CAB	ABCD BADC CDBA DCAB	ABCD BCDA CDAB DABC	ABCD BDAC CADB DCBA	ABCD BADC CDAB DCBA
5×5	**6×6**		**7×7**	
ABCDE BAECD CDAEB DEBAC ECDBA	ABCDEF BFDCAE CDEFBA DAFECB ECABFD FEBADC		ABCDEFG BCDEFGA CDEFGAB DEFGABC EFGABCD FGABCDE GABCDEF	

For randomization purpose two-way heterogeneity is eliminated by means of rows and columns and a latin square of order $m \times m$ is picked up from the table of Fisher and Yates. After picking the latin square its rows and columns are randomised by the help of random numbers and this randomised square is superimposed on the arranged square.

11.3 STATISTICAL ANALYSIS OF LSD

Let y_{ijk} (i, j, k = 1, 2, …, m) denote the response from unit (plot in the filed experimentation) in the i^{th} row, j^{th} column and receiving the k^{th} treatment. The triple (i, j, k) assumes only m^2 of the possible m^3 values of a LSD selected by the experiment. If S represents the set of m^2 values, then symbolically (i, j, k) belongs to S. If a single observation is made per experimental unit, then the linear additive model is:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \tau_k + e_{ijk}\,; \quad (i, j, k) \in S$$

where, μ is the general mean effect, α_i, β_j and τ_k are the constants effects due to the i^{th} row, j^{th} column and k^{th} treatment respectively and e_{ijk} is the error effect due to random component assumed to be normally distributed with mean zero and variance σ_e^2 i.e. e_{ijk} follows (i.i.d.) $N(0, \sigma_e^2)$.

If we write that

$G = y_{...}$ = Grand total of all the m^2 observations.

$R_i = y_{i..}$ = Total for m observations in the i^{th} row.

$C_j = y_{.j.}$ = Total of the m observations in the j^{th} column.

$T_k = y_{..k}$ = Total of the m observations in the k^{th} treatment.

Then heuristically, we get

$$\underset{(i,j,k)\in S}{\sum_i\sum_j\sum_k}\left(y_{ijk} - \bar{y}_{...}\right)^2 = \sum_i\sum_j\sum_k\Big[(\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})\Big]^2$$

$$= m\sum_i(\bar{y}_{i..} - \bar{y}_{...})^2 + m\sum_j(\bar{y}_{.j.} - \bar{y}_{...})^2 + m\sum_k(\bar{y}_{..k} - \bar{y}_{...})^2 + \sum_i\sum_j\sum_k(y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2$$

The product terms vanish since the algebraic sum of deviations from mean is zero. Thus

TSS = SSR + SSC + SST + SSE

where TSS is the total sum of squares and SSR, SSC, SST and SSE are sum of squares due to rows, columns, treatments and due to error respectively given by

$$\text{TSS} = \sum_{i,j,k\in S}(y_{ijk} - \bar{y}_{...})^2\,;$$

$$\text{SSR} = m\sum_i(\bar{y}_{i...} - \bar{y}_{...})^2 = S_R^2 \text{ (say)}$$

$$\text{SSC} = m\sum_j(\bar{y}_{.j.} - \bar{y}_{...})^2 = S_C^2$$

$$SST = m\sum_k (\bar{y}_{..k} - \bar{y}_{...})^2 = S_T^2$$

and $SSE = S_E^2 = TSS - SSR - SSC - SST$

Hence, the Total sum of squares is partitioned into three sum of squares, whose degree of freedom add to the degree of freedom of TSS.

ANOVA Table for LSD

Source of Variation	DF	SS	MSS	Variance Ratio(F)
Treatments	m– 1	S_T^2	$MSST = S_T^2/(m-1)$	$F_T = \frac{MSST}{MSSE}$
Columns	m– 1	S_C^2	$MSSC = S_C^2/(m-1)$	$F_C = \frac{MSSC}{MSSE}$
Rows	m– 1	S_R^2	$MSSR = S_R^2/(m-1)$	$F_R = \frac{MSSR}{MSSE}$
Error	(m– 1)(m– 2)	S_E^2	$MSSE = S_E^2/(m-1)(m-2)$	
Total	m^2-1			

Under the null hypothesis,

For row effects $H_{0\alpha}$: $\alpha_1 = \alpha_2 = \ldots = \alpha_m = 0$

For column effects $H_{0\beta}$: $\beta_1 = \beta_2 = \ldots = \beta_m = 0$ and

For treatment effects $H_{0\tau}$: $\tau_1 = \tau_2 = \ldots = \tau_m = 0$

against the alternative that all α's, β's and τ's are not equal, the test statistics F_T, Fc, F_R follow F distribution with [(m – 1), (m – 1) (m – 2)] df, under the above null hypothesis.

Thus, $F_\alpha = F_\alpha$ [(m– 1), (m– 1) (m– 2)] be the tabulated value of F distribution with [(m– 1), (m– 1) (m– 2)] df at the level of significance α. Thus, if $F_R > F_\alpha$ we reject the null hypothesis $H_{0\alpha}$, otherwise accept the null hypothesis. Similarly, we can test for $H_{0\beta}$ and $H_{0\tau}$.

Remark 1: Efficiency of LSD over RBD

There may be two cases to judge the relative efficiency of LSD over RBD:

1. Relative efficiency of LSD over RBD, when rows are taken as blocks is

$$= \frac{MSSC + (m-1)MSSE}{m \times MSSE}$$

2. Relative efficiency of LSD over RBD, when columns are taken as blocks is

$$= \frac{MSSR + (m-1)MSSE}{m \times MSSE}$$

Remark 2: Efficiency of LSD over CRD

Relative efficiency of LSD over CRD is given by

$$= \frac{MSSR + MSSC + (m-1)MSSE}{(m+1)MSSE}$$

Example 1: The example of petrol consumption by different makes of cars for illustrating randomised block designs has been converted to one with 5 makes of cars to illustrate latin square design. The effects of day and driver on consumption rate have been eliminated in addition to the effect of speed by suitable modification of the experimental situation. For this purpose, 5 drivers were chosen and each driver was used on one of 5 days. On that day, he drove 5 cars each of different make and each car with a different speed. The arrangement of the drivers, speeds and makes was as in the following table:

		Speeds in miles per hour				
		25	35	50	60	70
Drivers and Days	D_1	B(19.5)	E(21.7)	A(18.1)	D(14.8)	C(13.7)
	D_2	D(16.2)	B(19.0)	C(16.3)	A(17.9)	E(17.5)
	D_3	A(20.6)	D(16.5)	E(19.5)	C(15.2)	B(14.1)
	D_4	E(22.5)	C(18.5)	D(15.7)	B(16.7)	A(16.0)
	D_5	C(20.5)	A(19.5)	B(15.6)	E(18.7)	D(12.7)

Solution: Here, D_i (i = 1, 2, 3, 4, 5) denotes the i^{th} driver driving in the i^{th} day. A, B, C, D and E denote the 5 Makes of the cars. In the first cell of the table indicates that a car of Make B was driven by D_1 on this day with a speed of 25 miles per hour. The alphabets in the other cells have similar meaning. The number of miles covered by a gallon of petrol is shown in bracket in each cell.

The design adopted is actually a latin square design with the makes of cars as treatments and the drivers and speeds are the two controlled factors representing rows and columns. The observations of the miles per hour have been analysed below as appropriate for the design.

Correction Factor = 7638.76

Sum of Squares due to Speeds = 7719.49 –7638.76 = 80.73

Sum of Squares due to Drivers = 7640.12 – 7638.76 = 1.36

Sum of Squares due to Makes = 7704.18 – 7638.76 = 65.42

Total Sum of Squares = 7792.70 – 7638.76 = 153.94

Error Sum of Squares = 153.94 – 80.73 – 1.36 – 65.42 = 6.43

Analysis of Variance Table

Sources of Variation	DF	SS	MS	F Calculated	F Tabulated
Speeds	4	80.73	20.18	37.37**	3.26
Drivers	4	1.36	0.34	0.63	
Makes	4	65.42	16.35	30.28**	
Error	12	6.43	0.54		
Total	24	153.94			
** highly significant					

Mean numbers of miles per gallon for the different makes arranged in order

$\bar{E}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{D}$
19.98	18.42	16.98	16.84	15.18

$$SE = \sqrt{\frac{2\times MSSE}{5}} = \sqrt{\frac{2\times 0.54}{5}} = 0.33$$

CD at 1 per cent = 3.055 $\times 0.33$ = 1.42

The initial difference indicates that the Make E is significantly better than all the other Makes. Make A was better than B, C and D. Finally D is the worst.

Efficiency of Latin square

$$E\,(\text{Drivers}) = \frac{4\times 0.34 + 0.54\times 16}{20\times 0.54} = \frac{.34 + 0.54\times 4}{5\times 0.54} = 0.93$$

$$E\,(\text{Speeds}) = \frac{4\times 20.18 + 0.54\times 16}{20\times 0.54} = \frac{20.18 + 0.54\times 4}{5\times 0.54} = 8.27$$

The efficiency figures show that elimination of speed variation increased precision considerably while elimination of driver variation did not reduce error variance.

E1) Carry out ANOVA for the following design:

A 5	B 7	C 7	D 8	E 9
B 7	C 9	D 8	E 8	A 5
C 6	D 5	E 9	A 8	B 9
D 5	E 6	A 8	B 5	C 7
E 8	A 9	B 5	C 7	D 6

11.4 MISSING PLOTS TECHNIQUE IN LSD

As we have discussed in Section 10.4 of Unit 10, sometimes observations from one or more experimental units are not found (missing) due to some unavoidable causes. There may be some unforeseen causes for example in agricultural experiments damage by animal or pets, in animal experiment any animal may die or observations from one or more plot is excessively large as compared to other plots and thus accuracy of such observation is often in doubt. In such situations, these observations are omitted and treated as missing.

In case of missing observations, analysis is done by estimating the missing observation. This type of analysis was given by Yates (1937) and it is known as missing plot technique. As similar as in the RBD, we are now going to discuss the same in LSD in the following sub-section:

11.4.1 One Missing Plot

Suppose without loss of generality that in $m \times m$ latin square design the observation occurring in the first row, first column and receiving first treatment is missing. Let us assume that $y_{111}=Y$

R_1' = Total of all available (m – 1) observations in 1st row.

C_1' = Total of all available (m – 1) observations in 1st column.

T_1' = Total of all available (m – 1) observations receiving 1st treatment.

G' = Total of all available ($m^2 - 1$) observations.

On the basis of these totals we calculate different sum of squares as follows:

$$\text{Sum of Squares for Rows (SSR)} = \frac{(R_1' + Y)^2 + \sum_{i=2}^{m} R_i^2}{m} - \frac{(G'+Y)^2}{m^2}$$

$$\text{Sum of Squares for Columns (SSC)} = \frac{(C_1' + Y)^2 + \sum_{j=2}^{m} C_j^2}{m} - \frac{(G'+Y)^2}{m^2}$$

$$\text{Sum of Squares for Treatments (SST)} = \frac{(T_1' + Y)^2 + \sum_{k=2}^{m} T_k^2}{m} - \frac{(G'+Y)^2}{m^2}$$

$$\text{Total Sum of Squares (TSS)} = \underset{(i,j,k) \neq (1,1,1)}{\sum_i \sum_j \sum_k y_{ijk}^2} + Y^2 - \frac{(G'+Y)^2}{m^2}$$

Sum of Squares due to Error (SSE) = TSS – SSR – SSC – SST

$$SSE = Y^2 + \frac{2(G'+Y)^2}{m^2} - \frac{(R_1^{'}+Y)^2}{m}$$

$$- \frac{(C_1^{'}+Y)^2}{m} - \frac{(T_1^{'}+Y)^2}{m} + \text{Terms not involving Y}$$

For obtaining the value of Y, we minimize the sum of squares due to error with respect to Y. This is obtained by solving the equation

$$\frac{\partial(SSE)}{\partial Y} = 2Y + \frac{4(G'+Y)}{m^2} - \frac{2(R_1^{'}+Y)}{m} - \frac{2(C_1^{'}+Y)}{m} - \frac{2(T_1^{'}+Y)}{m} = 0$$

$$\Rightarrow Y + \frac{2Y}{m^2} - \frac{Y}{m} - \frac{Y}{m} - \frac{Y}{m} = \frac{R_1^{'}}{m} + \frac{C_1^{'}}{m} + \frac{T_1^{'}}{m} - \frac{2G'}{m^2}$$

$$\Rightarrow \frac{Y(m^2+2-3m)}{m^2} = \frac{m(R_1^{'}+C_1^{'}+T_1^{'}) - 2G'}{m^2}$$

$$\hat{Y} = \frac{m(R_1^{'}+C_1^{'}+T_1^{'}) - 2G'}{(m-1)(m-2)}$$

$\hat{Y}$ is the least square estimate of the yield of the missing plot. The value of Y is inserted in the original table of yield and ANOVA is performed in the usual way except that for each missing observation 1 df is subtracted from total and consequently from error df.

Example 2: In the following data, one value is missing. Estimate this value and analyse the given data.

Column / Row	**I**	**II**	**III**	**IV**	**Row Totals (R_i)**
I	A 12	C 19	B 10	D 8	49
II	C 18	B 12	D 6	A 7	43
III	B 22	D Y	A 5	C 21	48+Y
IV	D 12	A 7	C 27	B 17	63
Column Totals (C_j)	64	38+Y	48	53	203+Y

Solution: Here $m = 4, R_3^{'} = 48$, $C_2^{'} = 38, T_4^{'} = 26$, $G^{'} = 203$

Applying the missing estimation formula

$$\hat{Y} = \frac{m\left(R_3^{'} + C_2^{'} + T_4^{'}\right) - 2G'}{(m-1)(m-2)}$$

$$= \frac{4(48+38+26) - 2\times 203}{(4-1)(4-2)} = 7$$

Inserting the estimated value of Y, we get the following observations:

Column / **Row**	**I**	**II**	**III**	**IV**	**Row Totals (R_i)**
I	A 12	C 19	B 10	D 8	49
II	C 18	B 12	D 6	A 7	43
III	B 22	D 7	A 5	C 21	55
IV	D 12	A 7	C 27	B 17	63
Column Totals (C_j)	64	45	48	53	210

Correction Factor (CF) = $\frac{(210)^2}{16} = \frac{44100}{16} = 2756.25$

Raw Sum of Squares (RSS) = $(12)^2 + (18)^2 + \ldots + (21)^2 + (17)^2$ = 3432

Total Sum of Squares (TSS) = 3432– 2756.25 = 675.75

Row Sum of Squares (SSR) = $\frac{(49)^2 + (43)^2 + (55)^2 + (63)^2}{4} - CF$

$$= \frac{2401 + 1849 + 3025 + 3969}{4} - 2756.25 = 54.75$$

Column Sum of Squares (SSC) = $\frac{(64)^2 + (45)^2 + (48)^2 + (53)^2}{4} - CF$

$$= \frac{4096 + 2025 + 2304 + 2809}{4} - 2756.25 = 52.25$$

Treatment Sum of Squares (TSS) = $\frac{(31)^2 + (61)^2 + (85)^2 + (33)^2}{4} - CF$

$$= \frac{961 + 3421 + 7225 + 1089}{4} - 2756.25 = 417.75$$

Error Sum of Squares (SSE) = TSS– SSR – SSC – SST

= 675.75 – 54.75 – 52.25 – 417.75 = 151

ANOVA Table

Source of Variation	DF	SS	MSS	Variance Ratio		Conclusion
				Calculated	Tabulated	
Rows	4– 1=3	54.75	18.25	0.60	5.41	Insignificant
Columns	4– 1=3	52.25	17.42	0.58	5.41	Insignificant
Treatments	4– 1=3	417.75	139.25	4.61	5.41	Insignificant
Error	6– 1=5	151	30.20			
Total	15– 1 =14					

E2) Estimate the missing value in the following LSD and then carry out the analysis of variance test.

Row \ Column	I	II	III	IV
I	A 8	C 18	B 11	D 8
II	C 16	B 10	D 7	A Y
III	B 12	D 10	A 6	C 20
IV	D 10	A 9	C 28	B 16

11.5 SUITABILITY OF LSD

The latin square design is used when the experimental material is heterogeneous with respect to two factors and this two-way heterogeneity is eliminated by means of rows and columns. In fact LSD can be applied to all those cases where either the variation in the experimental material is not known or is known in two mutually perpendicular directions. Thus, LSD is successfully used in industry, animal husbandry, biological and social sciences, piggeries, marketing, medical and educational fields, where it is desired to eliminate the two factor heterogeneity simultaneously.

11.5.1 Advantages and Disadvantages of LSD

Advantages of LSD

1. Since total variation is divided into three parts namely rows, columns and treatments, the error variance is reduced considerably. It happens due to the fact that rows and columns being perpendicular to each other, eliminates the two-way heterogeneity up to a maximum extent.

2. LSD is an incomplete three way layout. Its advantage over the complete three way layout is that instead of m^3 units only m^2 units are needed. Thus, a 4×4 LSD results in saving 64 – 16 = 48 observations over a complete three way layout.

3. The analysis creates no problem even if a missing observation exists.

Disadvantages of LSD

1. The fundamental assumption that there is no interaction between different factors may not be true in general.

2. The main limitation of LSD is the equality of number of rows to that of columns and treatments. If the layout of experimental material is not of square design then LSD cannot be used.

3. RBD can be accommodated in any shape of field whereas for LSD field should perfectly be a square.

4. For smaller number of treatments, say less than 5, the degree of freedom for error is very small and thus the results are not reliable. Even in case of 2×2 LSD, degree of freedom for error becomes zero. In such situations, either the number of treatments should be increased or the latin square should be repeated.

5. On the other side, if the number of treatments increases the size of latin squares increases and this causes a disturbance in heterogeneity.

6. Analysis of LSD becomes very much complicated if complete row or complete column is missing. Analysis of RBD is quite easy in such situations.

11.6 SUMMARY

In this Unit, we have discussed:

1. The Latin Square design;
2. The layout of LSD;
3. The method of statistical analysis of LSD;
4. The missing plots technique in LSD; and
5. The advantages and disadvantages of LSD.

11.7 SOLUTIONS / ANSWERS

E1) The analysis of the given design is done by the method of analysis of variance. The computation results are given as follows:

Correction factor (CF) = 1239.04

Raw Sum of Squares = 1292

Total Sum of Squares = 52.92

Column Sum of Squares = 4.56

Row Sum of Squares = 4.96

Treatment Sum of Squares = 7.76

Error Sum of Squares = 35.68

Analysis of Variance Table

Sources of Variation	**DF**	**SS**	**MSS**	**F**
Rows	4	4.96	1.24	0.42
Columns	4	4.56	1.14	0.38
Treatment	4	7.76	1.94	0.65
Error	12	35.68	2.97	
Total	24	52.96		

Tabulated value of F (4, 12) = 3.26

Since the calculated value of F is much less than the tabulated value of F at 5% level of significance, we conclude that there is no significant difference between treatment means.

E2) Let the missing value is Y then we have

Column / **Row**	**I**	**II**	**III**	**IV**	**Row Totals (R_i)**
I	A 8	C 18	B 11	D 8	45
II	C 16	B 10	D 7	A Y	33 + Y
III	B 12	D 10	A 6	C 20	48
IV	D 10	A 9	C 28	B 16	63
Column Totals (C_j)	46	47	52	44 + Y	189 + Y

Here, m = 4, $R'_2 = 33$, $C'_4 = 44$, $T'_1 = 23$, $G' = 189$

Applying the missing estimation formula

$$\hat{Y} = \frac{m\left(R'_3 + C'_2 + T'_4\right) - 2G'}{(m-1)(m-2)}$$

$$= \frac{4(33 + 44 + 23) - 2 \times 189}{(4-1)(4-2)} = 3.66 \sim 4$$

Inserting the estimated value of Y, we get the following observations:

Column / Row	I	II	III	IV	Row Totals (R_i)
I	A 8	C 18	B 11	D 8	45
II	C 16	B 10	D 7	A 4	37
III	B 12	D 10	A 6	C 20	48
IV	D 10	A 9	C 28	B 16	63
Column Totals (C_j)	46	47	52	48	193

Correction Factor (CF) = $\frac{(193)^2}{16} = 2328.06$

Raw Sum of Squares (RSS) = $(8)^2 + (16)^2 + ... + (20)^2 + (16)^2 = 2895$

Total Sum of Squares (TSS) = 2895 – 2328.06 = 566.94

Row Sum of Squares (SSR) = $\frac{(45)^2 + (37)^2 + (48)^2 + (63)^2}{4} - CF$

$$= \frac{2025 + 1369 + 2304 + 3969}{4} - 2328.06 = 88.69$$

Column Sum of Squares (SSC) = $\frac{(46)^2 + (47)^2 + (52)^2 + (48)^2}{4} - CF$

$$= \frac{2116 + 2209 + 2704 + 2304}{4} - 2328.06 = 5.19$$

Treatment Sum of Squares (SST) = $\frac{(27)^2 + (49)^2 + (82)^2 + (35)^2}{4} - CF$

$$= \frac{729 + 2401 + 6724 + 1225}{4} - 2328.06 = 441.69$$

Error Sum of Squares (SSE) = TSS − SSR – SSC – SST

= 566.94 – 88.69 – 5.19 – 441.69

= 31.37

ANOVA Table

Source of Variation	DF	SS	MSS	Variance Ratio		Conclusion
				Calculated	Tabulated	
Rows	4−1=3	88.69	29.56	4.71	5.41	Insignificant
Columns	4−1=3	5.19	1.73	0.28	5.41	Insignificant
Treatments	4−1=3	441.69	147.23	23.48	5.41	Significant
Error	6−1=5	31.37	6.27			
Total	15−1 =14					

Since for treatment effect calculated value of F is greater than the tabulated value of F at 5% level of significance, so we conclude that the treatment effect is significant. For pairwise testing, find the standard error of difference of two treatment means.

$$\text{SE} = \sqrt{\frac{2\text{MSSE}}{m}} = \sqrt{\frac{2\times 6.27}{4}} = 1.77$$

Critical difference (CD) = SE $\times\, t_{\alpha/2}$ at error df

= 1.77 × 2.571 = 4.55

Treatment means

$\bar{A} = \frac{27}{4} = 6.75$, $\bar{B} = \frac{49}{4} = 12.25$, $\bar{C} = \frac{82}{4} = 20.5$ & $\bar{D} = \frac{35}{4} = 8.75$

Pair of Treatments	Difference	CD	Inference
A, B	$\lvert\bar{A}-\bar{B}\rvert = 05.50$	4.55	Significant
A, C	$\lvert\bar{A}-\bar{C}\rvert = 13.75$	4.55	Significant
A, D	$\lvert\bar{A}-\bar{D}\rvert = 02.00$	4.55	Insignificant
B, C	$\lvert\bar{B}-\bar{C}\rvert = 08.25$	4.55	Significant
B, D	$\lvert\bar{B}-\bar{D}\rvert = 03.50$	4.55	Insignificant
C, D	$\lvert\bar{C}-\bar{D}\rvert = 11.75$	4.55	Insignificant