UNIT 11 COMPACT STARS

Structure

11.1 INTRODUCTION

In Unit 10, you have learnt about the evolution of stars, that is, how they *live their lives* after being formed and take their respective positions on the H-R diagram. You know that the evolution of a star is governed by two competing forces, namely, the gravitational contraction and the radiation pressure due to energy generating thermonuclear fusion reactions. In spite of the huge mass of a star, the amount of its nuclear fuel is finite and it has to end ultimately. When it happens, a star can no longer be prevented from contraction due to gravity. The density of the star increases manifold and it turns into a compact object. In the present Unit, you will study about such compact objects, also called compact stars.

There are many interesting questions pertaining to compact stars, such as: Does the gravitational collapse of a star take place uninterrupted? If not, what is the mechanism which balances the force of gravity? Do all stars end their life by becoming compact stars of similar type? If not, what determine(s) the nature of the compact stars? You will discover the answers to these and other related questions as you study this Unit.

A compact star can become a white dwarf, a neutron star, or a black hole depending upon its initial mass. Further, the gravitational collapse of compact stars like white dwarf and neutron stars is halted by the degeneracy pressure of fermions – a quantum mechanical phenomenon. You will learn about the degeneracy pressure in Sec. 11.2. The theoretical analysis of the relation between the nature of a compact star and its mass was done by S. Chandrasekhar. This led him to predict a limiting mass for white dwarf stars. You will learn about the theory of white dwarfs in Sec. 11.3. In Sec. 11.4, you will study various characteristics of neutron stars and also understand why it is difficult to detect them optically. All the theoretical predictions about neutron stars could only be put to test when they were observed in the form of pulsars. In Sec. 11.5, you will learn about one of the most interesting objects, called the black hole, which physics has ever predicted. You will discover that the black hole signify the ultimate victory of gravity in the evolution of stars.

Objectives

After studying this unit, you should be able to:

- understand the role of mass in deciding whether a compact star becomes a white dwarf, a neutron star or a black hole;
- explain the concept of degeneracy pressure of fermions and its role in compact stars;
- discuss the concept of Chandrasekhar limit and obtain an expression for it;
- understand the formation of neutron stars and its internal structure;
- derive an expression for the gravitational red shift of the neutron stars;
- describe the detection of neutron stars in the form of pulsars and discuss its properties;
- explain the concept of Schwarzschild radius for black holes; and
- describe the geometry of space-time around the non-rotating black holes.

11.2 BASIC FAMILIARITY WITH COMPACT STARS

You may recall from Unit 10 that the gravitational force is balanced by the outward force due to pressure gradient inside a star. The pressure inside is generated due to thermonuclear reactions. With the passage of time, a star exhausts all its nuclear fuel by 'burning', that is, due to nuclear reactions. The star cannot support itself against gravity and begins to collapse. Eventually, the density of the star increases tremendously and the star turns into a very compact object. At this stage, you would like to know:

a) In the absence of radiation pressure due to nuclear reactions, what enables compact stars to withstand gravitational squeeze?

b) Are all the stars similar to one another after their collapse?

To address the first issue, recall from the course entitled Physics of Solids (PHE-13) that when atoms are very close to one another, their quantum energy states overlap and the electrons in those orbitals behave as if they are free from their parent atoms. A similar situation is obtained in compact stars where, due to very high pressure, all the electrons are separated from their parent atoms. This phenomenon is called **pressure ionisation**. *A compact star is, therefore, a collection of nuclei and free electrons.* You have studied in Unit 10 how the pressure of the degenerate gas of free electrons restrains the seemingly unstoppable gravitational contraction. This is because a quantum state cannot accommodate more than one fermion (e.g., electron, proton, neutron, etc.). This implies that, when the density of electrons is high, they are forced to occupy quantum states with higher energies because lower states are full. *In such a situation, the pressure of the gas depends only on the density and is independent of the temperature*. You have learnt that gas of free electrons in such a state is called **degenerate electron gas** and the pressure exerted by it is called **degeneracy pressure**.

So, the gravitational collapse of compact stars is balanced by the degeneracy pressure of electrons. It is, however, important to note that Pauli's exclusion principle holds for all fermions. You will learn later in this Unit that degeneracy pressure due to neutrons plays an important role in stabilizing some compact stars.

The answer to question (b) raised above is: No. The nature of the remains of a star after death depends on its mass. You may recall from Unit 10 that the mass of a star plays a crucial role in its evolution and determines its luminosity. Similarly, depending upon its mass, a dying star can turn into any one of the three kinds of compact stars, namely a white dwarf, neutron star or black hole. You will study about these compact stars later in the Unit. *Here, it should suffice to say that it was genius of* *the Indian astrophysicist, S. Chandrasekhar, who first showed that a degenerate star cannot have mass larger than a certain maximum mass.* He suggested, on the basis of theoretical calculations, that the degeneracy pressure of electrons will be able to stop further collapse of a star and its mass is less than a certain mass called Chandrasekhar limit. The resulting star is called a **white dwarf**. If the mass of a collapsing star is more than the Chandrasekhar limit, but less than 3 M_{Θ} then the degeneracy pressure of neutrons can halt the collapse. These stars are known as **neutron stars**. Further, if the mass of the collapsing star is even higher, there is no way that the collapse can be halted and the collapsing star becomes a **black hole**.

Thus, compact stars are simply the end products of ordinary stars and are characterised by smaller sizes and higher densities. To compare and contrast their sizes with that of the Sun, recall that the radius of the Sun is 7×10^{10} cm and its mass is $\sim 2 \times 10^{33}$g. A white dwarf of the same mass as that of the Sun would have a radius about 100 times smaller, that is, around 10^9 cm. A neutron star of similar mass may have radius of about 10 km only. And a mass equal to that of the Sun is too small for a black hole! Generally, it is believed that the minimum mass of a black hole is three times the mass of the Sun ($3M_{\Theta}$). You may ask: **What is the radius of a black hole?** *Well, that is a somewhat difficult concept.* We will talk about it in Sec. 11.5 of this unit.

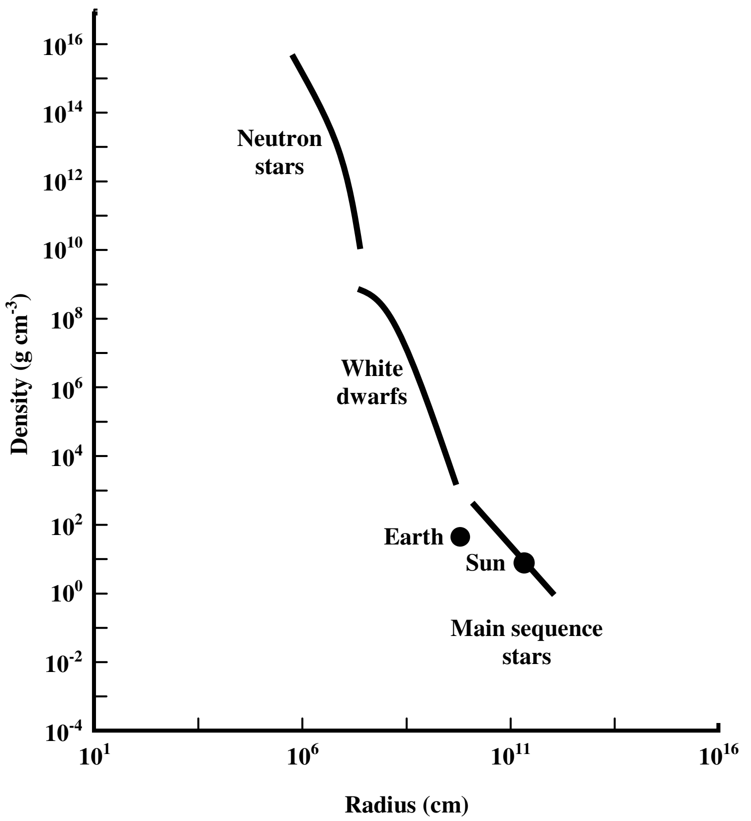


Fig.11.1: Typical range of densities of celestial objects as a function of their typical radii

To get an idea about the average densities of compact stars vis-à-vis their radii, refer to Fig. 11.1. Note that the main sequence stars, such as the Sun, have density ~ 1 g cm^{-3}. On the other hand, the density of a white dwarf could be ~ 10^7 to 10^8 g cm^{-3} and that of a neutron star ~ 10^{15}g cm^{-3}. A black hole, however, has no defined average density, as you will learn later in this Unit. Also, the radius of a black hole is a *theoretical construct* which means that if you come within this distance from the centre (where all the mass is concentrated), you cannot escape from the tremendous attraction of its gravitational force. Why only you? Even photons, the fastest particles, cannot escape.

You know from the course entitled Thermodynamics and Statistical Mechanics (PHE-06) that the behaviour of a gaseous system can be understood on the basis of its equation of state.

11.2.1 Equation of State and Degenerate Gas of Fermions

In the preceding paragraphs, you have learnt that the densities of white dwarfs and neutron stars are very high compared to the densities of objects we come across in everyday life, or even the densities of ordinary stars like the Sun. *Therefore, to know more about the compact stars, you should have some idea about how matter behaves when the density is extraordinarily high.* You may ask: **Why should matter behave differently?** It is because the equation of state of matter at high densities is different from the equation of state at ordinary densities. When the molecules of a gaseous system are few and far between, as in our room, or in the atmosphere, collision is the only way the molecules can interact with one another. Such a gas of molecules is called an **ideal gas** and its equation of state is given by:

$$PV = NRT, \qquad (11.1)$$

where P is the pressure, V is the volume, N is the total number of particles, R is the gas constant, and T is the temperature. It is evident from Eq. (11.1) that an equation of state relates different thermodynamic quantities (such as pressure, volume and temperature) of the gas. Such a relation is very useful in investigating the behaviour of gaseous systems.

The velocity distribution of the particles of an ideal gas is given by the well known Maxwell's velocity distribution law which you know already from the kinetic theory of gases.

When the density of the gaseous system becomes as high as in compact stars, the distance between two atoms/molecules becomes comparable to the size of the atoms/molecules or even the size of the nucleons (i.e., protons, neutrons)! *At such densities, other forces such as the Coulomb force and nuclear forces begin to influence the dynamical behaviour of the atoms/molecules.* As a result, the equation of state of an ideal gas cannot describe the behaviour of high density gases.

Now, to get an idea about the equation of state which can describe matter at high densities such as that in compact stars, we note that compact stars consist mainly of degenerate gas of electrons and neutrons. Further, you may recall from the course Thermodynamics and Statistical Mechanics (PHE-06) that we need to use statistics to describe the collective behaviour of a large number of particles (atoms or molecules) of a system. Ordinary gases obey the so-called Maxwell-Boltzmann (M-B) statistics. But, the fermions, which make up the compact stars, obey the so-called Fermi-Dirac (F-D) statistics. So, in the context of the equation of state, our problem reduces to know how F-D statistics influences the physical parameters such as pressure inside a compact star.

You may further recall from this course that the number density $\left(= \dfrac{dN}{d^3p\, d^3x}\right)$ of particles in phase space can be written as:

$$\frac{dN}{d^3p\, d^3x} = \frac{g}{h^3} f \qquad (11.2)$$

When the particle density is low and temperature is high, F-D as well as B-E (Bose-Einstein) distributions become identical to:

$$f(E) = e^{-(E-\mu)/kT}.$$

which is known as the Maxwell-Boltzmann distribution.

where p and x are the momentum and position variables of the particle, respectively, h is the Planck's constant and f is the distribution function. In Eq. (11.2), $d^3p\; d^3x$ is the volume element in the phase space, with d^3p containing the momentum elements and d^3x containing the position elements; g is called the statistical weight, i.e., number of states that a single particle can have for a given value of the momentum p. If S denotes the spin of particle, then $g = 2S + 1$. The distribution function f denotes the average occupation number of a cell in the phase space.

You may also recall that, for an ideal gas in equilibrium at temperature T, the F-D statistics gives the distribution function f as:

$$f(E) = \frac{1}{\exp[(E-\mu)/k_B T]+1} \quad (11.3)$$

where k_B is the Boltzmann constant, μ is the chemical potential and E is the energy. The energy distribution function given by Eq. (11.3) is valid for fermions such as electron, proton and neutron which have half-integral spins (i.e., 1/2, 3/2 ..). From Eq. (11.3), it is obvious that for $T \to 0, f(E)$ becomes a step function (Fig. 11.2), that is,

$$f(E) = 1 \quad \text{when} \quad E < \mu \quad (11.4a)$$

and

$$f(E) = 0 \quad \text{when} \quad E > \mu. \quad (11.4b)$$

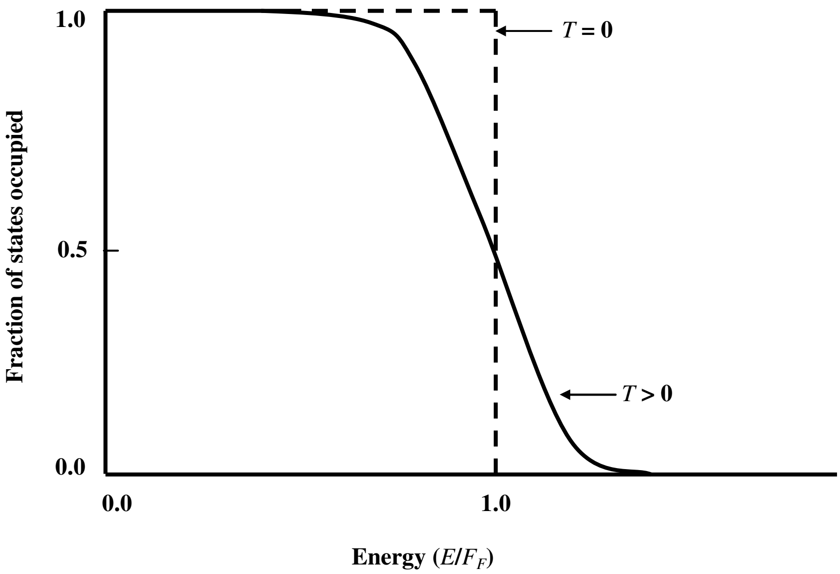


Fig.11.2: Fermi distribution function at $T = 0$ and at temperature, $T > 0$

When the distribution of fermions is similar to the $T = 0$ case shown in Fig. 11.2, the gas of fermions is called completely degenerate. Physically, this means that all energy states below a certain energy are fully occupied and the occupancy above this energy is zero. The energy up to which all states are occupied is called the Fermi energy, E_F and for fermions at $T = 0$, $E_F = \mu$.

SAQ 1

Spend 5 min.

Explain how Eqs. (11.4a) and (11.4b) are satisfied.

So, now you know the nature of f for a degenerate gas. The question is: **How can we use Eq. (11.2) to arrive at the equation of state for compact stars?** Suppose you want to know how many particles are present in a unit volume having all permissible values of momentum. You have to simply integrate the number density in phase space (left hand side of Eq. (11.2)) over all possible momenta, i.e.,

$$n = \int \frac{dN}{d^3p\,d^3x} d^3 p \quad (11.5)$$

Now, if you want to know the energy density ε of particles, you have to integrate over the energies of all the particles:

$$\varepsilon = \int E \frac{dN}{d^3 p\, d^3 x} d^3 p, \qquad (11.6a)$$

where E is the energy of the particle given by:

$$E = \left(m^2 c^4 + p^2\right)^{\frac{1}{2}} \qquad (11.6b)$$

m being the mass of each particle and c being the velocity of light. Similarly, the pressure of the gas defined as the rate of momentum exchanged across an ideal surface of unit area, can be written as:

$$P = \frac{1}{3}\int pv \frac{dN}{d^3 p\, d^3 x} d^3 p \qquad (11.7)$$

Actually, Eq. (11.7) shows that pressure can be expressed as the momentum flux of a gas. Since we are discussing only the isotropic pressure here, the said flux in any one direction (out of three) is one-third of the net momentum flux. That is why a factor of 1/3 has been put before the integral. Assuming that the motion of fermions in compact stars is non-relativistic, it can be shown (we have avoided giving the mathematical details) that Eq. (11.7) reduces to:

$$P = K \rho_e^{\frac{5}{3}} \qquad (11.8)$$

The planetary nebula, so called because of its planet-like appearance, is visually one of the most attractive astronomical objects.

where K is a constant and ρ_e is the density of electrons. Eq. (11.8) is the equation of state of a degenerate gas of electrons at high density. *Note that, unlike the ideal gas, the pressure of a degenerate gas is independent of temperature.* This implies that even when temperature of a degenerate gas of fermions is very very low, it can exert tremendous pressure.

Let us now discuss white dwarf stars in which only the electrons are degenerate.

11.3 THEORY OF WHITE DWARF

After a star has exhausted its nuclear fuel, it begins to collapse due to gravity. If the mass of the star is less than about 8 $M_{\odot}$, the gravitational contraction is accompanied by expulsion of matter from its outer envelope. The discarded matter forms a ring-like structure around the collapsing star and is called **planetary nebula** (see Fig. 11.3).

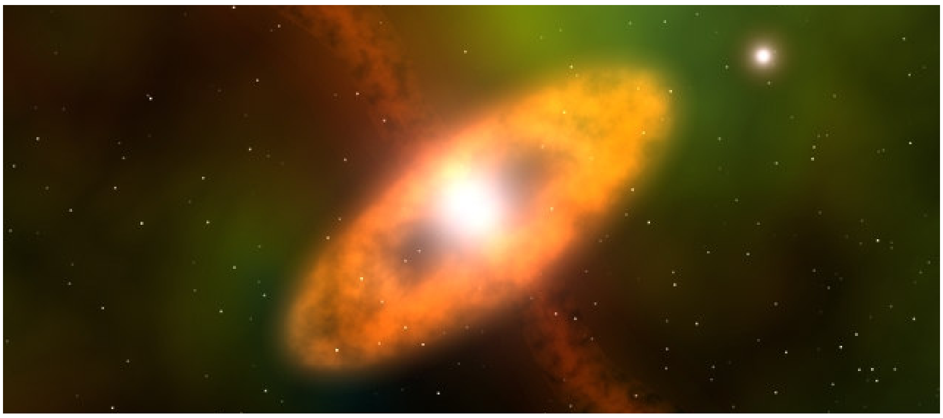


Fig. 11.3: A planetary nebula

The core of the star continues to contract till its density attains a value in the range $10^5 - 10^8$ g cm^{-3}. At this density, a new equilibrium sets in and these stars are called white dwarfs. The evolution of a medium mass star into a white dwarf is shown schematically in Fig. 11.4.

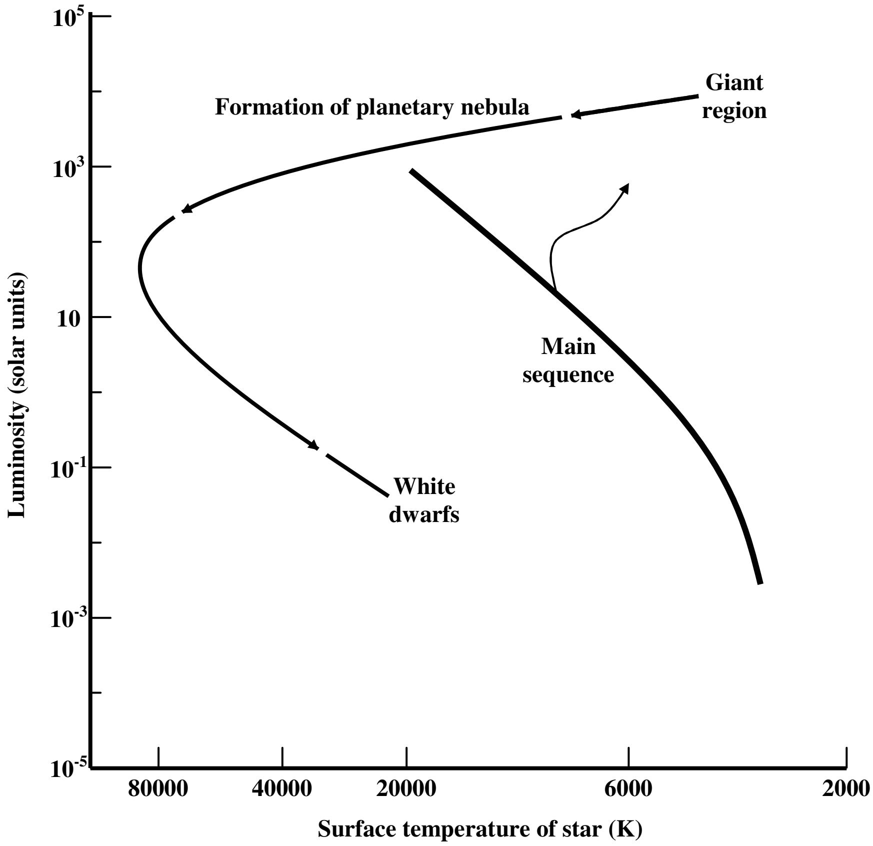


Fig.11.4: Evoluation of medium mass star leading to the formation of white dwarf

One of the interesting features of white dwarfs is that they have mass almost equal to the mass of the Sun, but their radii are only about five to ten thousand kilometres. The characteristic features of white dwarfs were explained theoretically by S. Chandrasekhar.

11.3.1 Chandrasekhar Limit

S Chandrasekhar investigated the effect of the gravitational field on a degenerate gas of electrons to estimate the mass-radius relation of white dwarfs. One of the characteristics of a degenerate gas is that its equation of state does not involve temperature. When particles are non-relativistic, the pressure exerted by such a gas is given by Eq. (11.8). Now, inside a star in hydrostatic equilibrium, the pressure must balance the gravitational pull towards the centre. For a star in hydrostatic equilibrium, (recall from Unit 8 that) the pressure gradient can be expressed as:

$$\frac{1}{\rho_e}\frac{dP_e}{dr} = \frac{GM}{R^2} \qquad (11.9)$$

where M and R are the mass and the radius of the star respectively. If we assume that the density of the star is uniform, we can write:

$$M = \frac{4}{3}\pi R^3 \rho_e \qquad (11.10)$$

Substituting for ρ_e from Eq. (11.10) in Eq. (11.9) and integrating from the centre to the surface, we get:

$$P_e \propto \frac{M^2}{R^4} \qquad (11.11)$$

But, from Eq. (11.8) and Eq. (11.10), we find that:

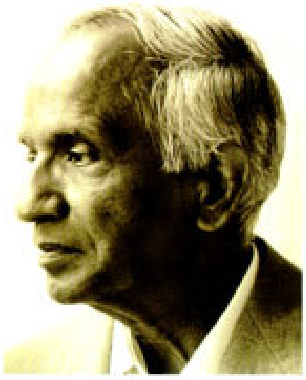
$$P_e \propto \frac{M^{\frac{5}{3}}}{R^5} \qquad (11.12)$$

Comparing Eqs. (11.11) and (11.12), we obtain:

$$R \propto \frac{1}{M^{1/3}} \qquad (11.13a)$$

We can also write Eq. (11.13a) as

$$\rho \propto M^2 \qquad (11.13b)$$



S. Chandrasekhar was awarded Nobel Prize in 1983 for his extensive theoretical work on white dwarfs.

Eq. (11.13) shows that, as the mass of the white dwarf increases, its radius decreases. The mass-radius relation is plotted in Fig. 11.5. The shrinking of radius is understandable because increase in mass would mean increase in the force of gravitational contraction.

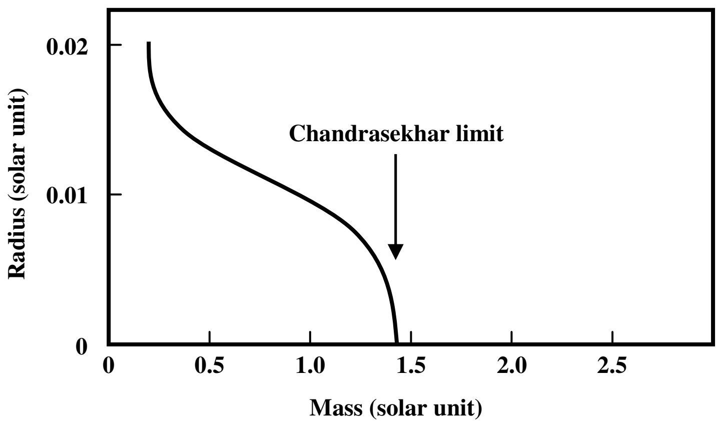


Fig.11.5: Mass-radius plot for white dwarfs

A logical question at this point is: **What will happen if we go on adding mass to a white dwarf?** Eq. (11.13a) indicates that if enough mass is added to a white dwarf, its radius would ultimately shrink to zero! **What is the value of this mass?** *Chandrasekhar showed that if mass of the white dwarf is about 1.4 times the solar mass, its radius will shrink to zero.* This is called the **Chandrasekhar limit**. If the mass of the white dwarf is more than this limiting mass, its gravitational contraction cannot be balanced by the degeneracy pressure of the electrons. Thus, a star of mass greater than 1.4 solar mass cannot become a stable white dwarf unless it ejects mass in some way. No white dwarf has been discovered which has a mass higher than Chandrasekhar limit.

In the absence of energy generating nuclear reactions, there is actually no source of energy left in a white dwarf. Therefore, these stars would go on shining by radiating their thermal energy and in the process, their temperature would decrease. Ultimately, the entire thermal energy will be lost. The star becomes a cold object.

Would you not like to know how long a white dwarf star takes to cool down? The internal temperature (say, T_{wd}), of a white dwarf is almost constant. Its total thermal energy can, therefore, be written as:

$$U = \frac{3}{2} N_e k_B T_{wd}$$

$$= \frac{3}{2} \frac{M}{\mu m_p} k_B T_{wd} \qquad (11.14)$$

where M is the mass of the white dwarf. Eq. (11.14) has been written by taking the energy of an electron at temperature T_{wd} as $(3/2)k_B T_{wd}$, $(1/2)k_B T_{wd}$ for each degree of freedom. In Eq. (11.14), μm_p is the mean molecular weight of nuclei inside the star.

For stars which consist entirely of heavy elements, $\mu = 2$. If we take $M = 1M_\Theta$, and $T_{wd} = 10^7$K, the total thermal energy of a white dwarf is ~ 10^{48} ergs! This is a very significant amount of energy. The question is: **How long will this amount of thermal energy enable a white dwarf to shine?** To know this, solve the following SAQ.

SAQ 2

Spend 5 min.

Suppose the luminosity of a white dwarf star of mass $1M_\Theta$ is $10^{-3}L_\Theta$. If the luminosity of the Sun, L_Θ is 4×10^{26} Js^{-1}, calculate the time for which the white dwarf will keep shining with its present luminosity.

Having solved SAQ 2, you know that the thermal energy of a white dwarf can keep it shining for billions of years! Sirius B, the companion of the 'Dog Star' Sirius A, is the earliest white dwarf star to be observed. Fig. 11.6 shows a white dwarf star.

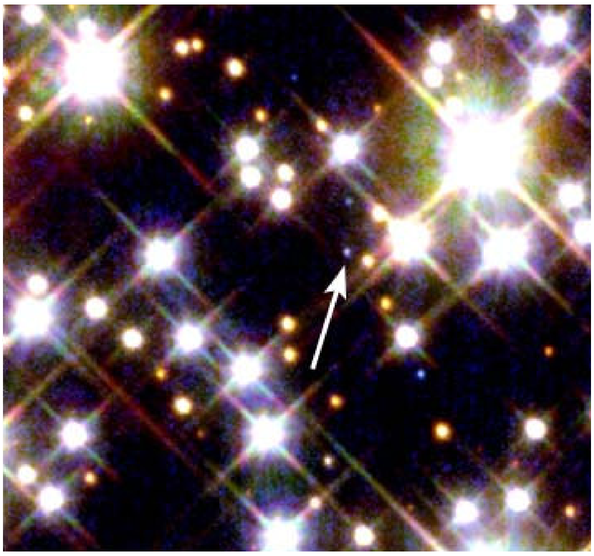


Fig.11.6: A white dwarf star shown by arrow

Fig. 11.7 shows the distribution of the observed white dwarfs on the H-R diagram. Different lines such as 0.89, 0.51 and 0.22 in the Figure indicate the masses of stars in terms of the solar mass. White dwarfs are dimmer compared to the main sequence stars of the same surface temperature because of their smaller size. The difference in absolute magnitudes of white dwarfs compared to main sequence stars is in the range 5 – 10.

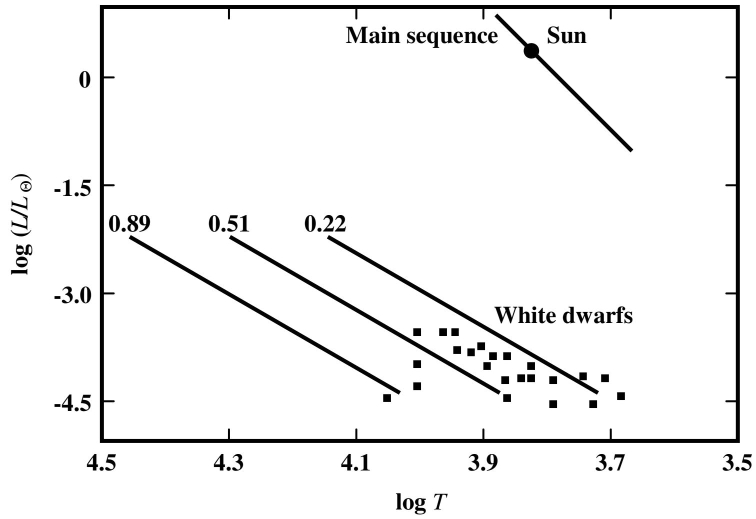


Fig. 11.7: Location of white dwarfs in the H-R diagram

As we have mentioned earlier, white dwarfs are compact stars resulting from the death of medium mass stars like the Sun. You may ask: **What are the remnant of stars which are much more massive than the Sun?** Such stars end their lives as neutron stars. You will learn about it now.

11.4 NEUTRON STAR

To understand the formation of neutron stars, you may recall from Unit 10 that when the nuclear reactions stop, the core of a massive star (~ 10 M_Θ to 100 M_Θ) collapses and a supernova explosion takes place. The supernova explosion blows away the outer shell of the star. The question is: **What is the remnant of a supernova explosion?** In 1934, Walter Baade and Fritz Zwicky suggested that the core of a supernova explosion could be a small, high density neutron star.

When the core of a star begins to collapse in the absence of nuclear reactions, its density increases and attains a value comparable to the density of white dwarfs. The degeneracy pressure due to electrons can balance the gravitational contraction only when the mass of the object is smaller than the Chandrasekhar limit. If the mass is more than this limiting value, the gravitational collapse continues and the density of the star increases further. When the density is in the range ~ $10^{14} - 10^{15}$ g cm^{-3}, the following two things happen:

a) protons and electrons combine to produce neutrons and neutrinos according to the following *inverse* β*-decay reaction*:

$$e + p \rightarrow n + v$$

b) atomic nuclei begin to disintegrate due to pressure ionisation.

Since neutrons are fermions, their degeneracy pressure halts the gravitational contraction and a stable neutron star is formed. It has been calculated that the neutron stars have radii of about 10 km and densities of their core is about 10^{14} to 10^{15}g cm^{-3}. Note that neutron stars are much denser than the white dwarfs.

You may ask: **Is there any limiting mass for neutron stars similar to the Chandrasekhar limit for white dwarfs**? Yes; but the precise value of the limiting mass for the stars is difficult to determine because the behaviour of matter at such high densities is not well understood yet. The estimated limiting mass is in the range 2 – 3 M_Θ.

11.4.1 Gravitational Red-shift of Neutron Stars

You have just learnt that the neutron stars are very compact. Their gravitational pull must be very strong indeed. **How strong is its gravitational pull?** To get an idea, we may estimate the value of gravitational acceleration, g, on a neutron star, say, of mass $M_n \sim 1.5M_\odot$ and radius $R_n \sim 10$ km. The expression for gravitational acceleration on a neutron star can be written as:

$$g_n = \frac{GM_n}{R_n^2}$$

Substituting the values of M_n and R_n, we get, $g_n \sim 2 \times 10^{14}$ cm s^{-2}. The value of g_n is $\sim 2 \times 10^{11}$ times higher than the gravitational acceleration on the Earth! That is, the gravitational force on a neutron star is 10^{11} times stronger than that due to the Earth.

Very often, we do thought experiments: the experiments which cannot be done for practical reasons, but *they could be done*, in principle.

Well, to get a feel for the perceptible effect of such a strong gravitational field, let us do a *thought experiment*. Suppose you are standing on the surface of the star which is emitting a yellow light at $\lambda = 5800$ Å. The light reaches your eyes approximately one metre above the ground. The interesting question is: **Will you see the light as yellow?** If you think you will, you are wrong! Let us find out the reason.

A photon leaving the surface of the neutron star has energy $h\nu$ and equivalent mass $h\nu/c^2$. The neutron star will attract this photon and, as a result, the photon has to work 'hard' against the star's gravity to reach your eyes (just as a stone thrown upward from the earth slows down as it goes higher). The work done by the photon to reach your eye is, $W = \frac{h\nu}{c^2} g_n H$, where $H \sim 100$ cm, is the height of your eyes from your feet. Further, ν can be calculated using the relation $\lambda = c/\nu$, i.e., $\nu \sim 5.172413 \times 10^{14}$ Hz. Now, to arrive at your eyes, the photon works against gravity and hence its energy, that is, frequency will decrease and wavelength will increase. To calculate the change in wavelength or frequency, we can use the energy conservation principle. Let ν' be the frequency of the light (photons) that you observe. Thus, from the energy conservation principle, we can write:

$$h\nu' = h\nu - W \tag{11.15}$$

$$= h\nu - \frac{h\nu}{c^2} g_n H$$

$$= h\nu\left(1 - \frac{g_n H}{c^2}\right)$$

In terms of the wavelength of photons, Eq. (11.15) can be written as:

$$\frac{d\lambda}{\lambda} \sim \frac{g_n H}{c^2} \tag{11.16}$$

where $d\lambda = \lambda' - \lambda$ and $\lambda' = c/\nu'$. Substituting the values of g_n, c and H in Eq. (11.16), we get:

$$\frac{d\lambda}{\lambda} \sim 2.2 \times 10^{-5} \tag{11.17}$$

Thus, yellow light of wavelength $\lambda = 5800\ \mathring{A}$ leaving your feet (the surface of the neutron star) will arrive at your eyes as light of

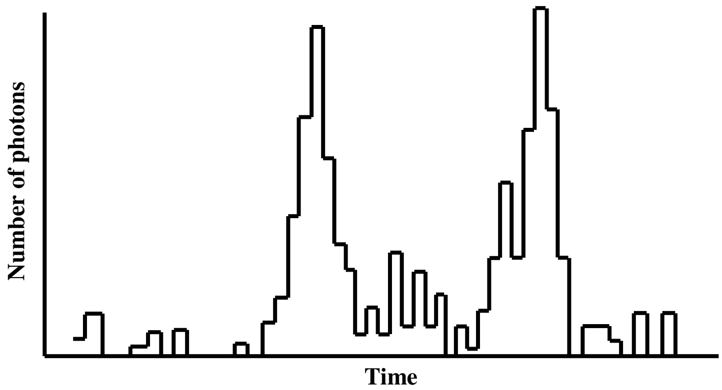
wavelength $\lambda' \left(= \lambda + \lambda \frac{g_n H}{c^2} \right) \sim 5800.13\ \mathring{A}$. Such differences in the wavelengths can be measured easily these days.

The increase in the wavelength of a photon when it comes out of a strong gravitational field is called **gravitational red-shift.** The significance of this phenomenon lies in the fact that scientists must make necessary corrections in the observed frequency of photon presumably coming from objects such as neutron stars in order to understand physical processes on their surface.

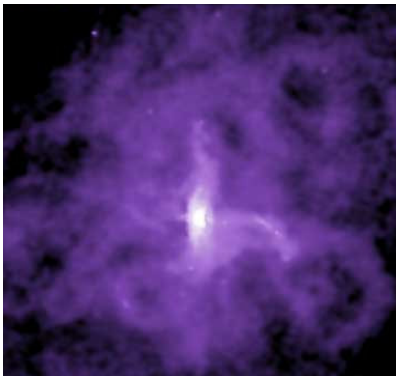
It is difficult to observe neutron stars optically because, being very small in size, these are very faint objects. However, with radio telescopes, neutron stars have been detected in the form of *pulsars*: the *stars which emit regular pulses of radiation, very often several times a second.*

11.4.2 Detection of Neutron Stars: Pulsars

In 1967, a remarkable discovery was made in the history of astronomy by a student named Jocelyn Bell who was doing her doctoral work under Prof. Anthony Hewish in Cambridge, England. She observed certain periodic pulses of radio waves coming from a certain direction in the sky which were repeated precisely every 1.337s (Fig. 11.8). Very soon, she discovered a few more such objects with different periodicities. These objects are known as pulsars. A year later, it was already clear that these pulses must have been emitted by **rotating neutron stars**. Periodicities of the pulsars have been found to be between 10^{-3}s and 4s.



(a)



(b)

Fig. 11.8: a) Detection of pulses of radio waves, an evidence of the existence of neutron stars; and b) pulsar at the centre of a nebula (Image credit: Chandra X-ray Observatory)

You may ask: **How can we be sure that a strong pulse, whose periodicity is about a second, is emitted by a neutron star and not by a white dwarf?** When a star rotates, each of its layers experiences centrifugal force directed outward as well as the pointing gravitational force directed inward. If Ω be the angular velocity and R be the radius of the rotating star, the two forces balance in a Keplerian orbit and we can write:

$$\Omega^2 R = \frac{GM}{R^2} \qquad (11.18)$$

For a white dwarf, we can write $\frac{M}{R^3} \sim \rho \sim 10^7\,\text{gcm}^{-3}$. So the typical value of Ω is:

$$\Omega \sim 2.58\ \text{s}^{-1}. \qquad (11.19)$$

Therefore, the time period can be written as:

$$T = \frac{2\pi}{\Omega}$$

$$\sim 2.433\text{s}. \qquad (11.20)$$

From Eq. (11.18), it is clear that for objects with lower densities, the corresponding Ω will also be lower, and hence the time period would be higher! Therefore, white dwarfs and other bigger celestial objects are too big, their densities are too low and they cannot emit pulses with periods as short as those observed in pulsars. In addition, it has been observed that there are pulsars having periods of only a few milliseconds! It is, therefore, almost impossible for white dwarfs to show such periodicities. So, the belief that pulsars must be neutron stars became even stronger. Now, before proceeding further, you should solve an SAQ.

SAQ 3

Spend 5 min.

Calculate the gravitational red shift for the yellow light $(\lambda = 5800\,\mathring{A})$ on the surface of Sirius B when the photon travels a distance of 1m. Take the mass of Sirius B as $M_{SiB} = 1M_{\Theta}$, and its radius as $R_{SiB} = 16000\,\text{km}$.

Emission mechanism and observed profiles

Regarding pulsars, a logical question could be: **What causes emission of pulses from a neutron star?** It is suggested that a neutron star is like a gigantic **light house**. In a light house, a powerful light-source rotates and ships see it periodically. In exactly the same way, emission of radiation takes place continuously, but due to rotation of the neutron star the emission is detected only periodically by an observer on the Earth (Fig. 11.9).

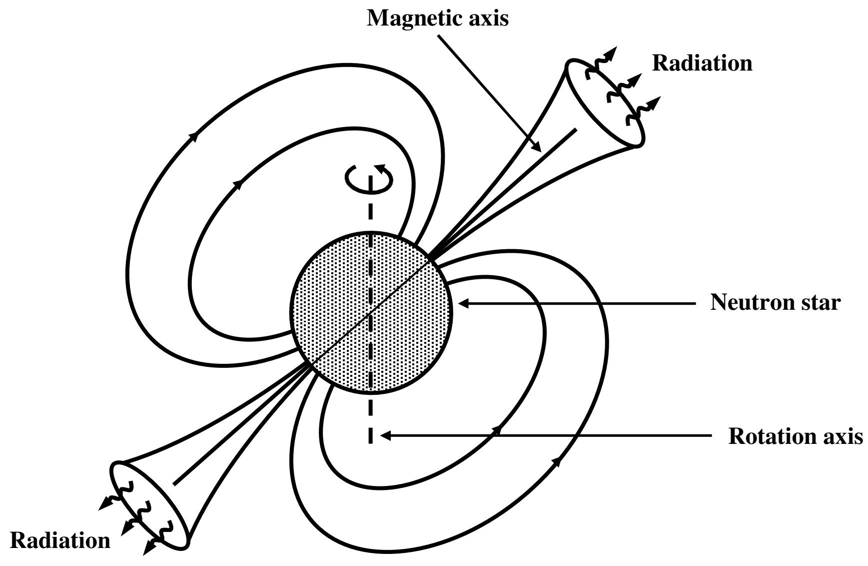


Fig. 11.9: Light-house model of pulsars

You may argue: **If emission takes place from the surface of a neutron star, it should have been detected all the time and not periodically!** Well, to understand the periodic emission, you need to know that it is linked to the rapid rotation and strong magnetic fields of neutron stars. Suppose a molecular cloud of radius, R_{mc} ~ 1 pc having a typical magnetic field, $B_{mc} \sim 10^{-6}$G collapses into a neutron star of size, R_n ~ 10 km. From Unit 8, you know that if the matter is highly conducting, the magnetic flux linked to it is conserved:

$$B\, r^2 \sim \text{constant}, \tag{11.21}$$

Thus, we can write:

$$B_n R_n^2 = B_{mc} R_{mc}^2 \tag{11.22}$$

Substituting the typical values of R_n, B_{mc} and R_{mc}, we get:

$$B_n \sim 9.5 \times 10^{18}\text{G}.$$

Thus, we find that the value of magnetic field associated with a neutron star is very high compared to the Earth's magnetic field which is only a fraction of a Gauss! The above estimate has been arrived at under the assumption that there is no loss of magnetic field and thus the value indicates the upper limit. In reality, after some dissipation in the process of the formation of a neutron star, the field is smaller, close to 10^{11} to 10^{14} Gauss or even less. Generally, this field is bipolar and the magnetic axis is not aligned with the spin axis of the star (Fig. 11.9).

As the neutron star spins at great speeds, its magnetic field induces a very strong electric field. As a result, electrons present in the atmosphere of the neutron star are accelerated and attain very high energies. These electrons then gyrate round the magnetic lines of force and emit radiation called **synchrotron radiation** which is directed along the lines of force. *Every time the neutron star rotates, each of the two radiating magnetic poles may point towards us* (Fig. 11.9) *and we may see two pulses of radiation per rotation of the star*. In many objects, you can see these two peaks very distinctly (Fig. 11.8). Thus, the lighthouse model tells us that pulsars are not mysterious objects at all; *they are rotating neutron stars*.

You may further argue: **If the neutron star is emitting radiation continuously, it would lose energy; in the absence of nuclear energy, what energy is it losing – the gravitational energy, the rotational energy or the magnetic energy?** If it loses gravitational energy, the star would collapse further and the spin period may go down. If it loses magnetic energy, the intensity of radiation will go down with time. However, if it loses rotational energy, the pulsar would slow down and its time period would increase. Observations support the argument that the most dominant component of the energy loss is the loss of rotational energy.

So far, you have learnt that stars having mass up to 1.4 $M_\odot$ stabilise as white dwarfs and those having mass up to 3 $M_\odot$ stabilise as neutron stars. Now, suppose that a star has mass greater than 3 $M_\odot$ and all thermonuclear reactions have ceased in it. The question is: What is the fate of such a star? What force will oppose gravity and prevent the *complete* gravitational collapse of such stars? Obviously, the degeneracy pressures due to electrons and neutrons are insufficient to halt the collapse because the stellar mass is greater than 3 $M_\odot$. In fact, there is no force which can halt the complete gravitational crunch of such a star. This *theoretical* collapse of a star into a singular point of zero volume and infinite density is called a black hole. You will learn about black holes now. But, before that, how about solving an SAQ?

SAQ 4

Spend 5 min.

Suppose the Sun shrinks to the size of a neutron star of radius 10^6 cm. Calculate the magnetic field strength at the surface of the neutron star. Take the radius of the Sun to be 10^{11} cm and the magnetic field at its surface equal to 1 gauss.

11.5 BLACK HOLE

When the mass of a star is more than the limiting mass (~3 M_Θ) for neutron stars and all nuclear reactions in its core have stopped due to the lack of nuclear fuel, there is no force which can stop its continuous gravitational collapse. Due to such an unhindered collapse, the object collapses to zero radius and infinite density! It is very difficult to visualise such an object physically. However, such an object has an interesting property. Its gravity is so strong that not even light can escape from it. Therefore, such a body is called a **black hole**.

The physics of black hole involves very sophisticated mathematics which is beyond the scope of this course. Nevertheless, we can apply simple principles of physics to conclude that black holes do exist. Let us ask ourselves: **For a given mass, what should be the size of a body so that even light (that is, photons - the fastest moving thing) cannot escape from it?** The answer to this question has great bearing on our understanding of black holes. The first theoretical attempt to address this question was made by Schwarzschild who used the principles of General Theory of Relativity given by Einstein. The main conclusions of Schwarzschild can also be derived using Newtonian mechanics and the concept of escape velocity. You may recall from school physics that, for a body of mass M and radius R, the escape velocity is given by:

$$v_{esc} = \sqrt{\frac{2GM}{R}} \qquad (11.23)$$

Since we are interested in a black hole and trapping of light by it, we put $v_{esc} = c$, the velocity of light. Then, Eq. (11.23) reduces to:

$$R_g = \frac{2GM}{c^2} \qquad (11.24)$$

R_g in Eq. (11.24) is called the **Schwarzschild radius**. Eq. (11.24) signifies that the size of an object of mass M must shrink to Schwarzschild radius to become a black hole. For example, if the object has mass equal to $1M_\Theta$, its radius must be 3 km if it has to behave like a black hole. In other words, the Sun must shrink to a radius of 3 km to be able to trap light and become a black hole!

SAQ 5

Spend 3 min.

Calculate the Schwarzschild radius for the Earth.

You may ask: **If black holes are point like objects, what is the physical significance of Schwarzschild radius?** Schwarzschild radius essentially means that any object (including a photon) which comes within R_g (= $2GM_{bh}/c^2$) of a black hole is trapped. This limiting value R_g is also called the **event horizon**, since no event that takes place within R_g from the centre of the black hole can be viewed by any observer at $R > R_g$. Why is it so? This is because the gravity of the point like object is so strong that even radiation (i.e. photons) cannot escape from the region inside the Schwarzschild radius.

Although the concept of Schwarzschild radius is helpful in visualising a black hole, you have to be careful in not stretching the meaning of escape velocity too far. This is because of Einstein's General Theory of Relativity (GTR) which treats gravity entirely differently from Newtonian gravity. *According to GTR*, *we cannot talk of escape velocity at all*. Einstein proposed that near a gravitating object, the photons move in a curved trajectory as seen by an observer at a large distance. The black hole being compact and massive, its gravitational force is very strong which bends the photon path so much that a photon trying to escape would immediately return back.

Bending of light close to a black hole

As you know from the well known Einstein mass-energy equation ($E = mc^2$), every form of energy E has an equivalent mass m. So, the photons with energy $E = h\nu$ will also have a *mass*, $m_{photon} = h\nu/c^2$. **This is not the true mass since photons are really massless**. Nevertheless, this mass can be attracted by any other massive body such as the Sun. This will bend the path of a photon. In fact, the bending of light was observed by the famous British astronomer Arthur Eddington in the year 1919. He observed a star during the total eclipse whose location in the sky was near the edge of the Sun and found that the apparent location of the star has been changed by a small angle. This convinced him that the light from that star must have been bent by the Sun. Thus, strong bending of light can be taken as an evidence that black holes exist.

Types and location of black holes

The exact process of the formation of a black hole is not known but there are several possibilities. Unlike the other types of compact stars, black holes do not have any narrow range of masses. There seem to be two populations of black holes: one population has a mass typically 6 to 14 times the mass of the Sun. It is formed due to gravitational collapse after a supernova explosion. These so-called *stellar mass* black holes could be detected all around the galaxy. Black holes of the second population are very massive, with masses ranging from a few times $10^6 M_\odot$ to a few times $10^9 M_\odot$. These seem to be located at the centres of galaxies.

Detection of a black hole

Since radiation cannot escape from a black hole, it cannot be observed / detected directly. However, it can be detected indirectly through its gravitational field. Any matter close-by is strongly attracted by it. Suppose that the black hole is a member of a binary. Then, it sucks matter from its companion. The matter flowing into the black hole gets heated to a very high temperature and emits *X*-rays. **Thus, to search for black holes, we should look for *X*-ray binaries**.

Now, let us summarise what you have learnt in this unit.

11.6 SUMMARY

- When the nuclear fuel of a star is exhausted completely, it contracts due to self-gravity and becomes a **compact star** having density much higher than a main sequence star.

- In the absence of radiation pressure due to nuclear reactions, the gravitational force in the compact stars is counter-balanced by the **degeneracy pressure** of fermions like electrons and neutrons.

- The nature of the remnants of a star after death **depends on its mass**; the dying star turns into any one of the three kinds of compact stars, namely, white dwarf, neutron star or black hole.

- If the mass of a dying star is less than the **Chandrasekhar limit** (~ $1.4M_{\Theta}$), it turns into a **white dwarf** in which the gravitational force is balanced by the degeneracy pressure of electron gas.

- If the mass of the dying star is more than the Chandrasekhar limit but less than another limiting value (~ $3M_{\Theta}$), it turns into a **neutron star** in which the gravitational force is balanced by the degeneracy pressure of neutrons. If the mass of the collapsing star is even higher, the gravitational collapse cannot be halted and the collapsing star becomes a **black hole**.

- The degeneracy pressure of a degenerate gas of electrons is given by:

$$P = K \rho_e^{\frac{5}{3}}$$

- Chandrasekhar showed that, for white dwarfs, the mass-radius relation is:

$$R \propto \frac{1}{M^{\frac{1}{3}}}$$

and the density-mass relation for such stars is:

$$\rho \propto M^2$$

- **Pulsars** are stellar objects which emit periodic radio frequency pulses. Pulsars are **rotating neutron stars**, because white dwarfs cannot emit pulses of this periodicity.

- Emission of pulses from a rotating neutron star is explained on the basis of **light-house model**.

- Black holes are objects of zero radius and infinite density. Such an object is difficult to visualise physically.

- The **Schwarzschild radius** is given as:

$$R_g = \frac{2GM}{c^2}$$

and it signifies that the size of an object of mass M must shrink to R_g to become a black hole.

- The Schwarzschild radius is also called the **event horizon** because an event that takes place within R_g from the centre of the black hole cannot be observed.

- The physics of black hole cannot be understood on the basis of Newtonian mechanics; we need to invoke Einstein's **General Theory of Relativity** (GTR) for this purpose.

11.7 TERMINAL QUESTIONS

Spend 30 *min.*

1. For a completely degenerate electron gas, the number density of particles with momenta in the range p and $p + dp$ is given by:

$$n(p)dp = \frac{8\pi}{h^3} p^2 dp \qquad p \le p_F$$

$$= 0 \qquad p > p_F$$

where p_F is the Fermi momentum (corresponding to Fermi energy). Show that p_F increases with n as $n^{\frac{1}{3}}$. Also show that the average momentum $< p >$ varies as $n^{\frac{1}{3}}$.

2. The masses and radii of a typical neutron star (NS), a typical white dwarf (WD) and a typical main sequence star (MS) are given below:

	Mass	**Radius**
NS	$1M_{\Theta}$	10 km
WD	$1M_{\Theta}$	10^4 km
MS	$1M_{\Theta}$	10^6 km

Calculate the rotational time periods in all these cases and show that only neutron stars satisfy the pulse time periods observed for pulsars.

3. Assume that the peak wavelength of X-rays coming from an X-ray binary is $1\overset{\circ}{\mathrm{A}}$. Calculate the temperature that the falling matter onto a black hole must attain to emit this radiation.

11.8 SOLUTIONS AND ANSWERS

Self Assessment Questions (SAQs)

1. At $T = 0$, the exponent $(E - \mu)/k_B T$ in the denominator of Eq. (11.3) goes to infinity. Thus, for $E > \mu$, the denominator of $f(E)$ becomes infinite and we have $f(E) = 0$.

   For $E < \mu$, the exponent $(E - \mu)/k_B T$ in the denominator of Eq. (11.3) becomes $-\infty$ at $T = 0$. Thus, the denominator of $f(E)$ becomes 1 and $f(E) = 1$.

2. On the basis of Eq. (11.14), we find that the thermal energy of a star of mass $1M_{\Theta}$ is 10^{48} erg. As per the problem, luminosity of the white dwarf star is:

$$L = 10^{-3} L_{\Theta}$$

$$= 10^{-3} \times (4 \times 10^{26}\ \mathrm{Js^{-1}})$$

$$= 4 \times 10^{30}\ \mathrm{erg.s^{-1}}$$

Thus, the time for which the white dwarf will keep shining can be written as:

$$t = \frac{10^{48}\ \mathrm{erg}}{4 \times 10^{30}\ \mathrm{erg.s^{-1}}}$$

$$= \frac{10^{18}}{4} \quad \frac{1}{3\times10^{7}} \text{ yr} \qquad \left[\because 1\,\text{yr} = 3\times10^{7}\text{ s}\right]$$

$$\cong 10^{10}\text{yr.}$$

3. On the basis of Eq. (11.16), we can write the expression for the change in wavelength on the surface of white dwarf (Sirius B) as

$$d\lambda = \lambda.\frac{g_{wd}H}{c^2}$$

$$= \frac{\lambda H}{c^2} \times \frac{GM_{wd}}{R^2}$$

$$= \frac{(5800\,\mathring{A}) \times (100\,\text{cm})}{(9\times10^{20}\text{ cm}^2\text{s}^{-2})} \times \frac{(6.67\times10^{-8}\text{ cm}^3\text{g}^{-1}\text{s}^{-2})\times(2\times10^{33}\text{ g})}{(1.6\times10^{9}\text{ cm})^2}$$

$$= \frac{5.8\times6.67\times2}{9\times1.6\times1.6}\times10^{-8}\ \mathring{A}$$

$$\cong 3.3\times10^{-8}\ \mathring{A} \quad \text{(Negligible)}$$

4. From Eq. (11.21), we can write the magnetic field linked to the Sun and to the neutron star (to which the Sun converts) as:

$$B_\Theta R^2{}_\Theta = B_{NS}\, R^2{}_{NS} = \text{Constant}$$

where B_{NS} and R_{NS} , respectively, are the magnetic field and radius of the neutron star. Thus,

$$B_{NS} = B_\Theta \frac{R^2{}_\Theta}{R^2{}_{NS}}$$

$$= 1 \text{ gauss} \times \frac{(10^{11}\text{ cm})^2}{(10^{6}\text{ cm})^2}$$

$$= 10^{10}\text{ gauss}$$

5. From Eq. (11.24), we can write the expression for Schwarzschild radius for the Earth as:

$$\left(R_g\right)_E = \frac{2GM_E}{c^2}$$

$$= \frac{2 \times (6.67 \times 10^{-8}\text{ cm}^3\text{g}^{-1}\text{s}^{-2}) \times (6 \times 10^{27}\text{ g})}{9\times10^{20}\text{ cm}^2\text{s}^{-2}}$$

$$= 0.9\text{ cm}$$

Terminal Questions

1. As per the problem,

$$n(p)\,dp = \frac{8\pi}{h^3}\,p^2\,dp$$

$$n = \frac{8\pi}{h^3}\int_0^{p_F} p^2\,dp$$

$$= \frac{8\pi}{h^3}\,\frac{p^3{}_F}{3}$$

Thus, we can write:

$$p_F \propto n^{\frac{1}{3}}$$

Now, the expression for average momentum can be written as:

$$< p > = \frac{1}{n}\int_0^{p_F} p\,n(p)\,dp$$

$$= \frac{8\pi}{h^3}\times\frac{1}{n}\int_0^{p_F} p^3\,dp$$

$$= \frac{8\pi}{h^3}\times\frac{1}{n}\times\frac{p_F{}^4}{4}$$

$$= \frac{8\pi}{h^3}\times\frac{1}{n}\times\frac{n^{4/3}}{4}$$

Thus, we can write:

$$< p > \propto n^{\frac{1}{3}}$$

2. From Eq. (11.18), we can write the expression for the angular velocity of a rotating star as:

$$\Omega = \sqrt{\frac{GM}{R^3}}$$

So, the time period is:

$$T = \frac{2\pi}{\Omega}$$

$$= 2\pi\sqrt{\frac{R^3}{GM}}$$

Thus, the time period for the neutron star is:

$$T_{NS} = 2\pi \sqrt{\frac{10^{18}\ \text{cm}^3}{(6.67\times10^{-8}\ \text{cm}^3\text{g}^{-1}\text{s}^{-2})\times(2\times10^{33}\ \text{g})}}$$

$$= 2\pi \times 10^{-4} \sqrt{\frac{10}{6.67\times2}}\ \text{s}$$

$$= 5.4\times10^{-4}\ \text{s}$$

Time period for white dwarf is:

$$T_{WD} = 2\pi \sqrt{\frac{10^{27}\ \text{cm}^3}{(6.67\times10^{-8}\ \text{cm}^3\text{g}^{-1}\text{s}^{-2}) \times (2\times10^{33}\ \text{g})}}$$

$$\approx\ 17\ \text{s}$$

And the time period for a main sequence star is:

$$T_{MS} = 2\pi \sqrt{\frac{10^{33}\ \text{cm}^3}{(6.67\times10^{-8}\ \text{cm}^3\text{g}^{-1}\text{s}^{2})\times (2\times10^{33}\ \text{g})}}$$

$$\approx\ 17\times10^{3}\ \text{s}$$

Thus, we find that the time period of the pulses emitted by a neutron star is of the same order as of the pulses from pulsars.

3. We know that,

$$\lambda_m T = 0.3\,\text{cm K}$$

As per the problem:

$$\lambda_m = 1\,\mathring{\text{A}} = 10^{-8}\,\text{cm}$$

Thus,

$$T = \frac{0.3}{10^{-8}}\,\text{K}$$

$$= 3\times10^{7}\ \text{K}$$