EXPERIMENT 2
SOME INVESTIGATIONS ON INTERFERENCE OF LIGHT

Structure

2.1 INTRODUCTION

As a child you may have enjoyed blowing soap bubbles and seeing bright rainbow colours reflected from them. On a rainy day you must have also observed brilliant, though irregular, colour patterns on the wet road surface due to a thin layer of oil spilt by a motor vehicle. You may have also realised that colour patterns change if you look at them from different angles. Have you ever looked at a fairly transparent piece of silk or polyester cloth from a distance? If you do so, you would observe patterns of bright and dark bands. All these patterns arise due to interference of light waves. The bright and dark bands so produced are known as **interference fringes**.

Two sources are said to be **coherent** if light waves emitted by them are of the same frequency and have a constant phase difference.

The simplest demonstration of interference of light waves was devised by Thomas Young. You must be familiar with it from your school physics curriculum. You would recall that in this setup, monochromatic light from a point source is made to give rise to two coherent sources by placing two closely spaced narrow slits across its path. The superposition of waves from these two coherent sources produces a clear interference pattern on a screen placed some distance away.

Instead of a double-slit arrangement, Fresnel used a biprism to produce coherent sources. In your experimental investigations here, you will learn to produce interference fringes using a biprism. You will then measure the distance between two dark (or bright) bands, known as **fringe width.** This will enable you to determine the wavelength of the incident monochromatic light.

Objectives

After performing this experiment you will be able to

- obtain interference fringes using a biprism
- determine the factors on which fringe width depends, and
- determine the wavelength of light from a monochromatic source.

2.2 WHAT IS INTERFERENCE?

You have learnt in your school physics course that two identical progressive waves travelling along the wire of a musical instrument in exactly opposite directions give rise to stationary waves. These waves are characterised by a succession of nodes and anti-nodes. The nodes are positions of minimum intensity whereas anti-nodes are positions of maximum intensity. In other words, there is redistribution of energy.

Under certain conditions light also exhibits a similar behaviour. When two or more light waves of same frequency and having well defined and constant phase relations between them are superimposed on each other, the intensity of the resultant wave gets modified. This is the phenomenon of **interference**. The interference pattern comprises of a series of regularly spaced maxima and minima. If the resultant intensity is zero, or in general, less than what we expect from individual waves, we have **destructive interference** (seen as dark bands). On the other hand, if resultant intensity is greater than intensities due to individual waves, we have **constructive interference** (seen as bright bands in the pattern). Let us briefly learn about this phenomenon now.

With the exception of laser sources, interference can be observed only when the waves come from the same source. A narrow slit S is illuminated by a monochromatic source of light which in turn illuminates two other narrow equidistant slits S_1 and S_2 separated through a distance d from each other. The interference pattern is obtained on a screen placed at a distance D from the double-slit and parallel to the plane containing these slits, as shown in Fig. 2.1.

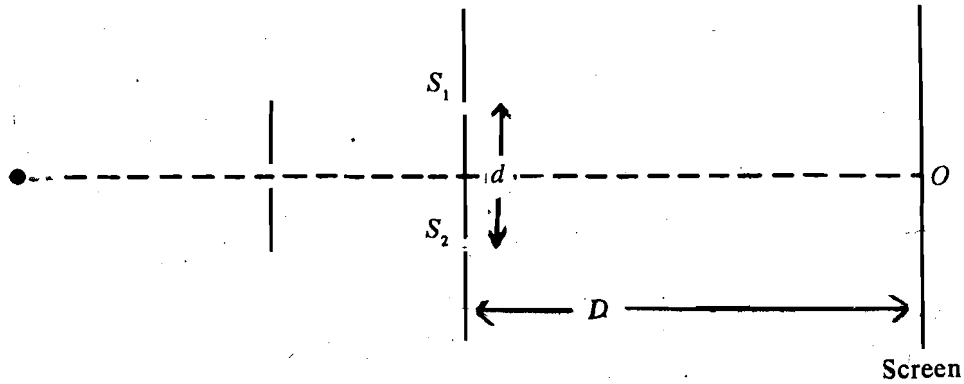

Fig.2.1: Cross-sectional view of the double slit arrangement to observe interference of light waves

Let us consider a point P on the screen which is the nearest maxima or minima from the origin. Suppose that the magnitude of electric fields of the two interfering waves at P are given by

$$E_1 = E_0 \sin \omega t \qquad (2.1a)$$

$$E_2 = E_0 \sin (\omega t + \phi) \qquad (2.1b)$$

where ϕ is the phase difference between the two waves and is constant. The phase difference arises because of the path difference. Since a path difference of one wavelength corresponds to a phase difference of 2π radians, we can write

$$\phi = \frac{2\pi}{\lambda} \times \text{(path difference)} \qquad (2.2)$$

The path difference between S_1P and S_2P is given by

$$S_2P - S_1P = d \sin \theta \qquad (2.3)$$

Thus we find that

$$\phi = \frac{2\pi}{\lambda} d \sin \theta \qquad (2.4)$$

The resultant magnitude of electric field at P is given by

$$E = E_1 + E_2 = 2E_0 \cos \left(\frac{\phi}{2}\right) \sin \omega t \qquad (2.5)$$

And the intensity is

$$I = 4E_0^2 \cos^2 \left(\frac{\phi}{2}\right). \qquad (2.6)$$

The intensity is maximum ($= 4E_0^2$ or four times the intensity of either wave) if

$$\frac{\phi}{2} = n\pi \qquad n = 0, 1, 2, \ldots \qquad (2.7a)$$

and minimum (in fact, zero) if

$$\frac{\phi}{2} = \left(n + \frac{1}{2}\right)\pi \qquad n = 0, 1, 2, \ldots \qquad (2.7b)$$

Thus, using Eq. (2.4), we can write **the condition for constructive interference** as

$$\sin\theta_n = \frac{n\lambda}{d} \qquad n = 0, 1, 2, 3, \ldots \qquad (2.8a)$$

and **destructive interference** as

$$\sin\theta_n = \left(n + \frac{1}{2}\right)\frac{\lambda}{d} \qquad n = 0, 1, 2, 3, \ldots \qquad (2.8b)$$

From Fig. 2.1, by letting $OP = y$, we can write

$$y = D\tan\theta \qquad (2.9a)$$

Thus the positions of alternate maxima and minima are given by

$$y_n = D\tan\theta_n \qquad (2.9b)$$

where θ_n is given by Eqs. (2.8a) and (2.8b) for constructive and destructive interference, respectively.

Finally, if the slit separation is much greater than the wavelength of light used ($d >> \lambda$), then for non-zero n, the value of $\frac{n\lambda}{d}$ will be very small. Therefore, it readily follows from Eq. (2.8a) that θ_n will be very small. Then in the small angle approximation, we can take

$$\sin\theta_n \approx \tan\theta_n \approx \theta_n$$

Hence

$$y_n = D\theta_n \qquad (2.10)$$

and

$$\theta_n = \frac{n\lambda}{d} = \frac{y_n}{D} \qquad (2.11)$$

That is, the nth bright fringe is located at

$$y_n = \frac{n\lambda D}{d} \qquad n = 0, 1, 2, 3, \ldots \qquad (2.12a)$$

and the n^{th} dark fringe is located at

$$y_n = \frac{\left(n + \frac{1}{2}\right)\lambda D}{d} \qquad n = 0, 1, 2, 3, \ldots \qquad (2.12b)$$

From these relations it readily follows that once we can measure the **distance between two consecutive bright or dark fringes,** i.e. **fringe width,** we can determine the wavelength of light emitted by the source.

So far we have confined to a double slit arrangement which enables us to obtain two coherent sources from a given source. You can make such an arrangement at home, by cutting very fine (razor thin) slits in black art paper. An ordinary lamp may be used as source of light. Make your observations and summarise them below.

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When you go to attend your physics laboratory–III course, you may like to discuss your findings with your counsellor after gaining experience of working with sophisticated equipment. The waves from coherent (virtual) sources are capable of interfering. Another arrangement of historic importance to obtain two coherent sources is due to Fresnel, who used a biprism. We will now discuss it briefly.

Fresnel's biprism

Refer to Fig. 2.1 again. If slits S_1 and S_2 are made very narrow, the amount of light available for forming the fringes will be very small and the fringes will be of feeble intensity. Moreover, you can argue that these slits diffract light and the observed pattern is composite. To avoid such a confusion it is better to replace the double slit by a biprism.

Refer to Fig. 2.2. S is a narrow vertical slit illuminated by monochromatic light. The light from S is made to fall symmetrically on the biprism having its refracting edges parallel to the slit. A biprism can be considered to be made up of two identical prisms of very narrow refracting angles placed base to base. The light incident on each half of the prism is deviated by the corresponding refracting edge. This gives rise to a virtual image of the slit on either side of it. These two virtual images (S_1 and

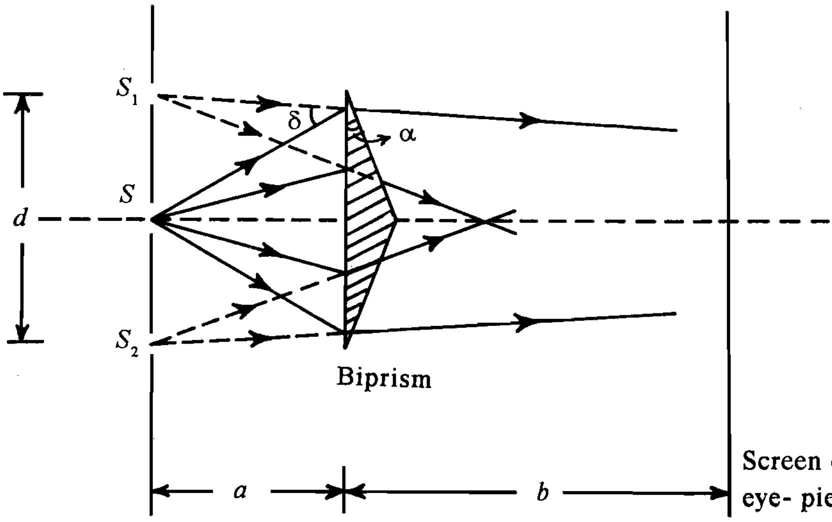

Fig. 2.2: Formation of virtual images S_1 and S_2 by the biprism

S_2) act as two coherent sources. S_1 and S_2 are fairly close to the source (S_1, S, and S_2 are in the same plane) as the angles of deviation are small. You can verify from Fig.2.2 that $SS_1 = SS_2 = a\delta$, where a is the distance between the source S and the biprism and δ is the angle of deviation. We know that for a prism

$$\delta = (n - 1)\,\alpha \qquad (2.13)$$

where n is the refractive index of the material and α is the refracting angle. Thus

$$S_1 S_2 = S_1S + SS_2 = d = 2a\,(n\text{-}1)\,\alpha \quad (2.14)$$

If the eye-piece is held at a distance b from the biprism anywhere in the region of overlap of the two refracted beams, the distance of the pair of sources from the plane of interference is

$$D = (a + b) \quad (2.15)$$

You may recall that the fringe width β is given by $\beta = \frac{D\lambda}{d}$. On substituting for d and D from Eqs. (2.14) and (2.15), respectively, we find that

$$\beta = \frac{(a + b)\,\lambda}{2a\,(n - 1)\,\alpha} \quad (2.16)$$

This result shows that we can calculate the wavelength of light once we have measured a, b, α and β, for a biprism of given refractive index. Alternatively, by using a light source of known wavelength, these measurements allow us to determine n. Since biprism is very thin, the angle α is very small ($= 6\times10^{-3}$rad) and it is not convenient to measure it. We, therefore, resort to an alternative scheme wherein d is connected to the separation between the virtual sources rather than α. And to measure these distances, we introduce a convex lens of short focal length ($f \sim 15$ cm to 20cm) between the biprism and the eye-piece. The eye-piece is kept at a large distance from the slit ($5f > D > 4f$). This condition on D minimises the error in the measurement of d.

The condition $D > 4f$ is a theoretical consideration but the condition $5f > D$ arises from the practical consideration of minimising the error in the measurement of d. The error consideration for the condition is as follows:

$$d = \sqrt{d_1\, d_2}$$

Taking logarithm of both sides, we get

$$\ln\, d = \frac{1}{2}\ln d_1 + \frac{1}{2}\ln d_2$$

On differentiation, we find that

$$\frac{\Delta d}{d} = \frac{\Delta d_1}{2d_1} + \frac{\Delta d_2}{2d_2}$$

so that if we denote $\delta d/d$ by e, then we can write

$$e = \frac{1}{2}(e_1 + e_2)$$

and

$$e_1 e_2 = \frac{\Delta d_1}{d_1} \cdot \frac{\Delta d_2}{d_2} = \frac{\Delta d_1 \Delta d_2}{d^2} = \text{constant}$$

Since $(e_1+e_2)^2 = (e_1-e_2)^2 + 4e_1e_2$, e_1+e_2 is minimum when $e_1 = e_2$ and $e_1 e_2$ is constant. This implies that

$$\frac{\Delta d_1}{d_1} = \frac{\Delta d_2}{d_2}$$

But $\Delta d_1 = \Delta d_2$

$\therefore$ $d_1 = d_2$

Thus d_1 should be as close to d_2 as possible.

The convex lens converges the two refracted beams. We can adjust its position to obtain clear well-defined images in the plane of the cross wires in the eye piece. In fact, while performing the experiment you will find that once positions of slit, biprism and the eye-piece are fixed, there are two positions of the lens, shown as L_1 and L_2 in Fig. 2.3, for which clear images of S_1 and S_2 are obtained in the eye-piece. In one of the positions (at L_1), we obtain a magnified image while in the other position (L_2), we obtain diminished images of the sources.

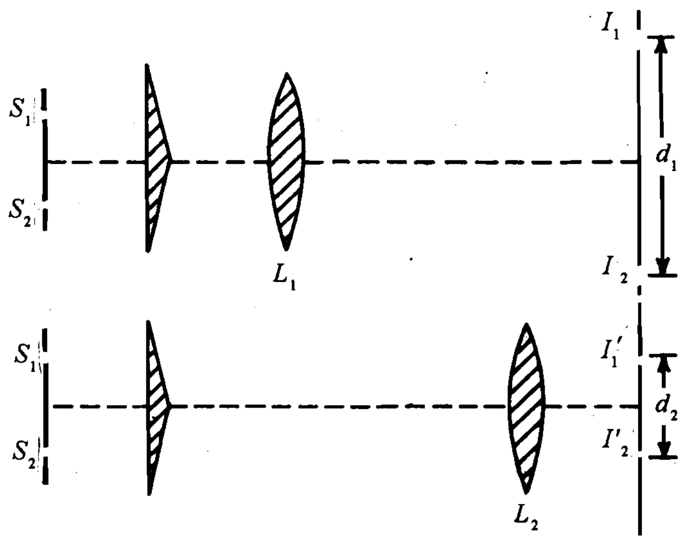

Fig.2.3: Two positions of the lens between the biprism and the eye-piece giving enlarged images I_1, I_2 and diminished images I_1', I_2' of S_1 and S_2

Suppose that the separation between the two magnified images is d_1. If d is the actual distance between the virtual sources S_1 and S_2, the magnification by the lens is given by

$$m_1 = \frac{d_1}{d} \quad (2.17a)$$

If d_2 is the distance between diminished images, the magnification is given by

$$m_2 = \frac{d_2}{d} \tag{2.17b}$$

For two conjugate positions of the image, we find that $m_1 = 1/m_2$. You must have got an opportunity to prove this result in Block-2 of our PHE-09 course on Optics. However, we are again giving it here for completeness. This equality enables us to correlate fringe width with measurable parameters.

You will agree that we can write

$$u + v = D$$

so that

$$u = D - v$$

From the lens formula we know that

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

On substituting for u, we find that

$$\frac{1}{v} + \frac{1}{D-v} = \frac{1}{f}$$

or

$$\frac{D - v + v}{v\,(D - v)} = \frac{1}{f}$$

or

$$\frac{D}{v\,(D - v)} = \frac{1}{f}$$

This can be rewritten as

$$v^2 - Dv + fD = 0$$

For real roots, we must have

$$D^2 - 4fD > 0$$

or $$D > 4f$$

and sum of the roots

$$v_1 + v_2 = D$$

But

$$u_1 + v_1 = u_2 + v_2 = D$$

Therefore, on substituting the value of $v_1 = D - v_2$ in this expression, we can write

$$u_1 + D - v_2 = D$$

or $$u_1 = v_2$$

We can similarly show that $u_2 = v_1$. Since

$$m_1 = \frac{v_1}{u_1}$$

we find that

$$m_1 = \frac{v_1}{v_2}$$

The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is equal to $-b/a$. Here $b = D$ and $a = 1$. Therefore, sum of roots $v_1 + v_2 = D$

Similarly, it readily follows that

$$m_2 = \frac{v_2}{u_2} = \frac{v_2}{v_1}$$

Hence, $m_1 \times m_2 = 1$

or $m_1 = \frac{1}{m_2}$

In terms of d, d_1 and d_2, this equality can be rewritten as

$$d = \sqrt{d_1 d_2}$$

so that the expression for fringe width takes the form

$$\beta = \frac{D\lambda}{d} = \frac{(a+b)\lambda}{\sqrt{d_1 d_2}}$$

From this result it is clear that once a, b, d_1, d_2 and β can be measured, λ can be easily calculated. We hope that now you appreciate the need of introducing a convex lens in this arrangement.

Before we outline the procedure for determination of wavelength of light, we list the necessary apparatus.

Apparatus

A biprism, optical bench with uprights, sodium vapour lamp and a convex lens of short focal length.

2.3 DETERMINATION OF WAVELENGTH OF LIGHT

The experimental procedure involves obtaining coherent sources using a biprism. For this you will have to adjust the apparatus to obtain interference fringes, measure the fringe width and the distance between the coherent sources. We now give the steps you have to follow to adjust the apparatus to obtain interference fringes.

2.3.1 Adjusting the Apparatus

1. Look at Fig. 2.4. It shows a sodium lamp, an optical bench with three uprights and an eyepiece.

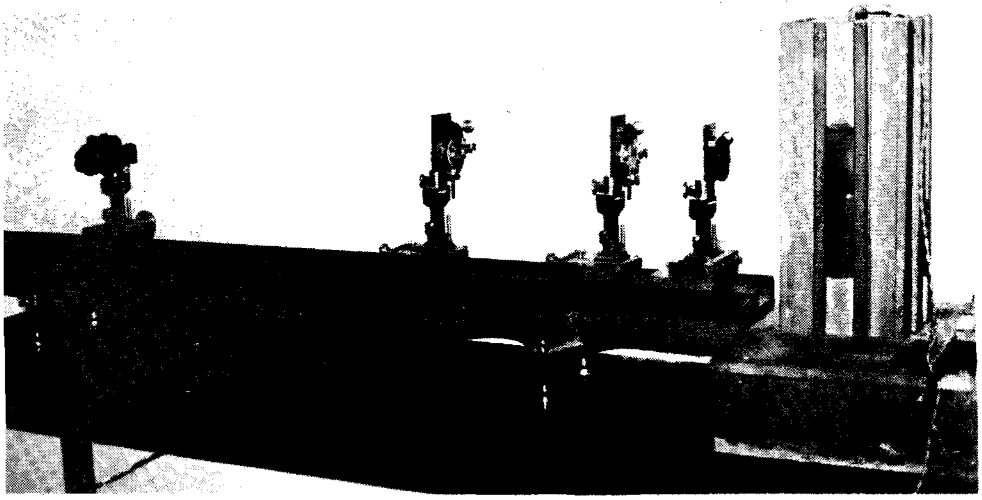

Fig. 2.4: The experimental setup for observing interference pattern due to Fresnel's biprism.

2. Arrange the sodium lamp, normally housed in a rectangular box and having a small rectangular opening on one side to allow light to pass through, on one end of the optical bench.

3. Arrange the uprights on the optical bench so that they are at the same height.

4. Mount a slit of adjustable width on the first upright, the biprism on the second upright and the micrometer eye piece on the third upright. You must note that the slit is provided with a screw to rotate it in its own plane. Using this screw, ensure that the slit is vertical. Keep the width of the slit as small as possible. Just as the slit can be rotated in its own plane, the frame on which the biprism is mounted can also be rotated in its own plane. Also make sure that the edge of the biprism is parallel to the slit.

5. Now view the slit (illuminated by sodium light) through the biprism. Move your eye sideways. What do you observe? Does one of the bright vertical lines appear and disappear suddenly? If it is so then you can be sure that the edge of the biprism is exactly parallel to the slit. If the bright line appears or disappears gradually from top to bottom, then the edge of the biprism is not parallel to the slit. Rotate the biprism in its own plane till it is exactly parallel to the slit. In doing so, remember to keep the slit and the biprism as close as possible.

6. Next put the micrometer eyepiece at about 15 to 20 cm from the biprism. Keep your eye just above the eyepiece and make sure that you see two images of the slit. If you do not, move the biprism or the eye-piece laterally. However, you should not disturb the vertical alignment. Next look through the eyepiece. You should see a number of vertical bright and dark fringes. These can be seen only if the slit and the edge of the biprism are exactly parallel to each other. *If you do not see sharp fringes in the field of view, narrow down the slit S* and slightly rotate the biprism in its plane. These two adjustments should enable you to obtain sharp fringes in the field of view.

7. The next step is to align the biprism and the eyepiece. For this you should move the eyepiece away from the biprism along the optical bench. Check whether the fringes first become broader without shifting to one side as a whole. *If you observe such a lateral shift of the pattern,* use the screw on the side of the upright and give a transverse motion to the biprism. This will shift the whole fringe pattern observed in the field of view laterally.

8. Now move the eyepiece forward and check whether the fringes just become narrower without a lateral shift. The above adjustments should be done alternately and repeatedly, till a longitudinal movement of the eyepiece on the optical bench gives rise to a side-ways shift of the whole fringe pattern.

Now your apparatus is set for measurements of fringe width. Let us learn to do so now.

2.3.2 Measurement of Fringe Width

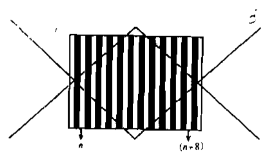

Fig. 2.5: Enlarged view of the fringe pattern. Note how the intersection of the cross-wires is placed on any (here n^{th} fringe) bright fringe.

1. Note the pitch and calculate the least count of the micrometer. Record it in Observation Table 2.1. Consider the *left extreme line on the pitch scale as zero.* As the head is rotated, the head scale readings as well as pitch scale readings should increase.

2. You can now measure the fringe width. Keeping the eyepiece at a distance of about 20 cm from the biprism, move the micrometer screw till the intersection of the cross wires falls on one of the bright lines (Fig. 2.5). Note the pitch scale as well as head scale readings and record them in your observation table.

Observation Table 2.1: Measurement of fringe width

Least count of micrometer = cm

Position of source slit on the optical bench = cm

Position of the biprism on the optical bench = cm

Position of eye piece	Reading of fringe		Shift	Mean	Mean
	nth	$(n+10)$th	10β	10β	β

3. Rotate the micrometer head to shift the cross-wires by 10 fringes and record the reading. The difference gives us the width of 10 fringes.

You must note that there is nothing sacrosanct regarding this number 10. It can be 8 or 12 as well. The larger the difference, the greater is the accuracy of measurement and the smaller becomes the error.

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In determining the fringe width β, the cross-wire is first placed on the nth fringe and then on $(n+10)$th fringe. In your set up, can you take readings on consecutive fringes? If so, is it desirable? Record your experiences and discuss with your counsellor.

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4. Repeat the above step at least five times by starting at different fringes along the pattern. Calculate the mean fringe width. Let us denote it by β_1.

5. Keeping the slit S and the biprism in the same position, move the eye-piece away by about 40 cm from the biprism. Following steps 2–4 above, compute the average value of fringe width. Let us denote it by β_2.

To determine (β/D), you were asked to measure the fringe widths β_1 and β_2 by putting the eyepiece at two different distances D_1 and D_2, respectively from the biprism. A better method of determining (β/D) would be to vary D in steps of 5 cm and finding corresponding values of β. Then the slope of the β vs D graph gives the desired result. Can you now appreciate why graphical method is better?

6. Find the approximate focal length f of the convex lens by focussing a distant object on a screen.

7. Put the micrometer eyepiece at a large distance from the source slit S (say more than $4f$). Next insert the convex lens between the biprism and the eyepiece. Adjust the centre of the lens to be in line with the slit and eyepiece. Move the lens along the bench till sharp enlarged images of the two virtual sources are seen in the plane of the cross wires. Measure the distance d_1 $(=S_2-S_1)$ between the images. Move the lens, towards the eye piece, till sharp diminished images of the two virtual sources are seen in the plane of the cross wires. Measure the distance d_2 $(=S_2'-S_1')$ between the images. Record your readings in Observation Table 2.2.

Calculate the source separation d using the relation

$$d = \sqrt{d_1 d_2}$$

Calculate the wavelength λ of sodium light using the formula

$$\lambda = d\left(\frac{\beta}{D}\right)$$

Observation Table 2.2: Measurement of separation between coherent sources

S.No.	Position of eyepiece (cm)	Magnified images (cm)			Diminished images (cm)			$d = \sqrt{d_1 d_2}$ (cm)
		S_1	S_2	$d_1=S_2-S_1$	S_1'	S_2'	$d_2=S_2'-S_1'$	
1.								
2.								
3.								
4.								
5.								

Result : The wavelength of light emitted by the sodium lamp is =.......... nm

A sodium vapour lamp gives out two wavelengths, which are very close to one another. (The wavelengths are 589.0 nm and 589.6 nm.) Therefore, strictly speaking the sodium lamp does not emit coherent waves. How does the wavelength measured by you compare with the actual value? You can change the distance between the source slits and the biprism. How does the fringe pattern change ? When the distance is very large, you should observe that fringes get crowded. Comment on the relationship between β and D.

If time permits and you can get to know the value of α from your Counsellor, calculate the refractive index n and comment on the material of the biprism.

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