BLOCK 1 NETWORK ANALYSIS AND DEVICES

In your School syllabus you have studied the various laws to deal with simple electric circuits like Ohm's law and Kirchoff's laws. Sametimes the circuits are so complicated that these can't be simplified using these laws or the calculations are very lengthy which can be done at the cost of a lot of time. The Radio, Television, Computer etc. contain very complicated circuits. To determine the current in any element is very complicated using above laws. In Unit 1 we will discuss some special kind of networks and theorems which are very useful in reducing the complicated circuits to simple form and then direct calculate the current in any circuit element.

In Unit 2, you will study RLC resonant circuits, fitters, and attenuators.

In Unit 3 you will study various electronic devices. Some of these are already familliar to you in the lower classes. These devices are the basic pillars of the foundation of electronics. Thus we will first recapitulate them before proceeding to the higher devices like transistors, FET, MOSFET etc. and their applications.

UNIT 1 CIRCUIT ANALYSIS

Structure

1.1 INTRODUCTION

Developments in the field of electronics can be considered to be one of the great success stories of this century. Beginning with crude spark-gap transmitters and 'cat's-whisker" detectors at the turn of the century we have passed through a vacuum tube era of considerable sophistication to a solid-stale era in which flood of stunning advances shows no signs of abating.

In order to understand the marvels of electronics, we must start with the study of the laws, rules of the thumb and tricks that constitute the att of electronics as we see it. It is necessary to begin with talk of voltage, currrent, power and component that make up electronic circuits. To visualise, in a simple manner, an electric circuit consists of three parts: (i) Energy Source such as battery or generator (ii) the load or sink such as lamp or motor and (iii) connecting wires, The purpose of this circuit Is to transfer energy from source to the load. Using this example, we can define an electric network as basically interconnection of two or more simple circuit elements viz voltage source, resistors, inductors or capacitors. If the network contains at least ooe closed path, it is called an electric circuit. In this unit, we shall be studying the basic laws which govern the transfer of energy from source to load, some important theorems md simplifications at which we arrive after going through these laws, in order to understand functioning of a given electric circuit, which forms basis of any modern electronic gadget,

Objectives

After studying this unit, you should be able to

- explain the concepts of voltage and current sources,
- apply Kirchoff's law, Thevenin and Norton theorem to simplify given network,
- state and apply Superposition, reciprocity and Maximum power transfer theorem to a given network.

1.2 CIRCUIT ELEMENTS

A network is a connection of elements to obtain a certain performance. In circuit analysis, we are interested in electrical networks in which passive elements such as resistors, inductors and capacitors are appropriately connected to voltage and current sources. These elements and sources are idealizations to actual elements and sources. The idealization enables an effective analysis of the network. The problem in network analysis is to analyse the given network and to determine the voltages and currents through various elements. Broadly, network element. may be classified into four groups:

(i) Active or passive

(ii) Unilateral or bilateral

(iii) Linear or nonlinear

(iv) Lumped or distributed

Active or passive: Energy sources (voltage or current sources) are active elements, capable of delivering power to some external device. Besides energy sources there are many other components used in electronic circuits which fall under the category of active elements. These components can be classified into two: tubes (both vacuum tubes and gas tubes) and semiconductor devices which include junction diode, transistor, field effect transistor, UJT, SCR, zener diode etc.

The passive elements are those which are capable only of receiving power like resistors, inductors, or capacitors. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element.

Bilateral or unilateral: We know that electrical characteristics of passive elements are best described by current-voltage relationship. In the bilateral elements, the voltage - current relation is the same for current flowing in either direction e.g. resistors. In contrast, a unilateral element has diffferent relation between voltage and current for the two possible direction of current e.g., diodes.

Linear or non-linear: An element is said to be linear, if it satisfies the linear voltage — current relationship implying that if the current through the element is scaled up by a factor, the voltage across the element also gets scaled up by the same factor. For example, the V - I relation for resistor is V = IR and is linear. All elements which does not satisfy this relation is called nonlinear elements such as Diode. The V - I relation for diode is given by

$$I = I_o\,[e^{qv/kt} - 1]$$

which is clearly non-linear.

Lumped or distribubuted : The elements which are separated physically are known as **Lumped elements like** resistors, **capacitors or inductors. Distributed** elements on the other hand are those which are not separable for analytical purposes for example a transmission line which has distributed resistance, inductance and capacitance along its length.

V–I relations:

(a) Resistor : See Fig 1.1.

(i) $V = IR \rightarrow I = GV$

where $G = I/R =$ conductance and V & I are time **independant** voltage and current.

(ii) $V(t) = i(t)R$

where V (t) and i (t) are **time** dependent voltage and current.

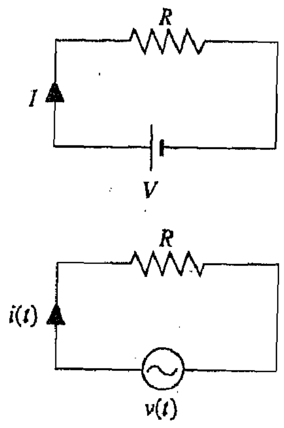


Fig. 1.1: V–I relationship lor resistor.

(b) Inductor : See Pig 1.2.

For inductor, the flux is given by

$$\phi \propto i$$

$$\rightarrow \phi = Li$$

where L is coefficient of self inductance. Differentiating above expression with time, we **get**

$$\frac{d\phi}{dt} = L\frac{di}{dt}$$

As per **Lenz's** law, rate of change of flux is potential,

$$\therefore \quad V = L\frac{di}{dt}$$

(i) For **time** independent voltage and **current,**

$$\frac{di}{dt} = 0$$

$$\therefore \; V(t) = 0$$

which implies that for DC **voltages and currents, under steady state (after** long **lapse** of time) conditions, the **voltage** across inductor will be zero implying, it will behavo as a short circuit.

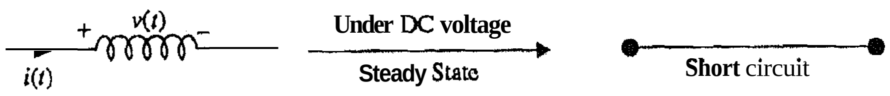


Fig. 1.21 V-I relationship for Inductor.

(ii) For time dependent voltage and current, the relationship is given by

$$V(t) = L\frac{di(t)}{dt}$$

(c) **Capacitors : See** Fig 1.3. For a capacitor Q $\propto$ V .

$$\rightarrow Q = CV$$

where C is called capacity of **capacitor.** Differentiating with respect to time, **we** get

$$i = \frac{dQ}{dt} = C\frac{dV}{dt}$$

(i) For time **independent current &** voltage **(i.e.,** for **dc)**

$$\frac{dV}{dt} = 0$$

This suggests that **for** time **independent** current and voltage, the **capacitor** under **steady state** (after long lapse of **time)** will behave **as** open circuit because i = 0.

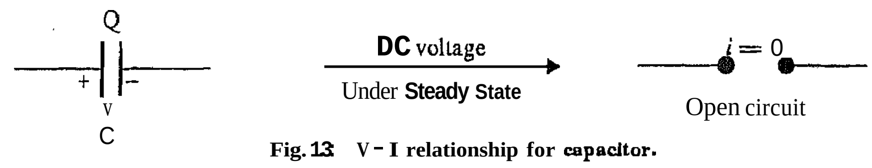


Fig. 1.3 V − I relationship for capacitor.

(ii) For time dependent current and voltages the current - voltage relationship is given by

$$i(t) = C\frac{dV(t)}{dt}$$

(d) Energy **sources:**

There are two categories of energy sources.

(i) Independent Energy Source

(ii) Dependent Energy Source

(i) Independent Energy Source:

As you know independent energy source can be of two types: Voltage sources and current sources. Independent voltage or current sources are one for which voltage. and current are fixed and are not affected by other parts of the circuit.

An ideal voltage source is a two terminal element in which the voltage V_a is completely independent of the current i_a through its terminals. The representation of ideal voltage source is shown in Fig. 1.4.

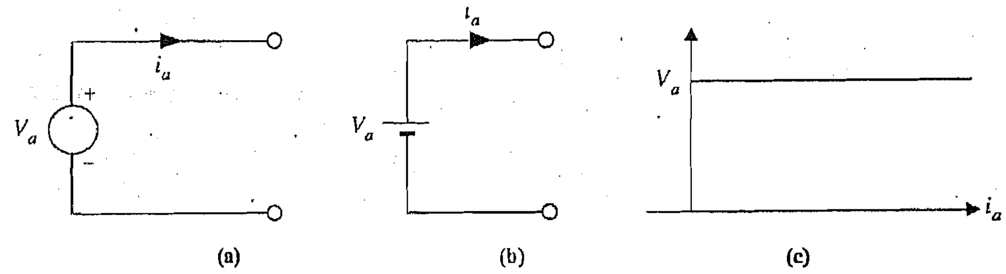


Fig. 1.4: Ideal voltage source.

If we observe V − I characteristics for an ideal voltage source as shown in Fig. 1.4(c), at any time, the value of the terminal voltage V_a is constant with respect to the current value i_a. But for a practical voltage source, its internal resistance is in series with the source as shown in Fig. 1.5 and its terminal voltage falls as the current through it increases. The terminal voltage depends on the source current because, $V_{terminal} = V_s - i_s R$.

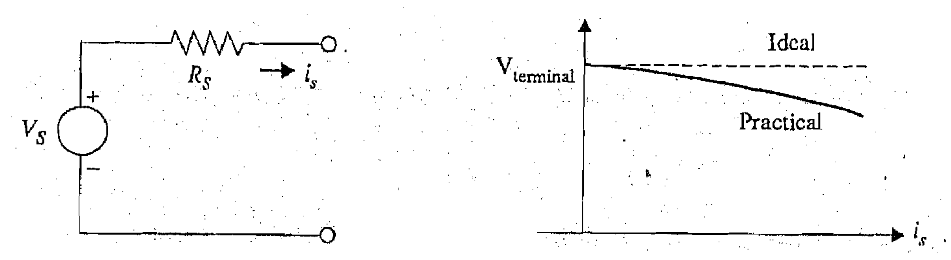


Fig. 1.5: Practical voltage source.

An ideal independent current source is a two terminal element in which the current is completely independent of the voltage V_s across its terminals. The representative of ideal current source is shown in Pig. 1.6,

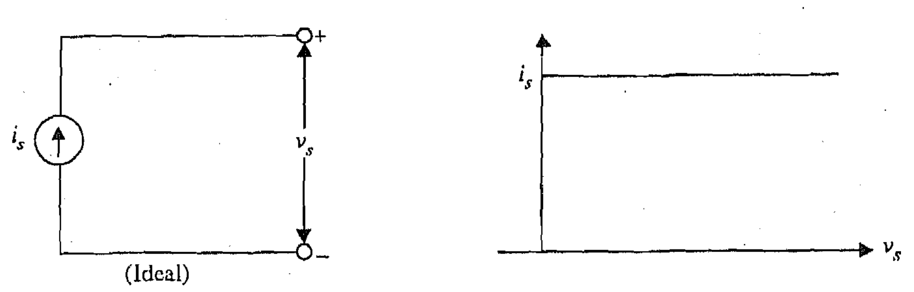


Fig. 1.6: Ideal current source.

In practical current sources, the internal resistance is shown in parallel with the source as shown in Fig.1.7. In this case, the magnitude of current falls, as the voltage across its terminals increases. The terminal current is given by

$$I_{terminal} = i_S - \left(\frac{V_S}{R}\right)$$

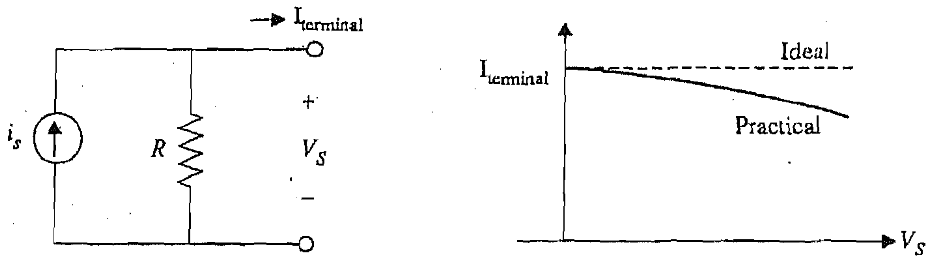
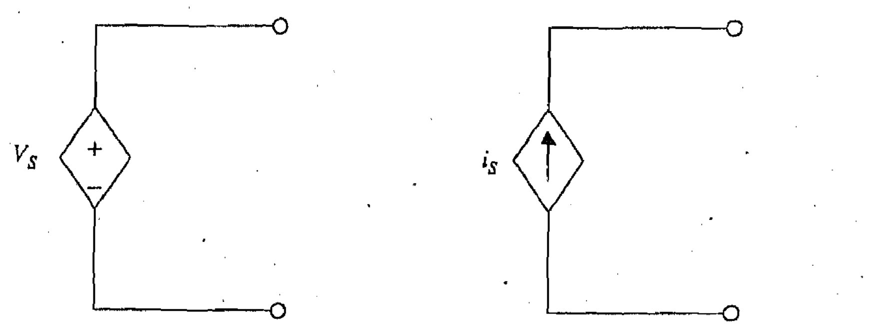


Fig. 1.7: Practical current source.

Dependent Sources

In case of dependent sources (voltage or current), the source voltage or current is not fixed but is dependent on the voltage or current existing at some other location in the circuit. These sources mainly occur in the analysis of equivalent circuits of active devices such as transistors. The symbols for dependent circuit voltage and current sources are shown in Fig. 1.8.



Dependent Voltage Source (a) Dependent Current Source (b)

Fig. 1.8: (a) Dependent voltage source (b) dependent current source,

1.3 KIRCHOFF'S LAWS

There are two fundamental laws which are satisfied by electrical networks. These are Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL). These form the basis of circuit analysis. You have already studied these laws at the school level. However, we will recapitulate them as they are very important for circuit analysis.

Kirchoff's Current Law (KCL)

It states that the algebraic sum of currents at a node (or junction) is equal to zero. Alternate form of KCL are:

Algebraic sum of currents entering a node = 0

Algebraic sum of currents leaving a node = 0

or sum of currents entering a node = sum of currents leaving a node

In order to apply KCL, we have to make a convention for currents because we have to take algebraic sum.

Convention : All entering currents are treated positive (+) and leaving currents as negative (–).

Example 1

In **Fig,** 1.9. write KCL for the various nodes (currents are marked).

Solution

Clearly 1, 2, 3, 4, 5 represents nodes, we shall now write KCL for ail the nodes.

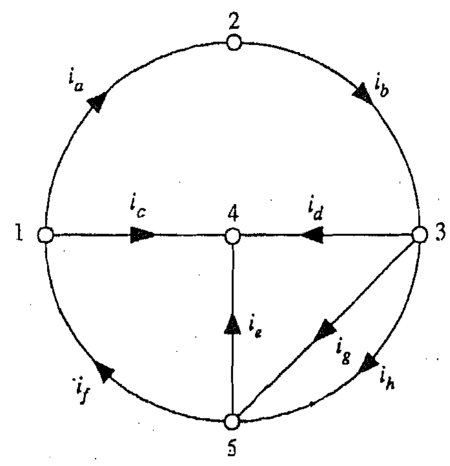


Fig. 1.9:

At node 1, $-i_a - i_c + i_f = 0$ [using conventions stated earlier]

At node 2, $i_a - i_b = 0$

At node 3, $-i_b - i_d - i_g - i_h = 0$

At node 4, $i_c + i_d + i_e = 0$

At node 5, $i_h + i_g - i_e - i_f = 0$

It is very interesting to note that the KCL at node 5 can be obtained simply by summing the KCL equations for all the other nodes (1 to 4). We observe the following from this exercise

(i) It is enough to write KCL for all nodes but one.

(ii) The KCL for the excluded node is not independent as it can be obatined from the KCL written for the other nodes.

Remember : It is sufficient to write KCL for (n – 1) nodes for a network having *n* nodes.

Kirchoff's Voltage Low (KVL)

It states that the algebraic (with proper sign) sum of voltages in a closed network is zero. In order to apply this **law,** we must **know** the magnitude and polarity of all the voltages in that closed network. In order to make our life simple, we follow a convention that for any passive element (R, L or C), if current is entering at a terminal, it will **have +ve** potential and the terminal where current is leaving is

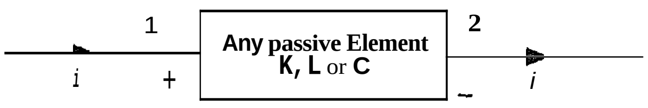


Fig .1.10. : Convention for KVL.

regarded as –ve potential. (See Fig. **1.10)**

As an example we apply **KVL** in a network shown in Fig. **1.11** with marked voltage polarity given along with their values :

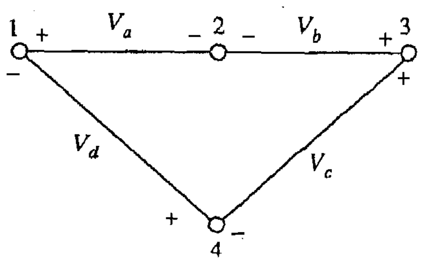


Fig. 1111

Applying **KVL,** we get,

$$V_a + (+V_d) + (V_c) + (-V_b) = 0$$

We have used V_a as reference voltage.

Remember :

1. Once you have chosen reference potential always come out from +ve **terminal** of that potential to move into other parts of network. Sometimes It may be clockwise or **anticlockwise.** You need not **worry** about it.

2. **In order** to determine the proper sign **of** voltages, you follow the following :

   While coming out from **+ve** terminal of ref. pot, if you encounter **first** the +ve terminal of next element, treat it as **–ve.** But **if** you **encouter** –ve terminal **of** the next element, treat it as **+ve.**

Simplifications **based** on **KCL, KVL**

In circuit analysis, the commonly used simplifications based on **KCL** and KVL ate as follows:

1. The same. current flows in elements connected in series.
2. The same voltage exists across elements connected in parallel.
3. A resistor R connected across a voltage source V_s has a voltage V_s and hence the current in that resistor element is V_s/R.
4. An element in series with a **current** source *I*, has a current *I*, flowing **through** it, so that the voltage across such a resistor element is $V = I_s R$.

Example 2

In the circuits shown in Fig. **1.12.** obtain the indicated **variable.** Use **simplifications** listed above.

Solution: .

(a) R_1 **and** R_2 are in series **and** hence, have the same current *I*. Apply **KVL,**

$$V_s - R_1 I - R_2 I = 0$$

$$\Rightarrow \quad V_s = (R_1 + R_2) I \qquad (1)$$

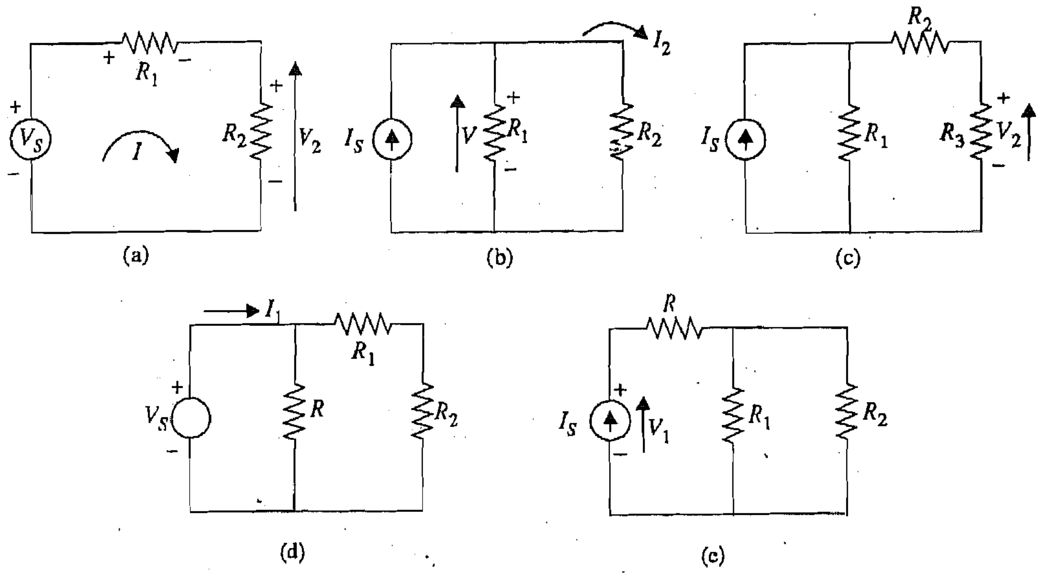


Fig. 1.12:

Also, $V_2 = R_2 I$ (2)

Solving (1) and (2), we get

$$V_2 = V_s \frac{(R_2)}{(R_1+R_2)}$$

This is 'voltage divider' action.

(b) Since R_1 and R_2 are in parallel, they have same voltage V across them. Apply KCL,

$$I_s = \frac{V}{R_1} + \frac{V}{R_2} \quad (3)$$

Also, $I_2 = \frac{V}{R_2}$ (4)

solving (3) and (4), we get

$$I_2 = I_s \frac{R_1}{(R_1 + R_2)}$$

This result is known as "current divider*' action.

(c) Notice that R_2 and R_3 *are in* series so that $(R_2 + R_3)$ is in parallel with R_1. **Hence,** the **current** in $(R_2 + R_3)$ can be found out using current divider principle and is given by

$$I = I_s \frac{R_1}{(R_1 + R_2 + R_3)}$$

Also, $V_2 = R_3 I$

$$= I_s \frac{R_1 R_3}{(R_1 + R_2 + R_3)}$$

(d) The **circuit** is the same as in (a) except for the resistor R across V_s. R draws a current $\frac{V_s}{R}$, Hence the current drawn from V_s is now :

$$I_1 = \frac{V_s}{R} + \frac{V_s}{(R_1 + R_2)}$$

(e) The circuit is same as in (b) except for the resistor R in series with I_s. Also R_1 and R_2 are in parallel.

$$\therefore \quad V_1 = I_s R + I_s \frac{R_1 R_2}{R_1 + R_2}$$

SAQ 1

Determine V for the circuit shown in Fig. 1.13.

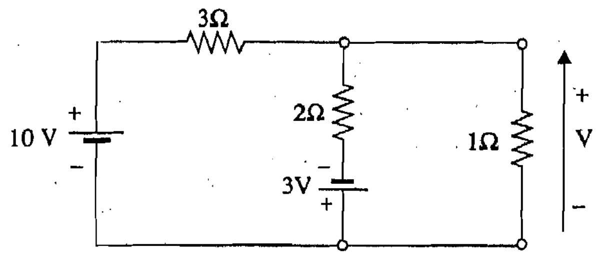


Fig. 1.13

SAQ 2

In the circuit shown in **Fig.** 1.14, determine R so that $i = 1$

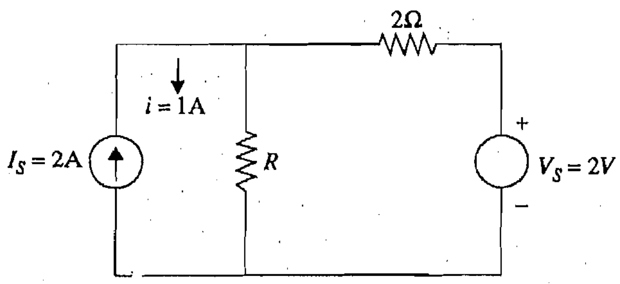


Fig. 1.14

1.4 COMPLEX IMPEDANCES

So far the discussions have been confined to resistive circuits. Resistance restricts the flow of current by opposing free electron movement. Each element has some resistance, for example, inductance has some resistance and capacitance also has some resistance. In resistive element, there is no phase difference between voltage and current whereas in inductive circuit, voltage leads over current and in capacitative circuit, current leads over voltage. The comlex impedance of a network is given by

$$Z = R + j\,X$$

where Z = Complex Impedance

R = Resistance

X = Reactance

$$|Z| = \sqrt{ZZ^*} = \sqrt{R^2 + X^2}$$

and $\angle Z = \frac{X}{R} = \frac{\text{Imaginary part}}{\text{Real part}}$ (as shown in Fig. 1.15)

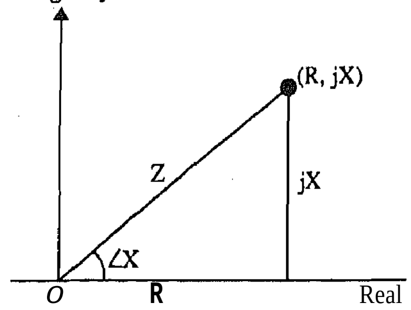


Fig. 1.15:

Example 3: In RL network as shown in Fig. 1.16

$$Z = R + j\omega L$$

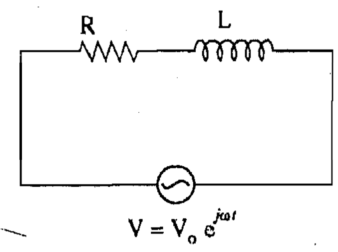


Fig. 1.16: *RL*-network

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

$$\angle Z = \frac{\omega L}{R}$$

Example 4: In RC network as shown in Fig. 1.17

$$Z = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \text{and} \quad \angle Z = -\frac{1}{\omega C R}$$

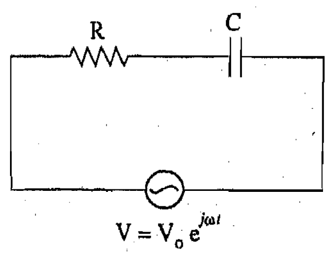


Fig. 1.17: *RC* network.

1.5 CURRENT-VOLTAGE SOURCE TRANSFORMATIONS

We have studied earlier that an independent energy source is either of voltage type or of current type. Moreover, a practical source is non-ideal in nature. We can represent a practical voltage source by an ideal voltage source having a series impedance (generally, a resistance), as shown in Pig. 1.18a. Also we can approximate a practical current source by an ideal current source with a shunt impedance [Fig. 1.18b].

These representations account for the fall in terminal voltage with an increase in output current (due to decrease of load impedance) for the voltage source, and for a fall in the output current with an increase of load resistance for the current source,

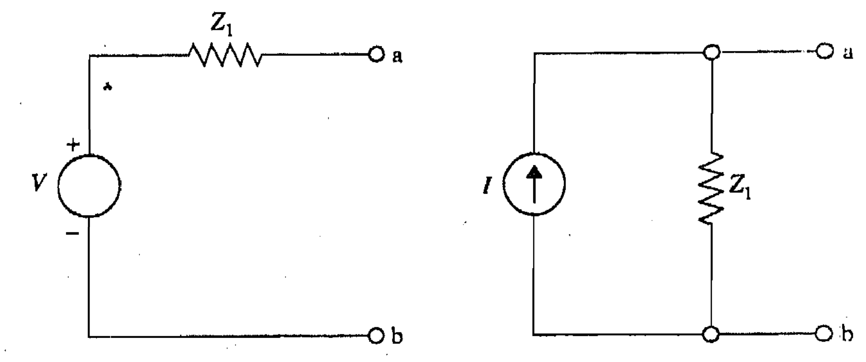


Fig. 1.18: A practical form of (a) voltage source (b) current source.

In many network analysis problems, it is found that the conversion, of voltage source into an equivalent current source or vice versa, **i.e.,** source **transformation,** makes the problems considerably simpler. In this section, we shall derive the **conditions** for the equivalence of practical **voltage** and current sources.

Let us consider Pig. **1.19a,** where a voltage source is shown connected to a load Z_l. **The current,** delivered to the load, is given by

$$I_{v,l} = \frac{V}{Z_V + Z_l} \qquad (1.1)$$

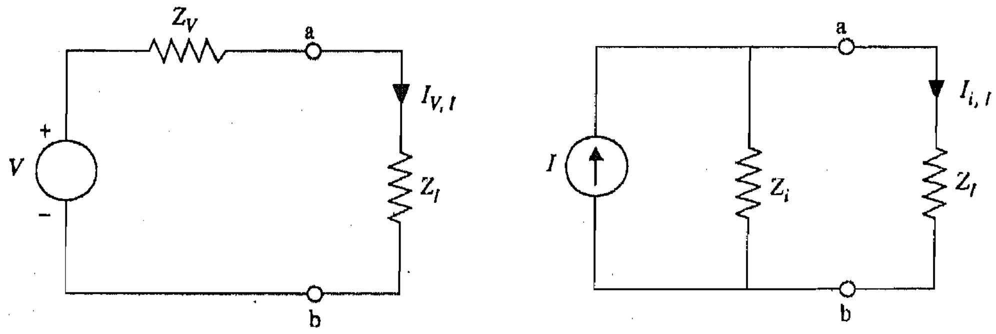


Fig. 1.19: (a) Voltage and (b) current **source with** load Z_l.

Now, if we connect the **same** load Z_l across the current source as shown in Fig. **1.19b** then the current delivered to the load is given by

$$I_{i,l} = \frac{Z_i}{Z_i + Z_l} I \qquad (1.2)$$

In case the two energy sources are equal, then the **respective** current, **delivered** to Z_l must be equal, **i.e.,** $I_{v,l} = I_{i,l}$. From Eqs. (1.1) and (1.2), we have

$$\frac{V}{Z_V + Z_l} = \frac{Z_i\, I}{Z_i + Z_l} \qquad (1.3)$$

Eq. (1.3) will be satisfied, if

$$I = \frac{V}{Z_i} \text{ and } Z_V = Z_i \qquad (1.4)$$

Statement of the Conversion Principle

A constant voltage source of voltage V and series impedance Z_V is equivalent to the constant current source of **current,** $I = V/Z_V$ and shunt impedance Z_V.

1.6 SUPERPOSITION THEOREM

The basic priciple of superposition states that if the effect produced in a system is directly proportional to the cause, then the overall effect, produced in the system due to the number of causes acting jointly, can be determined by superimposing (adding) the effects of each source acting separately. It is important for us to note that the above principle form the foundation of many engineering systems, such as the broadcasting system, audio system etc. For example, in a large orchestra, we would like the net response of the system to be the sum of individual responses of each instrument played separately. In other words, good fidelity of the system depends upon the validity of the superposition principle.

As the superposition principle is only applicable on linear networks and systems, it is important for us to first clearly understand the term 'linear' before proceeding with the formal presentation and proof of the superposition theorem.

We can call a device linear; if it is characterized by an equation of the form :

$$y = mx \qquad (1.5)$$

Where m is a constant. For example, a wire-wound inductor is linear device ; and its variables are V and i. Eq. (1.5) also implies that y is proportional to x, (You should clearly note that the general superposition principle stated earlier is dependent of proportionality). A Network / system, which contains only linear devices (or elements), is called a linear network / system.

1.6.1 Statement of Superposition Theorem

In a linear network, having several sources (which include 'the equivalent source due to initial conditions), the overall response, at any point in the network, is equal to the sum of individual responses of each source considered separately,' the other sources being made inoperative.

Remarks

(1) We observe that the theorem basically implies that the total response in a linear circuit is a proper summation of the partial responses of the sources, considered one at a time.

(2) By the term source, we include all the voltage and current sources and also the equivalent source constituted by the initial conditions in the network.

(3) We make the source inoperative by (a) short-circuiting the voltage sources and replacing them by their series impedances and by (b) open-circuiting the current sources and substituting them by their shunt impedances.

(4) The linear network comprises of independent sources, linear dependent sources and Linear passive elements like resistors, inductors, capacitors and transformers. Moreover, the components may either be time-varying or time-invariant.

(5) We find that the main advantage of superposition theorem lies in the fact that it allows solution of networks without the need of setting up large 'number of circuit equations. This is possible as only one source is considered at a time.

(6) The superposition theorem seems to be so apparent that one tends to apply it at places where it is not applicable.

1.6.2 Proof of Superposition Theorem

Let us consider a linear network N having L independent loops. The loop equations are :

$$Z_{11} I_1 + Z_{12} I_2 + \cdots + Z_{1L} I_L = E_1$$

$$Z_{21} I_1 + Z_{22} I_2 + \cdots + Z_{2L} I_L = E_2 \qquad (1.6)$$

$$\cdots\cdots\cdots\cdots$$

$$Z_{L1} I_1 + Z_{L2} I_2 + \cdots + Z_{LL} I_L = E_L$$

Ths solution of Eq. (1.6) is of the following form :

$$I_1 = Y_{11} E_1 + Y_{12} E_2 + \cdots + Y_{1L} E_L$$

$$I_2 = Y_{21} E_1 + Y_{22} E_2 + \cdots + Y_{2L} E_L \qquad (1.7)$$

$$\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots$$

$$I_L = Y_{L1} E_1 + Y_{L2} E_2 + \cdots + Y_{LL} E_L$$

Now we apply the superpostion theorem.

First we assume all E's, except E_1, to be zero. This will give current in loop 1 as $Y_{11} E_1$, in **loop** 2 as $Y_{21} E_1$, **etc.** Similarly, when all the sources except E_2 are made inoperative, the current in loop 1 is $Y_{12} E_2$, loop *2* is $Y_{22} E_2$, and so on. The procedure is repeated in tun for the remaining sources E_3, E_4, etc. On applying the superposition principle, the individual responses are added together to give the resulting responses. For example, we have for the first loop

$$I_1 = Y_{11} E_1 + Y_{12} E_2 + \cdots + Y_{1L} E_L$$

But this result is the same as that obtained by solving the simultaneous Eq. 1.7. A Similar proof, can be given based on junction method.

SAQ 3

Using the superposition theorem, **find** the current (I)in the branch AB of Fig. 1.20.

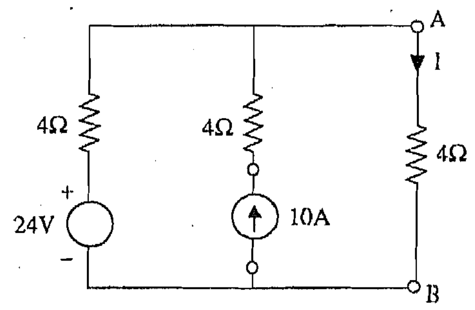


Fig. 1.20

1.7 RECIPROCITY THEOREM

The reciprocity phenomenon is commonly found in nature. Besides in electrical circuit we also observe this phenomenon in mechanics, acoustics, **etc.** When it is applicable to a physical system, we can interchange the excitation and the response **points** without changing the response, of the system, to a given excitation. It finds wide utility in the analysis and design of systems and also in the area of measurements. For a better **understanding,** let us consider the example of a telephone line **between** two **subscribers** X and *Y.* If the telephone circuit comprises permissible elements, then the reciprocity theorem **permits** us to **assume** that the **transmission** from X to Y is identical to that from Y **to** X. This greatly simplifies, in practice, the design and testing of **the** line.

1.7.1 Statement of Reciprocity Theorem

Reciprocity theorem states that if we consider two loops A and B of a reciprocal network N, and if an ideal voltage source, E, in loop A, produces a current I in loop B, then an identical source in loop B will produce the same current I in loop A.

The dual is also true. If we consider two node pairs AA' and BB' of a reciprocal network N and an ideal current source of I amp applied to the node - pair AA' produces a voltage V at the node - pair BB' then an identical current source at BB' will produce the same voltage V at AA'.

Remarks

(1) A reciprocal network comprises of linear time-invariant, bilateral, passive elements. It is applicable to resistors, capacitors, inductors (including coupled inductors) and transformers. However, both dependent and independent sources ate not permissible. Also, we are considering only the zero-state response by taking the initial conditions to be zero. In this sense, the theorem is more restrictive than the superposition theorem.

(2) The superposition theorem seems to be more obvious to most people and, hence, is easily acceptable. As pointed out earlier, we have to be more careful not to apply it at the wrong place. On the other hand, the reciprocity theorem is far less obvious.

1.7.2 Proof of Reciprocity Theorem

Let us consider a network N having only one energy source driving a voltage E_1. We can number the loop in which the source is present as 1 and that in which the response is to be determined as 2. The second Equation from Eq. (1.7) gives

$$I_2 = Y_{21} E_1 \quad (1.8)$$

Next we reduce E_1 to zero and place a voltage source E_2 in loop 2. Now, the first of Eq. (1.7) gives

$$I_1 = Y_{12} E_2 \quad (1.9)$$

In case the two sources are identical, i.e., $E_1 = E_2$, then I_1 will also be equal to I_2 provided that $Y_{12} = Y_{21}$. As $Y_{21} = Y_{12}$, in all linear, bilateral networks, we have proved the reciprocity theorem.

SAQ 4

In the circuit of Fig. 1.21, determine the current in the 4-ohm resistor. Verify the reciprocity therorem.

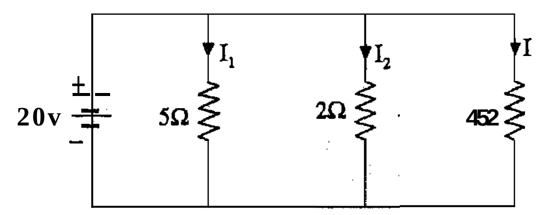


Fig. 1.21

Also, find the change in current in the 5-ohm resistor, when the points of excitation and response are interchanged.

1.8 THEVENIN'S THEOREM

Thevenin's theorem is one of the most important theorems in Circuit Theory. It provides us with a powerful technique of calculating the response in a complicated network, particularly when one part of the network (generally called the load) is varying, while the remaining parts **remain fixed.** In addition, it also helps us in getting a better insight of any linear network as seen from the terminals across which the load is connected.

1.8.1 Statement of Thevenin's Theorem

A two-terminal linear active network may be replaced by a voltage source in series with an impedance. The emf of the voltage source (V) is the open-circuit voltage at the terminals and the impedance (Z) is that viewed at the terminals when all the generators, in the network, have been replaced by their internal impedances.

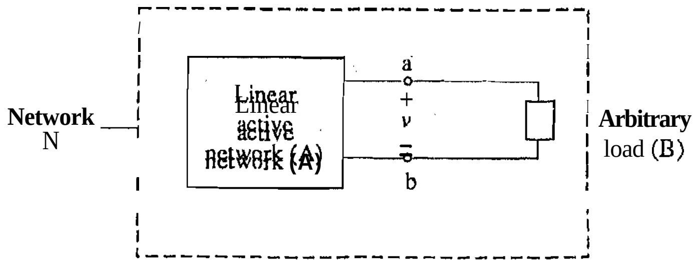


Fig. 1.22: Original network.

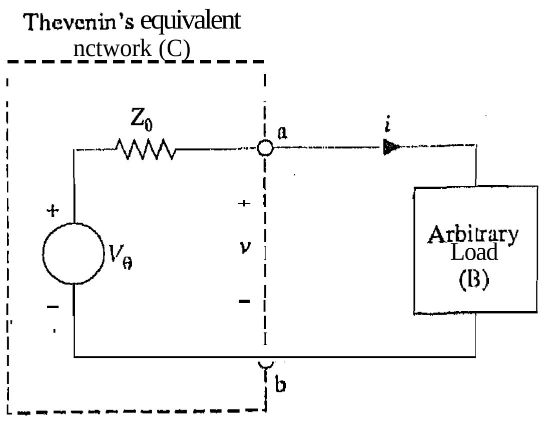


Fig. 1.23: Thevenin equivalent of Fig. 1.22.

Explanations and Remarks

Let us consider the network N shown in Pig. 1.22, It consists of two parts : a **linear** active network A and an arbitrary network B, called the load. We assume that the only interaction, between the two networks, Is through the load current (I), **i.e.,** no magnetic coupling or coupling through dependent source is present between A and B. According to the Thevenin's theorem, we can replace the active network **A by** the Thevenin's equivalent generator and a series impedance. The voltage of this generator (V_θ) is the open-circuit voltage across, a-b, when the load is disconnected. It is caused by the energy sources, in A, including the **initial conditions.** (This voltage is the one which a VTVM records across a-b). The equivalent series impendance (Z_θ) **is** the impedance across a-b **when** all the independent **sources** arc reduced to zero **by** replacing **tho voltage** sources **by** short-circuits, open-**circuiting** the current sources and setting the **intitial** conditions to zero. The dependent sources are left unaltered, (In fact, an impedance **bridge across a-b** will measure this impedance Z_θ). The Thevenin's equivalent of Fig. **1.22. is** shown in Fig. 1.23. In case the load B simply consists of an effective impendace Z_l , the load current is given by

$$I_i = \frac{V_\theta}{Z_\theta + Z_i} \tag{1.10}$$

we should take note of the following points.

(i) We only require network **A** to be linear : it may **include** time-varying elements.

(ii) We place no restriction on the load except that :

(a) it had no coupling with A **except** for the load current and

(b) the complete network had a unique solution.

1.8.2 Proof of Thevenin's Theorem

Let us consider **the** circuit of Pig. 1.24, where the active network A (having an open-circuit voltage V_0 is driving a current into a passive load B. We **assume** the load **current as I**, the effective impedance, offered by the load **as**, Z_0.

Next, a voltage **source** E_1 is introduced in **series** with the load, as shown in Fig. 1.24. It is adjusted such that the current **I** becomes zero. Under such a

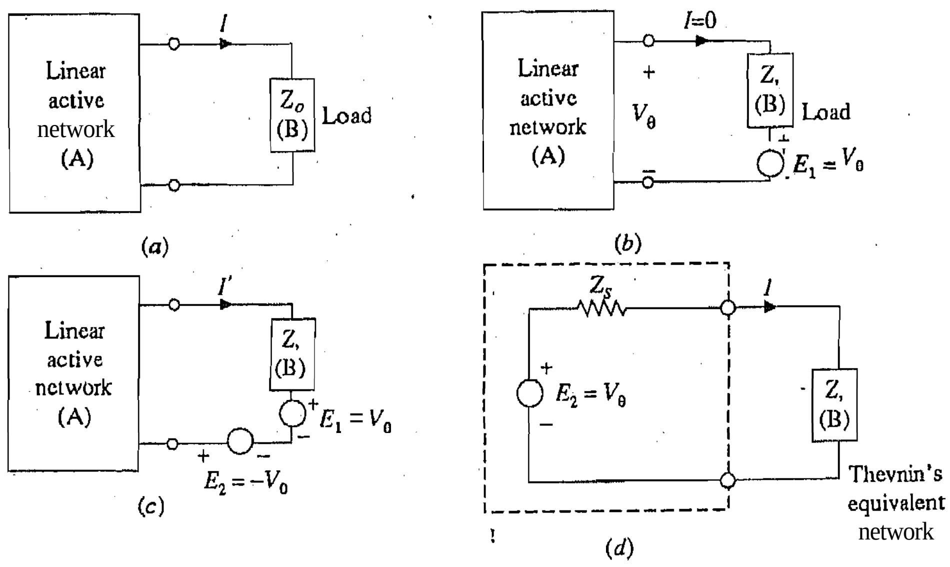


Fig. 1.24 Circuits required in the proof of Thenvenin's theorem.

condition, the network is practically under-open circuit condition : and the,. balancing voltage required is $E_1 = V_\theta$.

Now we place another voltage source E_2 in opposition **to** E_1 but equal **in** magnitude, **i.e.,** $E_2 = -V_\theta$. This results in a current I'. Since the net voltage across E_1 plus E_2, is zero, the circuits of Fig. **1.24a** and **1.24c** are **equivalent** : and $I' = I$. This also implies that the current, due to E_2 alone (after making the sources in A inoperative and replacing it by its **impedance** Z_θ), is given by

$$I = \frac{E_1}{Z_\theta + Z_o} = \frac{V_\theta}{Z_\theta + Z_o} \tag{1.11}$$

The process is depicted in Fig. **1.24d,** where the network A has been replaced by a voltage generator of voltage V_o and a series impedance Z_o. This proves Thevenin's theorem **for passive** load.

In case the load is active, Thevenin's theorem is still valid. The proof is based on the lines of passive load, with the difference that now we have to apply voltage source E_1 so as to balance the combined effects of V_θ and the voltage due to the external **network** and **make** the **resulting** load **current zero. This,** in turn implies **that** E_2 is either the sum or difference (in case V_θ and the voltage due to external network are opposed) of the open-circuit voltages of the networks A **and** B.

This shows that the Thevenin's equivalent network, i.e. V_o and Z_o, is the same whether the load is active or passive. Thus, the Thevenin's theorem is valid for an arbitrary load.

1.9 NORTON'S THEOREM

Norton's theorem provides the dual of Thevenin's theorem. In fact the only variation is that the thevenin's equivalent generator and the series impedance are replaced by an equivalent current generator and a shunt admittance. The replacement is in accordance with the source transformation. The transformed network is called the Norton's equivalent network. The various steps, in obtaining the Norton's equivalent network, are shown in Fig. **1.25.** We now give a formal statement of the theorem.

1.9.1 Statement of Norton's Theorem

A two-terminal linear active network may be replaced by a current source of value I_θ and a shunt admittance Y_θ. The current **I** is the short-circuit current between the terminals ab; and Y_θ $(-1/Z_\theta)$ is the admittance viewed at the terminals when the sources in the active network, including those due to initial conditions, arc made inactive and replaced by their internal impedances.

The Norton's equivalent circuit is obtained from the Thevenin's equivalent circuit with the following relationships:

$$I_\theta = \frac{V_\theta}{Z_\theta}$$

and

$$Y_\theta = \frac{1}{Z_\theta} \qquad (1.12)$$

Sometimes, the two theorems are heated as one and called the Thevenin-Norton Equivalent Network Theorem.

1.9.2 Proof of Norton's Theorem

Norton's theorem can be proved independently. However, here, we shall prove this theorem by using the results of Thevenin's theorem and source transformation.

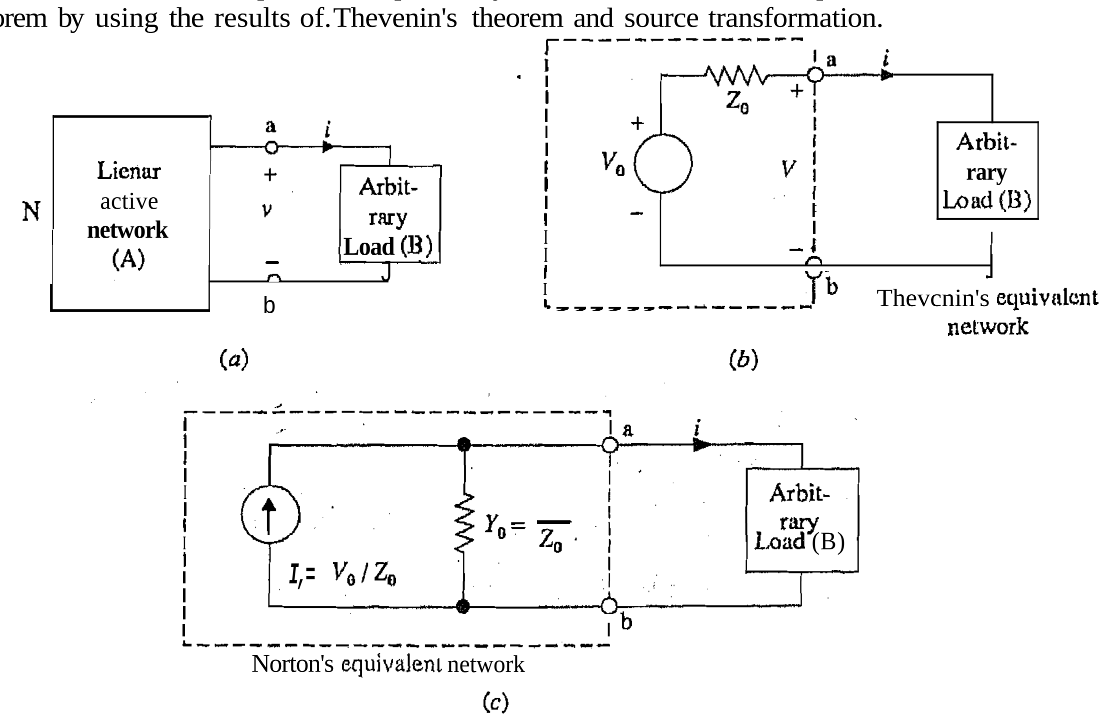


Fig. 1.25: (a) Original network N (b) Replacement of active nelwork A by the Thevenin's equivalent network (c) Replacement of active network by Norton's equivalent network.

Consider the network N of Fig. 1.25.

Its Thevenin's equivalent is obtained as outlined in Section 1.8 ; and the resulting circuit is shown in Fig. **1.25b.** Now we transform the Thevenin's equivalent network **into** a current source. The resulting current generator with shunt impedance is shown within the dotted lines in Fig. **1.25c** and satisfies 4.(1.12). This verify Norton's theorem.

Now, by **following** an approach **similar** to the **one used** for proving Thevenin's theorem, **try** to prove Norton's theorem independently.

SAQ 5

For the circuit shown in rig. 1.26 calculate the current in branch a-b when R_{ab} has the following values :

(i) 1 ohm (ii) 5 ohm

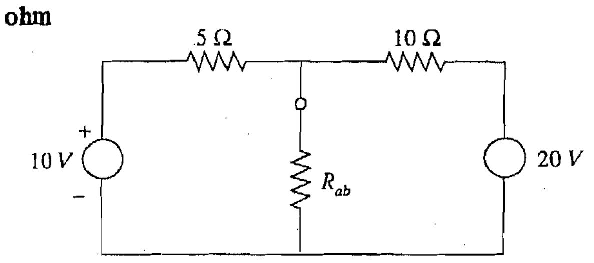


Fig. 1.26: Circuit for SAQ 5.

1.10 MAXIMUM POWER TRANSFER THEOREM

The problem of **transferring** maximum power to load is of great significance to Electronics and Communication Engineers. We have shown a general system, **where** V is the energy source, Z is the associated source impedance and Z_1 is the load to which the power is to be transferred. In fact, in complicated systems, V and Z represent the Thevenin's equivalent network on the source side. Some examples, of general systems, where **maximum** power transfer is significant, are broadcasting system, radar and space communication.

1.10.1 Statement and Proof of Maximum Power Transfer Theorem

The optimum load impedance Z_{im} for the maximum power transfer is equal to the complex conjugate of the source impedance Z_s, **i.e.,**

$$Z_{im} = Z_s^* \qquad (1.13)$$

Consider the steady state operation of the **system** shown in Fig. 1.27 at the **angular** frequency ω of the source V. For simplicity, **we** shall write Z_1 for $Z_i(j)$ and Z_s for $Z_s(j)$.

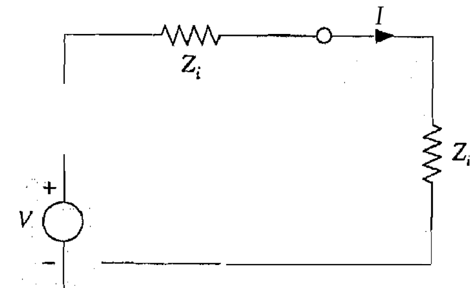


Fig. 1.271 Representation of a general system.

Also, let

$$Z_s = R_s = jX_s$$

and

$$Z_l = R_l = jX_l \quad (1.14)$$

where R_s and R_l are the real parts and X_s and X_l are the imaginary parts of Z_s and Z_l respectively. The average power, delivered to the load, is given by

$$P = [I_s]^2 R_l \quad (1.15)$$

Here the load current is given by

$$I_s = \frac{V_s}{Z_s + Z_l} \quad (1.16)$$

where V_s and I_s are r.m..s. values.

The substitution,of I_s in Eq. (1.15) gives

$$P = | V_s |^2 \frac{R_l}{(Z_s + Z_l)^2}$$

$$= | V_s |^2 \frac{R_l}{(R_s + R_l)^2 + (X_s + X_l)^2} \quad (1.17)$$

In this process, we are given V_s, R_s and X_s, and we have to select R_l and X_l so that the power transfer, to the load is maximum. Let us concentrate on the denominator of Eq. (1.17). We first consider the reactive term. It is evident that $(X_s + X_l)^2$ term will become zero when $X_l = -X_s$. This implies that, if X_s is inductive, then X_l must be capacitive. This choice reduces Eq. (1.17) to

$$P = | V_s |^2 \frac{R_s}{(R_s + R_l)^2} \quad (1.18)$$

Now in order to maximize Eq. (1.18), we determine the partial derivative of P with respect to R_l and equate it to zero, i.e.,

$$\frac{\partial P}{\partial R_l} = 0.$$

On using Eq. (1.18) we have

$$\frac{\partial P}{\partial R_l} = | V_s |^2 \frac{(R_l + R_s)^2 - 2\,(R_l + R_s)\, R_l}{(R_l + R_s)^4} = 0$$

On simplification of the above expression, we get

$$R_l = R_s\,.$$

Therfore, the condition, for the transfer of maximum power, is given by

$$R_l = R_s \text{ and } X_l = -X_s \quad (1.19a)$$

or

$$Z_{lm} = R_s - jX_l = Z_s^* \quad (1.19b)$$

1.11 SUMMARY

- **Kirchoff's** laws state that :

  (1) In an electric network the algebraic sum of the current at a junction point is zero.

(2) In a closed loop the algebraic sum of the potential difference across each circuit component is zero.

- The superposition theorem is valid only for Linear networks. It is useful when the network has a large number of energy sources as it makes possible to consider the effect of each source separately. The theorem states that, in a linear network, the overall response, including the equivalent of initial conditions, is equal to the sum of individual responses of each source considered separately.

- The reciprocity theorem states that in any passive linear; bilateral network containing bilateral linear impedances and sources of emf the ratio of a voltage E introduced in one mesh to the current I in any other mesh is the same as the ratio obtained if the position of E and I are interchanged.

Thevenin's theorem states that in any two terminal network of fixed resistance and constant sources of emf, the current in a load resistor connected to the output terminals is equal to the current that would exist in the same resistor if it were connected in series with (a) a simple emf whose voltage is measured at the open-circuited network terminals and (b) a simple resistance whose magnitude is that of the network looking back from the two terminals into the network with all the sources of emf replaced by their internal resistance.

- Norton's theorem states that any two-terminal linear network containing energy sources and impedances can be replaced by an equivalent circuit consisting of a current source I in parallel with an admittance *Y*.

The maximum power transfer theorem states that the condition for the transmission of maximum power to the load (optimum load matching), is that the load impedance must be the complex conjugate of the source impedance, i.e., $Z = Z_s^*$.

1.12 TERMINAL QUESTIONS

1. In Pig. 1.28, the equivalent circuit of an electronic amplifier is shown. Calculate its output voltage.

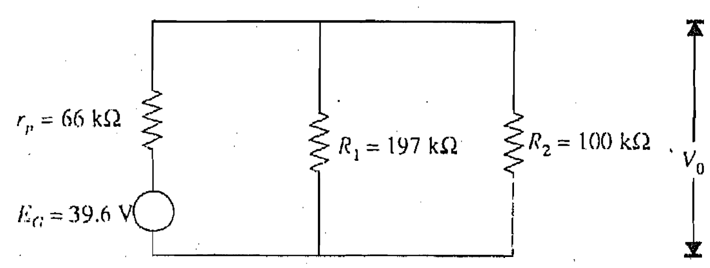


Fig. 1.28:

2. Using superposition theorem, determine the current in the given network shown in Fig. 1.29.

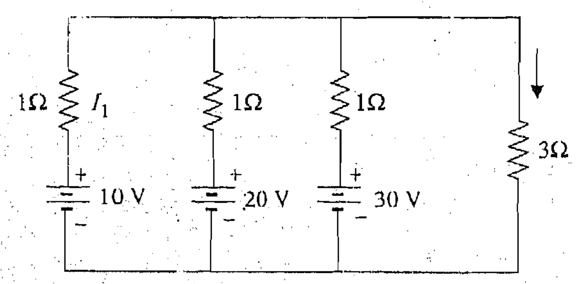


Fig. 1.29:

3. Solve the network shown in Pig. 1.30 for the branch current using Norton's theorem.

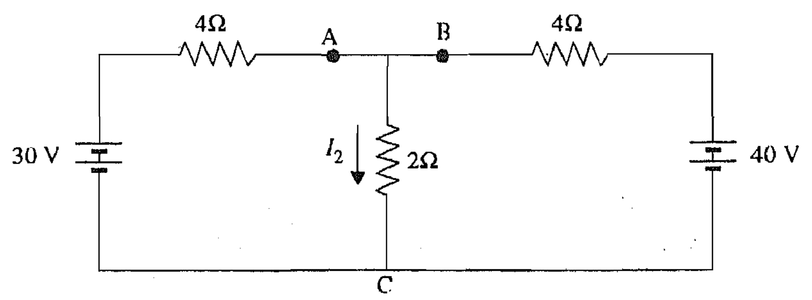


Fig 1.30:

4. A length of a uniform wire of resistance 10 ohm is bent into a circle and two points at a quarter of the circumference apart are connected with **battery** of internal resistance 1 ohm and emf 3 volt. Find the current in the different parts of the circuit.

1.13 SOLUTIONS AND ANSWERS

SAQ's

1. Writing **KCL** at node 1.

$$\frac{10-V}{3}=\frac{V+3}{2}+\frac{V}{1}$$

Solving this gives V = 1.

2. For **determining** the current i_1 due to the current source, we replace voltage source by a short circuit. **The** corresponding circuit is shown in Fig. **1.31(a)** and the current i_1 is given by

$$i_1=\frac{2}{2+R}\times 2=\frac{4}{2+R}$$

For determining **the** current i_2 due,to voltage source, the current source is **replaced** by open circuit. The corresponding circuit is shown in Fig. **1.31(b)** and the current i_2 is given by

$$i_2=\frac{2}{2+R}$$

Applying Superposition theorem, we **get**

$$i_1+i_2=i=1$$

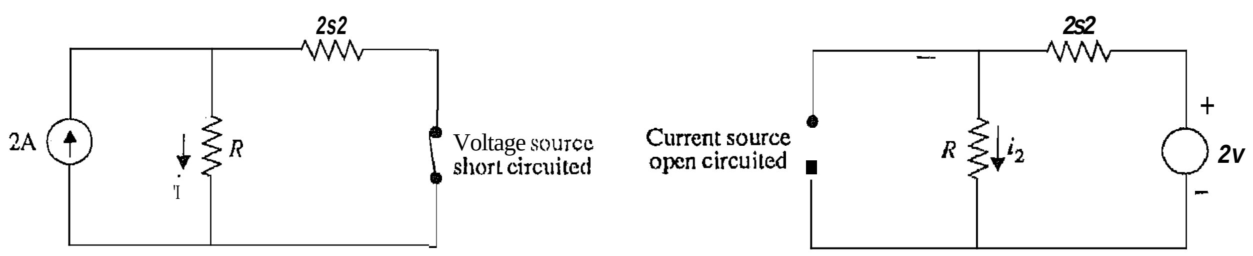


Fig. 1.31:

$$\Rightarrow \frac{4}{2+R} + \frac{2}{2+R} = 1$$

$$\Rightarrow \frac{6}{2+R} = 1$$

$$\Rightarrow 1 = 2 + R$$

$$\Rightarrow R = 4\,\Omega$$

3. Let us first consider the response **due** to the voltage source when the current source is rendered inoperative by **open-circuiting** it. The resulting circuit is shown in **Fig. 1.32(a).** The current I_v, due to the voltage source, is

$$I_v = \frac{24}{4+4} = 3\text{ A}$$

For **determining** the response only due to the current source, we replace the voltage source by a short-circuit. **The corresponding** circuit **is** shown in **Fig.1.32(b)** and the current in AB is

$$I_i = 10\frac{4}{4+4} = 5\text{ A}$$

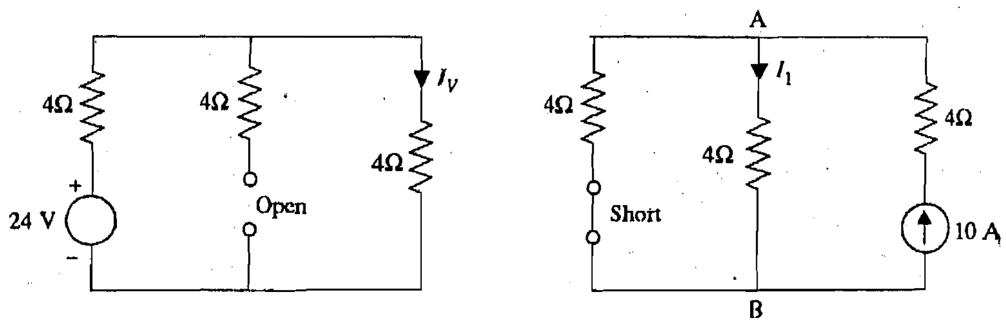


Fig. 1.32: Circuit after making (a) current source inoperative (b) voltage source inoperative.

Mow applying superposition **principle,** we have

$$I = I_v + I_i = 8\text{ A}$$

(We urge you to verify **the** result by the Nodal analysis).

4. The branch **current** I, I_1 and I_2 are :

$$I = \frac{20}{4} = 5\text{ A}$$

$$I_1 = \frac{20}{2} = 10\text{ A}$$

and

$$I_2 = \frac{20}{5} = 10\text{ A}$$

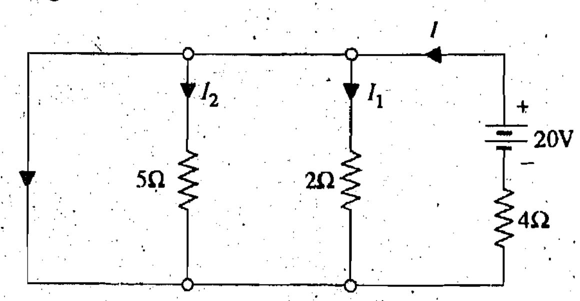


Fig. 1.33

For the verification of reciprocity theorem, we replace the original source by a short-circuit and introduce an equivalent source in the branch containing the 4 ohm resistor. The resulting circuit is shown in Pig. 1.33. The total current, drawn from the battery, is

$$I_T = \frac{20}{4} = 5 \text{ A.}$$

Now all the current will pass through the short-circuit, i.e.

$$I' = 5 \text{ A}$$

and

$$I_1 = I_2 = 0 \text{ A.}$$

As $I' = I = 5$ A, the reciprocity theorem is verified. Also, the changes, in currents in the 5- and 2-ohm resistors, are from 4A and 10A, respectively to zero amp (each).

5. We solve the problem by using Norton's theorem. The resistor R_{ab} is assumed to be the load and we determine the Norton's equivalent of the remaining circuit.

The current I_θ is obtained by finding the current in the short-circuit placed between points a and b. Therfore, from Fig. 1.34(a) we have

$$I_\theta = I_{ab} = \frac{10}{5} + \frac{20}{10} = 4 \text{ A}$$

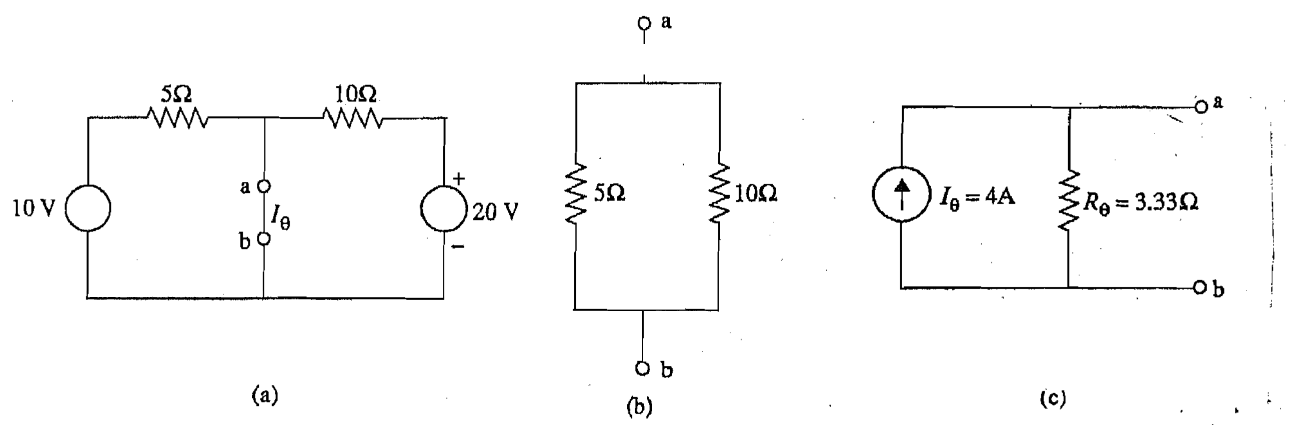


Fig. 1.34: Resulting Circuits at various steps in the analysis of SAQ 5.

We next calculate the resistance R_θ across the terminals a-b, after making the voltage sources inoperative by replacing them with short-circuits. The resulting circuits shown in Fig. 1.34(b). Now, we have

$$R_\theta = R_{ab} = \frac{5 \times 10}{15} = 3.33 \text{ ohm}$$

The Norton's equivalent network, along with the load, is shown in Pig. 1.34(c).

The currrent in the branch a-b is given by

$$I_{ab} = I_\theta \frac{R_\theta}{R_\theta + R_{ab}}$$

(i) When $R_{ab} = 1$ ohm,

$$I_{ab} = \frac{4 \times 3.33}{3.33 + 1.0} = \frac{4 \times 3.33}{4.33} = 3.076 \text{ A.}$$

(ii) When $R_{ab} = 5$ ohm.

$$I_{ab} = \frac{4 \times 3.33}{3.33 + 5.0} = \frac{4 \times 3.33}{8.33} = 1.6\,A.$$

Answer of TQs

1. In this problem, We are required to calculate the voltage at the output of the amplifier. As a first step, we replace the voltage source E_p and the series resistor r_p by an equivalent current generator, shown in Fig. 1.35. The current of the equivalent source is

$$I = \frac{E_p}{r_p} = \frac{39.6}{66 \times 10^3}\,A = 0.6\,mA.$$

As the three resistances are in parallel after the source trasformation. The equivalent resistance is given by

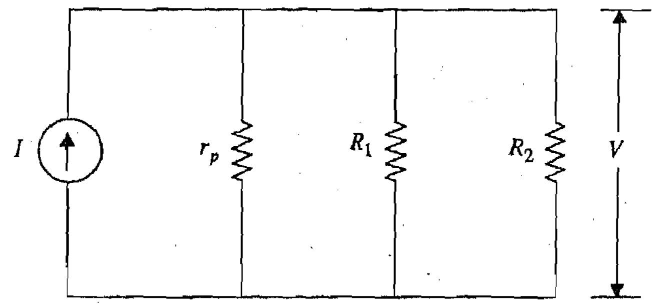


Fig. 1.35:

$$\frac{1}{R_{eq}} = \frac{1}{r_p} + \frac{1}{R_1} + \frac{1}{R_2}$$

On substituting the values and simplification, we get

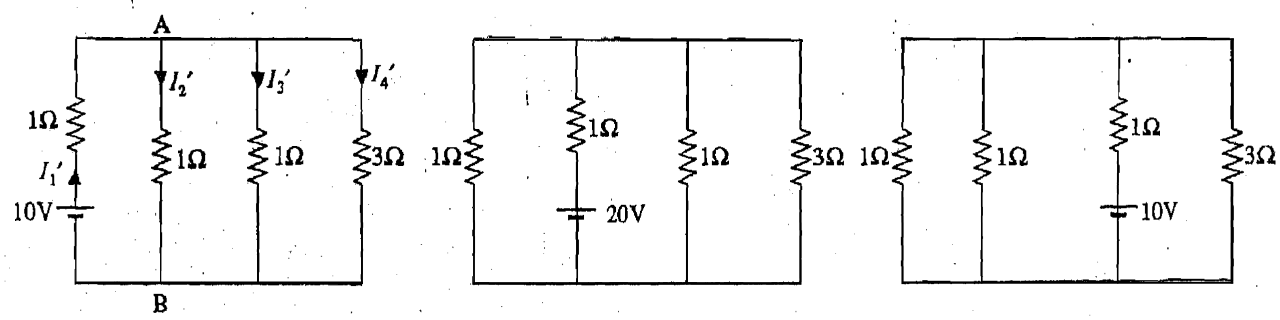
$$R_{eq} = 31.2\,k\Omega$$

The output voltage is

$$V_o = 0.6 \times 10^{-3} \times 31.2 \times 10^3$$

i.e., $$V_o = 18.72\,\text{volts}$$

2. Superposition theorem for three emf are as follow :



(b) (c)

Fig. 1.36: (a) First emf, (b) second emf, (c) Third emf.

the resistance between the point A and B is given as

$$\frac{1}{R_{AB}} = \frac{1}{1} + \frac{1}{1} + \frac{1}{3} = 2.33$$

$$R_{AB} = 0.429\,\Omega$$

The equivalent resistance across 10V battery in Fig. 1.36(a) is given as

$$R_T = 1 + R_{ab} = 1 + 0.429 = 1.429$$

using ohm's law, we can calculate current

$$I_1' = \frac{10}{R_T} = \frac{10}{1.429} = 7 \text{ A}$$

The voltage drop across AB is

$$V_{AB} = I_1' R_{AB} = 7.00 \times 0.429 = 3 \text{ V}$$

The branch currents in Fig. 1.36(a) are

$$I_2' = \frac{3}{1} = 3 \text{ A}$$

$$I_3' = \frac{3}{1} = 3 \text{ A}$$

$$I_4' = \frac{3}{3} = 1 \text{ A}$$

Similarly we can calculate the branch Current in Fig. 1.36(b). The branch current in Pig. 1.36(b) are

$$I_1'' = 6 \ A$$

$$I_2'' = 14 \ A$$

$$I_3'' = 6 \ A$$

$$I_4'' = 2 \text{ A}$$

and the branch current in Fig. 1.36(c) can be calculated in similar way, which will be as follows:

$$I_1''' = 12 \ A$$

$$I_2''' = 12 \ A$$

$$I_3''' = 30 \ A$$

$$I_4''' = 6 \text{ A}$$

Prom the above equation, we'get

$$I_1 = I_1' - I_1'' - I_1''' = 7 - 6 - 12 = -11 \text{ A}$$

$$I_2 = -I_2' + I_2'' + I_2''' = -3 + 14 - 12 = -1 \ A$$

$$I_3 = -I_3' - I_3'' + I_3''' = -3 - 6 + 30 - 21 \text{ A}$$

$$I_4 = I_4' + I_4'' + I_4''' = I + 2 + 6 = 9 \text{ A}$$

3. The Norton's equivalent is given by Fig. *1.37*

$$I = 7.5 + 10 = 17.5 \ A$$

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1 \quad \therefore \ R = 1 \ \Omega$$

$$I_2 = \frac{17.5}{2} = 8.75 \text{ A}$$

The voltage drop from C to A is $30 - 17.5 = 12.5$ V. *Then*

$$30 - 17.5 = 12.5 \text{ V}$$

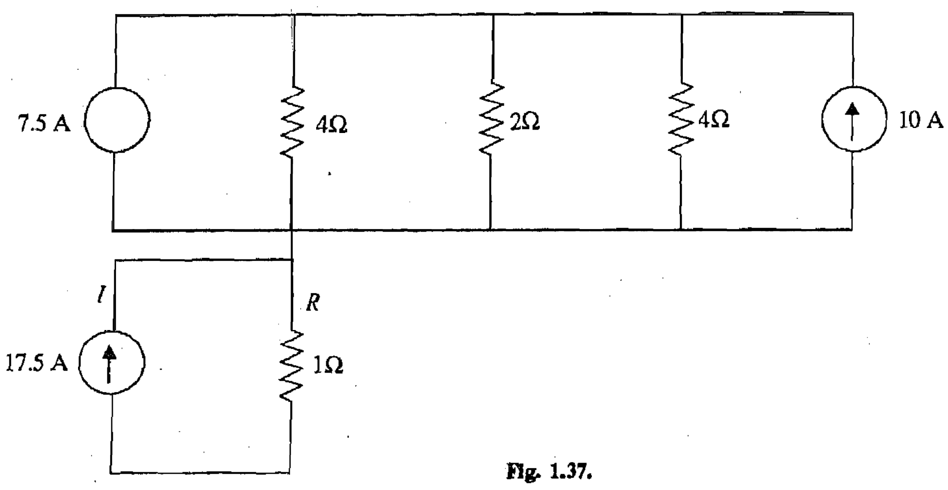


Fig. 1.37.

$$I_1 = \frac{12.5}{4} = 3.125\,A$$

and the voltage drop from C to B is 40 - 17.5 is 22.5 V. Then I_3 is given by

$$I_3 = \frac{22.5}{4} = 5.625\,A$$

4. See fig. 1.38 total resistance of the wire ABC = 10 ohm

Resistance of the portion AB, $r_1 = 10 \times \frac{1}{4} = 2.5$ and resistance of the portion ABC,

$$r_2 = 10 \times \frac{3}{4} = 7.5\,\Omega$$

These two resistance r_1 and r_2 are in parallel. Therefore their resultant resistance R is given by

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2.5} + \frac{1}{7.5}$$

$$= \frac{10}{2.5 \times 7.5} = \frac{8}{15}$$

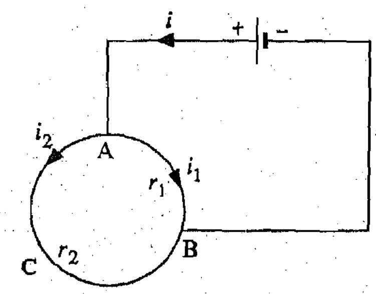


Fig. 1.38.

$$R = \frac{15}{8}\text{ ohm}$$

Internal resistance of battery

$$r_3 = 1\text{ ohm}$$

Total resistance in the circuit

$$R_T = R + r_3$$

$$R_T = \frac{15}{8} + 1 = \frac{23}{8} \text{ ohm}$$

Total current $i = \frac{V}{R_T} = \frac{3 \times 8}{23} = \frac{24}{23}$ A

Let the currents in the portion AB and ACB be i_1 and i_2 respectively. The potential difference between A and B is $V_A - V_B$

$\therefore \quad V_A - V_B = r_1 \times i_1 = r_2 i_2$

$\therefore \quad 2.5 i_1 = 7.5 i_2$ or $i_1 = 3i_2$

But $\quad i_1 + i_2 = i = \frac{24}{23}$ A

$\therefore \quad 3i_2 + i_2 = \frac{24}{23}$

or, $\quad i_2 = \frac{24}{23} \times \frac{1}{4} = \frac{6}{23}$ A

$\quad i_1 = 3i_2 = \frac{18}{23}$ A.