UNIT 12 COHERENCE

Structure

12.1 Introduction
Objectives
12.2 What is Coherence?
12.3 Temporal Coherence
Width of Spectral Lines
12.4 Spatial Coherence
Angular Diameter of Stars
Visibility of Fringes
12.5 Summary
12.6 Terminal Questions
12.7 Solutions and Answers

12.1 INTRODUCTION

In Unit 5 of this course, you studied about Young's double-slit interference experiment. We emphasized that for observing interference fringe pattern, the light from two sources must be coherent. By coherence, we mean that the light waves from two slits have a constant phase relationship, Can you recall how this condition of coherence is achieved? If you are unable to do so, you should refer back to relevant pages. Now the question arises why coherence is a prerequisite for observing interference? You will learn about coherence in detail now.

In Sec. 12.2, we elaborate the concept of coherence as applied to waves in general. Further, the most elementary definition of coherence says that the phases of the coherent waves have a predictable relationship at different points and at different times in space. This space and time predictability of the phase relationship of waves gives rise to two types of coherence, namely, spatial coherence and temporal coherence. The concept of temporal coherence, which refers to the phase relationship at different times at a point, has been discussed in Sec. 12.3, You will also learn about the correlation between the width of a spectral line and temporal coherence. In Sec. 12.4, we have discussed spatial coherence which relates to the coherence of two waves travelling side by side. The relationship between the visibility of fringe pattern with spatial coherence is also discussed in detail.

Objectives

After going through this unit, you should be able to

- explain the concept of coherence
- distinguish temporal coherence from spatial coherence
- relate temporal coherence with the width of spectral lines
- relate spatial coherence with the visibility of fringe pattern, and
- solve numerical problems based on coherence.

12.2 WHAT IS COHERENCE?

If you are asked what is coherence, you may say that it is the condition necessary to produce observable interference of light. And if you are asked what is interference, you may say it is connected with interaction of waves that are coherent. Well, nothing definite follows from such circular arguments! In fact, coherence is a property of light whereas interference is the effect of interaction of light waves. The crucial consideration in interference phenomenon is the relative phase of waves arriving at a given point from two or more sources. That is, in order to observe interference fringes, there must exist a definite phase relationship between the light waves from two sources. Hence, we may say that the necessity of having coherent sources for observing interference fringes essentially implies that the waves from the two sources must have a constant and predictable phase relationship. It is the absence of a definite phase relationship between light waves from ordinary sources that we do not obtain any observable interference fringe pattern.

Now, you may ask: Why there is no definite phase relationship between light waves from two ordinary light sources? Well, the basic mechanism of emission of light involves atoms radiating electromagnetic waves in the form of photons. Each atom radiates for a small time (of the order of 10^{-9} s). Meanwhile, other atoms begin to radiate. The phases of these emitted electromagnetic waves are, therefore, random; if there are two such sources, there can be no definite phase relationship between the light waves emitted from them.

In general, sources, and the waves they emit, are said to be coherent if they

(i) have equal frequencies,

(ii) maintain a phase difference that is constant in time;

If either of these properties is lacking, the sources are incoherent and the waves do not produce any observable interference.

Let us pause for a while and ask ourselves: Why it is a prerequisit for observing interference fringe pattern? To answer this question, let us consider the origin of the bright and dark fringes in the Young's experiment (Fig. 12.1). Let E_1 and E_2 be the electric fields associated with the light waves emanating from slits S_1 and S_2. These waves superpose and the combined electric field at any point on the screen is given by,

$$E = E_1 + E_2 \qquad (12.1)$$

You may recall that in the interference pattern, we observe the intensity of light, not the electric field. Since the average intensity of light is prorporlional to the time - averaged value of the associated electric field, we have

$$I \alpha < E^2 > \qquad (12.2)$$

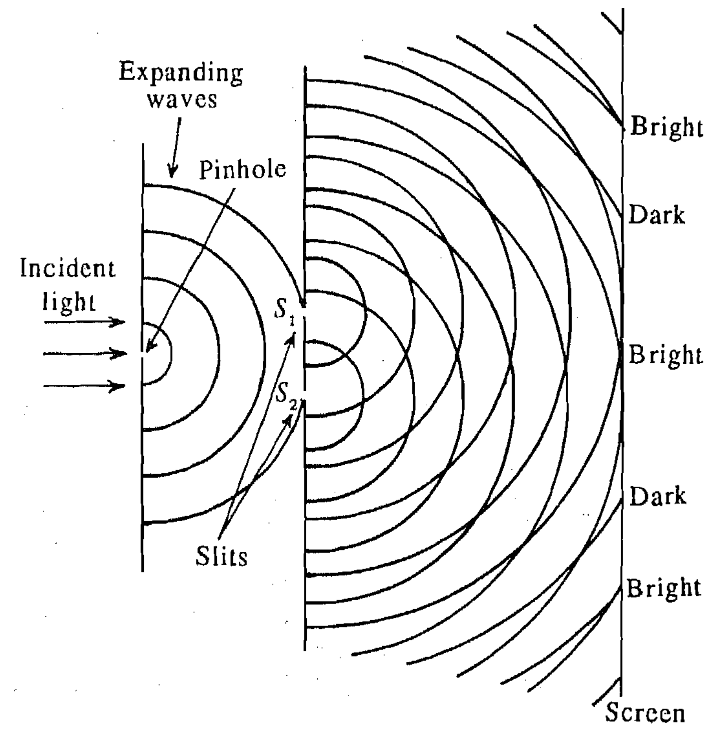


Fig.12.1: Young's Interference experiment.

Thus, we have, from Eq. (12.1) and (12.2),

$$I = \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2 \langle E_1 E_2 \rangle$$

$$= I_1 + I_2 + 2 \langle E_1 E_2 \rangle \qquad (12.3)$$

Eq. (12.3) shows that the resultant intensity on the screen is the sum of intensities I_1 and I_2 (due to individual slit sources) and an interference term $2 \langle E_1 E_2 \rangle$. The interference term is crucial because it determines whether the resultant intensity is an uniform illumination or a fringe pattern on the screen. The contribution of the interference term to the resultant intensity depends primarily on the phase relationship between the light waves emanating from the two slits.

Let us first consider the case when the light waves are in phase at one instance and are out of phase at another instance. In such a situation, the product $E_1 E_2$ will be positive at one instance and negative at the other. As a result, the time average of $E_1 E_2$ will be zero, i.e.

$$\langle E_1 E_2 \rangle = 0$$

Waves having this kind of phase relationship (varying with time) are said to be **incoherent** and the resultant intensity will be

$$I = I_1 + I_2 \qquad (12.4)$$

Thus, when light waves from two incoherent sources interfere, the resultant intensity will be the sum of individual intensities and the screen will be uniformly illuminated. To give you a simple example, when the headlights of a car illuminate the same area, their combined intensity is simply the sum of two separate intensities. The headlights are incoherent sources and there is no contribution of the interference term.

Now, what will happen if the light waves from two slits have a definite phase relationship i.e. a phase relationship which is constant in time. Source of light emiting such waves are coherent. When light sources are coherent, the resultant intensity is not simply the sum of individual intensities. It is so because in that situation, the interference term in equation (12.3) is non-zero. Let us see what is the form of the interference term when two coherent light waves superpose. There are two cases;

(a) When $E_1 = E_2$, that is, the two waves have same amplitude, frequency and phase. Thus,

$$I_1 = \langle E_1^2 \rangle = \langle E_2^2 \rangle = I_2$$

and

$$2 \langle E_1 E_2 \rangle = 2 \langle E_1^2 \rangle = 2I_1$$

The resultant intensity,

$$I = I_1 + I_2 + 2 \langle E_1 E_2 \rangle$$
$$= I_1 + I_1 + 2I_1$$
$$= 4I_1 \qquad (12.5)$$

Thus, the points on the screen where two interfering waves are in phase, the resultant intensity is four times that due to an individual source. These points will, therefore, appear bright on the screen.

(b) $E_1 = -E_2$, that is, the two waves have same amplitude and frequency but their phases differ by 180° which remains constant in time. In that case, the two waves are completely out of phase and the resultant wave amplitude and intensity will be zero.

$$E = E_1 + E_2 = 0$$
$$\Rightarrow I = 0$$

The points on the screen where the interfering light waves satisfy above condition will have zero intensity and hence they will appear dark.

Thus, the constant phase relationship between superposing light waves i.e. coherence, is a necessary condition for obtaining interference fringe pattern. When the phase relationship is not constant, the points where superposing light waves arrive in phase at one instant may receive light waves which are completely out of phase at another instance. This results in uniform illumination of the screen and no interference fringe pattern can be observed.

In the above discussion, you have studied about the necessity of having coherent sources for observing interference fringe pattern. As mentioned earlier, coherence, which is essentially a correlation phenomenon between two waves, can be with respect to time and/or space. Thus, for expediency, we distinguish two types of coherence: Temporal Coherence and Spatial Coherence. Temporal coherence, or the longitudinal spatial coherence (often called monochromaticity) applies to waves travelling along the same path. It refers to the constancy and predictability of phase relationship as a function of time. Spatial coherence, or transverse spatial coherence refers lo the phase relationship between waves travelling side by side, at a certain distance from one another. The further apart are the two waves, less likely they are to be in phase, and less coherent the light will be. You will study these two types of coherences in the following sections.

12.3 TEMPORAL COHERENCE

While studying interference and diffraction of light in the previous two blocks of this course, we assumed that electromagnetic waves remained perfectly sinusoidal for all time. This kind of electromagnetic waves are, however, practically impossible to obtain from ordinary light sources. Why is it so? It is because light emitted from an ordinary source consists of finite size wave trains. Each wave train is sinusoidal in itself and has a characteristic frequency (or wavelength) and phase. However, the collection of wave trains is not sinusoidal. Thus, light waves coming from an ordinary source can not have one single frequency (monochromatic). Instead, it has a range of frequencies; that is, it has a frequency bandwidth. For these reasons, the so called monochromatic'liglit, such as from gas discharge tube, is more appropriately called quasi- monochromatic.

This aspect of light (i.e. monochromaticity) refers to its temporal coherence. The temporal coherence can be identified qualitatively as the interval of time during which the phase of the wave motion changes in a predictable manner as it passes through a fixed point in space. And in wave motion corresponding to light from ordinary sources, a predictable phase relationship can be observed only within the average length of the wave trains on time scale.

To elaborate the concept of temporal coherence, let us consider a typical time variation of the amplitude of an electromagnetic wave as shown in Fig. 12.2.

You may notice from the figure that the electric field at time t and t + At will have a definite phase relationship if $\Delta t << \tau_c$ and will not have any phase relationship if At $>> \tau_c$ where τ_c represents the average duration of the wave trains. The time τ_c is known as **coherence** time of the radiation and the wave is said to be coherent for time τ_c.

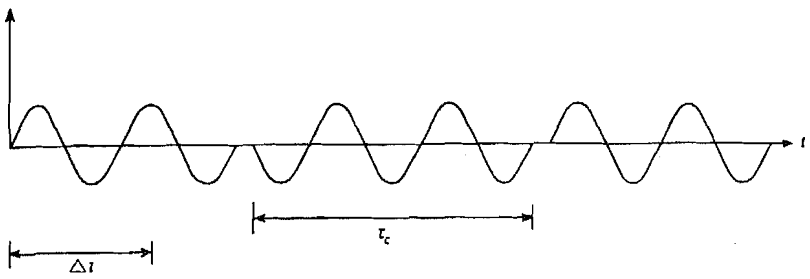
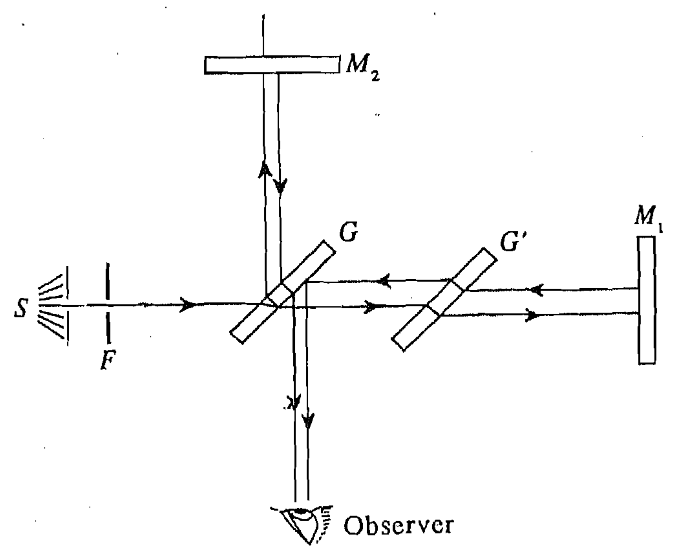


Fig.12.2: Typical variation of the amplitude of an electromagnetic wave with time. Three typical wave trains have been shown The coherence lime τ_c is the average durntion of the wave trains.

And the path length corresponding to τ_c, given as $L_c = c\,\tau_c$ is called the **coherence length** of the radiation, where c is the velocity of light.

In order to study the time-coherence of the radiation, let us re-consider Michelson's interferometer experiment. For completeness, we have reproduced the experimental arrangement in Fig. **12.3.** A nearly **monochromatic light source** is used in the investigation.

For the source *(S)* we may use a neon lamp in front of which we place a filter (F) so that radiation corresponding to $\lambda = 6328$ Å is allowed to fall on the beam-splitter G. Glass plate G' is the compensating plate. You may recall from Unit 7, if the eye is in the



Flg.12.3: Light paths for Michelson Interferometer.

position as shown in the figure, circular fringes are observed due to interference of the beams reflected from mirrors M_1 and M_2. You may also recall that for obtaining these circular fringes, the mirrors should be at right angle to each other and the path difference $(GM_2 - GM_1)$ should be small. If mirror M_2 is moved away from the beam splitter G, the visibility and hence the contrast of the interference fringes will become poorer, and, eventually, the fringe pattern will disappear. Why does it happen? Does disappearance of interference fringes has to do something with temporal coherence of light waves from neon lamp? Yes, it is so. The disappearance of the fringes is due to the following phenomenon. When mirror M_2 is moved through a distance d, an additional path $2d$ is introduced for the beam which gets reflected by M_2. As a result, the beam reflected from M_2 interferes with the one reflected from M_1 which had originated $(2d/c)$ s, where c is the velocity of light, earlier from the light source. Clearly, if this time delay $(2d/c)$ is greater than the coherence time (τ_c) of the radiation from the source, the waves reaching the eye after reflection from mirors M_1 and M_2 will not have any definite phase relationship. In other words, the waves reflected from mirrors M_1 and M_2 are incoherent. Thus, no interference fringes will be seen. On the other hand, if $(2d/c) << \tau_c$, a definite phase relationship exists between the two reflected waves and hence interference fringes with good contrast will be seen. It is so because in this case, we are superposing two wave trains (after reflection from mirrors M_1 and M_2) which are derived from the same wave train (from the source) and hence they are temporally coherent.

For the neon light (A = **6328 Å**), the disappearance of fringes occurs when path difference between the reflected waves from mirrors M_1 and M_2 is about a few cm. This path difference, $L_c = c\ \tau_c$ is known as coherence **length.** Hence for neon line, $\tau_c \sim 10^{-10}$ s. For commercially available lasers, the coherence length exceeds a few kilometers. Thus, if light beam from a laser be used in the above experiment, we can observe interference fringes for d as long as a few kilometers (provided, of course, we have such a big laboratory!).

In short, if the two paths, GM_1 and GM_2 in Fig. **12.3** are equal in length, the fringes have maximum contrast, hence a maximum temporal coherence. If they are not of equal length then the contrast is less. Hence temporal coherence is less. Temporal coherence is, therefore, inversely proportional to the magnitude of the path difference and directly proportional to the length of the wave train. The wave trains are of finite length; each containing only a limited number of waves. The length of a wavetrain is, therefore, the product of the number of waves, N, contained in a wave train and of its wave length A, so L_c = AN. Since visibility or the contrast of the interference fringes is directly proportional to the length of the wave train, it can also be taken as proportional to the product of N and λ. Further, for a given source of light, you can have some idea about its temporal coherence in terms of the path difference between two interfering waves of Michelson interferometer. You should now work out the following SAQ.

Spend 2 min

SAQ 1

If light of 660 nm wavelength has a wavetrain 20λ long, what is its (a) coherence length and (b) coherence time.

12.3.1 Width of Spectral Line

You might have studied in school physics course about the origin of spectral lines. You may recall that when an'atom undergoes a transition from an excited state to the ground state, it emits electromagnetic radiation. The energy (and hence frequency) of the radiation is equal to the difference in energies of the excited and the ground state. Each substance has a unique set of energy state to which its atoms can be excited. Each substance, therefore, has a characteristic set of energy values (and hence frequencies) for the emitted radiations. This set of frequency values constitutes the spectrum of the substance.

Due to one of the fundamental principles of quantum mechanics, namely, the uncertainty principle, about which you would learn in the course on Modern Physics (PHE-11), the lines in the spectrum are not sharp i.e. corresponding to each spectral line, there is a continuous distribution of frequency in a narrow frequency interval. This narrow frequency or wavelength interval is known as width of the spectral line. For example, for **Cd** red line, width of this interval is about 0.007 **Å.**

You may now be interested in knowing what determines the width of the spectral lines? Is width ofspectral lines related to temporal coherence? Yes, temporal coherence of the source of light is intimately related to the width of its spectral lines. To see how, let us again consider the interference fringes obtained by Michelson interferometer. You may recall from Unit 7 that Michelson's Interferometer can be used for the measurement of two closely spaced wavelengths. Let us consider a sodium lamp source which emits predominantly two closely spaced wavelengths, λ_1 = 5896 Å and λ_2 = 5890 Å. Now, you may recall from Unit 7 that near d = 0, the fringe patterns corresponding to both the wavelengths will overlapp. If the mirror is moved away from the plate G by a distance d, Fig. 12.3, the maxima corresponding to the wavelength λ_1 will not, in general, occur at the same angle as for λ_2. It is so because the spacing between the fringes for λ_1 and λ_2 will be different. Indeed, if the distance d is such that the bright fringe corresponding to λ_1 coincides with the dark fringe corresponding to λ_2, we have

If 'd' is the distance through which one of the mirrors has been moved, the effective path difference will be 2d. And the condition for bright fringe is

$$2d = m\lambda$$

and the condition for dark fringe is

$$2d = (m + 1/2)\lambda$$

where m is an integer.

$$2d = m\lambda_1 \Rightarrow m = 2d/\lambda_1$$

and

$$2d = (m + 1/2)\lambda_2$$

$$\Rightarrow m = (2d/\lambda_2) - 1/2$$

$$\therefore \frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = \frac{1}{2}$$

$$2d = m\lambda_1 \quad \text{(bright fringe)} \qquad (12.4a)$$

and

$$2d = \left(m + \frac{1}{2}\right)\lambda_2 \quad \text{(dark fringe)} \qquad (12.4b)$$

and the fringe system will disappear. The condition for disappearence of fringe pattern can, therefore, be expressed as (see margin remark)

$$\frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = \frac{1}{2}$$

$$\Rightarrow \quad 2d = \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{\lambda^2}{2(\lambda_1 - \lambda_2)} \qquad (12.5)$$

since $\lambda_1 \approx \lambda_2$.

Now, if we assume that the light beam consists of all wavelengths lying between λ and $\lambda + \Delta\lambda$, instead of two discrete values λ_1 and λ_2, fringes will not be observed if

$$2d \leq \frac{\lambda^2}{\Delta\lambda} \qquad (12.6)$$

To arrive at equation (12.6) you should solve the following SAQ.

SAQ 2

Spend 5 min

Starting from Eq. (12.5) which gives the path difference ($2d$), in terms of two distinct wavelengths λ_1 and λ_2, for which fringes will disappear, derive Eq. (12.6) which is for all wavelengths lying between λ and $\lambda + \Delta\lambda$.

Now, can you see the basic reason why fringe pattern disappears? Is it somehow related to the nonmonochromaticity of the light beam? Yes, it is so. If fact, the moment we consider that the light beam consists of all wavelengths lying between λ and $\lambda + \Delta\lambda$, we are essentially considering interference pattern produced by non-monochromatic light beam. You may notice from equation (12.6) that as the spread in the wavelength ($\Delta\lambda$) becomes small (more and more monochromatic), the path difference ($2d$) for disappearance of fringes becomes large. And as mentioned earlier, larger the value of path difference for which fringe pattern does not disappear., more temporally coherent the light beam is. In other words, monochromaticity or the sinusoidal nature of light beam is strongly related to its temporal coherence. The temporal coherence of the beam is, therefore, directly associated with the width of the spectral line. Since no fringe pattern is observed if the path difference, $2d$, exceeds the coherence length, L_c, we may assume that the beam consists of all the wavelengths lying between λ and $\lambda + \Delta\lambda$ with

$$\Delta\lambda = \frac{\lambda^2}{L_c} \tag{12.7}$$

This gives the relation between coherence length and spread in wavelength of a light beam. Further, since $\nu = c/\lambda$, the spread in frequency $\Delta\nu$ is

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$$

$$= c/L_c$$

And, the coherence time is defined as, $\tau_c = L_c/c$. Thereforc, we have

$$\Delta\nu \sim 1/\tau_c \tag{12.8}$$

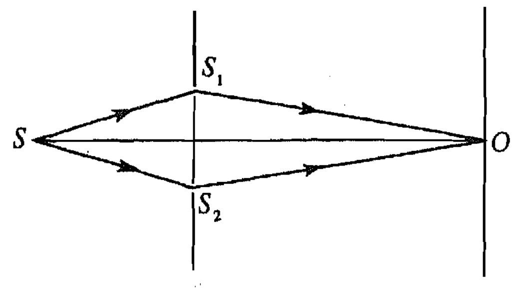
Thus the frequency spread of a spectral line is of the order of the inverse of the coherence time.

In this section, we discussed about temporal (or longitudinal spatial) coherence which relates the predictability or constancy of the phase relationship between two waves arriving at the same paint after traversing different optical paths. In other words, we talked about the constancy of phases of waves travelling along the same line. Light beam was considered as a series of wave trains. As per requirement of temporal coherence, if these wave trains are to produce observable interference fringe pattern, they must (a) have the same frequency and (b) overlap at the point of observation (i.e. path difference should be less than the coherence length). Now, what about the phase relationship between two waves travellingside by side at a certain distance from each other? Well, the constancy of the phase relationship of such waves relates to another type of coherence called spatial (or transverse spatial) coherence. This is the subject matter of the next section.

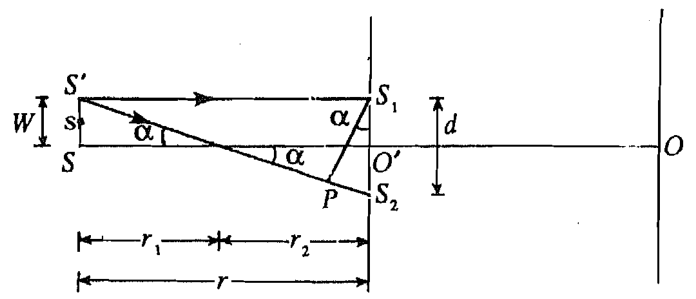
12.4 SPATIAL COHERENCE

In unit 6, you studied about Young's double-slit cxperiinent for obtaining interference fringe pattern. You may recall that one of the prerequisits for observing the interference pattern was that the source of light should be a point source. Can you say why this condition was imposed? What will happen if, instead of a point source, an extended source of light is used? These are some of the issues which relate to the spatial coherence about which we will study now.

You are aware that in an extended conventional source of light, the radiations emitted from different parts are independent of each other, and in that sense, such sources may be thought of as incoherent. But our interest is not so much in the nature of the source itself as in the quality of the illumination field it produces, for example, in a plane at some distance from the source. Thus, in Young's experiment we are interested in the extent to which there is a constant phase relationship between S_1, and S_2, Fig. 12.4a, so that interference effects can be observed. In other words, we are interested in examining the effect of the finite size of the source, S, on the interfcrcnce pattern.



(a)



(b)

Fig. 12.4: *(a)* **Young's** double slit experiment **with a** polnt **source,** *S*, (b) **Young's** double slit experiment with nn extended source S ' S,

In order to understand the effect of an extended source (and hence **of spatial** coherence) on the interference fringes, let us consider Young's double-slit experiment with an extended source. Fig. **12.4b** shows schematically the two slits S_1 and S_2 with an extended light source S' S of **width** W at a distancer. **Light** from some points in the source illuminates the slits, and **interference** fringes are produced on the screen. If the source consisted of just this single points (as in an idealised Young's experiment, Fig. **12.4a),** the fringes of maximum visibility would have been observed. A real source (such as S' S in Fig. **12.4b)** is, however, of finite size and the fringes produced by illumination from other points of the source are displaced relative to those due tos. Light from **the** extended source, therefore, produces a spread in fringes **with** a consequent reduction in the visibility of the fringe pattern.

In order to have some quantitative idea about the spatial coherence, let us assume that the two extreme points of the **extended** source (Fig. **12.4b),** S' and S act as **two** independent sources. Each source will produce its own interference pattern. Let us assume that $SS_1 = SS_2$ and the point O is such **that** $S_1O = S_2O$. Clearly the point source S will produce a **maximum** around O. On the other hand, intensity at O due to S' will depend on the path length (S' $S_2 - S'$ S_1). You **may** recall from Unit 6, that if this path difference

$$S' S_2 - S' S_1 = \lambda/2 \qquad (12.9)$$

the minima of interference pattern due to S will **fall** on the maxima of that due **to** S'. As a result, there will not be any observable interference pattern. **From** Fig. **12.4b,** we have

$$S' S_2 - S'S, = S_2P = \text{ad}$$

But $$\alpha = \frac{d/2}{r_2} = \frac{W}{r_1}$$

Thus, $$r = r_1 + r_2 = \frac{1}{\alpha}\left(W + \frac{d}{2}\right)$$

$$\alpha = \frac{W + (d/2)}{r}$$

Therefore,

$$S' S_2 - S' S_1 = \alpha d = \left(W + \frac{d}{2}\right)\frac{d}{r} = \frac{Wd}{r}$$

(neglecting d^2 term)

Thus, no fringes will occur if

$$\frac{Wd}{r} = \frac{1}{2}\lambda$$

$$W = \frac{r\lambda}{2d} \tag{12.10}$$

For every point on an extended source of extension r/d, there is a point at a distance r/2d which produces interference fringes separated by half a fringe width. Thus, for sources of such an extension, the visibility of the fringes would be poor.

We may, therefore, conclude that if we have an extended source whose linear dimension is $\sim\lambda r/d$, no interference fringe pattern will be observed. Equivalently, for a given source of width W, interference fringes will not be observable if the separation, d, between slits S_1 and S_2 is greater than $\lambda r/W$. If θ denotes the angle subtended by the source (S' S) at the point O' (midpoint of slits $S_1 S_2$), then $\theta = W/r$. So,

$$d = \frac{\lambda}{\theta} \tag{12.11}$$

which gives the maximum lateral distance between slits S_1 and S_2 such that the light beam from the extended source may be assumed to have some degree of coherence (i.e., the light waves from an extended source, after passing through slits S_1 and S_2 are able to produce interference fringes). The quantity λ/θ is known as Lateral (or transverse) Coherence Width and is denoted by l_w. You may note that the coherence width is linear in dimension and is approximately perpendicular to the direction of wave propagation. By contrast, the coherence length, introduced in relation to temporal coherence, is along the direction of wave propagation. For this reason, temporal coherence is sometimes called longitudinal coherence and spatial coherence is sometimes called lateral coherence.

Further, closely related to coherence width is a parameter called coherence area given as

$$a_c = \pi\,(l_w/2)^2$$
$$= \pi\,(\lambda/2\theta)^2 \tag{12.12}$$

The waves at any two points within coherence area are coherent. You may have noticed that Eqs. (12.11) and (12.12) apply to the case in which the extended source is essentially a uniform linear source. If the source is in the form of an uniform circular disc, the lateral coherence width is given as

$$l_w = 1.22\,\lambda/\theta \tag{12.13}$$

Well, in order to recaptulate what you have studied in this section, how about solving an SAQ!

SAQ 3

Spend 2 min

Suppose we set up Young's experiment with a small circular hole of diameter 0.1 mm in front of a sodium lamp ($\lambda \sim 589.3$ nm) source. If the distance from the source to the slits is 1m, how far apart will the slits be when the fringe pattern disappears?

12.4.1 Angular Diameter of Stars

Now, let us consider an application of the concept of spatial coherence. In the preceding paragraphs, we have seen that the angle subtended by the extended source at the midpoint of the slit separation is related to the lateral coherence width (l_w). Also for a critical value of l_w, the interference fringes will disappear. If, instead of an ordinary extended source of light, we consider a terrestrial extended source such as a star, you may like to know: Is it be possible to know its angular diameter (i.e. the angle subtended by the star on the slits) by observing the disappearence of fringes? Indeed, it is possible. For measuring the angular diameter of a star, Young's double slit experiment set-up needs modification. Modification in the experimental set-up is necessitated because, for such an arrangement, if we take a typical value of the angular diameter of a star as $\sim 10^{-7}$ radians, the distance d between the slits for which fringes disappear will be

$$d = \frac{1.22\lambda}{\theta} = \frac{1.22 \times 5 \times 10^{-7}}{10^{-7}}$$

$$\simeq 6\text{m}.$$

And for such a large value of d, the fringe width will be too small.

To overcome this difficulty, Michelson used an ingeneous technique. He achieved an effectively large value of d by using two movable mirrors M and M' as shown in Fig. 12.5. This modified interferometer is known as **Michelson's** Stellar Interferometer.

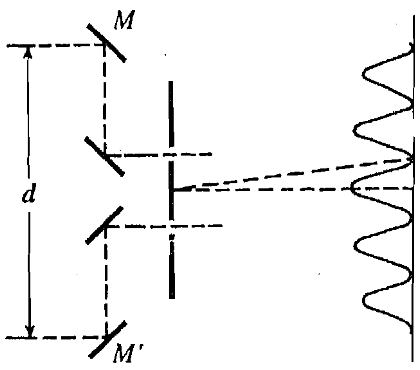


Fig. 12.5: Michelson's stellar interferometer.

Since you have studied in detail about the Michelson Steller interferometer in Unit 7, we would just mention here the results of one typical experiment. In a typical experiment the first disappearance of fringes occured when the distance between mirrors M and M' was about 7.3m which gave the angular diameter, θ as (taking $\lambda \sim 5 \times 10^{-7}$m)

$$\theta = 1.22\lambda/d$$

$$\theta = \frac{1.22 \times 5 \times 10^{-7}}{7.3}$$

$$= 8.4 \times 10^{-8} \text{ rad}$$

From the known distance of star and the value of its angular diameter, 8, we can estimate its diameter.

12.4.2 Visibility of Fringes

Till now, we have been discussing coherence and its importance for **observing** interference fringes. We have been talking about the disappearence of fringes under different circumstances. For example, in the Young's double slit experiment, interference fringes are seen on a screen with highly spatially coherent light. The fringes are rather distinct, their **visibility** is high. As the two slits are moved further apart the fringes are more closely spaced and will lose visibility. The degree of visibility, therefore, is the measure of spatial coherence.

The amount of radiation power incident per unit **area** is called **areana.**

Assume that two wave trains of light, each of **finite** length Δl, overlap to their full extent. Such complete overlap will result in distinct maxima and minima of highest degree of visibility. Even if the wave trains overlap partially, as in Fig. 12.6, interference is possible. However, the degree of visibility of the fringes **will** diminish depending on the extend of overlap. The question, therefore, is not how much the wave trains must overlap to produce interference; rather, the question is how much visibility we need to see a fringe pattern?

The definition of visibility is essentially a **matter** of comparison. Visibility, V, can be defined as the **ratio** of the difference between the maximum areana E_{max}, and minimum **areana** E_{min}, to the sum of the **areanas; i.e.**

$$V = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}; \qquad (12.14)$$

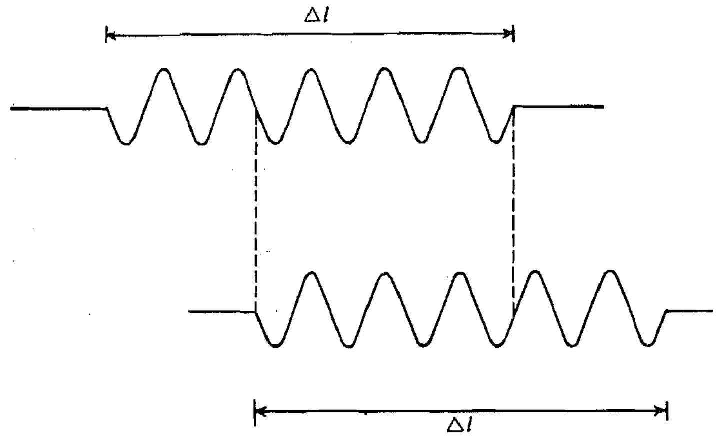


Fig. 12.6: Partial overlap of two wave trains.

Let us assume that E_{max} can take any arbitrary value but $E_{min} = 0$. Then visibility, V = 1. On the other hand, if $E_{max} = E_{min}$, V = 0, fringes cannot be seen. Thus, the visibility may assume any value between 0 and 1. Generally, a visibility of 0.8 is considered high, **but** a value 0.2 is barely visible.

Now, you may like to know whether visibility is related to coherence? Yes, it is. To see how, let two points on a distant screen be illuminated by two light sources that produce equal **areanas** E_o. Light waves from each consists of two parts-coherent (A) and incoherent (B). Areana due to the coherent part, A can be expressed as

$$E_A = \rho E_o$$

where, p is the degree of coherence,

and, the areana due to incoherent part is,

$$E_B = (1 - \rho) E_o$$

Interference fringes are observed because of part A. The coherent part forms fringes whose maxima have intensities. Areana of the maxima is four times as high as the individual contribution. Thus, the maximum areana, $(E_A)_{max}$, is $4\rho E_o$ and minimum is zero. Moreover, on this interference fringe pattern, due to the coherent part A, a uniform distribution due to incoherent part B, is superimposed. The areana of this distribution will be twice as high as the contribution E_B, because it comes from two sources. Hence,

$$(E_B)_{max} = 2E_B = 2(1-\rho)E_o = (E_B)_{min}$$

As a result, the areana in the maxima is

$$E_{max} = (E_A)_{max} + (E_B)_{max}$$

$$E_{max} = 4\rho E_o + 2(1-\rho)E_o$$

$$= 2(1+\rho)E_o$$

and the areana in the minima

$$E_{min} = (E_A)_{min} + (E_B)_{min}$$

$$= 0 + 2(1-\rho)E_o$$

Therefore, Eq. (12.14) for visibility of the fringes can be written as,

$$V = \frac{2(1+\rho)E_o - 2(1-\rho)E_o}{2(1+\rho)E_o + 2(1-\rho)E_o}$$

$= \rho$, the degree of coherence.

Thus, the degree of visibility (or the contrast) of the fringes produced by two light waves is equal to degree of coherence between them,

The highest visibility and hence highest degree of coherence will occur when the minimum areana in the expression for V is zero. In that case, both the visibility and the degree of coherence are unity. Although concievable in theory, but this is not possible in practice. Complete coherence is merely a theoretical result. However, with the development of laser, about which you would study in the next unit, it is now possible to have light beam of extremely high degree of coherence.

12.5 SUMMARY

Coherence is a property of light. A predictable phase relation exists between light waves passing through a point at different times.

- Temporal coherence or the longitudinal spatial coherence refers to the predictability of the phase of radiation as a function of time. In other words, temporal coherence can be identified as the interval of time during which the phase of the wave changes in a predictable manner as it passes through a fixed point in space. This time interval is known as coherence time, τ_c. And the path length corresponding to τ_c, given as $L_c = c\,\tau_c$ is called the coherence length of the radiation.

- Temporal coherence is related with the width of the spectral lines. The spread in wavelength is given as,

$$\Delta\lambda = \frac{\lambda^2}{L_c}$$

and the corresponding spread in the frequency of the spectral line is

$$\Delta\nu \sim 1/\tau_c$$

- Spatial coherence or transverse spatial coherence refers to the correlation between the pbases of two light waves travelling side by side. Use of point source in Young's double slit experiment is essentially to meet the requirement of spatial coherence.
- If an extended source of light of width W is used in Young's interference experiment, for observing interference fringe pattern following condition must be satisfied

$$W = \lambda/\theta$$

where, λ is the wavelength of the light, and θ is the angle subtended by the extended source on the slits.

The quantity (λ/θ) is known as lateral **(or transverse) coherence width, l_w.**

- For a circular extended source, the coherence width l_w is given as

$$l_w = \frac{1.22\lambda}{\theta}$$

- Visibility of an interference pattern is given as

$$V = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

where, E_{max} is maximum areana and E_{min} **isminimum** areana.

In terms of the degree of **coherencc, ρ**, the visibility is given as

$$V = \frac{2(1+\rho)E_o - 2(1-\rho)E_o}{2(1+\rho)E_o + 2(1-\rho)E_o}$$

where, E_0 is the areana produced on the screen by individual light **source.**

12.6 TERMINAL QUESTIONS

1. The sodium line at $\lambda = 5890$ Å, produced in a low-pressure discharge, has spread in wavelength, A $\lambda = 0.0194$ Å. Calculate (a) the coherence length and (b) line width in hertz.

2. If the visibility in an interference fringe pattern is 50 percent and the **maxima** receive 15 units of light, how much light does the minima receive?

12.7 SOLUTIONS AND ANSWERS

SAQs

1. The wavelength of the light, λ = 660 nm and N, the number olwaves in the wave train is 20.

(a) So, the coherence length

$$L_c = N\lambda$$

$$= 20 \times 660 \text{ nm}$$

$$= 13200 \text{ nm} = 13.2 \times 10^{-12} \text{m}$$

(b) Coherence time

$$\tau_c = L_c/c \text{ ; where } c = \text{velocityof light} = 3 \times 10^8 \text{ ms}^{-1}$$

$$\tau_c = \frac{13200 \times 10^{-9} \text{ m}}{3 \times 10^8 \text{ ms}^{-1}}$$

$$= 4400 \times 10^{-17} \text{s}$$

$$= 4.4 \times 10^{-20} \text{s}$$

2. Eq. (12.5), $2d = \lambda^2/2$ (A, $-1,$)gives the path difference for the disappearance of fringe pattern due to light of wavelengths I ,and λ_2. When this expression is to be used for the disappearance of the fringe pattern due to the light beam consisting of all wavelengths lying between A and A + A λ, we must divide the interval (width) into two equal parts of $\Delta\lambda/2$. Thus, the fringe pattern will bc produced by wavelength values

$$\lambda_1 = \lambda + (\Delta\lambda/2)$$

$$\lambda_2 = \lambda$$

With these values, Eq. (12.5) reduces to

$$2d = \frac{\lambda^2}{2((\lambda + \Delta\lambda/2) - \lambda)} = \frac{\lambda^2}{2(\Delta\lambda/2)} = \frac{\lambda^2}{\Delta\lambda}$$

which is Eq. (12.6)

Now, for each wavelength lying between I and $\lambda + \Delta\lambda/2$, there will be a corresponding wavelength lying between $\lambda + \Delta\lambda/2$ and $\lambda + \Delta\lambda$ such that the minima of one falls on the maxima of the other. Therefore, the fringe pattern will disappear.

3. Width (or the diameler) of the source

$$W = 0.1 \text{ mm} = 1 \times 10^{-4} \text{m}$$

And distance between the source and slits

$$r = 1\text{m}$$

Hence the angle subtended by the source on slits

$$\theta = \frac{W}{r} = \frac{1 \times 10^{-4}\text{m}}{1\text{m}} = 10^{-4} \text{ radian}$$

Wavelength of the light

$$I = 589.3 \times 10^{-9}\text{m}$$

The lateral coherence width for a circular extended source

$$l_w = \frac{1.22\lambda}{\theta} = \frac{1.22 \times 589.3 \times 10^{-9}\text{m}}{10^{-4}\text{rad}}$$

$$= 0.72 \text{ cm}$$

Thus, if the separation between the slits is more than 0.72 cm, the fringe pattern will disappear.

TQs

1. $\lambda = 5890\,\text{Å} = 5890 \times 10^{-10}\text{m}$

$\Delta\lambda = 0.0194\,\text{Å} = 0.0194 \times 10^{-10}\text{m}$

(a) From equation (12.7), we have

$$\Delta\lambda = \frac{I}{L_c}; \text{where, } L_c = \text{coherence length}$$

$$\Rightarrow \qquad L_c = \frac{\lambda^2}{\Delta\lambda} = \frac{(5890 \times 10^{-10})^2\,\text{m}^2}{0.01194 \times 10^{-10}\,\text{m}}$$

$$= 0.18\text{m}$$

(b) The spread in frequency $\Delta\nu$ (line width in hertz) and the coherence time τ_c is related as (equalion (12.8))

$$\Delta\nu = \frac{1}{\tau_c} = \frac{1}{L_c/c} = \frac{c}{L_c};$$

'where c = velocity of light $= 3 \times 10^8\,\text{m/s}$

$$\Rightarrow \Delta\nu = \frac{3 \times 10^8\,\text{m/s}}{0.18\,\text{m}}$$

$$= 1.6 \times 10^9\,\textbf{Hz.}$$

2. The visibility of an interference fringe pattern is given as

$$V = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

where

E_{max} is the maximum areana **i.e.** the amount of radiation power contained in the maxima of the fringe pattern; and E_{min} is the minimum areana.

From the problem, we have

$$V = 50 \text{ percent} = \frac{1}{2}\ ;\ E_{max} = 15 \text{ units}\ ;\ E_{min} = ?$$

So, **from** above equation for visibility, we have

$$\frac{1}{2} = \frac{15 - E_{min}}{15 + E_{min}}$$

$$\Rightarrow \quad (15 + E_{min}) = 2\,(15 - E_{min})$$

$$\Rightarrow \quad E_{min} = 5 \text{ units}$$

Hence, 5 units of light will be received in the minima of the fringe pattern.